Construction of Quantum Theory: S-matrix

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Abstract

The construction of S-matrix has been key challenge of quantum TGD from the very beginning when it had become clear that path integral approach and canonical quantization make no sense in TGD framework. My intuitive feeling that the problems are not merely technical has turned out to be correct. In this chapter the overall view about the construction of S-matrix is discussed. It is perhaps wise to summarize briefly the vision about S-matrix.

a) S-matrix defines entanglement between positive and negative energy parts of zero energy states. This kind of S-matrix need not be unitary unlike the U-matrix associated with unitary process forming part of quantum jump. There are several good arguments suggesting that that S-matrix cannot unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In TGD framework path integral formalism is given up although functional integral over the 3-surfaces is present.

c) Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. Feynman cobordism is the generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. This picture differs dramatically from that of string models. There is functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function.

d) Imbedding space degrees of freedom seem to imply the presence of factor of type I besides HFF of type II_1 for which unitary S-matrix can define time-like entanglement coefficients. Only thermal S-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to S-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature. S-matrices and thus also quantum states in zero energy ontology possess a semigroup like structure and in the product time and inverse temperature are additive. This suggests that the cosmological evolution of temperature as $T \propto 1/t$ could be understood at the level of fundamental quantum theory.

e) S-matrix should be constructible as a generalization of braiding S-matrix in such a manner that the number theoretic braids assignable to light-like partonic 3-surfaces glued along their ends at 2-dimensional partonic 2-surfaces representing reaction vertices replicate in the vertex.

f) The construction of braiding S-matrices assignable to the incoming and outgoing partonic 2-surfaces is not a problem. The problem is to express mathematically what happens in the vertex. Here the observation that the tensor product of HFFs of type II is HFF of type II is the key observation. Many-parton vertex can be identified as a unitary isomorphism between the tensor product of incoming *resp.* outgoing HFFs. A reduction to HFF of type II_1 occurs because only a finite-dimensional projection of S-matrix in bosonic degrees of freedom defines a normalizable state. In the case of factor of type II_∞ only thermal S-matrix is possible without

finite-dimensional projection and thermodynamics would thus emerge as an essential part of quantum theory.

g) HFFs of type III could also appear at the level of field operators used to create states but at the level of quantum states everything reduces to HFFs of type II_1 and their tensor products giving these factors back. If braiding automorphisms reduce to the purely intrinsic unitary automorphisms of HFFs of type III then for certain values of the time parameter of automorphism having interpretation as a scaling parameter these automorphisms are trivial. These time scales could correspond to p-adic time scales so that p-adic length scale hypothesis would emerge at the fundamental level. In this kind of situation the braiding S-matrices associated with the incoming and outgoing partons could be trivial so that everything would reduce to this unitary isomorphism: a counterpart for the elimination of external legs from Feynman diagram in QFT.

h) One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This turns out to be the case only in non-perturbative phase. The reason is that the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be regarded as negligible and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement.

i) In this picture light-like 3-surface would take the dual role as a correlate for both state and time evolution of state and this dual role allows to understand why the restriction of time like entanglement to that described by S-matrix must be made. For fixed values of moduli each reaction would correspond to a minimal braid diagram involving exchanges of partons being in one-one correspondence with a maximum of Kähler function. By quantum criticality and the requirement of ideal quantum-classical correspondence only one such diagram would contribute for given values of moduli. Coupling constant evolution would not be however lost: it would be realized as p-adic coupling constant at the level of free states via the $log(p)$ scaling of eigen modes of the modified Dirac operator.

j) A completely unexpected prediction deserving a special emphasis is that number theoretic braids replicate in vertices. This is of course the braid counterpart for the introduction of annihilation and creation of particles in the transition from free QFT to an interacting one. This means classical replication of the number theoretic information carried by them. This allows to interpret one of the TGD inspired models of genetic code in terms of number theoretic braids representing at deeper level the information carried by DNA. This picture provides also further support for the proposal that DNA acts as topological quantum computer utilizing braids associated with partonic light-like 3-surfaces (which can have arbitrary size). In the reverse direction one must conclude that even elementary particles could be information processing and communicating entities in TGD Universe.

1 Introduction

During years I have spent a lot of time and effort in attempts to imagine various options for the construction of S-matrix, and it seems that there are quite many strong constraints, which might lead to a more or less unique final result if some young analytically blessed brain decided to transform these assumptions to concrete calculational recipes.

The realization that configuration space spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 meant [A8, A9] a turning point also in the attempts to construct S-matrix. A sequence of trials and errors led rapidly to the generalization of the quantum measurement theory and re-interpretation of S-matrix elements as entanglement coefficients of zero energy states in accordance with the zero energy ontology applied already earlier in TGD inspired cosmology [A10]. This in turn led to the discovery that stringy formulas for S-matrix elements emerge in TGD framework.

The purpose of this chapter is to collect to single chapter various general ideas about the construction of S-matrix scattered in the chapters of books about TGD and often drowned into details and plagued by side tracks. My hope is that this chapter might provide a kind of bird's eye of view and help the reader to realize how fascinating and profound and near to physics the mathematics of hyper-finite factors is. I do not pretend of having handle about the huge technical complexities and can only recommend the works of von Neumann [21, 22, 23, 24], Tomita [27, 28, 29, 30], the work of Powers and Araki and Woods which served as starting point for the work of Connes [25, 26], the work of Jones [31, 32], and other leading figures in the field. What is may main contribution is fresh physical interpretation of this mathematics which also helps to make mathematical conjectures. The book of Connes [26] available in web provides an excellent overall view about von Neumann algebras and non-commutative geometry.

1.1 About the general conceptual framework behind quantum TGD

Let us first list the basic conceptual framework in which I try to concretize the ideas about S-matrix.

1.1.1 $N = 4$ super-conformal invariance and light-like 3-surfaces as fundamental dynamical objects

Super-conformal symmetries generalized from string model context to TGD framework are symmetries of S-matrix. This is very powerful constraint to S-matrix but useless unless one has precisely defined ontology translated to a rigorous mathematical framework. The zero energy ontology of TGD is now rather well understood but differs dramatically from that of standard quantum field theories. Second deep difference is that path integral formalism is given up and the goal is to construct S-matrix as a generalization of braiding S-matrices with reaction vertices replaced with the replication of number theoretic braids associated with partonic 2-surfaces taking the role of vertices.

The path leading to the understanding of super-conformal invariance in TGD framework was long but the final outcome is briefly described. The are two kinds of super-conformal symmetries.

1. The first super-conformal invariance is associated with light-cone boundary and is due to its metric 2-dimensionality putting 4-D Minkowski space in a unique position. The canonical transformations of $\delta H_{\pm} = \delta M^4_{\pm} \times CP_2$ are identified as isometries of the configuration space. The super-generators of super-canonical algebra correspond to the gamma matrices of configuration space.

2. Light-like partonic 3-surfaces X^3 are the basic dynamical objects and light-likeness is respected by the 3-D variant of Kac-Moody algebra of conformal transformations of imbedding space made local with respect to X^3. Ordinary 1-D Kac-Moody algebra with complex coordinate z replaced with a light-like radial coordinate r takes a special role and super Kac-Moody symmetry is associated with this. The conformal symmetries associated with X^2 are counterpart of stringy conformal symmetries but have a role analogous to the conformal symmetries of critical statistical systems.

   The light-likeness property allows Chern-Simons action for the induced Kähler gauge potential as the only possible action principle. The resulting almost topological conformal field theory has maximal $N = 4$ super-conformal symmetry with the inherent gauge group $SU(2) \times U(2)$ identified in terms of rotations and electro-weak symmetries acting on imbedding space spinors.

Fermionic dynamics is determined by the modified Dirac action fixed uniquely by the requirement of super-conformal symmetry. The generalized eigen modes of the modified Dirac operator D are in a fundamental role.

1. The first interpretation of the eigenvalues λ of D would be as conformal weights. Zero modes would give rise to Ramond type representations and non-zero modes to a generalization of N-S representations with ground state conformal weight $\Delta = 1/2$ replaced with $\Delta = 1/2 + i\sum_k n_k y_k$, $n_k \geq 0$, where $s_k = 1/2 + iy_k$ is zero of Riemann Zeta.

2. On the other hand, TGD based description of Higgs mechanism encourages the interpretation of λ as a complex square root of conformal weight (mass squared rather than mass corresponds to conformal weight). For this option $|\lambda|^2$ would correspond to the ground state conformal weight apart from a numerical constant equal to zero for zero modes and $1/4 + (\sum n_k y_k)^2$ for non-zero modes [A9].

This hypothesis makes sense only if one accepts the conjectured number theoretical universality of Riemann Zeta stating that both zeros s_k, the values of ζ, and the values of the basic multiplicative building blocks $1/(1-p^{-s})$ of ζ at points $s = \sum_k n_k s_k$, are algebraic numbers.

Generalized eigenvalues of the modified Dirac operator are actually functions expressible as the inverse $\zeta^{-1}(z)$ of Riemann Zeta with a suitably chosen argument z coordinatizing the projection of X^2 to the geodesic sphere $S^2_\pm$ of $\delta M^4_\pm +$. The points of X^2 satisfying $z = \zeta(\sum_k n_k s_k)$ define the number theoretical braids. Different branches of the inverse of ζ labelled by zeros of ζ correspond to various eigen modes. An analogous picture applies in super-canonical degrees of freedom with $S^2_\pm$ replaced with the geodesic sphere of CP_2. One can say, that these geodesic spheres serve as heavenly spheres providing geometric representation for conformal weights (and/or their square roots).

The constraints coming from p-adic mass calculations lead to the following overall view about the relationship between the two algebras. Mass squared is p-adic thermal expectation value of conformal weight meaning that four-momentum does not appear in the super-conformal generators: this option is excluded also by purely geometric considerations. p-Adic thermodynamics is justified by the fact that physical states are not annihilated by SKMV. Super-canonical Virasoro algebra (SCV) creates tachyonic ground states with vanishing conformal weight as null states annihilated by L_n, $n < 0$, and SC and SKM generate massless states to which p-adic thermodynamics in SKMV degrees of freedom applies. The commutators of SKM and SC algebras and their Virasoro counterparts annihilate the physical states.

1.1.2 S-matrix as a functor

Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. Feynman cobordism is the generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. This picture differs dramatically from that of string models. There is a functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function. Functor property generalizes the unitary condition and allows also thermal S-matrices which seem to be unavoidable since imbedding space degrees of freedom give rise to a factor of type I with $Tr(id) = \infty$.

1.1.3 S-matrix in zero energy ontology

Zero energy ontology allows to construct unitary S-matrix in fermionic degrees of freedom as unitary entanglement coefficients between positive and negative energy parts of zero energy state. The basic properties of hyper-finite factor II_1 are absolutely crucial. The inclusion of bosonic degrees of freedom lead to a replacement of HFF of type II_1 with HFF of type $II_\infty = II_1 \otimes I_\infty$. However, normalizability of the states allows only a projection of S-matrix to a finite-dimensional subspace of incoming or outgoing states. Hence the S-matrix is effectively restricted to $II_1 \otimes I_n = II_1$ factor so that at the level of physical states HFF of type II_1 results. This is absolutely crucial for the unitary of the S-matrix since it makes possible to have $Tr(SS^\dagger) = Tr(Id) = 1$. If factor of type I is present as a tensor factor, thermal S-matrix is the only possibility and later arguments in favor of the idea that thermodynamics is unavoidable part of quantum theory in zero energy ontology will be developed.

One can worry whether unitarity condition is consistent with the idea that fermionic degrees of freedom should allow to represent Boolean functions in terms of time-like entanglement. That unitary time evolution is able to represent this kind of functions in the case of quantum computers suggests that unitarity is not too strong a restriction. The basic question is whether only a "cognitive" representation of physical S-matrix in terms of time like entanglement or a genuine physical S-matrix is in question. It seems that the latter option is the only possible one so that physical systems would represent the laws of physics.

1.1.4 U-matrix

Besides S-matrix there is also U-matrix defining the unitary process associated with the quantum jump. S- *resp.* U-matrix characterizes quantum state *resp.* quantum jump so that they cannot be one and same thing.

1. There are good arguments supporting the view that U-matrix is almost trivial, and the real importance of U-matrix seems to be related to the to the description of intentional action identified as a transition between p-adic and real zero energy states and to the possibility to perceive states rather than only changes as quantum jumps leaving the state almost unchanged.

2. State function reduction corresponds to a projection sub-factor in TGD inspired quantum measurement theory whereas U process in some sense corresponds its reversal. Therefore U matrix might correspond to unitary isomorphism mapping factor to a larger factor containing it.

3. State function reduction must be consistent with the unitarity of S-matrix defining time-like entanglement. Since state function reduction means essentially multiplication by a projector to a sub-space it seems that state function reduction for both incoming and outgoing states are possible and would naturally correspond to projections to sub-factors of corresponding HFFs of type II_1.

1.1.5 Unitarity of S-matrix is not necessary in zero energy ontology

U-matrix is necessarily unitary. There are good reasons to believe that this condition combined with Lorentz invariance makes it almost trivial. In the case of S-matrix unitarity is not absolutely necessary.

The restriction of the time-like entanglement coefficients to a unitary S-matrix would conform with the idea that light-like partonic 2-surfaces represent a dynamical evolution at quantum level so that zero energy states must be orthogonal both with respect to positive and negative energy parts of the states. On the other hand, the light-like 3-surface can be chosen arbitrarily and its choice indeed affects S-matrix. Hence the theory cannot fully reduce to a 2-dimensional theory. The interpretation is that light-like 3-surfaces are in 1-1 correspondence with the ground states of super-conformal representations identifiable as light particles.

There are several arguments supporting the view that S-matrix need not be unitary. The simplest observation is that imbedding space degrees of freedom naturally give rise to a factor of type I so that only thermal S-matrix defines a normalizable zero energy state. S-matrix as functor from the category of Feynman cobordisms to the category operators defining entanglement coefficients implies that S-matrix in fermionic degrees of freedom for a product of cobordisms is product of the S-matrices for cobordisms. This implies that in fermionic degrees of freedom S-matrix is thermal S-matrix with time parameter replaced with complex time parameter whose imaginary part corresponds to inverse temperature. Also an argument based on the existence of universal thermal S-matrix with a complex time parameter for hyper-finite factors of type III_1 supports the view that unitarity is not necessary. A further argument is based on the finding that in dimensions $D < 4$ unitary S-matrix exists only if cobordism is trivial so that topology change would not be possible. This raises the fascinating possibility that thermodynamics - in particular p-adic thermodynamics - is an unavoidable and inherent property of quantum TGD.

1.1.6 Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.

2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.

3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [48] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."*

later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M} ...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of S-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. The defining condition for the variant of the Connes tensor product proposed here has the following equivalent forms

$$MN = N^*M \quad , \quad N = M^{-1}N^*M \quad , \quad N^* = MNM^{-1} \quad . \tag{1}$$

If M_1 and M_2 are two M-matrices satisfying the conditions then the matrix $M_{12} = M_1 M_2^{-1}$ satisfies the following equivalent conditions

$$N = M_{12} N M_{12}^{-1} \quad , \quad [N, M_{12}] = 0 \quad . \tag{2}$$

Jones inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ are irreducible which means that the operators commuting with $\mathcal{N}$ consist of complex multiples of identity. Hence one must have $M_{12} = 1$ so that M-matrix is unique in this case. For $\mathcal{M} : \mathcal{N} > 4$ the complex dimension of commutator algebra of $\mathcal{N}$ is 2 so that M-matrix depends should depend on single complex parameter.

The dimension of the commutator algebra associated with the inclusion gives the number of parameters appearing in the M-matrix in the general case.

When the commutator has complex dimension $d > 1$, the representation of $\mathcal{N}$ in $\mathcal{M}$ is reducible: the matrix analogy is the representation of elements of $\mathcal{N}$ as direct sums of d representation matrices. M-matrix is a direct sum of form $M = a_1M_1 \oplus a_2M_2 \oplus ...$, where M_i are unique. The condition $\sum_i; |a_i|^2 = 1$ is satisfied and *-commutativity holds in each summand separately.

There are several questions: Could M_i define unique universal unitary S-matrices in their own blocks? Could the direct sum define a counterpart of a statistical ensemble? Could irreducible inclusions correspond to pure states and reducible inclusions to mixed states? Could different values of energy in thermodynamics and of the scaling generator L_0 in p-adic thermodynamics define direct summands of the inclusion? The values of conserved quantum numbers for the positive energy part of the state indeed naturally define this kind of direct direct summands.

4. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation laws and automatic emergence of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

1.1.7 Symplectic variant of QFT as basic building block of construction

The latest discovery related to the construction of M-matrix was the realization that a symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-canonical (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

1.1.8 Quantum classical correspondence

Quantum classical correspondence states that there is a correspondence between quantum fluctuating degrees of freedom associated with partonic 2-surfaces and classical dynamics. The weakest form of this principle is that the ground states of partonic super-conformal representations (massless states which generate light masses observed in laboratory) correspond to the interior dynamics of space-time sheets containing the partonic 2-surfaces. At the space-time level there would be 1-1 correspondence with the maxima of Kähler function giving rise to the analog of spin glass energy landscape.

One could protest by saying that excited states of super-conformal representations have no space-time correlate in this picture. Quantum states are replaced with states in which the projection of S-matrix to a finite-dimensional space in bosonic degrees of freedom appears as time-like entanglement coefficients so that quantum classical correspondence is obtained in strict sense after all. These states states are formally analogous which raises the question whether an actual relationship exists. For HFFs of type III unitary time evolution and thermal equilibrium are indeed closely related aspects of states [26]. $I_\infty \rightarrow I_n$ cutoff in the bosonic degrees of freedom would naturally have the discretization represented by number theoretic braids as a space-time correlate.

The effective elimination of the degrees of freedom associated with the space-time interior implied by the 1-1 correlation would allow to forget 4-D space-time degrees of freedom more or less completely as far as calculation of S-matrix is considered and everything would reduce to Fock space level as it does in quantum field theories. The functional integral around the maximum of Kähler function would select a set of preferred light-like partonic 3-surfaces. Quantum criticality suggests that the functional integral can be carried out exactly.

1.1.9 How TGD differs from string models

An important detail which deserves to be mentioned separately is one crucial deviation from string model picture: the stringy decays of partonic 2-surfaces or 3-surfaces are space-time correlates for the propagation of particle via several different routes rather than genuine particle decay. Note that partonic 2-surfaces can have arbitrarily large size and the outer boundary of any physical system represents the basic example of this kind of surface. Particle reactions correspond to branchings of light-like partonic 2-surfaces so that incoming and outgoing partons are glued together along their ends. This picture makes sense because quantum TGD reduces to almost topological conformal QFT at parton level (only light-likeness brings in the notion of metric).

Quantum classical correspondence allows to interpret light-like partonic 3-surface either as a time evolution of a highly non-deterministic 2-D system or as a 3-D system. This state-dynamics duality was discovered already in [E9], where it was realized that topological quantum computation has interpretation either as a program (state) or running of program (dynamics). Complete reduction to 2-D dynamics is not possible since the light-like 3-surfaces associated with maxima of Kähler action define spin glass energy landscape such that each maximum corresponds to its own S-matrix.

In this picture particle reactions correspond classically to branchings of partonic 2-surfaces generalizing the branchings for lines in Feynman diagrams. The stringy vertices for decays of surfaces correspond in TGD framework to the classical space-time correlate for a particle travelling along different paths and the particle creation and annihilation is a generalization of what occurs in Feynman diagrams with vertices replaced with 2-dimensional partonic surfaces along which light-like partonic 3-surfaces meet.

1.1.10 Physics as a generalized number theory vision

TGD as a generalized number theory vision gives powerful constraints. New view about space-time involves p-adic space-time sheets as space-time correlates for cognitive representations in fermionic case and for intentions in the bosonic case. This leads to the notion of number theoretic braid belonging to the algebraic intersection of real and p-adic partonic surfaces obeying same algebraic equations.

The implication is that the data characterizing S-matrix elements should come from discrete algebraic points of number theoretic braids. The Galois groups for braids occupying regions of partonic 2-surface emerge as a new element and relate closely to the representations of braid groups in HFFs of type II_1. Number theoretic universality leads to the condition that S-matrix elements are algebraic numbers in the extension of rational defined by the extension of p-adic numbers involved.

1.1.11 The role of hyper-finite factors of type II_1

The Clifford algebra of configuration space ("world of classical worlds") spinors is very naturally a hyper-finite factor of type II_1. During the last few years I have gradually learned something about the magnificent mathematical beauty of these objects.

1. TGD inspired quantum measurement theory with measurement resolution characterized in terms of Jones inclusion and based on HFFs of type II_1 brings in non-commutative quantum physics and leads to powerful general predictions [A8, H2]. The basic idea is that complex rays of the state space are replaced with $\mathcal{N}$ rays for Jones inclusion $\mathcal{N} \subset \mathcal{M}$. $\mathcal{N}$ defines the measurement resolution in the sense that the group G leaving elements of $\mathcal{N}$ invariant characterizes the measured quantum numbers.

2. Hyper-finite factors have the property that they are isomorphic with their tensor powers. This makes possible the construction of vertices as unitary isomorphisms between tensor products of HFFs of type II_1 associated with incoming and outgoing states. The core part of S-matrix boils down to a unitary isomorphism between tensor products of hyper-finite factors

of type II_1 associated with incoming *resp.* outgoing partonic 3-surfaces whose ends meet at the partonic 2-surface representing reaction vertex.

3. The study of Jones inclusions leads to the idea that Planck constant is dynamical and quantized. The predicted hierarchy of Planck constants involving a generalization of imbedding space concept and an explanation of dark matter as macroscopic quantum phases [A9]. Here the special mathematical role of Jones inclusions with index $r \leq 4$ is crucial.

4. The properties of HFFs inspire also the idea that TGD based physics should able to mimic any imaginable quantum physical system defined by gauge theory or conformal field theory involving Kac-Moody symmetry. Thus the ultimate physics would be kind of analog for Turing machine. The prediction inspired by TGD based explanation of McKay correspondence [33] is that TGD Universe is indeed able to simulate gauge and Kac-Moody dynamics of a very large subset of ADE type groups. In fact, also much more general prediction that simulation should be possible for any compact Lie group emerges.

5. HFFs of type II_1 lead also to deep connections with number theory [33] and number theoretical braids can be interpreted in terms of representations of Galois groups assignable with partonic 2-surfaces in terms of HFFs of type II_1. Particle decay represents a replication of number theoretical braids and this together with p-adic fractality and hierarchy of Planck constants suggests strongly direct connections with genetic code and DNA.

1.1.12 Could TGD emerge from a local version of infinite-dimensional Clifford algebra?

A crucial step in the progress was the realization that TGD emerges from the mere idea that a local version of hyper-finite factor of type II_1 represented as an infinite-dimensional Clifford algebra must exist (as analog of say local gauge groups). This implies a connection with the classical number fields. Quantum version of complexified octonions defining the coordinate with respect to which one localizes is unique by its non-associativity allowing to uniquely separate the powers of octonionic coordinate from the associative infinite-dimensional Clifford algebra elements appearing as Taylor coefficients in the expansion of Clifford algebra valued field.

Associativity condition implies the classical and quantum dynamics of TGD. Space-time surfaces are hyper-quaternionic of co-hyper-quatenionic sub-manifolds of hyper-octonionic imbedding space HO. Also the interpretation as a four-surface in $H = M^4 \times CP_2$ emerges and implies $HO - H$ duality. What is also nice that Minkowski spaces correspond to the spectra for the eigenvalues of maximal set of commuting quantum coordinates of suitably defined quantum spaces. Thus Minkowski signature has quantal explanation.

1.2 Summary about the construction of S-matrix

It is perhaps wise to summarize briefly the vision about S-matrix.

1. S-matrix defines entanglement between positive and negative energy parts of zero energy states. This kind of S-matrix need not be unitary unlike the U-matrix associated with unitary process forming part of quantum jump. There are several good arguments suggesting that that S-matrix cannot be unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In TGD framework path integral formalism is given up although functional integral over the 3-surfaces is present.

2. Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. It is difficult to overestimate the importance of this result bringing category theory absolutely essential part of quantum TGD. One can assign to thermal S-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature. S-matrices and thus also quantum states in zero energy ontology possess a semigroup like structure and in the product time and inverse temperature are additive. This suggests that the cosmological evolution of temperature as $T \propto 1/t$ could be understood at the level of fundamental quantum theory. The most general identification of the time like entanglement coefficients would be as a "square root" of density matrix thus satisfying the condition $\rho^+ = SS^\dagger$, $\rho^- = SS^\dagger$, $Tr(\rho^{pm}) = 1$. $\rho^\pm$ has interpretation as density matrix for positive/negative energy states. Physical intuition suggest that S can be written as a product of universal unitary matrix and square root of state dependent density matrix.

3. S-matrix should be constructible as a generalization of braiding S-matrix in such a manner that the number theoretic braids assignable to light-like partonic 3-surfaces glued along their ends at 2-dimensional partonic 2-surfaces representing reaction vertices replicate in the vertex [C6].

4. The construction of braiding S-matrices assignable to the incoming and outgoing partonic 2-surfaces is not a problem [C6]. The problem is to express mathematically what happens in the vertex. Here the observation that the tensor product of HFFs of type II is HFF of type II is the key observation. Many-parton vertex can be identified as a unitary isomorphism between the tensor product of incoming *resp.* outgoing HFFs. A reduction to HFF of type II_1 occurs because only a finite-dimensional projection of S-matrix in bosonic degrees of freedom defines a normalizable state. In the case of factor of type II_∞ only thermal S-matrix is possible without finite-dimensional projection and thermodynamics would thus emerge as an essential part of quantum theory.

5. HFFs of type III could also appear at the level of field operators used to create states but at the level of quantum states everything reduces to HFFs of type II_1 and their tensor products giving these factors back. If braiding automorphisms reduce to the purely intrinsic unitary automorphisms of HFFs of type III then for certain values of the time parameter of automorphism having interpretation as a scaling parameter these automorphisms are trivial. These time scales could correspond to p-adic time scales so that p-adic length scale hypothesis would emerge at the fundamental level. In this kind of situation the braiding S-matrices associated with the incoming and outgoing partons could be trivial so that everything would reduce to this unitary isomorphism: a counterpart for the elimination of external legs from Feynman diagram in QFT.

6. One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This turns out to be the case only in non-perturbative phase. The reason is that the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be regarded as negligible and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement.

7. In this picture light-like 3-surface would take the dual role as a correlate for both state and time evolution of state and this dual role allows to understand why the restriction of time like entanglement to that described by S-matrix must be made. For fixed values of moduli each reaction would correspond to a minimal braid diagram involving exchanges of partons being in one-one correspondence with a maximum of Kähler function. By quantum criticality and the requirement of ideal quantum-classical correspondence only one such diagram would contribute for given values of moduli. Coupling constant evolution would not be however lost: it would be realized as p-adic coupling constant at the level of free states via the $log(p)$ scaling of eigen modes of the modified Dirac operator.

8. A completely unexpected prediction deserving a special emphasis is that number theoretic braids replicate in vertices. This is of course the braid counterpart for the introduction of annihilation and creation of particles in the transition from free QFT to an interacting one. This means classical replication of the number theoretic information carried by them. This picture provides a further support for the proposal that DNA acts as topological quantum computer utilizing braids associated with partonic light-like 3-surfaces (which can have arbitrary size) [E9]. In the reverse direction one must conclude that even elementary particles could be information processing and communicating entities in TGD Universe.

9. Symplectic variants of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. This leads to a concrete recursive formula for n-point functions using fusion rules.

1.3 Topics of the chapter

The goal is to sketch an overall view about the ideas which have led to the recent view about the construction of S-matrix. First basic philosophical ideas are discussed. These include the basic ideas behind TGD inspired theory of consciousness [10], the identification of p-adic physics as physics of cognition and intentionality forcing the central idea of number theoretic universality, quantum classical correspondence, and the crucial notion of zero energy ontology. The views about S-matrix as a characterizer of time-like entanglement and S-matrix as a functor are analyzed. The role of hyper-finite factors in the construction of S-matrix is considered with more detailed discussion left to the next chapter. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. The last section is devoted to the construction of unitary U-matrix characterizing the unitary process forming part of quantum jump. The basic arguments are U-matrix is almost trivial and that is highly desirable in zero energy ontology. The reader wishing for a brief summary of TGD might find the e three articles about TGD, TGD inspired theory of consciousness, and TGD based view about quantum biology helpful [16, 17, 18].

2 Basic philosophical ideas

The ontology of quantum TGD differs dramatically from that of standard quantum field theories and these differences play a key role in the proposed approach to the construction of S-matrix.

2.1 The anatomy of the quantum jump

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories rather than to a non-deterministic behavior of a single quantum history (understood as an evolution of Schrödinger equation). Therefore U-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of U-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

1. The classical time development determined by the absolute minimization of Kähler action.

2. The unitary "time development" defined by U associated with each quantum jump

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator. The hierarchy of U-matrices corresponding to hierarchy of durations of macro-temporal quantum coherence allows to resolve this problem and assign a definite time duration to the effective quantum jump and thus to U-matrix. At the limit of large number of quantum jumps (10^{39} quantum jumps per second of geometric time) the S-matrix as it is applied in elementary particle physics emerges.

3. The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness [10]. A crucial role is played by the classical non-determinism of Kähler action implying that the non-determinism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

2.1.1 Unitary process

U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration since there is *no* real time evolution or translation involved. It is not clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of super-string models is however something very different and could correspond to U-matrix.

Macro-temporal quantum coherence suggests strongly a fractal hierarchy of S-matrices defined for periods of macro-temporal quantum coherence consisting of sequences of quantum jumps and defined for selves. The hierarchy of these unitary S-matrices would not be only an approximation but provide exact descriptions consistent with the limitations of conscious experience. The duration of the macro-temporal quantum coherence would correspond to the time interval defining unitary time development. Also p-adic length scales would define similar hierarchy of S-matrices.

Physically it seems obvious that U matrix should decompose to a cosmological U-matrix representing dispersion in configuration space and U-matrix representing local dynamics: this indeed occurs thanks to the classical non-determinism of the Kähler action. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation. An important exception are the zero modes characterizing center of mass degrees of freedom of 3-surface which correspond to the isometries of $M^4_+ \times CP_2$. In these degrees of freedom localization does not occur. At the limit when 3-surfaces are regarded as point-like objects theory should obviously reduce to quantum field theory.

2.1.2 State function reduction

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parametrize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical.

This assumption does not yet quite imply standard quantum measurement theory. If U matrix in zero modes corresponds to a flow in some orthogonal basis for the configuration space spinor fields in the quantum fluctuating fiber degrees of freedom of the configuration space, then density matrix between quantum fluctuating degrees of freedom and zero modes is diagonal and quantum jump can be regarded as a quantum measurement of this density matrix serving as a universal observable.

The requirement that U-matrix induces is effectively a flow in zero modes is consistent with the effective classicality of the zero modes requiring that quantum evolution causes no dispersion. The one-one correlation between preferred outgoing quantum state basis in quantum fluctuating degrees of freedom and zero modes implies nothing but a one-one correspondence between outgoing quantum states and classical variables crucial for the interpretation of quantum theory. It is needless to emphasize, that the reduction of the standard quantum measurement theory, which is just a set of ad hoc rules, to fundamental quantum theory is a victory for TGD approach. The reduction involves in an essential manner the replacement of point-like particle with 3-surface and the degeneracy of the configuration space metric so that string model approach does not yield this kind of reduction. What is nice that the mere decomposition of configuration space to a union of symmetric spaces labelled by zero modes implies considerable part of measurement theory as a mathematical necessity.

In p-adic configuration space degrees of freedom one must assume that complete localization occurs so that the final state configuration space spinor fields are localized to space-time surfaces for which p-adic space-time sheets are completely fixed and superposition is possibly only for the real space-time sheets in quantum fluctuating degrees of freedom.

What are then the counter arguments against complete localization? First of all, one can imagine that the reduction could occur to a sub-space of zero modes consisting of a discrete points. Rational bound state entanglement in discrete sub-spaces of zero modes would be stable against state function reduction. Even more generally, the existence of symplectic structure in zero modes allows to consider a hierarchy of 2n-dimensional sub-manifolds in the space of zero modes with volume element defined by the n:th power of the symplectic form. State function reduction could occur to this kind of sub-manifold since at least the transition amplitude would be well-defined. Preferred sub-manifolds of this kind are sub-manifolds closed with respect to the action of $SO(3) \times SU(3)$ isometries such that only the coordinates associated with a finite number of super-canonical generators are non-constant.

2.1.3 State preparation

TGD inspired theory of consciousness inspires the hypothesis that the standard quantum measurement is followed by a self measurement inside self, which reduces entanglement between some subsystem and its complement in quantum fluctuating degrees of freedom. Again a measurement of the density matrix is in question. Self measurements are repeated until a completely unentangled product state of self results: the process is equivalent with the state preparation process, which is a purely phenomenological part of standard quantum measurement theory. In well defined sense state preparation corresponds to an analysis or decay process respecting only bound state entanglement.

The dynamics of self measurement is governed by Negentropy Maximization Principle (NMP, [H2]), which specifies which subsystems are subject to quantum measurement in a given quantum jump. NMP can be regarded as a basic law for the dynamics of quantum jumps and states that the information content of the conscious experience is maximized. In p-adic context NMP dictates the dynamics of cognition. In real context, self measurement makes possible for the system to fight against thermalization by self-repair at quantum level, and might be a crucial additional element besides the many-sheeted space-time concept needed to understand how bio-systems manage to be macroscopic quantum systems.

The hypothesis that bound state entanglement coefficients are in the hierarchy of extensions of rational numbers allows to use number theoretic definition of entanglement entropy. This allows to have also negative entropies and in this case NMP does not imply the reduction of entanglement in quantum jump so that there is no need to separately postulate the bound state entanglement is stable against NMP.

2.1.4 Classical space-time correlates for the basic steps of quantum jump

The classical space-time correlates for the basic notions of quantum measurement theory should be of crucial help in the construction of the S-matrix.

1. Space-time sheets correspond to coherence regions for various classical fields obtained by inducing various geometric structures of imbedding space to the space-time surface. They correspond also to the coherence regions of induced spinor fields. De-coherence means simply the decomposition of the 3-space sheet into pieces: emission of on mass shell photon by charged elementary particle is the simplest possible example here. The classical non-determinism of Kähler action and corresponding supersymmetrically related Dirac equation makes possible to have space-time correlates for the non-determinism of quantum jump sequence. Sequence of quantum jumps has as the space-time correlate the gradual increase of the psychological time defined by the temporal position for the front of the intention-to-action phase transition proceeding towards geometric future (this is basically due to the geometry of future light cone).

   At single particle level S-matrices are associated with space-time sheets. Virtual particles correspond to space-time sheets and one can distinguish between positive and negative energy space-time sheets having different time orientation. Negative energy space-time sheets are counterparts for negative energy virtual particles propagating in the internal lines of Feynman graphs. Since also the propagation towards geometric future is possible, second quantized S-matrix is obtained by Feynman rules in which propagators correspond to single particle unitary evolution and vertices are associated with the processes in which coherence is lost and 3-space-sheet decomposes to pieces representing particle and emitted particle(s).

2. The classical non-determinism of the Kähler action allows to represent state function reduction at classical level. The correlation between zero modes and quantum fluctuating degrees of freedom has correlation between space-time sheet and spinor mode as a correlate. Orthogonal spinor modes correspond to different space-time sheets. In particular, vacuum space-time sheets correspond to spinor fields with vanishing energy momentum and other currents.

   Double slit experiment serves as a good example of what could happen. Before the decision to measure which slit the particle propagates through, the space-time surface representing the particle is branched to two parts going through the slits and both branches contain classical spinor field. When the decision is made, p-adic space-time sheet representing the intention to make the measurement is transformed in quantum jump to real space-time sheets, most naturally negative energy topological light rays propagating to the geometric past and interacting with the spinor field and in such a manner that spinor field propagates only along the second branch of the space-time sheet. This is achieved if the interaction of negative energy topological light ray transforms space-time sheet to vacuum extremal for which also spinorial energy momentum tensor and various currents vanish identically. Presumably the absorption of negative energy nullifies the energy otherwise propagating along the branch in question. Conservation of various currents implies that the total probability defined by the spinor field goes to the second space-time branch.

3. Also state preparation and NMP should have space-time correlate. During state preparation process generation of de-coherence continues and involves maximal de-entanglement in quantum fluctuating degrees of freedom with the formation of bound states being exception. Since join along boundaries bonds are correlates for the entanglement, the process should correspond at space-time level to the splitting of join along boundaries bonds connecting 3-sheets. 3-surface quite literally decomposes into pieces. Negentropy Maximization Principle would most naturally correspond to the absolute minimization of Kähler action, which should imply the splitting of 3-surface into pieces. In [H7] I have proposed that Kähler action indeed has information theoretic interpretation. The non-determinism of NMP would correspond to the non-determinism of Kähler action.

2.1.5 The three non-determinisms

Besides the non-determinism of quantum jump, TGD allows two other kinds of non-determinisms: the classical non-determinism basically due the vacuum degeneracy of the Kähler action and p-adic non-determinism of p-adic differential equations due to the fact that functions with vanishing p-adic derivative correspond to piecewise constant functions.

To achieve classical determinism in a generalized sense, one must generalize the definition of the 3-surfaces Y^3 (belonging to light cone boundary) by allowing also "association sequences", that is 3-surfaces which have, besides the component belonging to the light cone boundary, also disjoint components which do not belong to the light cone boundary and have mutual *time like separations*. This means the introduction of additional, one might hope typically discrete, degrees of freedom (consider non-determinism based on bifurcations as an example). It is even possible to have quantum entanglement between the states corresponding to different values of time.

Without the classical and p-adic non-determinisms general coordinate invariance would reduce the theory to the light cone boundary and this would mean essentially the loss of time which occurs also in the quantization of general relativity as a consequence of general coordinate invariance. Classical and p-adic non-determinisms imply that one can have quantum jumps with non-determinism (in conventional sense) located to a finite time interval. If quantum jumps correspond to moments of consciousness, and if the contents of consciousness are determined by the locus of the non-determinism, then these quantum jumps must give rise to a conscious experience with contents located in a finite time interval.

Also p-adic space-time sheets obey their own quantum physics and are identifiable as seats of cognitive representations. p-Adic non-determinism is the basic prerequisite for imagination and simulation. The notion of cognitive space-time sheet as a space-time sheet having finite time duration is one aspect of the p-adic non-determinism and allows to understand how the notion of psychological time emerges. Cognitive space-time sheets simply drift quantum jump by quantum to the direction of geometric future since there is much more room there in the light cone cosmology.

The classical non-determinism is maximal for CP_2 type extremals for which the M^4_+ projection of the space-time surface is random light like curve. In this case, basic objects are essentially four- rather than 3-dimensional. The basic implication of the classical non-determinism is that quantum theory does not reduce to the light cone boundary. Secondly, U-matrix reduces to a tensor product of a cosmological U-matrix and local U-matrices relevant for particle physics. As a matter fact, an entire hierarchy of U-matrices defined in various p-adic time scales is expected to appear in the hierarchy. Thirdly, the classical non-determinism of CP_2 type extremals allows a topologization of the Feynman diagrammatics of quantum field theories and string models. Although localization in zero modes characterizing zitterbewegung orbit occur in quantum jump, there is integral over the positions of vertices which correspond to cm degrees of freedom for imbedding space, and this gives rise to a sum over various Feynman diagrams.

2.2 Quantum classical correspondence and consciousness theory

Quantum-classical correspondence is perhaps the deepest discovery inspired by consciousness theory. One can say that space-time surface provides a symbolic representation for quantum physics analogous to written or spoken language. Every quantum aspect of the theory must have classical space-time correlate. Even the sequence of quantum jumps defining contents of consciousness has classical space-time correlate made possible by the decomposition of the space-time sheets to strictly non-deterministic pieces. The systematic application of quantum classical correspondence has been the main tool in the development of various ideas. In particular, one can identify space-time correlates for the basic aspects of quantum jump (unitary process, state function reduction, state preparation) as well as for those related to the construction of S-matrix.

One important implication is that space-time sheets correspond to space-time correlates of selves which are particular kinds of sub-systems. The space-time sheet glued to a larger space-time sheet by CP_2-sized wormhole contacts does not however allow the application of standard notion of sub-system. The reason is that wormhole contacts have Euclidian signature of induced metric and thus are bounded by 3-dimensional boundaries with a degenerate metric which makes them effectively 2-dimensional. Therefore these surfaces having time-like

M^4_+ projection act as causal determinants. Space-time sheet cannot therefore correspond to a tensor factor of state space describing the space-time sheet at which it is topologically condensed. One implication is that sub-systems which are un-entangled can have subsystems which are entangled. This only requires that sub-system space-time sheets are connected by join along boundaries bonds whereas system space-time sheets are disjoint in this sense. At the level of conscious experience this means sharing of mental images and stereo consciousness. At the level of S-matrix this suggests strongly that S-matrix must be replaced by a hierarchy of S-matrices characterized by p-adic length scales, not only for practical limitations, but by the basic structure of existence at various levels.

That standard physics does not allow macroscopic and macro-temporal quantum coherence is the basic objection against quantum theories of consciousness. TGD suggests a solution of the problem. At the level of space-time correlates for single particle quantum mechanics the situation is almost trivial. Space-time sheets, which are correlates for selves, correspond to coherence regions of classical fields and of spinor fields. By quantum criticality, spin glass degeneracy, and p-adic fractality they form infinite hierarchy with increasing sizes and temporal durations. At configuration space level, the spin glass degeneracy implying enormous degeneracy of states allows to understand how bound states can have an exceptionally long subjecto-temporal duration. The same mechanism allows also to understand color confinement.

The effective binding of quantum jumps to single quantum jump is analogous to the formation of bound states from smaller units and actually precisely corresponds to this. Just as these bound states behave as elementary particles effectively, also the sequences of quantum jumps are expected to behave as single quantum jumps. At the level of S-matrix this suggest a hierarchy of S-matrices such that one can assign definite duration of subjective time to the duration of S-matrix as the number of quantum jumps involved and thus also a definite duration of geometric time.

2.3 New view about time and classical non-determinism

How the emergence of psychological time can be understood if quantum jumps occur between quantum histories, remained for a long time one of the mysteries plaguing quantum TGD approach. The non-determinism of the Kähler action gives hopes about the understanding of psychological time. One must allow unions of of disjoint 3-surfaces with time like separations such that disjoint components are extremely tightly but not totally correlated 3-surfaces. In ordinary strictly causal theory this is not possible. As a consequence, one cannot reduce the theory to the space of 3-surfaces at the boundary of the light cone.

It turns out that it is just this non-determinism which makes it possible to understand why our sensory experience seems to correspond to time=constant snapshot of quantum history rather than entire quantum history and how psychological time and its arrow emerge from the theory. To understand how this occurs however requires the introduction of p-adic numbers and TGD inspired theory of consciousness [H8]. Suffice it to say, the concept of psychological time forced by TGD forces radical generalization of the common sense thinking and that it took twenty years to to end up with this generalization.

2.4 p-Adic physics as physics of cognition and intentionality

The most crucial step was however the realization that p-adic physics is physics of cognition and intentionality and real physics is physics of the matter. Finite-p p-adic space-time regions and the associated p-adic spinor fields provide cognitive representations for the real regions so that also 'map is not territory' metaphor makes sense. Because of p-adic non-determinism and the fact that these regions do not contribute to the configuration space geometry, one can indeed identify p-adic regions as 'mind stuff', the geometric correlate of cognitive consciousness. It is however possible that p-adic regions contribute to fermion numbers and affect the statistics. Of course, p-adic regions are not conscious: consciousness is in the quantum jump between quantum histories. This realization led to a totally new view about p-adic physics and resolved many difficult interpretational issues: p-adic physics is a model for cognitive representations about real physics and doing p-adic physics means mimicking what Nature is doing all the time.

2.5 Zero energy ontology

2.5.1 Motivations for zero energy ontology

Zero energy ontology was first forced by the finding that the imbeddings of Robertson-Walker cosmologies to $M^4 \times CP_2$ are vacuum extremals. The interpretation is that positive and negative energy parts of states compensate each other so that all quantum states have vanishing net quantum numbers. One can however assign to state quantum numbers as those of the positive energy part of the state. In particular, gravitational four-momentum can be identified in this manner. At space-time level zero energy state can be visualized as having positive energy part in geometric past and negative energy part in geometric future. In time scales shorter than the temporal distance between states positive energy ontology works. In longer time scales the state is analogous to a quantum fluctuation.

Zero energy ontology gives rise to a profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

2.5.2 How the new ontology relates to the existing world view?

In the new rather Buddhistic ontology zero energy states are identified as experienced events and objective reality in the conventional sense becomes only an illusion. Before the new view can be taken seriously one must demonstrate how the illusion about positive energy reality is created and why it is so stable.

1. The very fact that the factorizing S-matrices are trivial apart from the changes in the internal degrees of freedom means that the event pairs are extremely stable once they are generated (how they are generated is an unavoidable question to be addressed below). Infinite sequences of transition between states with same positive energies and same initial energies occur. What is nice that this makes it possible to test the predictions of the theory by experiencing the transition again and again.

2. Statistical physics becomes statistical physics for an an ensemble consisting of zero energy states $|m_+n_-\rangle$ including also their time reversals $|n_+, m_-\rangle$. In the usual kinetics one deduces equilibrium values for various particle densities as ratios for the rates for transitions $m_+ \rightarrow n_+$ and their reversals $n_+ \rightarrow m_+$ so that the densities are given by $n(n_+)/n(m_+) = \sum_{n_+} \Gamma(m_+ \rightarrow n_+)/\sum_{n_+} \Gamma/(n_+ \rightarrow m_+)$. In the recent situation the same formula can be used to *define* the particle number densities in kinetic equilibrium using the proposed identification of the transition probabilities.

3. Because of the stability of the zero energy states, one can construct many particle systems consisting of zero energy states and can speak about the density of zero energy states per volume. Also the densities $n_{+,i}$ ($n_{-,i}$) of initial (final) states of given type can be defined and $n_{+,i}$ can be identified as densities of positive energy states. Also the densities for particles contained by these states can be defined. It would seem that the new ontology can reproduce the standard ontology as something which is not necessary but to which we are accustomed and which does not produce too much harm.

4. The sequence of quantum jumps between negative energy states defines also a sequence between initial (final) states of quantum jumps and, as far as momentum and color degrees of freedom are considered, this sequence represents rather immutable reality if S-matrix is factorizing.

5. Ordinary scattering experiment involves the measurement of the quantum numbers of particles in initial and final states. In the zero energy ontology this would mean that one can perform separate quantum measurements

for the observables associated with positive and negative energy components of zero energy states. The idea about observer as a space-time sheet drifting quantum jump by quantum jump towards geometric future at larger space-time sheet suggests that the measurements are carried out first for the positive energy component of the state. The scattering event would result from the quantum measurements of observables assignable to positive and negative energy components of the state. Quantum measurement would naturally correspond to a projection to a sub-factor $\mathcal{N} \subset \mathcal{M}$ so that there would be no hope of measuring precisely the initial and final states. Hence infinite sequences of measurements yielding sequences of states representing a sequence of S-matrices S_{N_i} are possible. This picture might apply even in momentum degrees of freedom.

That the world of our sensory perception represents a continuous evolution of relatively static structures rather than a sequence of uncorrelated big bangs followed by big crunches must be also understood.

1. A reasonable conclusion is that the temporal distance between positive and negative energy particles ('particles' defined as the contents of light cones) is sufficiently long as compared to the time scale of the personal conscious experience. This of course applies only to what we can directly perceive. At elementary particle level the situation might be different.

2. The geometric duration of quantum jump would naturally correspond to the typical temporal distance between positive and negative energy particles so that the experience about a flow of time, certainly the key feature of conscious experience, would correlate with the basic structure of zero energy states.

3. The standard arrow of the subjective time could be assigned to the states for which negative energy particles are in the geometric future of positive energy particles. The situation would be reversed for phase conjugate matter.

4. The quantum jumps giving rise to our conscious experience would occur at dark matter levels and correspond to large values of Planck constant and hence to large values of integer n defining the quantum phase q. Dark matter would make possible the illusory ontology of the standard world view. Note that creation and doomsday myths would represent excellent metaphors for the zero energy ontology.

1. How does the quantum measurement theory generalize?

There are also important questions related to the quantum measurement theory. The zero modes associated with the interior degrees of freedom of space-time surface represent classical observables entangled with partonic observables and this entanglement is reduced in quantum jump. Negentropy Maximization

Principle [H2] is the TGD based proposal for the variational principle governing the statistical dynamics of quantum jumps. NMP states that entanglement negentropy tends to be maximized in the reduction of entanglement. Number theoretic variants of Shannon entropy making sense for rationally or even algebraically entangled states can be positive so that NMP can also lead to generation of this kind of entanglement and gives rise to a highly stable bound state entanglement.

Does this picture generalize to the new framework in which zero energy states become physical states? Factorizable S-matrices describe partonic dynamics and should be responsible for generating entanglement in the partonic degrees of freedom. One should understand also the S-matrix generating entanglement between zero modes and partonic degrees of freedom and quantum classical correspondence is the only guideline in the recent situation.

The obvious objection is that the replacement of a universal S-matrix coding entire physics with a state dependent unitary entanglement matrix means giving up entirely the greatest dream of theoretician. The situation could be saved if the S-matrices have fractal structure. The quantum criticality of TGD Universe indeed implies fractality. The possibility of an infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras expresses this fractal character algebraically. Thus one can hope that the S-matrix appearing as entanglement coefficients is more or less universal in the same manner as Mandelbrot fractal looks more or less the same in all length scales and for all resolutions. Whether this kind of universality must be posed as an additional condition on entanglement coefficients or is an automatic consequence of unitarity in type II_1 sense is an open question.

2. Understanding quantum computation in the new ontology?

The understanding of what really happens in quantum computation, in particular topological quantum computation [E9], is a challenge for the recent framework since the theory of quantum computation relies heavily on the Hamiltonian time evolution which cannot be an exact description in the new ontology. The basic element is entanglement between positive and negative energy states. It is generated by time evolution in standard framework whereas in the recent framework the creation of the quantum computer program and its realization reduces to the creation of a zero energy state realizing this entanglement. Note that also entanglement between positive energy states can be used for quantum computational purposes.

The problem is obvious: the creation of quantum computer program requires a creation of a zero energy state realizing the program. Can one allow quantum jumps creating zero energy states representing the desired program? The extreme stability of the zero energy states against evolution defined by a factorizing S-matrix does not allow the popping up of zero energy states from vacuum since four-momenta are in this case vanishing. Must be accept that we are passive spectators who just observe the already existing zero energy states representing quantum computer programs as we drift towards geometric future along larger space-time sheet?

It seems that this is not necessary. p-Adic physics as a physics of intentionality and cognition however suggests how the obstacle could be overcome at the level of principle. For zero energy states, p-adic-to-real transitions and vice versa are in principle possible and I have in fact proposed a general quantum model for how intentions might be transformed to actions in this manner [A8]. In the second direction the process corresponds to formation of cognitive representation of a zero energy physical state.

In the degrees of freedom corresponding to configuration space spinors situation is very much like for reals. Rational, and more generally algebraic number based physics applies in both cases. p-Adic space-time sheets however differ dramatically from their real counterparts since they have only rational (algebraic) points in common with real space-time sheets and p-adic transcendentals are infinite as real numbers. The S-matrix elements for p-adic-to-real transitions can be formulated using n-point functions restricted to these rational points common to matter and mind stuff. If this picture is not terribly wrong, it would be possible to generate zero energy states from vacuum and the construction of quantum computer programs would be basically a long and tedious process involving very many intentional acts.

One can of course, make a further question. What about the generation of intentions: can p-adic space-time sheets and quantum numbers pop up spontaneously from vacuum? What kind of p-adic space-time sheets and quantum numbers assignable to their partonic 2-surfaces can do so spontaneously?

Here an interesting aspect of the p-adic conservation laws passes a helping hand. p-Adic integration constants are pseudo constants in the sense that a quantity having vanishing (say) time derivative can depend on a finite number of pinary digits t_n of the time coordinate $t = \sum_n t_n p^n$. Could one think that quantum jumps can generate from vacuum exact vacuum states as vacuum tensor factors of the configuration space spinor, and that in subsequent quantum jumps factorizing p-adic S-matrix conserving quantum numbers only in p-adic sense transforms this state into a non-trivial zero energy state which then transforms to real state in intentional action? Note that if conserved quantum numbers are integers they are automatically pseudo constants. p-Adic conservation laws could allow also the p-adic zero energy states to pop up directly from vacuum.

Real-to-p-adic transitions would represent transformation of reality to cognition and would be also possible and mean destruction of real universe. The characteristic and perhaps the defining feature of living matter would be its highly developed ability to reconstruct reality by performing p-adic-to-real transitions and their reversals.

2.5.3 U-matrix and S-matrix

Contrary to the original hopes U-matrix associated with the unitary process seems to be almost trivial and does not allow interpretation as S-matrix. This is actually a highly desirable result since sensory perception can be described in terms of quantum jumps which induce practically no changes in physical states. Hence it is possible to perceive in practice zero energy quantum state rather than only its change. For p-adic-to-real transitions the almost triviality of U-matrix is highly desirable property since it means faithful realization of intentional action.

In the case that the U-matrix for zero energy states decomposes to a tensor product of U-matrices for positive and negative energy states sensory perceptions can give conscious information only about positive or negative energy part of the state. This would help to understand why we tend to perceive world as obeying positive energy ontology.

S-matrix has interpretation as time-like entanglement coefficients between positive and negative energy parts of zero energy states and characterizes therefore quantum states rather than Universe. On of the nice outcomes of this view is that unitary conditions can be weakened thermodynamic becomes part of quantum theory and time like entanglement coefficients have identification as thermal S-matrix assignable formally to a complex time parameter.

An important distinction between U-matrix and S-matrix is that if algebraic universality is accepted, U-matrix elements are algebraic numbers so that in zero energy ontology U-matrix can have elements between different number fields. This makes possible to assign U-matrix to quantum jumps transforming intention to action and the almost triviality of U-matrix turns out into a blessing. U-matrix characterizes also sensory perception as a quantum jump which does not change the perceivable properties of zero energy state (and thus its positive energy part) appreciably.

In case of S-matrix defines entanglement between positive and negative energy states. This entanglement does not make sense between different number fields in the general case. The Feynmann cobordism connecting p-adic and real partonic 2-surfaces does not make sense. Even the notion of zero energy state formed from real and p-adic parts is well-defined only if the sum of real and p-adic quantum numbers is well-defined only if the quantum numbers are algebraic numbers. The additivity should hold true also for the classical conserved charges if defined at all in the p-adic case.

2.6 Quantum measurement theory and the structure of quantum jump

TGD inspired theory of consciousness leads to a basic philosophy which could be summarized as statement 'Painting is the landscape'. Configuration space spinor fields representing physical states are the physical systems: there is no need to assume that world and the theory about the world are two different things since subjective experiences in any case are in the quantum jumps between quantum histories.

Quantum jump has a detailed anatomy decomposing into a unitary process U characterized by unitary U-matrix which however does not have interpretation as S-matrix, the counterpart of state function reduction involving localization in zero modes, and state preparation process minimizing all entanglement and respecting only bound state entanglement. Negentropy Maximization Principle governs the state preparation process and the number theoretic notion of entropy plays key role in the understanding of the process.

One of the deep inputs from TGD inspired theory of consciousness is that if quantum jumps between quantum histories are local quantum measurements in zero modes degrees of freedom of configuration, complete localization in zero modes or at least in a resolution determined by p-adic length scale must occur in each quantum jump. In p-adic configuration space degrees of freedom complete localization is necessary for purely mathematical reasons (p-adic configuration space integral does not exist): so that spatial cognition is purely classical in space-time degrees of freedom. p-Adic spinors can however remain entangled in quantum jump if number theoretic entanglement entropy is negative. This means a dramatic simplification in the construction of U-matrix and there are real hopes of constructing U-matrix solely on basis of symmetry arguments.

3 S-matrix as time-like entanglement coefficients in zero energy ontology

The original hope was that S-matrix could be constructed as a tensor product of factorizing S-matrices. The almost triviality of these matrices spoiled this hope. This forced the interpretation of particle reactions as detection of zero energy states, which are also forced by the cosmological considerations.

3.1 S-matrix as characterizer of time-like entanglement between positive and negative energy components of zero energy state

The idea about giving up the notion of unitary S-matrix in the standard sense of the word might seem too radical and there is actually no fundamental reason forcing this in the conceptual framework provided by hyper-finite factors of type II_1. Just the opposite, the freedom to construct zero energy states rather freely could be restricted by the unitarity of the matrix determined by the entanglement coefficients. There are however both mathematical and physical reasons to believe that entanglement coefficients give rise to a thermal S-matrix which is counterpart of ordinary S-matrix but for complex time parameter.

Before continuing, it must be added that S-matrix identified as entanglement coefficients between positive and negative energy parts of zero energy states would characterize zero energy *states* and would be something totally different from the U-matrix describing unitary process associated with the *quantum jump*. The almost triviality of U-matrix would guarantee that world looks highly classical and is irreversible, and would allow in a good idealization to experience the quantum states. And an injection of healthy realism is also in order. The argument that U-matrix is expressible in terms of tensor products of factorizing S-matrices is only an argument and, taking into account the immense infinite-dimensional complexity of quantum TGD, might be wrong at the level of details at least.

3.1.1 Unitarity in zero energy ontology

Quantum classical correspondence combined with the number theoretical view about conformal invariance could fix highly uniquely the dependence of S-matrix on cm degrees of freedom and on net momenta and color quantum numbers associated with CH_h. The corner stone of the interpretation is what might be called zero energy ontology applied already earlier in classical TGD. Zero energy ontology provides two different views but for both views o S-matrix characterizes the structure of zero energy states.

Unitary of S-matrix is possible for zero energy ontology in case of HFFs of type II_1. The interpretation of the condition $Tr(SS^{\dagger}) = Tr(Id) = 1$ as a normalization condition stimulates the hope that the entanglement between positive and negative energy states in zero energy states is coded by a unitary S-matrix in the conceptual framework provided by hyper-finite type II_1 factors so that states would represent dynamics in their structure.

It must be however emphasized that unitarity is by no means obvious in zero energy ontology.

1. What can be measured are basically ratios of scattering rates since one must always use a clock and clock corresponds to some standard scattering occurring with rate defining the time unit used.

2. In category theoretic approach only the assumption that S-matrix for product of Feynman cobordisms is product of S-matrices for individual cobordisms is natural and allows thermal S-matrices and thermodynamics at level of quantum states. In particular, p-adic thermodynamics crucial for understanding of particle massivation could emerge in this manner. This is the most general manner to state the idea that it is possible to assign an effective time development to the S-matrix.

It is not obvious whether unitarity is even possible.

1. For TQFTs with S-matrix functor from category of ordinary cobordisms, unitary S-matrix is assignable only to trivial cobordisms for $D < 4$. The situation might be same also for Feynman cobordisms but one might hope that thermal S-matrix exists for more general Feynman cobordisms meaning that thermodynamics and p-adic thermodynamics follow from fundamental principles somewhat like black hole temperature emerges as a property of black hole horizon. Note that for U-matrix the unitarity is necessary and in this case the almost triviality would be a blessing.

2. There is a further argument in favor of thermal S-matrix. It is quite possible that HFF of type II_1 is replaced with II_∞ factor which is a tensor product factors of type II_1 and type I. In the case of configuration degrees of freedom super-conformal symmetry might guarantee that HFF

of type II_1 is in question. Imbedding space degrees of freedom however seem to give rise to factor of type I via the representations of Poincare group and color partial waves and there seems to be no natural manner to avoid this. Only thermal S-matrix would define a normalizable state so that thermodynamical states would be genuine quantum states rather than only a useful fiction of theorist.

3. The detailed construction of zero energy states plus the requirement of p-adicability imply that zero energy states correspond to primary fields of rational conformal field theories so that their number is finite. Unitarity or its possible reduction to thermal S-matrix implies a further restriction and a connection with Jones inclusions emerges. The finite number of primary fields is also consistent with the interpretation in terms of Jones inclusion making the number of quantum fields finite. The representability of the primary fields of rational conformal field theories in terms of ordered exponentials of the stringy fields $X(z)$ in turn implies that the amplitudes for generating zero energy state from vacuum have have under very general assumptions the typical stringy form. Lorentz invariance is manifest at the level of the scattering amplitudes and mass squared formula follows.

3.1.2 The procedure leading from S-matrix to scattering rates in zero energy ontology

In standard QFT the procedure leading from from S-matrix to scattering rates breaks all rules of mathematical aesthetics. The ugliest step in this procedure involves the identification of the 4-dimensional momentum space delta function $\delta^4(0)$ as a 4-D reaction volume. Encouragingly, zero energy ontology allows to get rid of this feature and also provides a clear physical interpretation for it.

1. In standard ontology the conservation of energy does not allow localization in time direction so that in time direction the reaction volume is necessarily infinite. The conservation of four-momentum is a property of states in zero energy ontology and momentum space delta functions emerge only in the restriction of the four-momentum of positive energy states to a precise value. There is however no need to make this kind of restriction in zero energy ontology since for zero energy states the localization in M^4 center of mass degrees of freedom does not lead to a conflict with Uncertainty Principle. The conclusion is that there is nothing against the restriction of the value of center of mass coordinates of zero energy states to a finite M^4 volume making in turn possible normalized states in total four-momentum degrees of freedom.

2. p-Adic length scales $L_p(n)$ and their scaled up variants corresponding to scaled up values of Planck constants would naturally define a hierarchy of characteristic sizes for reaction volumes. Only in the idealization that the four-momentum of the initial state is precisely determined the square of

$\delta^4(0)$ would appear and a similar limiting procedure as in the usual case would be needed but would have a clear physical interpretation.

3. For finite reaction 4-volume the usual density of states description in finite volume would apply. The question is whether the density of states description with a finite resolution could be replaced as a Jones inclusion corresponding to a restriction of initial four-momentum and also initial 3-momenta of partons to a finite volume of momentum space. p-Adic length scale hierarchy is a good candidate for a hierarchy of Jones inclusions with increasing value of p defining an improved momentum resolution.

4. Finite length scale resolution would mean at the level of super conformal algebras to a cutoff n_{cr} for the values of conformal weight and thus mass squared. p-Adic mass calculations suggests that the value of conformal weight for which the mass of the state becomes equal to Hagedorn temperature fixes n_{cr} and predicts $n_{cr} \sim log_2(p)$ [F4]. Combining this with p-adic length scale hypothesis ($p \simeq 2^k$, k integer with primes favored) would encourage the hypothesis $n_{cr} = k$. The finite truncations of super conformal algebras obtained by replacing the integers n labelling the states with integers in Z/kZ would define physically natural Jones inclusions and prime values of k would correspond to the replacement of Z finite finite field $G(k)$.

3.2 About the construction of zero energy states

In the following it will be found that super conformal invariance for zero energy states implies under rather general assumptions that rational conformal field theories characterize the situation at partonic boundary components.

3.2.1 How to build tensor products of Super Kac-Moody representations?

Concerning the construction of the tensor product for Kac-Moody representations the most obvious problem is that the central extension parameter is additive for the ordinary tensor product. Second problem is that for dual of the representation central extension parameter is of opposite sign.

It is known that the tensor product based on fusion rules can be defined in a category consisting of representations with given level k and Virasoro central extension parameter c [51] and the construction works for non-negative values of k if integrable representations are in question.

This is however not enough in the recent situation since all values of (k, c) are expected to be possible for positive and negative energy states separately. The net values of these charges should however vanish for zero energy states and this observation suggests an elegant solution of the problem in TGD framework.

1. Positive and negative energy partons in a given zero energy state correspond to opposite values of k. The positive and negative energy par-

tons entering to a given partonic 2-surface of vertex 3-surface X^3_V correspond to the same value of $|k|$ and $|c|$. Negative energy particles naturally correspond to a representation obtained by the Hermitian conjugation $T^A_n \rightarrow (-1)^n T^A_{-n}$, $\omega(c) = -c$: here T^A corresponds to the Lie algebra element and c to Kac Moody central extension. Similar automorphism can be applied to Virasoro algebra.

2. The total Kac-Moody and Virasoro central charges should vanish for zero energy states and thus for the tensor product of positive and negative energy representations so that the category of physical states would correspond to a category with $(k, c) = (0, 0)$ naturally closed under tensor product whatever its precise definition is. These conditions would replaced the usual Super Virasoro conditions and there would be no need to compensate super-conformal weight with mass squared value.

3. One should define tensor products in zero energy category by a generalization of fusion rules. It is natural to assume that positive/negative energy states can belong to arbitrary integrable representations of Kac Moody and Virasoro algebras so that the usual restrictions on the highest weights would emerge. The restrictions on the tensor product of positive and negative energy representations would be given simply by Super Kac Moody, super-canonical and Super Virasoro conditions for the zero energy states and would generate correlations responsible for the non-triviality of S-matrix. Essentially fusion rules in which the outcome is $(c, k) = (0, 0)$ representation would be in question. Using the language of form factor bootstrap program, S-matrix would correspond to the form factor defined by identity operator between positive and negative energy states.

3.2.2 Constraints from p-adic thermodynamics

In p-adic thermodynamics the average value of the conformal weight determines the value of mass squared.

The only convincing picture is based on the existing vision about the roles of super-canonical (SC) and super Kac-Moody (SKM) algebras supported by p-adic mass calculations. It suggests that for elementary particles null states correspond to tachyonic ground states with *negative* conformal weight rather than physical states with positive conformal weight. Of course, also the above mentioned states would be possible.

i) In case of positive energy state S(uper)C(anonical) Virasoro (SCV) generators L_n carry negative conformal weights and SKM Virasoro (SKMV) generators positive conformal weights [C1] for $\delta H_+ = \delta M^4_+ \times CP_2$ and vice versa for δH_-. SCV creates the tachyonic ground state needed in p-adic mass calculations and this state is a null state annihilated by L_{-n}, $n > 0$: otherwise one would have tachyonic thermodynamics. What is especially nice that this condition reduces dramatically the number of ground states and therefore also the number of massless states.

ii) Massless ground state is generated by applying SC generators (Hamiltonians and their super counterparts) with conformal weights $1/2+iy$ and SKM generators.

iii) Massless state is thermalized with respect to SKMV with thermal excitations created by generators L_n, $n>0$.

3.2.3 Super-conformal invariance for zero energy states and p-adicization lead to rational conformal field theories

In the following the considerations are for simplicity restricted to the case when one has only Virasoro algebra. The conditions for Super Virasoro algebra and Super Kac-Moody algebras have however same general form.

1. Assume that positive and negative energy states correspond to same Super Virasoro representations characterized by central charge c and the conformal weight Δ of vacuum state. Assume also that the Hermitian generators generators $L^+_n + L^-_{-n}$, $n<0$ annihilate the ground states which are possibly tachyonic in the sense having ground state conformal weight $N_+ = N = -N_- \leq 0$ and from which massless ground states are created by applying other that Super Virasoro generators.

2. Assume that the state does not contain conformal weights smaller than $N_+ \leq 0$. From the vanishing of net conformal weight the states are therefore of the general form

$$|phys\rangle_N \quad = \quad \sum_{n=0}^{N}\sum_{\alpha_n}\sum_{\beta_n} C(n,\alpha_n,\beta_n)|n,\alpha_n\rangle \otimes |-n,\beta_n\rangle \ . \tag{3}$$

Virasoro conditions reduce to those for $L^+_{-1}+L^-_1$ and $L^+_{-2}+L^-_2$ and give linear conditions for the coefficients $C(n,\alpha_n,\beta_n)$. Here α_n and β_{-n} index the states of same conformal weight n. The numbers $n(N)$ of the states at level N is given by the generating functional

$$\prod_{n\geq 1}(1-x^n)^{-1} \quad = \quad \sum_{N\geq 0} n(N)x^N \ . \tag{4}$$

Consider now the implications of these conditions.

1. For the component $n=N<0$ the conditions imply that the tachyonic positive energy state $|N\rangle$ is annihilated by Virasoro generators L_n, $n<0$. Same result is obtained in negative energy sector for $n=-N$. The states with $n<N$ states are thus null norm states with respect to the natural inner product for Virasoro modules having $c\neq 0$ and therefore not physical states. The simplest solutions do not require the mixing of different conformal weights and are just products $|N\rangle|-N\rangle$.

2. From the tachyonic ground state a massless state with a vanishing conformal weight is generated by applying generators of super algebras. The generators $L_n \otimes L_{-n}$, $n > 0$, acting on tensor products of positive and zero energy states generate p-adic thermal excitations of these states so that one obtains naturally p-adic thermodynamics and particle massivation.

3. The existence of null norm state implies that the vanishing of the determinant of the metric defined by the inner product. The classical result of Kac characterizing the values of ground state conformal weights Δ, and the weight N of the null norm state for a given value of c in the case of ordinary conformal algebra generalizes to super-conformal algebras.

3.2.4 p-Adicization favors rational values for central extension parameter and vacuum conformal weights

p-Adicization strongly suggests that the vacuum conformal weights and central extension parameter are rational numbers. Also algebraic numbers could in principle considered too: this would not give any conditions if square root allowing algebraic extension of p-adic numbers are used.

1. $N = 0$ case

For ordinary conformal algebra the null states are characterized by the conditions

$$\begin{aligned} \Delta_{mm'} &= \Delta_0 + \frac{1}{4}(\alpha_+ m + \alpha_- m')^2 \ , \ m \ , \ m' \geq 1 \ , \\ N &= mm' \ , \\ \Delta_0 &= \frac{1}{24}(c-1) \ , \\ \alpha_\pm &= \frac{\sqrt{1-c} \pm \sqrt{25-c}}{\sqrt{24}} \ . \end{aligned} \tag{5}$$

Thus arbitrarily high conformal weights N are possible in the construction. For $c \in (1, 25)$ the conformal weights are complex.

For ordinary conformal algebra rationality implies that the ground state conformal weight satisfies

$$\Delta_{mm'} = \frac{(mp' - m'p)^2 - (p' - p)^2}{4pp'} \ , \ 0 < m < p \ , \ 0 < m' < p' \ . \tag{6}$$

A more elegant expression for the central charge and weights reads as

$$\begin{aligned} c &= 1 - \frac{6}{Q(Q+1)} \ , \\ \Delta_{mm'} &= \frac{1}{4Q(Q+1)} \left[(Q(m - m' + m)^2 - 1 \right] \ , \end{aligned}$$

$$Q = \frac{p}{p'-p} \ . \tag{7}$$

These conditions also imply also that the fusion rules close for a finite number of primary fields in the corresponding conformal field theory.

For $p' = p+1$ the minimal model is unitary. In this case one has $Q = p$ is integer $n \geq 3$. This range of integers characterizes also the allowed values of quantum phase characterizing Jones inclusions. Furthermore, Q is related to Kac-Moody central extension in $SU(2)_k$ theories by $Q = k+2$.

The ground state conformal weight corresponds to $m = m' = 1$ and vanishes. The null norm state however possesses the conformal weight $mm' \geq 1$ and is therefore massive. The tachyon of string theories with conformal weight 1 is transformed in TGD framework to the absence of massless states in full accordance with the breaking of conformal invariance. $Q = p = n$ corresponds naturally to the integer labelling Jones inclusion defining both UV and IR cutoffs with respect to conformal weight. For $c = 0$ representation without breaking of conformal invariance all states are null norm states and the spectrum contains also massless particles. These representations correspond to $n = \infty$ case for Jones inclusions and to full Kac-Moody symmetry and ordinary string theory in accordance with the general picture.

Since minimal conformal field theories are in question, the number of primary fields is restricted by the conditions $0 < m < p$ and $0 < m' < p' = p+1$. By the symmetry $\Delta_{mm'} = \Delta_{p-m,p'-m'}$. If corresponding primary fields can be identified, one has $0 < m < m' < p'(= p+1)$ and $0 < m < p$.

2. Rationality for $N = 1, 2$ super-conformal algebras

The previous considerations apply on Virasoro algebra. These considerations generalize to the case of Kac-Moody algebra and also to corresponding Super algebras. In case of Super Virasoro algebra rationality requirement gives rise to different conditions on the values of c and Δ_{mn} depending in the value of N. $N = 1$ super-conformal algebra corresponds to one real super charge and one real super field and is non-physical in TGD framework. $N = 2$ case corresponds to single complex super charge and one complex super-field. In this case the Super Virasoro algebra involves also U(1) Kac-Moody algebra as inherent algebra. If these algebras are important in TGD framework, it would be natural to assign these algebras to quark and lepton type gamma matrices.

The values of the central extension parameter and conformal weights for $N = 0, 1, 2$ for unitary rational field theories at sphere are summarized by the following table [40].

c_k	$1 - \frac{6}{(k+2)(k+3)}$	$\frac{3}{2}(1 - \frac{8}{(k+2)(k+4)})$	$3(1 - \frac{2}{k+2})$
$\Delta_{mm'}$	$\frac{\left[(k+2)m-(k+1)m'\right]^2-1}{4(k+2)(k+3)}$	$\frac{\left[(k+4)m-(k+2)m'\right]^2-4}{8(k+2)(k+4)}$	$\frac{m(m+2)-m'^2}{4(k+2)}$ $(q = \frac{mm'}{k+2}$
m, m'	$1 \leq m \leq k+1$ $1 \leq m' \leq k+2$	$1 \leq m \leq k+2$ $1 \leq m' \leq k+4$	$0 \leq m \leq k$ $-m \leq m' \leq m$

$$ (8) $$

It must be stressed that the conformal weights assignable to zero energy states are given by $\Delta_{m,m'} + mm'$ whereas in conformal field theories physical states have conformal weights $\Delta_{m,m'}$. For partonic 2-surfaces with handles modular invariance poses additional constraints since primary fields must form a closed set also under modular transformations [40]. In the table above $q = m'/(k+2)$ corresponds to U(1) charge.

3. Rationality for $N = 4$ SCA

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry defines the fundamental partonic super-conformal symmetry in TGD framework. In the case of SKM algebra the groups would act on induced spinors with $SU(2)_+$ representing spin rotations and $SU(2)_- \times U(1) = U(2)_{ew}$ electro-weak rotations. In super-canonical sector the action would be geometric: $SU(2)_+$ would act as rotations on light-cone boundary and $U(2)$ as color rotations leaving invariant a preferred CP_2 point.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [57]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$
\begin{aligned}
k_{\pm} &\equiv k(SU(2)_{\pm}) \ , \\
k_1 &\equiv k(U(1)) = k_+ + k_- \ .
\end{aligned} \tag{9}
$$

The central extension parameter c is given as

$$ c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{10} $$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$
\begin{aligned}
k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\
c &= \frac{6(k+1)}{k+2} \ .
\end{aligned} \tag{11}
$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. Central extension would be trivial in rotational degrees of freedom but non-trivial in $U(2)_{ew}$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$. A possible interpretation is in terms of electro-weak symmetry breaking with $k_+ > 0$ signalling for the massivation of electro-weak gauge bosons.

An interpretation consistent with the general vision about the quantization of Planck constants is that k_+ and k_- relate directly to the integers n_a and

n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants via the formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$. This would require $k_\pm \geq 1$ for G_i a finite subgroup of $SU(2)$ ("anyonic" phases). In stringy phases with $G_i = SU(2)$ for $i = a$ or $i = b$ or for both, k_i could also vanish so that also $n_i = 2$ corresponding to A_2 ADE diagram and $SU(2)$ Kac-Moody algebra becomes possible. In the super-canonical sector $k_+ = 0$ would mean massless gluons and $k_- = k_1$ that $U(2) \subset SU(3)$ and possibly entire $SU(3)$ represents an unbroken symmetry.

3.3 The amplitudes for creation of zero energy states from vacuum have stringy structure

The possibility to represent primary fields of rational conformal field theories as exponentials of stringy fields implies under very general assumptions that the amplitudes for creating zero energy states from vacuum have a structure similar to string amplitudes.

3.3.1 General assumptions

Let us list first the general assumptions leading to stringy scattering amplitudes.

1. Quantum criticality of TGD would suggest that, as far as conformal invariance is considered, all details about the microscopic dynamics can be forgotten and the amplitudes for the generation of zero energy states from vacuum can be expressed as vacuum expectation values of the products of primary fields of a rational conformal field theory at partonic 2-surfaces. The primary fields in question do not directly correspond to the M^4 local versions of fundamental super-conformal algebras creating states at the intersections of partonic causal determinants with $\delta M^4_+ \times CP_2$. Rather, they would describe the states created by these operators and possessing conformal weights consistent with rationality. Hence one can completely forget the detailed anatomy of these states and only the values of c and $\alpha = \Delta_{mn}$ matters.

2. Since the conformal weights of primary fields are non-negative, mass squared identified as conformal weight using CP_2 mass as unit is non-negative and no problems with tachyons are encountered. The deeper reason for the non-negativity of conformal weights would be that the super-canonical and Kac-Moody contributions to conformal weight sum up to a non-negative net result. It is important to notice that the vertex operators $V(z)$ representing Kac-Moody generators used to construct stringy scattering amplitudes have positive conformal weight $\Delta = mm'$ for $c \neq 0$ case and, as is clear on basis of Sugawara representation, they would correspond to a negative mass squared in stringy models. This would correspond to the convention $m^2 = kL_0$, $k < 0$ rather than $k > 0$, in TGD framework. It must be added that TGD mass formula is definitely not consistent with that of string models.

3. The first guess is that the expressions for the amplitudes for creating zero energy state generalize as such an could be expressed in terms of the vacuum expectation values of n-point functions for the primary fields of rational conformal field theories. Stringy form would be obtained by the integration of the arguments over a circle of the partonic 2-surface and by using standard arguments one could fix 3 of the arguments z_i to $z = 0, 1, \infty$ in case of sphere. Apart from the normalization constant the resulting amplitude would have the general form

$$
\begin{aligned}
A(\alpha_1, ..., \alpha_n) &= \int \prod_{i=4}^{n} dz^i \langle \phi_{\alpha_1}(0), \phi_{\alpha_2}(1), \phi_{\alpha_3}(\infty) \phi_{\alpha_4}(z_4) ..... \phi_{\alpha_n}(z_n) \rangle \ , \\
\sum_n \alpha_n = 0 \ . & \qquad (12)
\end{aligned}
$$

Note that the conformal weights of negative energy particles are negative.

3.3.2 Free field representation of rational conformal field theories gives stringy amplitudes

Rational conformal field theories allow a representation of the primary fields in terms of exponentials of massless free fields $X(z)$ [40] with the energy momentum tensor

$$
T(z) = -\frac{1}{4} : [\partial X(z)]^2 : \qquad (13)
$$

The correlation functions of $X(z)$ and $\partial X(z)$ are

$$
\begin{aligned}
\langle X(z) X(\zeta) \rangle &= -2 log(z - \zeta) \ , \\
\langle \partial X(z) \partial X(\zeta) \rangle &= -\frac{2}{(z-\zeta)^2} \ . \qquad (14)
\end{aligned}
$$

$X(z)$ has the stringy expansion

$$
\begin{aligned}
X(z) &= \sqrt{2} \left(q - ip \times log(z) + i \sum \frac{a_n}{n} z^n \right) \ , \\
[q, p] &= i \ , \ [a_n, a_m] = n \delta_{n+m,0} \ . \qquad (15)
\end{aligned}
$$

There is of course no need to assume that strings are the underlying dynamical objects and z corresponds to the complex coordinate of the partonic 2-surface in TGD context.

The normal order exponentials of the free field

$$V_\alpha(z) = :exp(i\alpha X(z):$$
$$= exp(i\sqrt{2}\alpha q)exp(i\sqrt{2}\alpha p)exp\left(\sqrt{2}\alpha\sum_{n>0}\frac{a_{-n}}{n}z^n\right)exp\left(-\sqrt{2}\alpha\sum_{n>0}\frac{a_n}{n}z^n\right) . \quad (16)$$

are also primary fields of conformal weight α^2. All primary fields of minimal models can be represented in this manner apart from possible factors relating to internal quantum numbers. For $\alpha^2 = 1$ one obtains representation for the charged generators of ADE type Kac-Moody Lie-algebras in this manner.

The n-point function for these fields can be deduced by using Campbell-Hausdorf formula

$$:exp(i\alpha X(z)) :: exp(i\alpha X(\zeta)) := (z-\zeta)^{2\alpha\beta} : exp(i\alpha X(z) + i\beta X(\zeta)) : , \quad (17)$$

and is given by

$$\langle V_{\alpha_1}(z_1)V_{\alpha_2}(z_2)....\phi_\alpha(z_n)\rangle = \prod_{i<j}(z_i - z_j)^{2\alpha_i\alpha_j} \quad (18)$$

for $\sum \alpha_i = 0$ and vanishes otherwise. Thus conformal invariance of zero energy states follows from mere internal consistency. Thus rational CFT:s and obviously also ($c = h = 0$) case, would give the basic stringy expression for the amplitudes for creating zero energy states from vacuum.

Consider now whether and how four-momenta could appear in this formula.

1. The number theoretic $M^4 = M^2 \times E^2$ decomposition and quantum classical correspondence are in accordance with the assignment of Kac-Moody generators with E^2 degrees of freedom. The physical interpretation would be in terms of deformations of partonic 2-surface restricted to $\delta M^4_\pm$ with one light-like coordinate so that only two degrees of freedom remain since light-like direction corresponds to Super Virasoro symmetries in the construction of configuration space geometry. The generator of Kac-Moody algebra with zero norm would naturally correspond to the light-like direction along M^4_+ for super-canonical algebra and along light-like partonic surface for Kac-Moody algebra.

   One could wonder whether both of these zero norm generators could be included to the extended Dynkin diagram so that twisted affine Lie-algebra would result ($A_2^{(2)}$, $A_{2l}^{(2)}$ with $l \geq 2$, $A_{2l-1}^{(2)}$ with $l \geq 3$, $D_{l+1}^{(2)}$ with $l \geq 2$, and $E_6^{(2)}$ are possible [40]).

2. Suppose therefore that the formula generalizes to 4-D case simply by assigning to each component p^k of four-momentum its own quantized M^4 coordinate X^k such that oscillator operator contribution is absent in M^2 degrees of freedom, and requiring $p^k p_k = \alpha^k \alpha_k = \alpha^2$ in suitable units: α^2 is the conformal weight of the primary field. The identification of the mass squared value as conformal weight would follow automatically using this ansatz. The interpretation would differ from that adopted in string models since only the counterparts of tachyonic scattering amplitudes would be allowed as is indeed natural in zero energy ontology.

3. If CP_2 mass is the unit of quantization the mass unit would be about 10^{-4} Planck masses. This mass scale should apply to the fundamental representations associated with the symmetries of the imbedding spaces. Physical intuition would suggest that p-adic mass squared defines the natural unit of quantization and that hadronic mass squared could be quantized in this manner. This quantization might occur for the secondary Kac-Moody representations defined by ADE series in the case of $q \neq 1$ Jones inclusions and extended ADE series in the case of $q = 1$ Jones inclusions suggested in [A9] to occur for large values of $\hbar$. The generation of multiplets of ADE quantum groups and ADE Kac Moody algebra could be made possible by the multiple coverings of M^4 defined by the space-time sheets for which points covering given point of M^4 are related by a discrete subgroup of $G_a \times G_b \subset SL(2,C) \times SU(2)$ (where one has $SU(2) \subset SU(3)$) defining the Jones inclusion. Thus one could say that TGD universal in the sense of being able to represent the quantum dynamics associated with any ADE type quantum group or Kac-Moody group.

4. The alert reader probably recalls the fact that four-momenta cannot appear in the definition of super generators of super-canonical and Kac-Moody algebras at the fundamental configuration space level. If the representations associated with the minimal super-conformal models are only a secondary concept, the situation changes for $N = 2$ algebra relevant for TGD. The point would be that gamma matrices in M^2 degrees of freedom could be complexified while keeping momenta real.

3.3.3 Is stringy perturbation theory needed?

The basic hypothesis have been that stringy perturbation theory is un-necessary but one cannot take this hypothesis as a blind belief.

1. In string models unitarity condition for the S-matrix leads to stringy perturbation theory, and it might be that exactly the same requirement forces a perturbative expansion in TGD framework by the iteration of $i(T - T^\dagger) = TT^\dagger$ condition. In string models the perturbation theory involves summation over different genera of closed string word sheets in the case of Euclidianized closed strings.

2. Also in TGD framework the functional integral over the vertex 3-surfaces X^3_V can involve summation over different genera for partonic 2-surfaces. Path integral is replaced with a well-defined functional integral in TGD framework. Also the physical situation is completely different since fermions correspond to states localized to single genus apart from topological mixing providing a first principle explanation of CKM mixing [F4]. For bosons a delocalization with respect to genus is implied by the fact that gauge couplings are same for various fermion families: the reason is presumably that for bosons p-adic thermodynamics gives extremely small genus dependent contribution to the mass so that bosons of different genera have very nearly identical masses and can mix with each other. One can of course ask whether the strong mixing of neutrinos could be due to the same reason.

3.4 What about configuration space degrees of freedom?

If the proposed picture is even roughly correct, also super-canonical sector involving functional integration over configuration space degrees of freedom could be treated exactly. As far as conformal invariance is considered, everything would boil down to the correlation functions of primary fields of $N = 4$ super-conformal field theories serving as representations of physical states. If this picture really works, it would be a demonstration of the immense power of the quantum criticality hypothesis.

The considerations based on quantum classical correspondence, in particular the argument relating the weakness of gravitational interaction and gauge boson masses to the properties of corresponding CP_2 type extremals, make it clear that configuration space degrees of freedom must play a highly non-trivial role.

The challenge is to disentangle the contribution of these degrees of freedom from the contributions of super-canonical degrees of freedom represented by a functional integral around maxima of Kähler function. An optimistic guess would be that the maxima of Kähler function for X^3_V alone code for this contribution and that in practice everything reduces to the average value of Kähler function for CP_2 type extremals representing virtual particles.

It is quite possible that configuration space degrees of freedom give rise to a tensor factor of type I unless super-conformal symmetry somehow manages to transform this tensor factor to HFF of of type II_1. In this case thermal S-matrix in configuration space degrees of freedom would be necessary.

3.5 Zero energy ontology and Witten's approach to 3-D quantum gravitation

There is an interesting relationship to the recent yet unpublished work of Witten related to 3-D quantum blackholes [63] which allows to get additional perspective.

1. The motivation of Witten is to find an exact quantum theory for black-

holes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [64, 63] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [65]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that gravitational radiation is not possible. HenceAdS_3 theory in Witten's sense might define this conformal field theory.

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory.

2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.

3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD [C3], gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and

have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

2. Also the fermionic counterpart of Chern-Simons action for the induced spinors whose form is dictated by the super-conformal symmetry is present. Furthermore, partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [A9, C3]. Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined

by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [A9]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [C3, E11].

6. To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   i) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   ii) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   iii) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D lightlike surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about S-matrix implied by the zero energy ontology.

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the

category of Feyman cobordisms and matrices have groupoid like structure (see discussion below). Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because S-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D lightlike surfaces as submanifolds (analogs of blackhole horizons and lightlike boundaries) [B2, B3]. Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the modified Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [B4].

4 S-matrix as a functor

John Baez's [72] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to imbedding space would conform with category theoretic thinking.

4.1 The *-category of Hilbert spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T^*_Ψ mapping Hilbert space to C, inner products can be defined as morphisms $T^*_\Phi T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the imbedding space in TGD.

4.2 The monoidal *-category of Hilbert spaces and its counterpart at the level of nCob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional imbedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

4.3 TQFT as a functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

1. Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?

2. Should one give up the unitarity condition and require that the theory

predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?

3. What is the relevance of this result for quantum TGD?

4.4 The situation is in TGD framework

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

4.4.1 Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naive idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [61] only the exotic diffeo-structures modify the situation in 4-D case.

4.4.2 Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

4.4.3 Feynmann cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of 2→2 reaction open string is pinched to a point at vertex. 1→2 vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

4.4.4 Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy. Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

The new element would be quantum measurements performed separately for observables assignable to positive and negative energy states. These measurements would be characterized in terms of Jones inclusions. The state function reduction for the negative energy states could be interpreted as a detection of a particle reaction.

4.4.5 Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

1. There is U-matrix between zero energy states. This is expected to be rather trivial but very important from the point of view of description of intentional actions as transitions transforming p-adic partonic 3-surfaces to their real counterparts.

2. The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics. Note that also the presence of factor of type I coming from imbedding space degrees of freedom forces thermal S-matrix.

4.4.6 Time-like entanglement coefficients as a square root of density matrix?

All quantum states do not correspond to thermal states and one can wonder what might be the most general identification of the quantum state in zero energy ontology. Density matrix formalism defines a very general formulation of quantum theory. Since the quantum states in zero energy ontology are analogous to operators, the idea that time-like entanglement coefficients in some sense define a square root of density matrix is rather natural. This would give the defining conditions

$$\begin{aligned} \rho^+ &= SS^\dagger \ , \rho^- = S^\dagger S \ , \\ Tr(\rho^\pm) &= 1 \ . \end{aligned} \tag{19}$$

$\rho^\pm$ would define density matrix for positive/negative energy states. In the case HFFs of type II_1 one obtains unitary S-matrix and also the analogs of pure quantum states are possible for factors of type I. The numbers $p^+_{m,n} = |S^2_{m,n}|/\rho^+_{m,m}$ and $p^-_{m,n} = |S^2_{n,m}|/\rho^-_{m,m}$ give the counterparts of the usual scattering probabilities.

A physically well-motivated hypothesis would be that S has expression $S = \sqrt{\rho}S_0$ such that S_0 is a universal unitary S-matrix, and $\sqrt{\rho}$ is square root of a state dependent density matrix. Note that in general S is not diagonalizable in the algebraic extension involved so that it is not possible to reduce the scattering to a mere phase change by a suitable choice of state basis.

What makes this kind of hypothesis aesthetically attractive is the unification of two fundamental matrices of quantum theory to single one. This unification is completely analogous to the combination of modulus squared and phase of complex number to a single complex number: complex valued Schrödinger amplitude is replaced with operator valued one.

4.4.7 S-matrix as a functor and the groupoid structure formed by S-matrices

In zero energy ontology S-matrix can be seen as a functor from the category of Feynman cobordisms to the category of operators. S-matrix can be identified as a "square root" of the positive energy density matrix $S = \rho_+^{1/2} S_0$, where S_0 is a unitary matrix and ρ_+ is the density matrix for positive energy part of the zero energy state. Obviously one has $SS^\dagger = \rho_+$. $S^\dagger S = \rho_-$ gives the density matrix for negative energy part of zero energy state. Clearly, S-matrix can be seen as matrix valued generalization of Schrö;dinger amplitude. Note that the "indices" of the S-matrices correspond to configuration space spinors (fermions and their bound states giving rise to gauge bosons and gravitons) and to configuration space degrees of freedom (world of classical worlds). For hyperfinite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [62]. Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^\dagger = f\rho_{g,+}$ and $f^\dagger fg = \rho_{f,-}g$, and the conditions $ff^\dagger = \rho_+$ and $f^\dagger f = \rho_-$ are satisfied. Here $\rho_\pm$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^\dagger\rho_{f,+}^{-1}$ satisfies $ff_L^{-1} = Id_+$ and $f_R^{-1} = \rho_{f,-}^{-1}f^\dagger$ satisfies $f_R^{-1}f = Id_-$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ-matrix. Since the interaction with Kea's M-theory blog at http://kea-monad.blogspot.com/ (M denotes Monad or Motif in this context) was led ot the realization of the connection with density matrix, also M-matrix might be considered. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough? Certainly it would inspire feeling of awe!

5 HFFs and S-matrix

In this section following topics are discussed.

1. A general master formula for the construction of S-matrix is proposed assuming that the generalization of duality symmetry of old-fashioned string models implying the reduction of diagrams to diagrams involving only single vertex makes sense. The S-matrix elements would be obtained by replacing ordinary tensor product for free fields with Connes tensor product so that a hierarchy of S-matrices parameterized by Jones inclusions would result.

2. It must be emphasized that this is just a starting hypothesis and the

considerations of the next section suggest a profound modification of the ontology so that zero energy states become the physical states and S-matrix for them gives rise ordinary S-matrix as a kind of statistical average over undetected degrees of freedom.

3. The hierarchy of Jones inclusions is shown to lead to a hierarchy of S-matrices in which Feynman diagrams of previous level can be said to represent states of the next level. As a matter, the S-matrix between zero energy states at first level would give rise to the ordinary S-matrix.

5.1 Von Neumann algebras and TGD

The realization that so called hyper-finite factors of type II_1 are an inherent property of quantum TGD meant a breakthrough in the understanding of the mathematical structure of quantum TGD.

5.1.1 TGD emerges from the localization of infinite-dimensional Clifford algebra

A crucial step in the progress was the realization that TGD emerges from the mere idea that a local version of hyper-finite factor of type II_1 represented as an infinite-dimensional Clifford algebra must exist (as analog of say local gauge groups). This implies a connection with the classical number fields. Quantum version of complexified octonions defining the coordinate with respect to which one localizes is unique by its non-associativity allowing to uniquely separate the powers of octonionic coordinate from the associative infinite-dimensional Clifford algebra elements appearing as Taylor coefficients in the expansion of Clifford algebra valued field [A8].

Associativity condition implies the classical and quantum dynamics of TGD. Space-time surfaces are hyper-quaternionic of co-hyper-quatenionic sub-manifolds of hyper-octonionic imbedding space HO. Also the interpretation as a four-surface in $H = M^4 \times CP_2$ emerges and implies $HO - H$ duality. What is also nice that Minkowski spaces correspond to the spectra for the eigenvalues of maximal set of commuting quantum coordinates of suitably defined quantum spaces. Thus Minkowski signature has quantal explanation.

5.1.2 Quantization of Planck constants

The geometric and topological interpretation of Jones inclusions led to the understanding of the quantization of Planck constants assignable to M^4 and CP_2 degrees of freedom (identical in "ground state"). The Planck constants are scaled up by the integer n defining the quantum phase $q = exp(i\pi/n)$ characterizing the Jones inclusion, which in turn corresponds to subgroup G of $SL(2,C) \times SU(2)_L \times U(1)$ in the simplest situation. The quantum phase can be assigned also to q=1 inclusions in which case second quantum phase can be associated with the monodromies of the corresponding conformal field theory.

The scaling of $M^4_\pm$ Planck constant by n_a means scaling of CP_2 metric by n_a, where n_a is the order of the maximal cyclic subgroup of G_a. And vice versa. The observed Planck constant must correspond to $\hbar_{eff}/\hbar_0 = n_a/n_b$ from the fact that only the ratio $\hbar(M^4_\pm)/\hbar(CP_2)$ appears in Kähler action. CP_2 can therefore have arbitrarily large size: hyper space travel might not be unrealistic after all! For infinite subgroups such as $G = SU(2) \subset SU(3)$ the situation is somewhat different. The variants of imbedding space can meet each other if either M^4 or CP_2 factors have same value of Planck constant so that a fan (or rather tree-) like structure results. Analogous picture emerged already earlier from the gluing of the p-adic variants of imbedding space along common rationals (or algebraics in more general case). The phase transitions changing the Planck constant have purely topological description.

An important outcome was the interpretation of McKay correspondence: one can assign to the ADE diagram of $q \neq 1$ Jones inclusion the corresponding gauge group. The $n(G)$-fold covering of $M^4_\pm$ points by a finite number of CP_2 points makes possible to realize the multiplets of gauge group purely geometrically in terms of G group algebra. In the case of extended ADE diagrams assignable to $q = 1$ Jones inclusions the group is Kac-Moody group. This picture applies both in M^4 and CP_2 degrees of freedom.

1. In CP_2 degrees of freedom this framework allows to understand anyonic charge fractionization and raises the question whether fractional Hall effect corresponds to the integer valued quantum Hall effect with scaled up Planck constant and whether free quarks could be integer charged and have fractional charges only inside hadrons.

2. In $M^4_\pm$ degrees of freedom this picture has fascinating cosmological consequences and leads to a possible explanation for the quantization of cosmic recession velocities in terms of lattice like structures (tesselations) of light-cone proper time constant hyperboloid defined by infinite subgroups of Lorentz group and consisting of dark matter in macroscopically quantum coherent phase.

5.1.3 Relationship to super-strings and M-theory

The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2+2+2+2$ structure of the Cartan algebra, which together with the notions of the configuration space and generalized coset representation formed from super Kac-Moody and super-canonical algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

5.1.4 Should one accept spontaneous breaking of Lorentz and color symmetries at the level of S-matrix?

One can assign to each sector of generalized imbedding space preferred quantization axis. This suggests that S-matrix identified as entanglement coefficients breaks Lorentz symmetry and color symmetry. This is natural if one accepts S-matrix as characterizer of zero energy state rather than of Universe. This symmetry breaking could be interpreted as a space-time correlate for the selection of the Cartan sub-algebra of the isometry group in quantum measurement situation and would thus represent an inherent property of quantum theory, something much deeper than a trouble produced by a gauge choice as in string models. Since the interior degrees of freedom of the space-time sheets correspond to those assignable to the measurement apparatus, the breaking of Lorentz and color symmetries at space-time level would provide a space-time correlate for this symmetry breaking.

There are several instances where this symmetry breaking indeed makes itself manifest.

1. The possibility to assign almost topological quantum numbers to M^4 and CP_2 degrees of freedom (see the appendix of the book or [D7]) involves a selection of Cartan sub-algebra of the isometry group.

2. A very general solution ansatz for the field equations based on Hamilton-Jacobi coordinates discussed in [D1] involves a local $M^2 \times E^2$ decomposition of M^4.

3. The Abelian holonomy for the classical color fields could be interpreted in terms of the reduction of color symmetries to Cartan algebra.

From the foregoing it is clear that also momentum space discretization requires breaking of Lorentz invariance. An interesting question, is whether the breaking of Lorentz symmetry is already encountered in the hadronic scattering in quark model description, which involves the reduction of Lorentz group to $SO(1,1) \times SO(2)$ corresponding to longitudinal and transverse momenta. The selection of quantization axis in astrophysical length scales together with gigantic value of gravitational Planck constant is an especially fascinating possibility whose implications have been discussed in [D6].

5.1.5 What about Lorentz invariance?

For $c = 0$ representations of $N = 4$ SCA critical dimension $D = 4 + 4$ should guarantee Lorentz invariance: this is indeed expected since the situation corresponds to Jones inclusion with trivial group $G = \{e\}$. One cannot however exclude the breaking of the full Lorentz and color symmetries for $c \neq 0$ representations of $N = 4$ SCA, which at the level of Jones inclusion means a change of the geometry and topology of the imbedding space and space-time.

The loss of Lorentz invariance would not be a catastrophe since S-matrix is a property of state rather than that of Universe in TGD framework. Only U-matrix need to be unitary. The interpretation would be in terms of quantum measurement theory selecting a preferred Cartan subgroup for observables. This kind of breaking of course happens in the realistic experimental situation and if state describes also the measurement situation, the breaking is expected. For the scattering of zero energy states Lorentz invariance is obtained in a statistical sense.

This relates interestingly to the claimed uniqueness of super-string model if one requires unitarity and Lorentz invariance. Super string theorists might be right: only 10-D super strings might give rise to a unitary and Lorentz invariant S-matrix in perturbative sense although the perturbation series does not converge. They might be wrong in their belief that S-matrix is property of the Universe.

One could interpret $M^4 \rightarrow M^2 \times E^2$ symmetry breaking as a vanishing of the Kac-Moody central charge k in M^2 factor so that non-broken gauge invariance results. This conforms with the fact that factorizing S-matrices correspond to finite-dimensional representations of loop group.

Whether Lorentz invariance is achieved for the stringy S-matrix characterizing entanglement between positive and negative energy states, depends on the assumptions one is ready to make about states and about what happens in state function reduction. The light cone quantization of string models involves $M^2 \times E^2$ decomposition interpreted now as a gauge choice and the scattering amplitudes are Lorentz invariant in the critical dimension. Due to the selection of preferred quantization axes the sectors of the configuration space are not Lorentz invariant. If zero energy states are identified as Lorentz invariant superposition of Lorentz transforms of a state in a given sector Lorentz invariance is achieved. Without this assumption it is not clear whether Lorentz invariance is achieved since zero energy ontology implies that the net Poincare quantum numbers assignable to the S-matrix elements vanish but does not imply Lorentz invariance. Similar conclusions apply in case of color quantum numbers.

5.2 Finite measurement resolution: from S-matrix to M-matrix

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum S-matrix for which elements have values in sub-factor of HFF rather than being complex numbers. It is still possible to satisfy generalized unitarity condition but one can also consider the possibility that only probabilities are conserved.

5.2.1 Jones inclusion as characterizer of finite measurement resolution at the level of S-matrix

Jones inclusion $\mathcal{N} \subset \mathcal{M}$ characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalized. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) \quad = \quad Tr_{\mathcal{M}/\mathcal{N}}(Tr_{\mathcal{N}}(X)) \ . \tag{20}$$

   Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle \quad = \quad \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{21}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M}:\mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M}:\mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{22}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \rightarrow r_2) \quad = \quad \mathcal{M}:\mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M}:\mathcal{N} \times Tr(P_N) = 1 \tag{23}$$

5. Unitary at the level of $\mathcal{M}/\mathcal{N}$ is obtained if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$.

5.2.2 Quantum S-matrix

The description of finite measurement resolution in terms of Jones inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full S-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum S-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full S-matrix can be expressed as S-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the S-matrix.

Quantum S-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

5.2.3 Quantum fluctuations and Jones inclusions

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of imbedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of imbedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For S-matrix in $\mathcal{M}/\mathcal{N}$ quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the S-matrix. The properties of the number theoretic braids contributing to the S-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

4. Accepting number theoretical vision, quantum criticality would mean that super-canonical conformal weights and/or generalized eigenvalues of the modified Dirac operator correspond to zeros of Riemann ζ so that the points of the number theoretic braids would be mapped to fixed points of G_a and G_b at geodesic spheres of $\delta M^4_+ = S^2 \times R_+$ and CP_2. Also weaker critical points which are fixed points of only subgroup of G_a or G_b can be considered.

5.3 Does Connes tensor product fix the allowed M-matrices?

Hyperfinite factors of type II_1 and the inclusion $\mathcal{N} \subset \mathcal{M}$ inclusions have been proposed to define quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ and with complex rays of state space replaced with $\mathcal{N}$ rays. What this really means is far from clear.

1. Naively one expects that matrices whose elements are elements of $\mathcal{N}$ give a representation for M. Now however unit operator has unit trace and one cannot visualize the situation in terms of matrices in case of $\mathcal{M}$ and $\mathcal{N}$.

2. The state space with $\mathcal{N}$ resolution would be formally $\mathcal{M}/\mathcal{N}$ consisting of $\mathcal{N}$ rays. For $\mathcal{M}/\mathcal{N}$ one has finite-D matrices with non-commuting elements of $\mathcal{N}$. In this case quantum matrix elements should be multiplets of selected elements of $\mathcal{N}$, **not all** possible elements of $\mathcal{N}$. One cannot

therefore think in terms of the tensor product of $\mathcal{N}$ with $\mathcal{M}/\mathcal{N}$ regarded as a finite-D matrix algebra.

3. What does this mean? Obviously one must pose a condition implying that $\mathcal{N}$ action commutes with matrix action just like C: this poses conditions on the matrices that one can allow. Connes tensor product [48] does just this. Note I have proposed already earlier the reduction of interactions to Connes tensor product (see the section *"Could Connes tensor product...."* later in this chapter) but without reference to zero energy ontology as a fundamental manner to define measurement resolution with respect time and assuming unitarity.

5.3.1 The argument demonstrating almost uniqueness of M-matrix

The starting point is the Jones inclusion sequence

$$\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}...$$

Here $\mathcal{M} \otimes_N \mathcal{M}$ is Connes tensor product which can be seen as elements of the ordinary tensor product commuting with $\mathcal{N}$ action so that $\mathcal{N}$ indeed acts like complex numbers in $\mathcal{M}$. $\mathcal{M}/\mathcal{N}$ is in this picture represented with $\mathcal{M}$ in which operators defined by Connes tensor products of elements of $\mathcal{M}$. The replacement $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ corresponds to the replacement of the tensor product of elements of $\mathcal{M}$ defining matrices with Connes tensor product.

One can try to generalize this picture to zero energy ontology.

1. $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ would be generalized by $\mathcal{M}_+ \otimes_{\mathcal{N}} \mathcal{M}_-$. Here $\mathcal{M}_+$ would create positive energy states and $\mathcal{M}_-$ negative energy states and $\mathcal{N}$ would create zero energy states in some shorter time scale resolution: this would be the precise meaning of finite measurement resolution.

2. Connes entanglement with respect to $\mathcal{N}$ would define a non-trivial and unique recipe for constructing M-matrices as a generalization of S-matrices expressible as products of square root of density matrix and unitary S-matrix but it is not how clear how many M-matrices this allows. In any case M-matrices would depend on the triplet $(\mathcal{N}, \mathcal{M}_+, \mathcal{M}_-)$ and this would correspond to p-adic length scale evolution giving replacing coupling constant evolution in TGD framework. Thermodynamics would enter the fundamental quantum theory via the square root of density matrix.

3. The defining condition for the variant of the Connes tensor product proposed here has the following equivalent forms

$$MN = N^*M \quad , \quad N = M^{-1}N^*M \quad , \quad N^* = MNM^{-1} \quad . \tag{24}$$

If M_1 and M_2 are two M-matrices satisfying the conditions then the matrix $M_{12} = M_1 M_2^{-1}$ satisfies the following equivalent conditions

$$N = M_{12} N M_{12}^{-1} \quad , \quad [N, M_{12}] = 0 \ . \tag{25}$$

Jones inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ are irreducible which means that the operators commuting with $\mathcal{N}$ consist of complex multiples of identity. Hence one must have $M_{12} = 1$ so that M-matrix is unique in this case. For $\mathcal{M} : \mathcal{N} > 4$ the complex dimension of commutator algebra of $\mathcal{N}$ is 2 so that M-matrix depends should depend on single complex parameter. The dimension of the commutator algebra associated with the inclusion gives the number of parameters appearing in the M-matrix in the general case.

When the commutator has complex dimension $d > 1$, the representation of $\mathcal{N}$ in $\mathcal{M}$ is reducible: the matrix analogy is the representation of elements of $\mathcal{N}$ as direct sums of d representation matrices. M-matrix is a direct sum of form $M = a_1 M_1 \oplus a_2 M_2 \oplus ...$, where M_i are unique. The condition $\sum_i; |a_i|^2 = 1$ is satisfied and *-commutativity holds in each summand separately.

There are several questions. Could M_i define unique universal unitary S-matrices in their own blocks? Could the direct sum define a counterpart of a statistical ensemble? Could irreducible inclusions correspond to pure states and reducible inclusions to mixed states? Could different values of energy in thermodynamics and of the scaling generator L_0 in p-adic thermodynamics define direct summands of the inclusion? The values of conserved quantum numbers for the positive energy part of the state indeed naturally define this kind of direct direct summands.

It must be of course noticed that reducibility and thermodynamics emerge naturally also in another sense since a direct sum of HFFs of type II_1 is what one expects. The radial conformal weights associated light-cone boundary and X^3_l would indeed naturally label the factors in the direct sum.

4. Zero energy ontology is a key element of this picture and the most compelling argument for zero energy ontology is the possibility of describing coherent states of Cooper pairs without giving up fermion number, charge, etc. conservation and automatic emerges of length scale dependent notion of quantum numbers (quantum numbers identified as those associated with positive energy factor).

To sum up, interactions would be an outcome of a finite measurement resolution and at the never-achievable limit of infinite measurement resolution the theory would be free: this would be the counterpart of asymptotic freedom.

5.3.2 How to define the inclusion of $\mathcal{N}$ physically?

The overall picture looks beautiful but it is not clear how one could define the inclusion $\mathcal{N} \subset \mathcal{M}$ precisely. One must distinguish between two cases corresponding to the unitary U-matrix representing unitary process associated with the quantum jump and defined between zero energy states and M-matrix defining the time-like entanglement between positive and negative energy states.

1. In the case of U-matrix both $\mathcal{N}$ and $\mathcal{M}$ corresponds to zero energy states. The time scale of the zero energy state created by $\mathcal{N}$ should be shorter than that for the state defined naturally as the temporal distance t_{+-} between the tips of the light-cones $M^4_\pm$ associated with the state and defining diamond like structure.

2. In the case of M-matrix one has zero energy subalgebra of algebra creating positive or negative energy states in time scale t_{+-}. In this case the time scale for zero energy states is smaller than $t_{+-}/2$. The defining conditions for the Connes tensor product are analogous to crossing symmetry but with the restriction that the crossed operators create zero energy states.

Quantum classical correspondence requires a precise formulation for the action of $\mathcal{N}$ at space-time level and this is a valuable guideline in attempts to understand what is involved. Consider now the definition of the action of $\mathcal{N}$ in the case of M-matrix.

1. In standard QFT picture the action of the element of $\mathcal{N}$ multiplies the positive or negative energy parts of the state with an operator creating a zero energy state.

2. At the space-time level one can assign positive/negative energy states to the incoming/outcoing 3-D lines of generalized Feynman diagrams (recall that in vertices the 3-D light-like surfaces meet along their ends). At the parton level the addition of a zero energy state would be simply addition of a collection of light-like partonic 3-surfaces describing a zero energy state in a time scale shorter than that associated with incoming/outgoing positive/negative energy space-time sheet. The points of the discretized number theoretic braid would naturally contain the insertions of the second quantized induced spinor field in the description of M-matrix element in terms of N-point function.

3. At first look this operation looks completely trivial but this is not the case. The point is that the 3-D lines of zero energy diagram and those of the original positive/negative energy diagram must be assigned to *single connected* 4-D space-time surface. Note that even the minima of the λ are not same as for the original positive energy state and free zero energy state since the minimization is affected by the constraint that the resulting space-time sheet is connected.

4. What happens if one allows several disconnected space-time sheets in the initial state? Could/should one assign the zero energy state to a particular incoming space-time sheet? If so, what space-time sheet of the final state should one attach the *-conjugate of this zero energy state? Or should

one allow a non-unique assignment and interpret the result in terms of different phases? If one generalizes the connectedness condition to the connectedness of the entire space-time surface characterizing zero energy state one would bet rid of the question but can still wonder how unique the assignment of the 4-D space-time surface to a given collection of light-like 3-surfaces is.

5.3.3 How to define Hermitian conjugation physically?

Second problem relates to the realization of Hermitian conjugation $\mathcal{N} \rightarrow \mathcal{N}^*$ at the space-time level. Intuitively it seems clear that the conjugation must involve M^4 time reflection with respect to some origin of M^4 time mapping partonic 3-surfaces to their time mirror images and performing T-operation for induced spinor fields acting at the points of discretized number theoretic braids.

Suppose that incoming and outgoing states correspond to light-cones M^4_+ and M^4_- with tips at points $m^0 = 0$ and $m^0 = t_{+-}$. This does not require that the preferred sub-manifolds M^2 and S^2_{II} are same for positive/negative energy states and inserted zero energy states. In this case the point $(m^0 = t_{+-}/2, m^k = 0)$ would be the natural reflection point and the operation mapping the action of $\mathcal{N}$ to the action of $\mathcal{N}^*$ would be unique.

Can one allow several light cones in the initial and final states or should one restrict M-matrix to single diamond like structure defined by the two light-cones? The most reasonable option seems to be an assignment of a diamond shape pair of light-cones to each zero energy component of the state. The temporal distance t_{+-} between the tips of the light-cones would assign a precise time scale assigned to the zero state. The zero energy states inserted to a state characterized by a time scale t_{+-} would correspond to time scales $t < t_{+-}/2$ so that a hierarchy in powers of 2 would emerge naturally. Note that the choice of quantization axes (manifolds M^2 and S^2_{II}) could be different at different levels of hierarchy.

This picture would apply naturally also in the case of U-matrix and make the cutoff hierarchy discrete in accordance with p-adic length scale hypothesis bringing in also quantization of the time scales t_{+-}. In the case of U-matrix $\mathcal{N}$ would contain besides the zero energy algebra of M-matrix also the subalgebra for which the positive and negative energy parts reside at different sides of the center of the diamond.

5.3.4 How to generalize the notion of observable?

The almost-uniqueness of M-matrix seems too good to be true and in this kind of situation it is best to try to find an argument killing the hypothesis. The first test is whether the ordinary quantum measurement theory with Hermitian operators identified as observables generalizes.

The basic implication is that M should commute with Hermitian operators of $\mathcal{N}$ assuming that they exist in some sense. All Hermitian elements of $\mathcal{N}$ could be regarded not only as observables but also as conserved charges defining symmetries of M which would be thus maximal. The geometric counterpart for this would be the fact that configuration space is a union of symmetric spaces having maximal isometry group. Super-conformal symmetries of M-matrix would be in question.

The task is to define what Hermiticity means in this kind of situation. The super-positions $N + N^*$ and products N^*N defined in an appropriate sense should Hermitian operators. One can define what the products MN^* and NM mean. There are also two Hermitian conjugations involved: $\mathcal{M}$ conjugation and $\mathcal{N}$ conjugation.

1. Consider first Hermitian conjugation in $\mathcal{M}$. The operators of $\mathcal{N}$ creating zero energy states on the positive energy side and $\mathcal{N}^*$ acting on the negative energy side are not Hermitian in the hermitian conjugation of $\mathcal{M}$. If one defines $MN^* \equiv N^*M$ and $NM \equiv MN$, the operators $N + N^*$ and N^*N indeed commute with M by the basic condition. One could label the states created by $\mathcal{M}$ by eigenvalues of a maximally commuting sub-algebra of $\mathcal{N}$. Clearly, the operators acting on positive and negative energy state spaces should be interpreted in terms of a polarization $\mathcal{N} = \mathcal{N}_+ + \mathcal{N}_-$ such that $\mathcal{N}_{+/-}$ acts on positive/negative energy states.

2. In the Hermitian conjugation of $\mathcal{N}$ which does not move the operator from positive energy state to negative energy state there certainly exist Hermitian operators and they correspond to zero energy states invariant under exchange of the incoming and outgoing states but in time scale $t_{+-}/2$. These operators are not Hermitian in $\mathcal{M}$. The commutativity of M with these operators follows also from the basic conditions.

5.3.5 Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+M^\dagger P_-M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

5.3.6 How generalized braid diagrams relate to the perturbation theory?

Many steps of progress have occurred in the understanding of TGD lately.

1. In a given measurement resolution characterized by the inclusion of HFFs of type II_1 Connes tensor product defines an almost universal M-matrix apart from the non-uniqueness due to the facts that one has a direct sum of hyper-finite factors of type II_1 (sum over conformal weights at least) and the fact that the included algebra defining the measurement resolution can be represented in a reducible manner. The S-matrices associated with irreducible factors would be unique in a given measurement resolution and the non-uniqueness would make possible non-trivial density matrices and thermodynamics.

2. As explained in [C1], Higgs vacuum expectation is proportional to the generalized position dependent eigenvalue of the modified Dirac operator and its minima define naturally number theoretical braids as orbits for the minima of the universal Higgs potential: fusion and decay of braid strands emerge naturally. Thus the speculation [C6] about a generalization of braid diagrams to Feynman diagram likes objects, which I already began to think to be too crazy to be true, finds a very natural realization.

In [C1] I explained how generalized braid diagrams emerge naturally as orbits of the minima of Higgs defined as a generalized eigenvalue of the modified Dirac operator. The association of generalized braid diagrams to incoming and outgoing 3-D partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

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The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

5.3.7 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A

deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

5.3.8 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

5.3.9 Some further comments about Connes tensor product

Below some further comments related to Connes tensor product.

1. M-matrix as an anti-unitary operator

The proposed form of Connes tensor product cannot be correct if M is a linear operator. The point is that if the conditions hold true for operator N and M then they cannot hold true for iN and M. One could restrict Connes tensor product to only Hermitian operators of $\mathcal{N}$ so that M-matrix would have $\mathcal{N}$ as symmetries. Another equivalent way to cope with the difficulty is to assume that M is anti-unitary operator. This assumption is natural since negative energy states are identified as hermitian conjugates of positive energy states so that entanglement matrix is interpreted as matrix multiplication plus conjugation acting on (say) negative energy states.

In both cases the interpretation is that the Hermitian operators of $\mathcal{N}$ act as symmetries of M-matrix. Quite generally, the interpretation would be in terms of symmetries of $U(n)$ or its subgroup. This conforms with the earlier view that finite measurement resolution allows to have any compact group as the group of dynamical symmetries. This might also relate to the connection between between Jones inclusions and Dynkin diagrams of ADE groups.

2. Connes tensor product in finite-D case

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple interpretation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn} W_{nr} e^{mn} \otimes e^{nr}$$

   in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now.

For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

3. Connes tensor product in positive/negative energy sector

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement. Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

4. The Lie-algebra of symmetries of M-matrix defines also Jordan algebra

Hermitian operators of $\mathcal{N} \subset \mathcal{M}$ act as maximal symmetries of M-matrix. The linear combinations of Hermitian operators with real coefficients are Hermitian and define an algebra under the product $A \circ B = (AB + BA)/2$, which is commutative and non-associative but satisfies the weaker associativity condition $(xy)(xx) = x(y(xx))$. A so called Hermitian Jordan algebra - introduced originally as a formalization of the algebra of observables - is in question [66]. Now Jordan algebra would characterize measurement resolution.

There exists four infinite families of Jordan algebras plus one exceptional Jordan algebra. The finite-dimensional real, complex, and quaternionic matrix algebras with product defined as above are Jordan algebras. Also the Euclidian gamma matrix algebra defined by Euclidian inner product and with real coefficients is Jordan algebra and known as spin factor: now the commutativity is not put in by hand. The exceptional Jordan algebra consists of real linear space of Hermitian 3×3 matrices with octonionic coefficients and with symmetrized product.

The maximal symmetries of M-matrix mean that the Hermitian generators of the algebra define a generalization of finite-dimensional Jordan algebra. The condition that all Hermitian operators involved are finite-dimensional brings in mind the definition of the permutation group S_∞ as consisting of finite permutations only and also the definition of infinite-dimensional Clifford algebra. Thus the natural interpretation of the algebra in question would be as maximal possible dynamical gauge symmetry implied by the finite measurement resolution. The active symmetries would be analogous to global gauge transformations and act non-trivially on all tensor factors in tensor product representation as a tensor product of 2×2 Clifford algebras.

Quaternionic Jordan algebra is natural in TGD framework since 2×2 Clifford algebra reduces to complexified quaternions and contains as sub-algebras real and complex Jordan algebras. Also Clifford algebra of world of classical worlds is a generalized Jordan algebra.

What about the octonionic Jordan algebra? There are intriguing hints that octonions might be important for TGD.

1. U(1), SU(2), and SU(3) are the factors of standard model gauge group and also the natural symmetries of minimal Jordan algebras relying on complex numbers, quaternions, and octonions. These symmetry groups relate also naturally to the geometry of CP_2.

2. 8-D Clifford algebra allows also octonionic representation.

3. The idea that one could make HFF of type II_1 a genuine local algebra analogous to gauge algebra can be realized only if the coordinate is non-associative since otherwise the coordinate can be represented as a tensor factor represented by a matrix algebra. Octonionic coordinate means an exception and would make 8-D imbedding space unique in that it would allow local version of HFF of type II_1.

4. These observations partially motivate a nebulous concept that I have christened HO-H duality [E2]- admittedly a rather speculative idea - stating that TGD can be formulated alternatively using hyper-octonions (subspace of complexified octonions with Minkowskian signature of metric) as the imbedding space and assuming that the dynamics is determined by the condition that space-time surfaces are hyper-quaternionic or co-hyper-quaternionic (and thus associative or co-associative). Associativity condition would determine the dynamics.

The question is therefore whether also 3×3 octonionic Jordan algebra might have some role in TGD framework.

1. Suppose for a moment that the above interpretation for the Hermitian operators as elements of a sub-factor $\mathcal{N}$ defining the measurement resolution generalizes also to the case of octonionic state space and operators represented as octonionic matrices. Also the direct sums of octonion valued matrices belonging to the octonionic Jordan algebra define a Jordan algebra and included algebras would now correspond to direct sums for copies of this Jordan algebra. One could perhaps say that the gauge symmetries associated with octonionic $\mathcal{N}$ would reduce to the power $SU(3)_o^n = SU(3)_o \times SU(3)_o \times ...$ of the octonionic $SU(3)$ acting on the fundamental triplet representation.

2. Triplet character is obviously problematic and one way out could be projectivization leading to the octonionic counterpart of CP_2. Octonionic scalings should not affect the physical state so that physical states as octonionic rays would correspond to octonionic CP_n. It is not however possible to realize the linear superposition of quantum states in CP_n. The octonionic (quaternionic) counterpart of CP_2 would be 2×8-dimensional and $U(2)_o$ would act as a matrix multiplication in this space. Realizing associativity (commutativity) condition for 2×8 spinors defined by octonionic CP_2 by replacing octonions with quaternions (complex numbers) would give 2×4-dimensional (2×2-dimensional) space.

3. This gives rise to two questions. The first question is whether CP_2 as a factor of imbedding space could somehow relate to the octonionic Jordan algebra. Could one think that this factor relates to the configuration space degrees of freedom assignable to CP_2 rather than Clifford algebra degrees of freedom? That color does not define spin like quantum numbers in TGD would conform with this. Note that the partial waves associated S^2 associated with light-cone boundary would correspond naturally to SU(2) and quaternionic algebra.

   Second question is whether the HFF of type II_1 could result from its possibly existing octonionic generalization by these two steps and whether the reduction of the octonionic symmetries to complex situation would give $SU(3) \times SU(3)$... reducing to $U(2) \times U(2) \times$ ....

6 Number theoretic constraints

6.1 Number theoretic constraints on S-matrix

6.1.1 Number theoretic universality

Number theoretical universality leads to the hypothesis that S-matrix elements must be algebraic numbers. This is achieved naturally if the definition of S-matrix elements involves only the data associated with the number theoretic braid. This leads naturally to a connection with braiding S-matrices also in the case of real-to-real transitions. Also the concept of number theoretic string emerges. This picture becomes highly predictive if one accepts number theoretic universality of Riemann Zeta to be discussed at the end of the article.

The partonic vertices appearing in S-matrix elements should be expressible in terms of N-point functions of almost topological $N = 4$ super-conformal field theory but with the p-adically questionable N-fold integrals over string replaced with sums over the strands of a braid: spin chain type string discretization could be in question . Propagators, that is correlations between partonic 2-surfaces, would be due to the interior dynamics of space-time sheets which means a deviation from super string theory. Another function of interior degrees of freedom is to provide zero modes of metric of WCW identifiable as classical degrees of freedom of quantum measurement theory entangling with quantal degrees of freedom at partonic 3-surfaces.

Number theoretic universality poses very strong conditions on the theory. For instance, it is far from clear whether the notion of 4-D p-adic space-time sheet makes sense.

1. The p-adic variants of 4-D field equations associated with Kähler action make sense but the meaning notion of preferred extremal in p-adic context is not well-defined. Kähler action and thus also Kähler function are ill-defined notions since p-adic definite integral does not exist. Same applies to classical charges.

2. The functional integral over configuration space can be defined only as an algebraic extension of real functional integral around maximum of Kähler function if the theory is integrable and gives as result an algebraic number. One might hope that algebraic p-adicization makes sense for the maxima of Kähler function.

3. Ordinary perturbation series based on Feynman diagrams does not make sense if p-adicization is required and there are strong reasons to believe that in TGD framework there is no need for this kind of perturbation series.

4. p-Adic variants for space-time sheets could be however defined by re-interpreting real space-time sheets in p-adic context. This is possible if they are characterized by algebraic equations involving only rational or algebraic coefficients. In this case also Kähler action and classical charges could exist in some extension of p-adic numbers.

6.1.2 p-Adic coupling constant evolution at the level of free field theory

The generalized eigen modes of the modified Dirac operator, the structure of which is fixed completely by super-symmetry, assign to a partonic 3-surface a unique value of p-adic prime p with $log(p)$ appearing as a scaling factor of eigenvalue spectrum. This allows a first principle formulation of renormalization group equations for p-adic coupling constant evolution at the level of "free theory" rather than in terms of radiative corrections.

Also Dirac determinant is well defined and involves a product over sub-determinants defined as products of eigenvalues at the points of a number theoretic braid defined as subset in the intersection of real partonic 3-surface and its p-adic counterpart obeying same algebraic equations. Algebraicity is indeed possible at parton level due to the almost topological QFT nature of dynamics. Algebraicity condition could well imply that the number of eigenvalues belonging to the extension of p-adic numbers is finite so that Dirac determinant would be a finite algebraic number as required by p-adicization program.

Physical intuition suggests that the transition $M \rightarrow \mathcal{M}/\mathcal{N}$ replacing spinors of WCW with quantum spinors implies that the anti-commutators of induced spinor fields vanish only for a discrete point set defining the number theoretic braids in the algebraic intersection of real and p-adic variants of the partonic 2-surface.

6.2 S-matrix and the notion of number theoretic braid

The work with topological quantum computation [E9] stimulated the idea that S-matrix could be seen as a generalization of braiding S-matrix by allowing splitting of the strands of braid. This work led to the vision about how super Kac Moody symmetries and super-canonical symmetries relate to each other, and about connections between configuration space geometry and spinor structure, von Neumann algebras, quantum- and braid groups, and topological quantum field theories. Perhaps the most fascinating and concrete outcome was the realization that the zeros of Riemann Zeta and polyzetas determine the conformal weights for the generating elements of the super-canonical algebra so that Riemann Zeta and also polyzetas become key elements of fundamental physics in TGD Universe and Riemann hypothesis has deep physical content.

The explicit realization of the braids and braiding dynamics had however to wait for the development of number theoretic ideas inspired by the requirement that S-matrix should exist also for transitions between different number fields. Discreteness is suggestive since the integrals appearing in stringy amplitudes do not make sense p-adically. The vision about quantization of Planck constants in turn forced to ask how to define S-matrix for the transitions changing the values of Planck constants. Also now number theoretic braids provide the necessary discretization.

6.2.1 The notion of number theoretic braid

Number theoretical constraints lead to the idea that S-matrix could be constructed in terms of a number theoretic braid defined as a subset of the intersection of the real partonic 2-surface and its p-adic counterpart obeying same algebraic equations. The intersection consists of points existing in the algebraic extension of the p-adic number field considered. Super-conformal invariance in turn suggests that S-matrix elements are expressible in terms n-point functions of a super-conformal field theory associated with the partonic 2-surface so that stringy formulas for S-matrix elements involving integrals of n-point functions over arguments at circle should be replaced with their discrete versions. Thus S-matrix would relate very closely to the braiding S-matrix utilized in the model of topological quantum computation. Already in [E9] it become clear that the braiding S-matrices are naturally associated with light-like 3-surfaces having at elementary particle level identification as partons.

Number theoretical approach inspires the formulation of quantum criticality purely algebraically. The hypothesis is that string model duality generalizes for the TGD counterparts of Feynman diagrams in the sense that loop diagrams are equivalent with tree diagrams. This hypothesis can be formulated using axioms of a category which can be regarded as a generalization of so called ribbon categories. In quantum field theory language the hypothesis would state the vanishing of loop corrections.

An open question is whether also the diagonal S-matrices representing transitions between quantum states in the same number field should be formulated in terms of number theoretic braids. If fermionic partons appear as pairs of real and corresponding p-adic 3-surface obeying same algebraic equation, as cognitive-number theoretic arguments suggest, the number theoretic braid would indeed be unique for fermionic partons.

In the case of bosonic partons the situation is not quite clear. There are however arguments encouraging the belief that it is possible to assign a unique prime to the generalized eigen modes of the modified Dirac operator with the factor $log(p)$ appearing as a scaling factor of the eigen values so that the value of p-adic prime would make it visible in the real quantum dynamics [C1]. Hence the spectrum of the modified Dirac operator would allow to assign a unique p-adic prime to bosonic partons too. These considerations involve the conjecture about the number theoretic universality of Riemann Zeta stating that the zeros of ζ as well as the values of ζ and its building blocks $1/(1-p^{-s})$ appearing in its product decomposition are algebraic numbers at points $s = \sum_k n_k s_k$, $n_k \geq 0$. Also the definition of Dirac determinant proposed in [C1] requires always a selection of a discrete set of points of the partonic 2-surface and this set defines define number theoretic braid in a natural manner.

It is of course possible that one can distinguish between a phase for which S-matrix elements are expressible as multiple integrals and a phase for which a sum over the point of number theoretic braid is involved. The latter phase could correspond to phases for which quantum phases $q = exp(i\pi/n_i)$ associated with M^4 and CP_2 degrees of freedom are non-trivial.

6.2.2 Number theoretic braids and phase transitions changing the values of Planck constants

The realization that so called hyper-finite factors of type II_1 are an inherent property of quantum TGD meant a breakthrough in the understanding of the mathematical structure of quantum TGD.

The geometric and topological interpretation of Jones inclusions led to the understanding of the quantization of Planck constants assignable to M^4 and CP_2 degrees of freedom (identical in "ground state"). The Planck constants are scaled up by the integer n defining the quantum phase $q = exp(i\pi/n)$ characterizing the Jones inclusion, which in turn corresponds to subgroup G of $SL(2,C) \times SU(2)_L \times U(1)$ in the simplest situation. The quantum phase can be assigned also to q=1 inclusions in which case second quantum phase can be associated with the monodromies of the corresponding conformal field theory.

The scaling of $M^4_\pm$ Planck constant by n_a means scaling of CP_2 metric by n_a, where n_a is the order of the maximal cyclic subgroup of G_a. And vice versa. The observed Planck constant must correspond to $\hbar_{eff}/\hbar_0 = n_a/n_b$ from the fact that only the ratio $\hbar(M^4_\pm)/\hbar(CP_2)$ appears in Kähler action. The space-time sheets are $n(G_b)$-fold coverings of $M^4_\pm$ by points of CP_2. Analogous statement applies in CP_2 degrees of freedom. For infinite subgroups such as $G_b = SU(2) \subset SU(3)$ the situation is somewhat different. The variants of imbedding space can meet each other if either $M^4_\pm$ or CP_2 factors correspond to the same subgroup so that a fan- (or rather, tree-) like structure results. Analogous picture emerged already earlier from the gluing of the p-adic variants of imbedding space along common rationals (or algebraics in more general case). The phase transitions changing the Planck constant have purely topological description.

The phase transitions changing values of Planck constants should allow description in terms of S-matrix. This makes sense if the S-matrix is expressible only in terms of points common to the two sectors of imbedding space involved. This also leads naturally to a formulation in terms of number theoretic braids.

6.2.3 Finite measurement resolution implies number theoretic braids

The transition $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ interpreted in terms of finite measurement resolution should have space-time counterpart. The simplest guess is that the functions, in particular the generalized eigen values, appearing in the generalized eigen modes of the modified Dirac operator D become non-commutative. This would be due the non-commutativity of some H coordinates, most naturally the complex coordinates associated with the geodesic spheres of CP_2 and $\delta M^4_+ = S^2 \times R_+$. Stringy picture would suggest that these complex coordinates become quantum fields: bosonic quantization would code for a finite measurement resolution. Their appearance in the generalized eigenvalues for the modified Dirac operator would reduce the anti-commutativity of the induced spinor fields along 1-dimensional number theoretic string to anti-commutativity at the points of the number theoretic braid only. The difficulties related to general coordinate invariance would be avoided by the fact that quantized H-coordinates transform linearly under $SU(2)$ subgroup of isometries.

The detailed argument runs as follows.

1. Since the anti-commutation relations for the fermionic oscillator operators are not changed, it is the generalized eigen modes of D (with zero modes included) which must become non-commutative and spoil anti-commutativity except in a finite subset of the number theoretic string identifiable as a number theoretic braid. This means that also the generalized eigenvalues become non-commuting numbers and should commute only at the points of the number theoretic braid. This would provide a physical justification for the proposed definition of the Dirac determinant besides mere number theoretic arguments and finiteness and well-definedness conditions.

2. Functions of form $p^{\zeta^{-1}(z(x))a(x)}$, where $a(x)$ is ordinary matrix acting on H- spinors appear as spinor modes. z is the complex coordinate for the geodesic sphere S^2 of either CP_2 or $\delta M^4_+ = S^2 \times R_+$. $z(x)$ is obtained as a projection of X^3 point x to S^2 and an excellent candidate for a non-commutative coordinate. In the reduction process the classical fields $z(x)$

and $\overline{z}(x)$ would transform to $\mathcal{N}$-valued quantum fields quantum fields $z(x)$ and $z^\dagger(x)$.

3. The reduction for the degrees of freedom in $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ transition must correspond to the reduction of number theoretic string to a number theoretic braid belonging to the intersection of the real and p-adic variants of the partonic 3-surface. At the surviving points of the number theoretic string the situation is effectively classical in the sense that quantum states can be chosen to be eigen states of $z(x_k)$ in the set $\{x_k\}$ of points defining the number theoretic braid for which the commutativity conditions $[z(x_i), z^\dagger(x_j)] = 0$ hold true by definition. The quantum states in question would be coherent states for which the description in terms of classical fields makes sense.

4. The eigenvalues of $z(x_i)$ should be algebraic numbers in the algebraic extension of p-adic numbers involved. Since the spectrum for coherent states a priori contains all complex numbers, this condition makes sense. The generalized eigenvalues of Dirac operator at these points would be complex numbers and Dirac determinant would be well defined and an algebraic number of required kind. Number theoretic universality of ζ would fix the eigen value spectrum of z to correspond to $\zeta(s)$ at points $s = \sum_k n_k s_k$, $n_k \geq 0$, $\zeta(s_k = 1/2 + iy_k) = 0$, $y_k > 0$. Note however that this condition is not absolutely essential.

5. If a reduction to a finite number of modes defined at the number theoretic string occurs then $\left[z(x), z^\dagger(y)\right]$ can vanish only in a discrete set of points of the number theoretic string. The situation is analogous to that resulting when the bosonic field $z(\phi)$ defined at circle has a Fourier expansion $z(\phi) = \sum_m a_m exp(m\phi/n)$, $m = 0, 1, ...n-1$, $\left[a_m^\dagger, a_n\right] = \delta_{m,n}$. $[z(\phi_1), z^\dagger(\phi_2)]$ is given by $\sum_m exp(im\phi/n)$, $\phi = \phi_1 - \phi_2$ and in general non-vanishing. The commutators vanish at points $\phi = k\pi$, $0 < k < n$ so that physical states can be chosen to be eigen states of the quantized coordinate $z(\phi)$ at points $z_k = k\pi$. One can ask whether n could be identified as the integer characterizing the quantum phase $q = exp(i\pi/n)$. The realization of a sequence of approximations for Jones inclusion as sequence of braid inclusions however suggests that all values of n are possible.

6. This picture conforms with the heuristic idea that the low energy limit of TGD should correspond to some kind of quantum field theory for some coordinates of imbedding space and provides a physical interpretation for the quantization of bosonic quantum field theories. $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ reduction is analogous to a construction of quantum field theory with cutoff. The replacement of complex coordinates of the geodesic spheres associated with CP_2 with time shifted copies of δM^4_+ defining a slicing of M^4_+ would define the quantization of H coordinates. This notion of quantum field theory fails at the limit of continuum.

7 Could Connes tensor product allow to gain a more detailed view about S-matrix?

A more detailed consideration of the master formula for S-matrix reveal quite non-trivial constraints coming from quantum classical correspondence. These considerations lead to a more precise formulation of TGD based view about quantum measurement theory, give a precise meaning for somewhat nebulous concept of effective 2-dimensionality and explain why it is necessary, allow to understand how a more conventional picture about S-matrix in terms of particle exchanges emerges, and reveal a closed connection with string models and with with topological quantum field theories defined by Chern-Simons action. There are good reasons to believe that Connes tensor product reduces to the familiar fusion rules of conformal field theories with Kac Moody symmetry. A basic test for the theory is the understanding of the weakness of the gravitational interaction and the resulting explanation refines the earlier one.

A warning to the reader is in order. I already considered briefly the implications of Connes tensor product to the construction of generalized S-matrix (M-matrix) in zero energy ontology at the general level (see the section "HFFs and S-matrix"). This section represents only an intermediate step in a process which is still going on (and I have not yet made this explicit in the text everywhere). The constructions are still based on the implicit assumption that S-matrix between positive energy states exists. In TGD Universe however only zero energy states are possible (all net values of conserved quantum numbers vanish). Therefore only an S-matrix between zero energy states is what one can hope to exist and this S-matrix should give rise to the ordinary S-matrix as an approximate concept. The considerations of the next section based on the attempt to identify S-matrix as a tensor product of factorizing almost-trivial S-matrices actually forced to take zero energy ontology seriously also in the construction of S-matrix since it resolves elegantly the problems caused by the almost-triviality and turns defeat into victory.

7.1 An attempt to construct S-matrix in terms of Connes tensor product

An explicit first principle formula for S-matrix has remained an unfulfilled dream although a lot of progress have been made: mention only the notion of generalized Feynman diagram involving a generalization of the duality symmetry [C6].

In the following it will be shown that the replacement of the ordinary tensor product with Connes tensor product for the states created by free fields leads to a very elegant proposal for the general formula for S-matrix. The dependence of the quantum S-matrix on Jones inclusion characterizing quantum measurement raises the interaction of the observer with the measured system in a central role. This dependence is already present in standard quantum field theory via the dependence of the S-matrix on ultraviolet and infrared cutoffs.

The proposal is beautiful mathematically but it has been however impossible to prove unitarity and by previous arguments unitarity is neither not necessary in zero energy ontology and is impossible if one has HFF of type II_∞. The assignment of S-matrix to a physical state is nothing new: already in elementary wave mechanics physical state can represent S-matrix (consider only scattering in a spherically symmetric potential). Hence one can hope that the original interpretation is very near to truth.

7.1.1 The challenge

The construction of S-matrix for a single space-time surface was discussed already in [C6] by introducing the notion of generalized Feynman diagram. Although this treatment seems to apply only to what happens inside single 3-D external line of Feynman diagrams the general picture is rather near to what seems to be correct one.

1. Lines correspond to 3-D light-like causal determinants (CDs) X_i^3 at four-dimensional space-time sheets representing incoming particles. They can correspond to boundary components of a space-time sheet (such as boundaries of magnetic flux tubes) but can also serve as horizons separating maximal non-deterministic regions within a space-time sheet.

2. Vertices correspond to 2-D partonic surfaces at which the light-like CDs X_i^3 branch like a lines of Feynman diagrams. X_i^3 and also space-time surfaces would be singular as manifolds but only apparently as will be found. The 2-surfaces representing vertices need not have any singularities, say pinch like singularities appearing in stringy diagrams.

3. There is rather close analogy with the branes in the sense that the intersection of space-time surfaces 7-D light-like CDs $X_\pm^7 = \delta M_\pm^4 \times CP_2$ of imbedding space provide a natural gauge fixing for 4-D general coordinate invariance. Hence the incoming partons X_i^2 correspond to intersections $X_i^3 \cap X_\pm^7$. Future (past) oriented light-cone corresponds to incoming (outgoing) particles.

In this framework the basic challenges would be following.

1. Construct explicitly the unitary evolution operators associated with the lines possibly defining the analogs of propagators. These operators should be fixed to a high degree by the dynamics of the second quantized induced spinor fields as has been suggested in [C6]. The existence of a universal unitary von Neumann algebra automorphism Δ^{it} suggesting itself as a candidate for this unitary evolution operator. The identifiability of this automorphism as that defined by the modified Dirac operator is also suggestive.

2. Δ^{it} represents scaling operation but is a mere inner automorphism for factors of type II_1, which suggests that both internal and external lines represent on mass shell particles in the sense that Virasoro conditions

hold true and the automorphism represents braiding S-matrix. This in turn inspires the hypothesis that the Feynmann graphs can have only on mass shell particles as internal lines: by unitarity the S-matrix elements reduce to diagrams having only single vertex.

3. Vertices are in principle fixed as vacuum expectation values for the product of operators creating the incoming and outgoing states at the vertices. Connes tensor product suggests itself strongly. These operators are constructible from oscillator operators associated with the generalized eigen modes of the modified Dirac operator acting on the second quantized induced spinor fields, whose quantization is fixed by the requirement that the super-canonical charges constructed in terms of oscillator operators define configuration space gamma matrices having super-symmetrized symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries. Intuitively it looks obvious that the product of the operators creating states at the lines defines vertex as its vacuum expectation. The challenge is to imbed the fermionic oscillator operator algebras associated with incoming lines to same oscillator algebra.

7.1.2 Master formula for S-matrix

The possibility to interpret configuration space Clifford algebra elements as analogs of conformal fields in M^4 suggests that the notion of n-point function could serve as a useful starting point in the attempts to understand the general structure of S-matrix.

1. The general formula should reproduce generalized Feynman diagrams for which lines are space-time sheets whose ends meet at vertices which are 3-surfaces. The lines should correspond to the solutions of field equations having interpretation as generalized Bohr orbits so that classical theory should be an exact part of the construction of S-matrix.

2. The generalization of the duality symmetry of the old fashioned string model requires that all Feynman diagrams should be equivalent with a diagram involving single vertex from which all incoming and outgoing lines emanate. This picture is analogous to the description of scattering matrix elements in terms of effective action so that each connected n-point function corresponds to a diagram with a single vertex. This picture would suggest that one should not start the construction from incoming and outgoing particles and continue adding all possible collections of vertices between them as in perturbative quantum field theory. Rather, one should do just the reverse by starting from the vertex and gluing to it space-time sheets leading to the initial states at the boundaries of light cones assignable to the arguments of n-point function.

General coordinate invariance has turned out to be extremely powerful guiding principle in the construction of TGD and comes in rescue also now.

1. The reduction to a diagram which single vertex suggests that particle reactions are essentially processes of creation of particles from vacuum which propagate classically to the boundaries of light cones associated with the arguments of n-point function. Hence S-matrix elements would be basically expressible as amplitudes for creating from vacuum a 3-surface X^3 at which one can assign a product of elements of configuration space Clifford algebra creating from the vacuum state with well-defined fermionic and other quantum numbers. A creation of $N_{in} + N_{out}$ particles from vacuum is in question.

2. The resulting N_{out} positive energy (N_{in} negative energy) particles travel to the boundaries of future (past) light cones associated with points m_i appearing in the n-point function. General coordinate invariance implies that the values of configuration space spinor fields at the two ends of a given line are identical and thus expressible using their values at the corresponding light cone boundaries. This means enormous simplification since same basis of configuration space spinor fields in vibrational degrees of freedom can be used for all light cones involved and only the dependence on the coordinate characterizing the position of light cone complicates the situation. Lines would indeed be 4-surfaces, whose ends meet at the vertex. Crossing symmetry would be an automatic consequence in this picture.

The rules would be formally rather simple for the construction of n-point function with given points of M^4.

1. Assign to each 3-surface X^3 (possibly consisting of disjoint components) incoming and outgoing space-time surfaces ending at the boundaries of the light-cones involved and satisfying classical field equations guaranteing generalized Bohr orbit property.

2. Form the Connes tensor product of Clifford algebra elements associated with corresponding light cones and thus depending on their positions m_i, and calculate its vacuum expectation. The dependence on m_i and the possibility of vertices with arbitrarily high number of incoming and outgoing lines trivially guarantee that S-matrix is non-trivial.

3. The tensor product appearing in the vertex would be Connes tensor product $\otimes_N$ and vacuum expectation would be defined as the manifestly finite trace. The factors in the tensor product would be infinite-dimensional Clifford algebras M_i associated with the lines of the graph and $\mathcal{N}$ could be identified by the condition that $\mathcal{M}_\rangle/\mathcal{N}$ is the quantum variant of the Clifford algebra of H. Also the quantum variants of S-matrix corresponding to various groups $G = G_a \times G_b \subset SL(2,C) \times SU(2)$, $SU(2) \subset SU(3)$ would result from the same formula.

4. Perform ordinary functional integral over 3-surfaces X^3 defined by the exponent of Kähler function, which by the non-locality of the Kähler function as a functional of 3-surface and by the Ricci flatness of the configuration

space geometry should be free of divergences. There are good reasons to hope that the radiative corrections to this integral sum up to zero around maxima of Kähler function. Super-symmetry raises the hope that also configuration space degrees of freedom correspond to a hyper-finite type II_1 factor and could be treated very much like fermionic degrees of freedom.

This description is not the only one can imagine. More complex tree diagrams with propagator lines would be obtained by allowing several vertices connected by internal lines represented by space-time sheets. The generalization of the duality symmetry says that these diagrams provide an alternative description equivalent with the minimal one. The classical non-determinism of Kähler action indeed forces to consider this possibility (recall that for CP_2 type extremals representing elementary particles the M^4 projection is light-like random curve which implies classical Virasoro conditions).

7.1.3 How to understand the unitarity of S-matrix

It should be possible to understand the unitarity of S-matrix (possible at least in fermionic degrees of freedom defining HFF of type II_1) in a simple manner from the proposed master formula for the S-matrix.

1. Connes tensor product is responsible for the non-triviality of S-matrix

The basic observation is that the presence of M^4 coordinates dependence makes configuration space gamma matrices analogous to quantum fields. Gamma matrices represent free fields as in string models and conformal field theories. S-matrix is obtained using ordinary inner product and by replacing the ordinary tensor product with Connes tensor product. Thus Connes tensor product would represent interactions and would be essential for the non-triviality of S-matrix. The beauty of this picture is that the quantum number spectrum of free field theory is preserved as such and there is no need to introduce the notion of virtual states or interaction terms.

Connes tensor product reduces degrees of freedom from the ordinary tensor product and there is a close analogy with the dynamics of space-time surfaces. Imbedding space metric, gamma matrices, and gauge fields are non-dynamical but their projection to the space-time surface makes them dynamical. Gamma matrices are free gamma matrix fields with trivial S-matrix but the restrictions posed by the Connes tensor make the dynamics non-trivial. Connes tensor product thus represents something analogous to the restriction of Clifford algebra to its sub-algebra.

2. Unitary S-matrix as a representation of crossing operation?

In the proposed picture states with positive and negative energies are created from vacuum. By expressing S-matrix elements as amplitudes for the generation of a state containing incoming particles as positive energy particles and outgoing particles as negative energy particles, unitarity condition can be transformed by crossing symmetry to orthogonality condition of negative (positive) energy states assuming their completeness. Therefore S-matrix would represent unitary crossing operation transforming positive energy bra to negative energy ket. Unitarity would state that crossing operation and its conjugate produce a trivial outcome.

3. Connes tensor product as a representation for the unitary crossing operation?

Vacuum expectation of the Connes tensor product of localized gamma matrices creating a zero energy states would be equal to the inner product of outgoing and incoming states created by the ordinary tensor products of the same gamma matrices. Connes tensor product should guarantee unitarity of S-matrix.

Gamma matrices would behave as free fields with respect to the ordinary tensor product and S-matrix would be trivial. Super Virasoro conditions would give the mass spectrum. $\mathcal{N}$ reducing to a unit matrix would define a trivial S-matrix. Free field property is essential for the finiteness of the theory since Connes tensor product and finite trace cannot induce infinities.

Connes tensor product would make the S-matrix non-trivial. Any $\mathcal{N}$ would define non-trivial S-matrix via Connes tensor product and a hierarchy of S-matrices would result. In what sense the S-matrix is unitary and whether it is so is of course not obvious. Since $\mathcal{N}$ takes the role analogous to complex coefficient field in quantum mechanics one is forced to ask whether a reduction of single particle degrees of freedom to $\mathcal{M}_k/\mathcal{N}$ occurs so that unitarity holds true in these degrees of freedom. In what sense it holds true is not quite obvious. The mathematical challenge is to prove that Connes tensor product indeed gives rise to a unitary S-matrix in the proposed framework and it has been already proposed that one must replaced unitarity with its statistical variant.

7.1.4 Jones inclusion as a representation of quantum measurement

The number of observable degrees of freedom is finite in any experiment. Since the number of degrees of freedom for the particle is infinite, the experimental situation must somehow leave almost all of these degrees of freedom undetected. $\mathcal{N} \subset \mathcal{M}_k$ must represent the interaction of the observer with the measured system. Finite-fractal dimensional $\mathcal{N}$ module $\mathcal{M}_k/\mathcal{N}$ would represent those gamma matrices which define the observable degrees of freedom.

There are two interpretations for this.

1. $\mathcal{N}$ represents those degrees of freedom in which there are no correlations between measurement system and measured system and experimenter entangles with $\mathcal{M}_k/\mathcal{N}$ degrees of freedom. The division by $\mathcal{N}$ could be also interpreted as being due to a finite measurement accuracy implying a thinning of those degrees of freedom in which the state function reduction can occur. It is not clear whether $\mathcal{N}$ must be same for all particles and one might argue that this need not be the case. The entanglement leading to state function would occur only in finite number of degrees of freedom characterized by $\mathcal{M}_k/\mathcal{N}$.

2. A completely opposite interpretation is that $\mathcal{N}$ represents Clifford subalgebra shared by particles and observed with the property that entanglement in this degrees of freedom is stable in the time scale of the experiment. This would leave only $\mathcal{M}_k/\mathcal{N}$ degrees of freedom as those for which state function reduction occurs in the time scale of the scattering experiment. This option would explain naturally why $\mathcal{N}$ is same for all particles.

3. Also the combination of above views is possible. Degrees of freedom in which dynamics is very rapid *resp.* slow would correspond to the case a *resp.* b).

Some further remarks are in order.

1. The fractal dimension $\mathcal{M} : \mathcal{N}$ tells that the correlations due to non-commutativity reduce their effective number.

2. If all degrees of freedom could be measured $\mathcal{N}$ would reduce to an algebra containing only unit and S-matrix would become trivial. The non-triviality of S-matrix would is thus due to the interaction between the experimenter and the system studied. In quantum field theories length and time scale cutoffs would represent this fact in a rough manner.

3. Γ matrices are not Hermitian since they are essentially superpositions of fermionic oscillator operators. Fourier transforms of Gamma matrices could be used to define occupation number operators in the momentum space as natural observables.

7.1.5 Precise definition of the notion of unitarity for Connes tensor product

The previous physical picture helps to characterize the notion of unitarity precisely for the S-matrix defined by Connes tensor product. For simplicity restrict the consideration to configuration space spin degrees of freedom.

1. $Tr(Id) = 1$ condition implies that it is not possible to define S-matrix in the usual sense since the probabilities for individual scattering events would vanish. Connes tensor product means that in quantum measurement particles are described using finite-dimensional quantum state spaces $\mathcal{M}/\mathcal{N}$ defined by the inclusion. For standard inclusions they would correspond to single Clifford algebra factor $C(8)$. This integration over the unobserved degrees of freedom is nothing but the analog for the transitions from super-string model to effective field theory description and defines the TGD counterpart for the renormalization process.

2. The intuitive mathematical interpretation of the Connes tensor product is that $\mathcal{N}$ takes the role of the coefficient field of the state space instead of complex numbers. Therefore S-matrix must be replaced with $\mathcal{N}$-valued S-matrix in the tensor product of finite-dimensional state spaces. The

notion of $\mathcal{N}$ unitarity makes sense since matrix inversion is defined as $S_{ij} \rightarrow S^{\dagger}_{ji}$ and does not require division (note that i and j label states of $\mathcal{M}/\mathcal{N}$). As also the generalization of the hermiticity: the eigenvalues of a matrix with $\mathcal{N}$-hermitian elements are $\mathcal{N}$ Hermitian matrices so that single eigenvalue is abstracted to entire spectrum of eigenvalues. Kind of quantum representation for conceptualization process is in question and might have direct relevance to TGD inspired theory of consciousness. The exponentiation of a matrix with $\mathcal{N}$ Hermitian elements gives unitary matrix.

3. The projective equivalence of quantum states generalizes: two states differing by a multiplication by $\mathcal{N}$ unitary matrix represent the same ray in the state space. By adjusting the $\mathcal{N}$ unitary phases of the states suitably it might be possible to reduce S-matrix elements to ordinary complex vacuum expectation values for the states created by using elements of quantum Clifford algebra $\mathcal{M}/\mathcal{N}$, which would mean the reduction of the theory to TGD variant of conformal field theory or effective quantum field theory.

4. The probabilities P_{ij} for the general transitions would be given by

$$P_{ij} = N_{ij}N^{\dagger}_{ij} \ , \tag{26}$$

and are in general $\mathcal{N}$-valued unless one requires

$$P_{ij} = p_{ij}e_{\mathcal{N}} \ , \tag{27}$$

where $e_{\mathcal{N}}$ is projector to $\mathcal{N}$. N_{ij} is therefore proportional to $\mathcal{N}$-unitary matrix. S-matrix is trivial in $\mathcal{N}$ degrees of freedom which conforms with the interpretation that $\mathcal{N}$ degrees of freedom remain entangled in the scattering process (option b)).

5. If S-matrix is non-trivial in $\mathcal{N}$ degrees of freedom, these degrees of freedom must be treated statistically by summing over probabilities for the initial states. The only mathematical expression that one can imagine for the scattering probabilities is given by

$$p_{ij} = Tr(N_{ij}N^{\dagger}_{ij})_{\mathcal{N}} \ . \tag{28}$$

The trace over $\mathcal{N}$ degrees of freedom means that one has probability distribution for the initial states in $\mathcal{N}$ degrees of freedom such that each state appears with the same probability which indeed was von Neumann's guiding idea. By the conservation of energy and momentum in the scattering this assumption reduces to the basic assumption of thermodynamics.

6. An interesting question is whether also momentum degrees of freedom should be treated as a factor of type II_1 although they do not correspond directly to configuration space spin degrees of freedom. This would allow to get rid of mathematically unattractive squares of delta functions in the scattering probabilities.

7.1.6 Conformal invariance and field theory and stringy phases

$\mathcal{M} : \mathcal{N} < 4$ assigns a unique minimal conformal field theory to the inclusion and this should give important information about the vertex. A priori the inclusions $\mathcal{N} \subset \mathcal{M}_k$ can have different values of $\mathcal{M}_k : \mathcal{N}$ determining the quantum phases q_i. Both physical intuition and anyonic statistics encourage to think that the values of quantum phases q_i are identical.

It seems conceivable that $\mathcal{M} : \mathcal{N} < 4$ vertices correspond physically to the low energy phase symmetry broken phase possible describable using renormalized field theory. Ordinary QFT would corresponds to $\mathcal{M} : \mathcal{N} \rightarrow 4$ limit whereas $\mathcal{M} : \mathcal{N} = 4$ phase with Kac-Moody symmetry would correspond to the "stringy" phase of the theory. The low energy limit would transform from an approximate theoretical description to an actual physical phase. In this phase massivation of massless particles would occur by p-adic thermodynamics [F2] whereas ultra-heavy particles would drop from the spectrum.

Dimensional regularization with complex space-time dimension $D = 4 - \epsilon \rightarrow 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for the a quantum Clifford algebra. An entire sequence of counterparts of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ is predicted.

As will be discussed, the evolution of Jones index could correspond to renormalization group evolution for phase resolution characterized by the value $\hbar$.

In $\mathcal{M} : \mathcal{N} = 4$ case all ADE type $k = 1$ conformal theories are in principle possible. An open question is whether actually the conformal field theory defined by the Kac-Moody algebra characterizing TGD is possible or whether the idea about interfaces as able to emulate any string model is more appropriate.

7.1.7 Braiding and S-matrix

The S-matrices associated with braiding were the inspiration leading to the new view about Feynman diagrams which I have attempted to formulate in [C6] in terms of bi-algebras. The trace of S-matrix associated with braid defines knot invariant. Each compact Lie group gives to its own S-matrix and one can assign to each strand of the braid its own representation of the group. Same applies to knotted links in 3-space to which one can assign the trace of non-integrable phase factor. Functional integral average using 3-dimensional Chern-Simons action defines the topological quantum field theory allowing to calculate the associated invariants.

1. *Braiding can appear in several manners*

The natural expectation is that these braiding matrices emerge in the proposed framework. A natural looking idea is that the incoming and outgoing lines parton lines represented by light like causal determinants could carry out these braids. The closed partonic 2-surfaces would define the 2-spaces carrying the anyons. This interpretation would be very natural since anyonic statistics is indeed possible only in 2-dimensional context. This would require the generalization of the proposed formula: it would not be possible to get completely rid of external lines since unitary S-matrices representing braiding should be associated with every lightlike causal determinant. Whether these unitary matrices have interpretation in terms of Connes tensor product is an open question.

Braiding could emerge also in a second manner. The 3-surface representing the N-vertex over which the functional integral is performed could also represent the braiding. It would seem natural to assume that these 3-surfaces are space-like. For instance, the boundary component of the 3-surface could be constructed from that having spherical topology by building handles as threadlike wormholes (think of an apple!) connecting punctures at the boundary of spherical boundary component. These wormholes could get linked and knotted would define the braiding naturally. In these cases the physical states would represent braiding S-matrix as entanglement coefficients between states localized to the punctures would represent the S-matrix. This S-matrix would depend on the state since the representation of the gauge group G could be chosen freely for each puncture and the choices could be different for initial and final states. The contractions of oscillation operators in Connes tensor product would give quantum traces of $Tr_q(S_1S_2^\dagger)$ in the S-matrix element and thus the braid invariants would appear in S-matrix.

The third manner that braiding can emerge is discussed in detail in the next section. Number theoretical considerations support the view that S-matrix can be constructed as a tensor product of 2-D factorizing S-matrices [40]. These S-matrices are however physically almost trivial. For the scattering of zero energy states this is not catastrophe since it implies the consistency of the ontology based on zero energy states with the ontology in which matter is identified as positive energy particles. The factorizing S-matrices are constructible in terms of Yang-Baxter S-matrix representing 2-particle scattering and this matrix reduces at the limit when the particles are at rest with respect to each other to braiding S-matrix.

2. How the braidings and Jones inclusions relate?

Braid S-matrix emerges in topological quantum field theory defined by Chern-Simons action. Only topological degrees of freedom and the moduli space of flat connections defines the genuine dynamical variables. The obvious questions relate to the interpretation of the braid S-matrix in TGD: is it associated with either space-like or lightlike braidings and is it associated with Jones inclusions labelled by finite subgroups $G \subset SU(2)$ or with Jones inclusions with $G = SU(2)$?

1. Quantum traces appears in the invariant would which would suggest that

for $q < 1$ the theory should be assigned to the Jones inclusion $R_0^G \subset R^G$. If so q would actually correspond to the quantum counterpart of the subgroup of $SU(2)$ defining a minimal conformal field theory. ADE correspondence would however assign with it gauge group $\hat{G}$ and the proposed construction of multiplets of gauge group would provide the correspondence with flesh and bones. The ADE diagram for $SU(2)$ is not allowed for Jones inclusions. $SU(2)$ is however the minimal $n = 5$ option allowing universal topological quantum computation using braids. Hence this option would be more naturally associated with the braids assignable to space-like 3-surface defining the vertex and would give rise to various topological invariants.

2. The inclusions could also correspond to $q = 1$ and $G = SU(2)$. All simply laced ADE groups are possible label the inclusion. The theory would be conformal field theory with Kac-Moody symmetry assignable to ADE group $\hat{G}_b$. As already explained, also now subgroups of $SU(2)$ could appear naturally in TGD framework but $G_b \subset SU(2) \subset SU(3)$ would collect points of geodesic sphere S^2 of CP_2 to multiplets allowing to represent the group algebra of G_b allowing to realize the representations of corresponding Lie group $\hat{G}_b$ defining the Kac-Moody group. Also $\hat{G}_b = SU(2)$ would be possible and braiding would be realized at the geodesic sphere of CP_2. Also in this case the subgroup G would assign quantum group parameter q naturally to the conformal field theory although it is not associated with Jones inclusion directly.

The simplest realization would be in termsof cosmic strings $X^2 \times S^2$, with S^2 perhaps depending on point of X^2. For ordinary cosmic strings the enormous string tension and would make this option energetically impossible. The situation is not change by the change of values of $\hbar$ since mass squared operators are invariant under the scaling of Planck constants associated with M^4 and CP_2 degrees of freedom.

This realization could be naturally assigned to the light-like causal determinants associated with the external lines to which ordinary Kac Moody symmetry is naturally associated in TGD framework. The space-time surface reduces to $X^1 \times S^2$ at the light-like causal determinants with X^1 a light like geodesic of M^4. The ends of the bosonic strings indeed move with light velocity. In the interior of X^4 there is no need to required $X^2 \times S^2$ decomposition which looks too strong a condition. Note that S^2 would represent topological magnetic monopole appearing in the model of high T_c superconductivity in TGD Universe [J1, J2, J3].

The identification of the gauge group associated with light-like causal determinants is naturally based on standard model gauge symmetries. Interestingly, the outcome of p-adic mass calculations depends on the number of tensor factors of Super Virasoro representation only with no dependence on what the actual Kac Moody groups associated with the factors are.

3. Holonomy of $\mathcal{N} \subset \mathcal{M}$ and unitarity of S-matrix and braid statistics

The reduction of Δ^{it} to inner automorphism for type II_1 factors need not mean that they would not have any role in the theory. The first thing coming in mind would be that the modular S-matrices assignable to the braids assignable to external lines and the 3-surface defining vertex could reduce to Δ^{it} for some value of parameter t.

Assume that $\mathcal{N}$ is imbedded into each factor $\mathcal{M}_k$ appearing in external line of the S-matrix. The automorphisms induced by $\Delta_{\mathcal{M}_k}$ assignable to the strand of the braid defines a closed path of $\mathcal{N}$ in $\mathcal{M}$. The action on the automorphism on individual points of $\mathcal{N}$ could be non-trivial and would have interpretation as a holonomy group defining a unitary action of $\mathcal{M}$ on $\mathcal{N}$.

Single particle S-matrix would represent this action. This action could relate to the 2-dimensional braid statistics defined by quantum group. For braids the 2π braid rotation of $(k+1)^{th}$ strand around k^{th} strand induces a non-trivial action on the state and this action could correspond to $\Delta_{\mathcal{M}_k}$ holonomy on $\mathcal{N}$. This interpretation would predict that the S-matrix elements for diagrams differing by braidings of partons inside incoming and outgoing lines (3-D light-like causal determinants) are not identical.

7.2 Effective 2-dimensionality and the definition of S-matrix

The metric of the configuration space is highly degenerate. Besides the vacuum degeneracy there are strong reasons to expect a degeneracy of metric associated with the interior degrees of freedom of space-like 3-surfaces due to the fact that configuration space Clifford algebra algebra is associated with 2-D intersections X^2 of 3-D light-like causal determinants (CDs) with the space-like 3-surfaces. This degeneracy would be dual to the degeneracy implied by the super-conformal invariance at light-like CDs implying that the data determining configuration space metric reduces to 2-dimensional partonic surfaces in CDs intersections with $\delta M^4_{\pm} \times CP_2$. Effective 2-dimensionality leads to a string model like conformal field theory as a natural description for the construction of vertices. The dimension of space-time is however visible in the vacuum functional defined as exponent of Kähler action for space-time sheet. The mystery is why there must be dynamics associated with the interior of the 3-surfaces.

In the following it will be found that the effective 2-dimensionality is forced also by the non-triviality of S-matrix and that quantum classical correspondence and that quantum measurement theory requires the dynamics in the interior degrees of freedom to be a classical dual for the quantum dynamics at partonic boundary components.

7.2.1 Implications of quantum classical correspondence

One can start from the rough vision that space-time sheets representing incoming and outgoing particle meet at the 3-surface X^3_V representing N-vertex and that S-matrix is defined as a functional integral over X^3_V with weighting defined by vacuum functional defined by the exponent of Kähler action for X^3_V and possibly also the vacuum functionals for the space-time sheets meeting at the vertex. N-vertex should somehow decompose to ordinary vertices and propagators if TGD is to be a physical theory.

Quantum classical correspondence poses very strong constraints on the space-time sheets. In particular, the classical conserved charges should coincide with quantum numbers corresponding to a maximal number of commuting observables. This correlation between classical interior degrees of freedom and quantum numbers in boundary degrees of freedom would provide a concrete realization of the quantum measurement theory in accordance with the original hypothesis that zero modes are in one-one correlation with the quantum fluctuating degrees of freedom and entangle with these degrees of freedom in quantum measurements.

One can imagine several options for how to realize this picture.

Option I: Could space-time sheets be general extremals of Kähler action?

Quantum classical correspondence for incoming and outgoing states could be satisfied easily if general extremals of Kähler action could be used. For instance in the case of scattering by graviton exchange X^3_V would describe the interacting systems connected by a 3-D light like causal determinant associated with topological condensed CP_2 type extremal describing graviton.

This option is in principle mathematically sound. Space-time sheets could be regarded as generalized Bohr orbits since a subset of classical conserved charges would co-incide with the values for a maximal set of commuting quantum observables. The generalized Bohr orbit would not be dictated by the knowledge of X^3 alone but by the quantum state whereas the definition of generalized Bohr orbit based on the absolute minimization of Kähler action and its variants [E2] implies that 3-surface X^3 alone dictates the generalized Bohr orbit $X^4(X^3)$ as is indeed required by the identification of Kähler action as Kähler function $K(X^3)$. Therefore it would seem that this option cannot be correct. On the other hand, $X^4(X^3)$, in other words X^3, should be able to represent maximal set of commuting quantum observables as conserved classical charges.

Option II: Is classical non-determinism enough?

It seems obvious that one cannot give up the Bohr orbit property in the sense that $X^4(X^3)$ is more or less unique and basically determined by the properties of X^3 alone. One might hope that the classical non-determinism of Kähler action serving as a space-time correlate of quantal non-determinism could save the situation. What would be required is double Bohr orbit property: $X^4(X^3)$ and the space-time surface X^4 fixed by the eigen values of the maximal set of commuting quantum observables should coincide.

The requirement that space-time sheets representing incoming and outgoing particles meet along their common ends is certainly impossible to satisfy without a strong failure of the classical determinism: without the failure only single space-time surface $X^(X^3_V)$ would be associated with X^3_V.

1. The situation might be different in the case of CP_2 type extremals representing elementary particles. The random zitterbewegung with light

velocity allows to assign to CP_2 type extremal arbitrary inertial four-momentum as average of the non-conserved gravitational momentum [D2, F6]. Classically the average of the non-conserved gravitational 4-momentum of CP_2 type extremal identifiable as inertial four-momentum is time like unless one allows the gradient dm^0/ds of M^4 time coordinate $m^0(s)$ as a function of CP_2 coordinate s to change its sign. There seems to be no reason forbidding this. If so, also space-like momenta can be described classically. If external particles are elementary particles there are therefore hopes that they can be represented classically and also virtual particles appearing in the decomposition of vertices to ordinary tree diagrams might be possible.

2. One must also notice that the CP_2 type extremals associated with elementary particles have suffered topological condensation at pieces of M^4 which are vacuum extremals before condensation. In the vicinity of wormhole contact the dimension $D(CP_2)$ of CP_2 projection is at least $D(CP_2) = 3$ so that the space-time sheet is a non-vacuum extremal. This is indeed necessary to have inertial momentum equal to the average gravitational four-momentum. Vacuum space-time sheets are highly non-deterministic and allow large number of non-vacuum deformations but the claim that they could make possible to realize quantum classical correspondence at the level of observables is not very convincing.

3. An additional requirement comes from the condition that X^3_V somehow represents the momentum exchanges between scattered particles. In 2-particle scattering this condition requires that X^3_V can be continued to a 4-surface which represents the momentum exchange between the scattered particles. If X^3_V is a 3-D portion of CP_2 type extremal this is possible since CP_2 type extremal can have arbitrary virtual momentum. But this would pose an additional strong condition of the 3-surfaces that can contribute to a given scattering event so that hopes about quantum classical correspondence look rather ill-founded.

4. For particles represented by other kinds of 3-surfaces situation would be different and their behavior would be extremely classical since scattering could occur only for very special kinematic configurations. This need not be a catastrophe but could provide a first principle explanation for why the world looks so classical above elementary particle length scales.

Option III: Could the ends of space-time sheets meet X^3_V along 3-D space-like portions?

If one modifies the idea that particle reaction corresponds to a creation of zero energy state from vacuum by allowing partial overlaps of incoming and outgoing particles with X^3_V, one can imagine more flexible options. The surfaces X^4_i could meet X^3_V only partially at its 3-dimensional space-like sections. One could decompose X^3_V to space-like parts at which incoming particles meet it and to light-like CDs representing particle exchanges in the reaction. This however still fails to guarantee that classical conserved charges coincide with commuting quantal charges for arbitrary incoming and outgoing systems.

A further problem is that the configuration space integral over quantum fluctuating degrees of freedom associated with at least partonic components cannot be carried out freely since there is no guarantee that deformations of space-like portion of X^3_V can represent the quantum numbers of the state classically even if X^3_V were able to do this.

Option IV: *Could space-time sheets meet X^3_V along 2-D partonic surfaces?*

Even more radical approach is suggested by the effective 2-dimensionality associated with the Clifford algebra of the configuration space. Since interior degrees of freedom of X^3 do not contribute to the configuration space metric, one can argue that also they represent one aspect of quantum non-determinism. Hence one could assume that the space-time sheets X^4_i representing incoming and outgoing particles meet at X^3_V only along some 2-dimensional partonic space-like sections X^2_V of the light-like causal determinants $X^3_{L,i}$ associated with them so that the interiors of space-like portions of X^3_V would not be fixed. The dynamics of particle space-time sheets would be determined by the generalized Bohr orbit property and the requirement that classical conserved charges represent commuting quantum charges at CDs.

1. This option gives a precise meaning and excellent motivation for the effective 2-dimensionality at the level of quantum dynamics and quantum measurement theory. The close connection with string models is obvious.

2. This option does not distinguish so strongly between elementary particles and macroscopic space-time sheets as other options. CP_2 type extremals represent unique candidates for virtual particles also for this option so that the explanation for why quantal effects are so strong in elementary particle length scales is not lost.

3. There is consistency with the proposed quantum descriptions for p-adic-to-real and real-to-padic phase transitions in which the 3-surfaces representing initial and final states have overlap consisting of only rational or algebraic points of the imbedding space. Also the phase transitions leading to phases with Planck constant differing from its ordinary value could involve only a discrete overlap in the generic case since different scaled up variants of M^4 and CP_2 can have only origin in common. For instance, the imbedding spaces $M^4 \times CP_2$ corresponding to different scaled up variants of CP_2 meet along M^4 so that space-time surfaces in these two sectors can meet in the generic case along along discrete points. Thus book like structures seem to appear very naturally both at the level of space-time and imbedding space. For these reasons only this option is discussed in the sequel.

7.2.2 Stringy formulation and analogy with Chern-Simons theory

For option IV one would have functional integrals over 3-surfaces X_i^3 associated with incoming and outgoing particles and over the vertex X_V^3 constrained by the condition that the 3-surfaces in question intersect along some partonic boundary components and X_i^3 represent classically the quantum observables. This kind of conditions are realized naturally as boundary conditions on the allowed deformations of various 3-surfaces appearing in the vertex. Quantum classical correspondence can be realized in strong form since for every deformation of partonic boundary components the interior can be chosen in such a manner that X_i^3 carries correct classical charges.

As a consequence, the functional integral over the partonic degrees of freedom reduces to Euclidian variant of a stringy functional integral. The values of Kähler function $K(X^3)$ assignable to X_i^3 and X_V^3 plays same role as Chern-Simons action and depends on the deformations of boundary components. The first order contribution vanishes since the normal components of conserved currents at boundaries must vanish. The dream is that this functional integral could be carried out explicitly using the gigantic symmetries of the configuration space metric and the fact that contravariant configuration space metric defines the propagator. In Gaussian approximation the exponents of Kähler action associated with with the incoming and outgoing lines cancel each other.

7.2.3 Decomposition of N-particle vertices to tree diagrams

For option IV it is easy to understand how N-particle vertices decompose to the analogs of ordinary tree diagrams.

1. Assume that X_V^3 representing vertex contains both space-like and light-like portions with incoming particles attached to X_V^3 along 2-dimensional partonic surfaces X_V^2.

2. In 2-particle scattering vertex the light-like parts of X_V^3 would represent exchanges of particles between reactants represented by different partonic CDs suggesting a connection with stringy diagrams. X_V^3 would in the simplest situation correspond to a CP_2 type extremal representing classically the exchange of an elementary particle between interacting particles. As found, the non-determinism of CP_2 type extremals allows to assign to them arbitrary of mass shell momenta. Hence the light-like portion of X_V^3 would represent propagator and the contraction with incoming and outgoing states using Connes tensor product creating the desired correlations could give rise to a genuine propagator factor.

This picture generalizes to N-particle case so that X_V^3 would consist of space-like segments representing vertices connected by light-like 3-surfaces representing the internal lines of a tree diagram. At the ends of these lines would appear the vertex functions constructible from n-point functions of a conformal field theory.

One can worry whether the functional integral over X^3_V could bring in bosonic loops in some sense. For instance, could one imagine a modification of CP_2 type extremals by allowing the extremal to branch temporarily? The conservation of light-like four-momentum makes this kind of decays impossible unless the final momenta are collinear in which case the resulting CP_2 type extremals would coincide. Furthermore, this process, even if it could occur, does not mean particle decay in TGD framework but just the propagation of spinor amplitude along two different routes like in double slit experiment so that loops in standard sense are not in question. The hypothesis that the possibly existing loop diagrams of this kind are equivalent with tree diagrams would mean that these degrees of freedom can be regarded as gauge degrees of freedom. Alternatively, if configuration space spinor field has same value for all these loopy configurations as for tree configuration, it is possible to take single one of them as a representative for all of them.

To conclude, it seems that option IV is consistent with the basic properties of S-matrix.

1. The free fields in M^4 defined by the physical states created by the Clifford algebra assignable to parton have mass spectrum determined by super conformal invariance and the integration over M^4 yields the desired momentum conserving delta functions. The fundamental cluster decomposition of S-matrix and crossing symmetry emerge automatically in the proposed picture.

2. N-vertices have natural decomposition to tree diagrams. In quantum field theories typically only vertices with at most 4 external lines appear. In conformal field theories the fusion rules reduce typically to fusion rules for pairs of primary fields implying the desired representation involving 3-vertices and analogs of propagators.

3. The new element is that classical behavior in macroscopic length scales is not a result of de-coherence but of the fact that non-deterministic propagation with arbitrary virtual momenta is possible only for CP_2 type extremals.

4. 3-surfaces X^3_V represent the change of quantum state in quantum jump whereas the ends of the incoming and outgoing 3-surfaces represent quantum state. Thus both quantum states and their changes giving rise to conscious experience in TGD inspired theory of consciousness have space-time correlates.

7.3 Connes tensor product and vertices

Connes tensor product means that $\mathcal{N}$ effectively replaces complex numbers as coefficient field of quantum theory. Connes tensor product generates correlations between the incoming and outgoing particles and therefore acts like a propagator and can be represented pictorially as a disk having incoming and outgoing particles along its boundary. The interpretation in terms of the simplest stringy tree diagram is suggestive and the absence of higher diagrams would mean that stringy perturbation theory is not needed.

7.3.1 Is the reduction of vertices to tree diagrams consistent with Connes tensor product?

According to the previous arguments two kinds of Connes tensor products appear in the vertices: the Connes tensor product for Clifford algebra elements associated with particles entering same partonic 2-surface X_V^2 and the gamma matrices at different partonic 2-surfaces but connected by light like CD. Since all 2-D cross sections of a light-like 3-D CD are conformally equivalent, it seems fair to conclude that these Connes tensor products are equivalent apart from the effects caused by the braiding along the light-like portion of X_V^3 which brings in also the modular S-matrix characterizing the braiding and playing a key role in TGD based model for topological quantum computation [E9]. Since the distance along the propagator line vanishes one can say that the propagator line is equivalent with a line contracted to a point, which would be a representation for the reduction of tree diagram to a diagram having single N-vertex.

7.3.2 Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an S-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of CH_{+-} (4-surfaces associated with 3-surfaces at the boundary of light cone at $m \in M^4$), extended to local fields in M^4 with gamma matrices acting on configuration space spinors assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [48] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [47].

Fusion rules are indeed something more intricate that the naive product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k

would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [51]. For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naive tensor product with something more intricate. The naivest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with CH_m. This is certainly too naive an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem is to construct an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

7.3.3 Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [49].

1. The light-like portions of X^3_V defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the

S-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [E9]. Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [M3].

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [49]. If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say three-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CD:s and connected by a piece of CP_2 type extremal.

7.4 Generalized Feynman diagrams

During I have developed several variants for the idea that perturbation theoretic expansion of S-matrix is not needed in TGD framework. The basic prerequisite for this is the p-adic coupling constant evolution coded by the $log(p)$ scaling factor of the modified Dirac operator already at the level of free field theory.

1. The first version of the idea states that radiative corrections to the functional integral over configuration space degrees of freedom (world of classical worlds) vanish. This condition is necessary if one wants number theoretic universality. The assumption that the functional integral reduces to a sum over maxima of Kähler function for fixe values of arguments of N-point functions (points of imbedding space) would leave only single generalized Feynman diagram for given maximum of Kähler function. If the dependence of the Kähler function on arguments of N-point function is weak, exponent of Kähler function cancels out from S-matrix elements in a good approximation and perturbation theory applies.

2. Second variant of the idea is based on category theoretic generalization of Feynman diagram notion of duality and states that loop diagrams are equivalent with tree diagrams or more generally, a minimal generalized Feynman diagram with loop. The loop does not however involve any integration. This view might be equivalent with the assumption that S-matrix is a functor from the category of Feynman cobordism to the operators in Hilbert space allowing to decompose themal S-matrix parametrized by argument $\tau = t + i\beta$ to a product of S-matrices with various values of τ satisfying $\sum_n \tau_n = \tau$ assuming that time reflection corresponds to Hermitian conjugation mapping positive energy states to negative energy states (bras to kets).

7.4.1 What the equivalence of loop diagrams with tree diagrams could mean?

The generalization of the duality of old-fashioned string models leads, not only to the equivalence of loop diagrams with tree diagrams but to the equivalence with diagrams involving only single N-vertex. This outcome leads then to the master formula of S-matrix in terms of Connes tensor product in which the original principle does not anymore seem to play any role. The question is whether the generalization of duality should be given up as obsolete or whether it has some non-trivial meaning as believed originally [C6].

7.4.2 Cancellation of loop corrections for Feynman diagrams

One can ask whether the cancellation loop corrections for ordinary Feynman diagrams in some sense could provide an alternative to state the proposed equivalence. The triviality of the automorphism Δ^{it} for hyperfinite factors of type II_1 serving as a universal candidate for an object defining a propagator has a natural interpretation as an on mass shell property of all internal lines of ordinary Feynman diagrams with arbitrarily high N-particle vertices allowed.

One can indeed allow all possible diagrams if this condition is posed. The sum of all intermediate states resulting in the decay of the particles of internal lines to on mass shell particles and their scattering to the original state yields just $SS^\dagger = 1$ factor on internal line or set of lines and produces unity. Hence one can say that $i(T - T^\dagger) - TT^\dagger$ represents on mass shell loop corrections which vanishes by unitarity. This interpretation is the only possible one in the proposed framework.

7.4.3 The equivalence of loop diagrams with tree diagrams for generalized braid diagrams

The interpretation is that a diagram with loop is equivalent with diagram with no loop. Diagram in this sense cannot correspond to ordinary Feynman diagram, in particular not the Feynman diagram with only on mass shell loops. Conditions for the equivalence in this sense have been formulated in algebraic terms for the generalization of ribbon algebras [C6]. The foregoing argument leaves the possibility to assign the equivalence of loop diagrams with tree diagrams to generalized braid diagrams so that also in this case diagrams could be reduced to a diagram with single vertex. If Universe is mimicking itself by using generalized braid S-matrix to emulate the proper S-matrix this equivalence would provide a representations for the cancellations of loop corrections for proper S-matrix.

1. What the equivalence of loop diagrams with tree diagrams could mean

Since the orbits of light like causal determinants are determined by field equations, only the failure of classical determinism can allow diagrams with loops or more complex diagrams involving what would be interpreted as particle creation in string model context. The equivalence would state that non-determinism has interpretation as a kind of gauge symmetry. In the case of 3-surfaces appearing as vertices this interpretation would state that the S-matrix assignable to the space-like braid is same for all 3-surfaces obtained from each other by this equivalence. All the considerations of [C6] would relate to these diagrams.

The stringy loops in which partonic 2-surface decays temporarily to two partonic 2-surfaces do not correspond in TGD framework to particle decays but to a single particle propagation along two different paths simultaneously. This picture leads to a generalization of the quantum measurement theory and explanation [20] for the findings of Shahriar Afshar relating to double slit experiment challenging Copenhagen interpretation [77]. For instance, in double slit experiment the measurement of the particle aspect of photon would reduce the branched photon path in such a manner that second branch corresponds to a vacuum extremal having a vacuum line representing identity operator as its algebraic counterpart. In this case the equivalence of loop diagrams with tree diagrams looks obvious.

The equivalence of loop diagrams with tree diagrams in this sense means that one can move the end of any internal line until it becomes a tadpole loop which must represent vacuum line and can be eliminated. This means that all diagrams are equivalent to a simplicial complex representing the homology of planar disk D^2 with the ends of the external lines at the boundary circle of the disk and having Euler characteristic $E + F - L = -2$.

The equivalence implies also that any tree diagram containing N-vertices with arbitrary values of N can be transformed to a single M-vertex, where

M is the number of incoming lines (for convenience al lines are regarded as incoming). The diagram can be also transformed to a diagram containing only 3-vertices. Number theoretic vision about the role of classical division algebras in TGD [E2] suggests that this symmetry is closely related to the octonionic triality reflecting itself also as the existence of 3 8-dimensional representations of $SO(8)$.

2. Equivalence of loop diagrams with tree diagrams and Jones inclusions

Consider now how the equivalence could be understood in terms of Jones inclusions. The argument below is a simplification of the algebraic conditions formulated in [C6] guaranteing also the possibility to move the ends of the lines around the graph.

1. If each incoming line is thought of as being imbedded in the same manner to a II_1 factor $\mathcal{M}$ then each incoming line of the vertex can be characterized by the same value of $\mathcal{M} : \mathcal{N}$, and one can assign to each line emanating from a vertex a representation of the same quantum group or Kac Moody group. The notions of product, co-product, and bi-algebra are well-defined [C6].

2. Assume that it is possible to transform the diagram to a diagram containing only 3-vertices by moving around the ends of the lines: this possibility should relate closely to octonionic triality [E2] underlying the vertex construction. As a consequence, all loops reduce to self-energy loops. Assume that the operator in the third line of the vertex is product of the operators associated with other two lines and the operator associated with two lines is a co-product of the operator in the third line. Under this assumption products and co-products in the self energy loops compensate each other and they are trivial.

3. Could the equivalence with tree diagrams imply unitarity of the generalized braid S-matrix?

It would be easier to take seriously the reducibility of generalized Feynman diagrams to tree diagrams if it would guarantee the unitarity of the generalized braid S-matrix. The following heuristics indicates that this could be the case.

1. The equivalence with tree diagrams allows to carry out two operations for the generalized Feynman diagrams.
   i) It is possible to transform diagrams representing S-matrix elements to diagrams involving single vertex with M incoming lines and N outgoing lines. Incoming lines start from the boundary of a future directed light cone $X^7_+ = \delta M^4_+ CP_2$ and outgoing lines end at the boundary of a past directed light-cone $X^7_- = \delta M^4_+ \times CP_2$ having its tip inside X^7_+.
   ii) It is possible to move the position of $M+N$ vertex arbitrarily near to the initial moment. At space-time level this means that the M partonic orbits intersect at the partonic 2-surface already at X^7_+ and decay to N partonic 2-surfaces.

2. Unitarity conditions should reduce to the statement that the initial states M_1 and M_2 are orthogonal. The sum over intermediate states in the unitarity relation involves a sum over number N of outgoing lines. Assume that it can be transformed by the completeness of states to a form in which a delta function appears stating that the values t_i are equal for N lines and their conjugates. If this is the possible, the unitary automorphisms Δ^{it_i} and their conjugates compensate each other for each outgoing N line.

3. If only the condition i) is assumed, the unitarity condition reduces to a condition stating the orthogonality of the images of the states $\hat{M}_1$ and $\hat{M}_2$ obtained from M_1 and M_2 by assigning the S-matrices S_i to the lines M_1 and their conjugates $S_j^\dagger$ to the lines of M_2. A reaction in which both incoming and outgoing partons belong to the boundary of the same future light-cone X_+^7 is in question. M particles travel to future, react in $M+N$ vertex and produce N particles, which are reflected back to the past.

4. If also the Feynman diagrams characterizing the S-matrix obtained by replacing the automorphisms associated with outgoing lines with their time reversals satisfy the equivalence with tree diagrams, this S-matrix is trivial without further conditions since the lines to future and back can be contracted to points. If also the condition ii) is assumed, $\hat{M}_i = M_i$ and unitarity conditions reduce to the ordinary orthogonality conditions.

8 Are both symplectic and conformal field theories be needed?

Symplectic (or canonical as I have called them) symmetries of $\delta M_+^4 \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M_+^4 \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [C1] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

8.1 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [**?**]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \quad . \tag{29}$$

Here the coefficients c_{kl}^m are constants and $A(s_1, s_2, s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1, s_2, s)) I d d\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1, s_2, s)) d\mu_s \quad . \tag{30}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1, s_2, s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

8.2 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{31}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that

ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\begin{aligned} \langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle &= c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s \\ &= \quad (32) \end{aligned}$$

$$c_{kl}^r c_{rm} \int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \quad . \quad (33)$$

Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(\Delta A(s_1,s_2,s)) d\mu_s \quad (34)$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

8.3 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars

obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time [E10].

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory

in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since

n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately.

8.4 Still more detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

8.4.1 Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections to two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

8.4.2 Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactification) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [C1].

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continuable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

8.4.3 What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

8.4.4 What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^\dagger/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^\dagger$. One can interpret the fermion number carrying M^4 gamma matrices as those of the complexified quaternion space appearing naturally in number theoretical framework.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in a massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation.

The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

9 Could 2-D factorizing S-matrices serve as building blocks of U-matrix?

A very explicit construction of U-matrix using as building bricks 2-dimensional factorizing S-matrices satisfying Yang-Baxter equations and generalizing braiding S-matrices emerges from number theoretical vision. First it became clear that this construction applies in imbedding space degrees of freedom but then it became clear that it might apply also in configuration space degrees of freedom and define the basic architecture of the U-matrix.

The construction led also to painful sequence of trials to circumvent the fact that the resulting U-matrix expressible in terms of factorizing 2-D S-matrices seemed to be almost trivial physically, and eventually forced the realization that the ontology based on zero energy states applied in classical TGD for years, must be applied also at the level of S-matrix.

The first number theory inspired idea was that the U-matrix could in a well-defined sense be a sum of basic building blocks defined in M^2 and the presence of conformal symmetries suggest a connection with 2-D integrable models with factorizing S-matrices for which particles with different masses suffer only phase shifts and particles with identical masses can permute the momenta [40]. In fact the treatment generalizes to CP_2 degrees of freedom using intriguing observations related to 6-vertex models. The real connection to physics turned out however to be quite different from the naive expectations.

9.1 U-matrix for the scattering of zero energy states

In the following the possibility that the almost trivial factorizing S-matrices define building blocks of a U-matrix scattering of zero energy states is considered and shown to lead to what might be called zero energy ontology. This does not however mean that there could not exist a physical process more or less consistent with the interpretation as scattering of positive energy states.

9.1.1 U-matrix describes scattering of zero energy states representing particle reactions

In TGD framework one can say that particle reaction becomes a zero energy states and that is scattering of these zero energy states representing particle reactions that represent our perceptions about world as a state rather than change.

1. What is observed are not particles in the initial and final states but particle reactions identified as zero energy states consisting of positive and negative energy particles usually identified as initial and final states of the particle reaction.

   What comes in mind is the construction of a factorizing U-matrix for zero energy states consisting of the particles of initial and final states having arbitrary momenta. This scattering would describe scattering between zero energy states interpreted in terms of ordinary particle reactions.

2. The basic property of factorizing S-matrices is that they affect only the internal degrees of freedom: therefore the scattering between zero energy states does not affect the initial and final momenta of positive and negative energy states so that the curses of factorizing S-matrices become blessings. The quantum jumps describing scattering makes it possible to experience these reactions consciously and affects only internal degrees of freedom which are not detected.

3. One should be even ready to give up the cherished Lorentz invariance and color symmetries since Jones inclusions associated with the scattering experiment could mean symmetry breaking caused by the selection of subgroups $SO(1,1) \times SO(2)$ $SO(1) \times SO(3)$, and $U(2) \subset SU(3)$. In the recent picture these choices affect even the geometry and topology of imbedding space and they reflect directly the effect of experimenter to the measurement situation, which simply cannot neglected as is usually done

axiomatically. One could also speak about number theoretic breaking of symmetries induced by the requirement that fundamental commutative sub-manifolds of imbedding space are 2-dimensional.

4. One can generalize the construction of factorizing S-matrices [40, 50] without difficulties to the case in which incoming and outgoing states are zero energy states representing particle reactions. For instance, U-matrix for zero energy states could decomposes to a tensor product of U-matrices for positive and negative energy states. For this option sensory perceptions can give conscious information only about positive or negative energy part of the state. This would help to understand why we tend to perceive world as obeying positive energy ontology.

   Pass-by of particles in M^2 defines the counterpart of particle exchange in braiding for factorizing S-matrices. This pass-by might be possible also for negative and positive energy states if pass by is considered for the projections of rapidities on $i\pi - \eta_i$ to real plane. The construction goes through as such for the other tensors. Also the crossing symmetry and other symmetries of factorizing S-matrices generalized in obvious manner.

5. The physical states in the negative energy tensor factor are annihilated by fermionic creation operators. This guarantees that the fermionic operators creating zero energy states anti-commute and no terms representing the annihilation of a fermion pair to vacuum appear in the pass-by process.

9.1.2 Could U-matrix define S-matrix in case of HFFs of type II_1?

The original hope was that one might be able to derive ordinary unitary S-matrix from the U-matrix. In the case of HFFs of type II_1 this is indeed possible but one must however admit that U-matrix cannot give rise to a realistic S-matrix. In the following U-matrix for HFF of type II_1 is formally treated as a matrix with discrete indices. A rigorous treatment would be by replacing indices representing 1-D projections by projections to infinite-dimensional sub-factors having non-vanishing trace.

The unitarity conditions for the scattering of zero energy states read formally as

$$\sum_{\hat{m}_+\hat{n}_-} S_{m_+n_-\rightarrow\hat{m}_+\hat{n}_-} S^*_{r_+s_-\rightarrow\hat{m}_+\hat{n}_-} = \delta_{m_+,r_+}\delta_{n_-,s_-} \ . \tag{35}$$

The sum over the final zero energy states can be also written as a trace for the product of matrices labelled by incoming zero energy states.

$$Tr(S_{m_+n_-}S^*_{r_+s_-}) = \delta_{m_+,r_+}\delta_{n_-,s_-} \ . \tag{36}$$

One can put $s_- = n_-$ on both sides and perform the sum over n_- to get

$$\sum_{n_-} Tr(S_{m_+n_-}S^*_{r_+n_-}) \quad = \quad \delta_{m_+,r_+}\sum_{n_-}\delta_{n_-,n_-} \quad . \tag{37}$$

This can be written as

$$\frac{1}{Tr(Id)}\sum_{n_-} Tr(S_{m_+n_-}S^*_{r_+n_-}) \quad = \quad \delta_{m_+,r_+} \quad . \tag{38}$$

For HFFs of type II_1 the sum $\sum_{n_-}\delta_{n_-,n_-}$ is equal to the trace $Tr(Id) = 1$ of the identity matrix so that one obtains

$$\sum_{n_-} Tr(S_{m_+n_-}S^*_{r_+n_-}) \quad = \quad \delta_{m_+,r_+} \quad . \tag{39}$$

This could be interpreted as a unitarity condition for positive and negative energy parts of the zero energy state are interpreted as incoming and outgoing state.

This result allows to consider the possibility that U-matrix between zero energy states could define also S-matrix for HFFs of type II_1. The almost triviality of U-matrix however suggests that this is not a good idea. The construction of S-matrix as time-like entanglement coefficients allowing to understand thermodynamics as part of quantum theory provides further support for this belief.

The interpretation of the result would be as a thermal expectation value of the unitarity condition in the sense of hyper-finite factors of type II_1. This averaging is necessary if one does not have any control over the scattering between zero energy states: this scattering is just a means to become conscious about the existence of the state we usually interpret as change of state. What looks a very non-physical feature of factorizing S-matrices turns into a victory since the trace is only over final states which are characterized by the same collection of momenta and same particle number and uncertainties relate only to the internal degrees of freedom which one cannot measure and whose basic function is to make it possible to consciously perceive the particle reaction as a zero energy state.

9.1.3 U-matrix can have elements between different number fields

The argument for the number theoretical universality applies as such only to the matrix elements of U-matrix between different number fields. By the preceding arguments, U-matrix could be almost trivial for real-to-real transitions. For p-adic-to-real transitions the notion of almost-triviality could mean that the set of real momenta and their p-adic counterparts are identical for the allowed transitions. This would mean a rather tight correspondence between intention and corresponding action. Discretization however suggests that the correspondence need not so tight.

Among other things discretization would mean quantization of the rapidities of particles meaning that $sinh(\eta)$ and $cosh(\eta)$ for hyperbolic rotation angle are rational numbers. Hyperbolic counterparts of Pythagorean triangles would be in question. Obviously, the (1+1)-D Lorentz group consisting would reduce to its subgroup of rational Lorentz transformations.

For S-matrix identified as unitary entanglement coefficients between positive and negative energy parts of zero energy state situation is more problematic. One can speak about entanglement between positive and negative energy states belonging to different number fields only if various quantum numbers are universal and make sense in all number fields. In this case also the notion of number theoretic braid emerges naturally. The universality of quantum number spectrum could however mean that the difference between number fields has no observable consequences. Therefore there is a temptation argue that S-matrix must be diagonal with respect to the number field whereas U-matrix can be non-trivial between different number fields because since it has only elements between zero energy states. Intentionality would be related to U-matrix to intentionality and free will and S-matrix to particle reactions.

This raises the worry that number theoretic universality and the representation in terms of number theoretic braids is lost for diagonal transitions. This need not be the case. The eigenvalue spectrum of the modified Dirac operator assigns a p-adic prime p to the partonic 3-surface so that the algebraic intersection of the real surface and its p-adic counterpart is uniquely defined. The natural definition of the Dirac determinant is as a product of sub-determinants defined by products of eigenvalues of the modified Dirac operator at algebraic points satisfying the additional condition that $p^{i\zeta^{-1}(z)}$ is algebraic number (z defines the projection to the geodesic sphere of CP_2). Hence the notion of number theoretic braid emerges naturally also in the case of S-matrix.

Also finite resolution of quantum measurement implies the notion number theoretic braid and justifies the heuristic identification of the number theoretic braid as a braid associated with Jones inclusion. The finite measurement resolution for the physically observable quantum S-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ consisting of finite-dimensional matrices with $\mathcal{N}$ valued matrix elements leads to the idea that the complex coordinates z for geodesic spheres of $\delta M^4_\pm$ and CP_2 become $\mathcal{N}$-valued bosonic quantum fields obeying standard bosonic commutation relations. Since the generalized eigenvalues of the modified Dirac operator depend on z this means that also these become non-commutative. Hence the anti-commutativity of induced spinor field along number theoretic string ceases to hold true and can be satisfied in a discrete subset of string only.

If quantum coordinate z has only a finite number of modes, the reduction to $\mathcal{M}/\mathcal{N}$ suggests, the commutators of z and its Hermitian conjugate vanish in a discrete set of points defining the number theoretic braid but now as a consequence of finite measurement resolution (rather natural). The finite number for the modes of z might result from the condition that the modes belong to the algebraic extension of p-adic numbers with p dictated by $log(p)$ scaling of the generalized eigen values.

An important class of Jones inclusions or their finite-dimensional approximations might be induced by the inclusions for algebraic extensions of p-adic numbers. For instance, the hierarchy of N-strands approximating Jones inclusions as inclusions of Temperley-Lieb algebras might correspond directly to a hierarchy of algebraic extensions for p-adic numbers containing the phases $exp(i\pi/k)$, $k \leq N$.

Physical states can be chosen to be coherent states that is eigen states $z(x_i)$, where x_i are position coordinates for the strands of the number theoretic braid. $z(x_i)$ can possess the proposed number theoretical spectrum expressible in terms of zeros of ζ since a priori all complex numbers are possible eigen-values.

9.2 Factorizing 2-D S-matrices and scattering in imbedding space degrees of freedom

Int this subsection the view that the scattering in imbedding space degrees of freedom could be described using a tensor product of 2-D factorizing S-matrices associated with at most 2-D subspaces of $M^4 \times CP_2$ is developed.

9.2.1 Factorizing S-matrix in M^2 as a building block of the full U-matrix

1. Why factorizability?

The known exact S-matrices in 1+1-dimensional space time are factorizing. According to [50] there exists a strong evidence that all exact S-matrices in 1+1 dimensions are factorizing, do not allow particle production, and that the sets of the initial and final state momenta are identical.

Exactness certainly follows from infinite number of conservation laws associated with integrable systems but also finite number of them is enough. Infinite number of conservation laws are expected also in TGD since Kac Moody type symmetries are present. The conserved charges of form

$$Q_a^n = exp(n\theta_i)Q_a \ , \tag{40}$$

where n is Lorentz spin completely analogous to conformal weight imply the factorizability [40]. These charges have interpretation as loop group generators of conformal weight n in the defining representation (where these generators are proportional to m^n) evaluated at the ray η_i of M^2 representing momenta as the positions for tips of light-cones. In the case of E^2 one obtains $exp(i\phi_n)$ where ϕ_n represent directions of momenta classically.

2. Yang-Baxter equations and Zamolodchikov algebra

Arranging the scattering particles in M^2 with respect to rapidities η_i (hyperbolic angles) such that the fastest particle is leftmost and slowest one rightmost (this is possible by the crossing symmetry and by assuming Yang-Baxter equations), the scattering can be described as a sequence of events in which particles pass by each other and can be therefore interpreted as a braiding like process with the additional feature that particles move with different velocities.

The pass-by event is described by a 2-particle S-matrix depending only on the difference $\eta_{12} = \eta_1 - \eta_2$ of their rapidities. By Uncertainty Principle, the position of the particle world line should not matter so that the world line of any particle can be shifted parallel to itself without affecting the S-matrix. This however affects the braiding. This symmetry gives rise to the celebrated Yang-Baxter equations

$$S(\eta_{12})S(\eta_{13})S(\eta_{23}) \quad = \quad S(\eta_{23})S(\eta_{13})S(\eta_{12}) \ . \tag{41}$$

N-particle S-matrix reduces to braiding S-matrix expressible in terms of S-matrices describing 2- particle scattering.

One can abstract the conditions on S-matrix algebraically to give what is known as Zamolodchikov algebra [40] so that S-matrix describes the pass-by process as a generalization of the exchange operation in braiding. Posing the conditions that 2-particle S-matrix approaches unit matrix at the limit $\eta_{12} \rightarrow 0$, unitarity stating $S(\eta)S(-\eta) = 1$, real analyticity $S^{\dagger}(\eta) = S(-\overline{\eta})$ and crossing symmetry $S_{ij}^{kl}(\eta) = S_{j\bar{l}}^{\bar{i}k}(i\pi - \eta)$, one achieves axiomatization for the algebra. Sine-Gordon theory provides a basic example of an integrable system whose S-matrix satisfying these constraints.

If one poses the restriction that light cone tips belong to M^1 situation simplifies still since all particles defined by the contents of light cones would be at rest relative to each other and the S-matrix reduces to a trivial braiding matrix obtained by putting $\eta_{ij} = 0$ in above equation. The limit $\eta_{ij} \rightarrow \pm\infty$ when taken in a somewhat delicate manner gives rise to the standard form of the non-unitary braiding matrix appearing in quantum group representations as shown by Jimbo [40].

3. Could factorizing S-matrix as tensor factor of full S-matrix make sense in TGD framework?

In a genuinely 2-D context this kind of system is of course physically somewhat uninteresting. In TGD framework the situation is different. By quantum classical correspondence the rapidities could be interpreted as M^2 projections of the 4-momenta of the particles created in the vertex. Since each light-cone can contain arbitrary many partons, the rapidities could be interpreted as M^2 projections of the total four-momenta of groups of particles associated with the light cones and behaving like a single particle. Therefore particle number conservation would not trivialize the S-matrix.

The reduction to a factorizing S-matrix convoluted with single particle configuration space spinor fields would mean a powerful constraint reducing the U-matrix effectively to 2-particle scattering for particle groups associated with the light cones. The fundamental scattering matrix would thus be almost trivial in longitudinal momentum projections but would not depend on the transversal momenta at all. The integration over all possible choices of M^2 plane would guarantee Lorentz invariance might destroy unitarity. Also the triviality in longitudinal momentum degrees for freedom looks non-physical.

Later it will be found that these properties are highly desirable if physical states are identified as zero energy states so that the TGD counterpart of ordinary S-matrix is coded by the structure of the zero energy states.

9.2.2 Factorization of U-matrix in CP_2 degrees of freedom

Center of mass degrees of freedom are unavoidable also in CP_2 degrees of freedom since Jones inclusions defined by the subgroups $G \subset SU(2) \subset SU(3)$ select preferred origin so that $CH^{\pm}_{m\in M^4}$ must be generalized to $CH^{\pm}_{h\in H}$: the tip of light cone means also tip in CP_2. It is natural to ask whether the complex CP_2 coordinates should be replaced with a quaternionic coordinate in such manner that the restriction to a geodesic sphere of CP_2 would be the Euclidian analog of the restriction to M^2 meaning restriction to scattering in compactified complex plane and commutativity of generalized n-point functions.

That this should and can can be done is suggested by an intriguing observations. The observation that for six-vertex model the solutions of Yang-Baxter equation are parameterized by CP_2 [40] was one of the first intriguing observations [E9] leading to the evolution of ideas the role of quantum groups and von Neumann algebras in TGD.

1. CP_2 parameterizes R-matrices

In six-vertex model the R-matrices (counterparts of S-matrices above) have slightly different form than the S-matrices. For weak (or color) isospin 1/2 case which is fundamental also now, R-matrix is parameterized by 3 complex parameters

$$R(a,b,c) = \begin{pmatrix} a & & & \\ & b & c & \\ & c & b & \\ & & & a \end{pmatrix} . \tag{42}$$

The matrices differing by a complex scaling are physically equivalent so that a, b and c can be interpreted as complex components of fundamental representation of $SU(3)$. c can be fixed to $c = isin(\gamma)$, where $exp(i\gamma)$ can in fact be identified as the quantum phase $q = exp(i\pi/n)$ and a and b can be identified as complex CP_2 coordinates (ξ^2, ξ^2) transforming linearly under $U(2) \subset SU(3)$.

The restriction of a an b to represent points of a geodesic sphere of CP_2 going through origin implies that the matrices $R(a,b,c)$ commute. The condition for commutativity reads as

$$\begin{aligned} \Delta(a,b,c) &= \Delta(a',b',c') , \\ \Delta &= \frac{a^2+b^2-c^2}{2ab} . \end{aligned} \tag{43}$$

The solution of Yang-Baxter equation for three R-matrices reduces to the condition

$$\Delta(a,b,c) \quad = \quad \Delta(a',b',c') = \Delta(a'',b'',c'') \ . \tag{44}$$

Commutativity (in the sense of S-matrices rather than with respect to the product appearing in Yang-Baxter equations) means that the three points of CP_2 belong to the geodesic sphere identifiable also as a maximal commuting submanifold of CP_2 interpreted as a space obtained by gluing together three copies of quaternionic space H along sphere S^2 representing compactified complex plane for the second quaternionic space glued together just like S^2 is obtained by gluing together two complex planes along real line compactified to circle.

A canonical parameterization satisfying the commutativity conditions is given by

$$\begin{aligned} a(u) &= sinh(u+i\gamma) \ , \\ b(u) &= sinh(u) \ , \\ c(u) &= isin(\gamma) \ , \end{aligned} \tag{45}$$

where u is a complex coordinate. Using u as coordinate the Yang-Baxter equations have the same additive form as in case of M^4. In other words, one has $u'' = u' - u$. Unitarity is achieved when u is real.

These observations made already earlier [E9] suggest that the construction for M^2 generalizes to CP_2 degrees of freedom representing its Euclidian version obtained by the replacement $M^2 \subset M^4 \rightarrow S^2 \subset CP_2$. The commutativity of R-matrices in the case of S^2 would have interpretation in terms of space-like metric whereas in the case of M^4 Minkowski signature implies correlations and non-commutativity.

2. Factorizable U-matrix associated with a geodesic sphere of CP_2

Quantum classical correspondence can be applied also now as a guide line. Before continuing it is however useful to restate some facts about CP_2 and introduce notations. Assume for definiteness that CP_2 is identified as the space of right cosets $gU(2)$ of $SU(3)$ so that the natural action of $SU(3)$ is left action. The orbits of $SU(2)_L$ and $U(2)_L$ are homologically non-trivial geodesic spheres S^2 and the double coset space $SU(3)/SU(2)_L \times SU(2)_R$ of these spheres is 2-dimensional.

Also the geodesic circles $S^1_\perp$ orthogonal to a given point of S^2 are interesting as analogs of M^1 in $M^1 \times S^2$ decomposition. By the symmetry of $SU(3)/SU(2)_L \times SU(2)_R$ the actions of both $SU(3)_L$ and $SU(3)_R$ in this space are well defined, and the natural idea is that $U(1)_R$ action defines the geodesic circles $S^1_\perp$ so that electro-weak symmetry group would have a geometric counterpart.

Both $SU(2)_L \subset SU(3)$ and weak $SU(2)_L$ are represented by 4×4-dimensional R-matrices acting on fundamental fermions. The coordinate u parameterizing commuting R-matrices corresponds to the geodesic sphere $S^2 \subset CP_2$.

1. $SU(2)_L \subset SU(3)$ quantum numbers replace M^2 momentum. Indeed, color is in TGD framework not a spin like quantum number but completely analogous to four-momentum and orbital angular momentum.

2. The complex coordinates ξ^i, $i = 1, 2$, of CP_2 have a phase $exp[i(\pm\phi + \psi)/2]$ with ϕ assignable to isospin and ψ to hypercharge. $\xi^2 = 0$ geodesic sphere thus represents $I_3 + Y$ rotation. The classical representation for the quantization of angular momentum suggests that the direction of the total $I_3 + Y$ associated with a particular $\delta M^4_+ \times CP_2$ defines a point at S^2 parameterized in standard manner by (θ, ϕ). This fixes the value of θ via the condition

$$cos(\theta) = \frac{I_3 + Y}{\sqrt{I(I+1) + Y^2}} \qquad (46)$$

   when $\xi^2 = 0$ is selected as the representative geodesic sphere.

3. The angle ϕ is the Euclidian counterpart of rapidity η so that that the classical model for the scattering would be in terms of particles rotating with different velocities along the circumference of circle. The momenta would be replaced with isospins $(I^3 + Y)_k$ ordered from left to right along the circumference such that one has $(I^3 + Y)_1 \geq (I^3 + Y)_2 ..... \geq (I^3 + Y)_n$ having $\phi_1 \leq \phi_2 .... \leq \phi_n$. Unitarity requires that the parameter u is real and $\gamma_i = \phi$ identification is suggestive.

   i) In the case of M^2 the values of rapidities can be fixed by four-momenta but in the recent case there are no four-momenta and Uncertainty Principle does not encourage the fixing of the phases so that one must simply integrate over all possible values. Most naturally the convolution of the scattering amplitude with color partial waves for center of mass degrees of freedom defines this integration naturally.

   ii) On the other hand, the existence of phases in algebraic extension of p-adic numbers would suggest that ϕ_i can come only as multiples of the angle π/n defining the quantum phase q so that circle would be discretized to a circular lattice. The values of color isospin J would be restricted to $J \leq n/2$ for even n since for J and $J + n$ the wave functions differ only by a sign. For odd n one has $J \leq n$. This conforms with the fact that for the finite-dimensional representations of quantum groups associated with $q = exp(i\pi/n)$ the action of raising and lowering operators $J^n_\pm$ reduces to a multiplication by a complex number [40], which can also vanish so that cyclic or semicyclic representations besides counterparts of ordinary finite-dimensional representations are obtained. Also the possibility of only $j \leq n/2$ representations of Kac Moody group fits with this picture. The groups $G \subset SU(2)$ for which n is the order of the maximal cyclic subgroup would naturally define as their orbits discrete analogs of the geodesic sphere allowing p-adicization and discrete versions of spherical harmonics. Physically the appearance of finite subgroups of SU(2) would be a direct analog for the presence of discrete subgroups of translation groups in solid state physics.

4. This construction would allow to fix the dependence of S-matrix on the center of mass coordinates and on total color quantum numbers and the integration over the orbifold $SU(3)/SU(2)_L \times SU(2)_R$ of geodesic spheres of CP_2 would restore the exact color invariance broken by Jones inclusion.

5. Just as the M^4 coordinates of the arguments of n-point function can be restricted to M^1, their CP_2 coordinates can be restricted to geodesic circle $S^1 \subset S^2 \subset CP_2$ implying the reduction of S-matrix to braiding S-matrix.

9.2.3 What about Yang-Baxter type scattering in transversal degrees of freedom?

One could also consider construction of a Yang-Baxter type scattering matrix in transversal degrees of freedom. This S-matrix cannot give rise to momentum transfers. One could argue that this is not in spirit with the basic number theoretic idea. One could however modify the idea. E^2 as the complement of hyper-complex plane in hyper-quaternionic space ($z = xiJ + yiK$) can be mapped to complex plane by $z \rightarrow iJz = x + yI$) and one can construct S-matrix for scattering in this plane. Similar argument applies in CP_2 degrees of freedom.

1. Factorizable U-matrix E^2 degrees of freedom

It is straightforward to modify the construction for CP_2 to construct S-matrix in transversal degrees of freedom. The angles ϕ_i characterizing the directions of transversal momenta would replace rapidities and particles could be ordered with respect to these angles and the intersections of projections of orbits to E^2 would define the interaction vertices. The commuting S-matrices applied in case of CP_2 parameterized by the values of u and γ could be used to define S-matrix. The values of ϕ coming as multiples of quantum angle π/n suggest themselves in p-adic context as intersections of p-adic E^2 with real one.

2. Factorizable U-matrix in $S^1 \perp S^2$ degrees of freedom

If one allows pass-by events in E^2, one must allow them also for the counterpart of E^2 in CP_2. Only the geodesic sub-manifolds representing commuting sub-algebra of quaternions and orbit of subgroup of color group are possible. This leaves only geodesic circles of CP_2 orthogonal to geodesic sphere S^2 into consideration. The reduction would be completely analogous to $M^1 \times S^2$ decomposition in the case of M^4. As noticed, the action of $U(1)_R$ groups in the space of geodesic spheres is well defined and generates these geodesic circles.

The reduction of $SU(2)_L \times SU(2)_R \subset SU(3)$ to $SU(2)_L \times U(1)_R$ obviously correlates with the structure of the electro-weak gauge group.

The four-fold decomposition of H is analogous to the decomposition of 8-D spinors to four-fold tensor product of 2-D spinors. M^2 (E^2) represents classically hyperbolic (ordinary) rotations. $\xi^2 = 0$ geodesic sphere S^2 represents $I_3 + Y$ rotations and $S^1_\perp$ represents $I_3 - Y$ rotations. Every commuting isometry charge of $SO(3,1) \times SU(3)$ would thus correspond to its own tensor factor in the factorizing S-matrix.

9.2.4 Factorization for Kac-Moody representations

An interesting question relates to whether one should use finite-dimensional or infinite-dimensional representations of quantum Kac-Moody algebra to construct S-matrix as braiding matrix in Kac-Moody algebra. In both cases the counterpart of complex coordinate z restricted to unit circle (hyper-quaternionic M^2 coordinate restricted to $m \cdot m = 1$ hyperboloid) brings in angle (rapidity) variable.

1. Finite-dimensional representations are obtained from those of quantum group and have vanishing central charge $k = 0$ and appear naturally in integrable 1+1-dimensional quantum theories so that the Yang-Baxter matrices are finite-dimensional, typically 2×2 matrices acting on quantum spinors. The infinite number of conservation laws have a natural interpretation in terms of elements of this algebra. Since Lorentz invariance in longitudinal degrees of freedom should not be broken by the central extension, one might argue that finite-dimensional representations are natural in this case. Also the idea that Connes tensor product makes the situation finite-dimensional fits with this interpretation. On the other hand, the breaking of Lorentz invariance might be a property of zero energy states and reflect the measurement situation as will be found and one must be cautious here.

2. The braiding matrix for infinite-dimensional Kac-Moody representations was found by Drinfeld [40] and has exponential form bringing in mind an exponent of Hamiltonian. The representation involves also Virasoro generator L_0. Presumably the generalization to the case of super Kac-Moody algebras exist. Neither the Kac-Moody- or quantum group R-matrix is unitary. I do not know whether a unitary R- matrix for Kac-Moody algebras is exclude by some deep reason.

The following arguments support the view that only finite-dimensional representations appear in S-matrices between zero energy states which seem to be the only possibility in TGD framework.

1. The universality of the R-matrix for affine algebras encourages the guess non-unitarity is a universal property of Kac-Moody R-matrices containing

only single continuous parameter and that unitary and thus trivial R-matrix is possible only in $q = 1$ case. This would conform with the fact that $q = 1$ also corresponds to extended ADE diagrams for Jones inclusions assignable to Kac-Moody representations.

Notice however that the non-unitary braiding R-matrix

$$\begin{pmatrix} q & & & \\ & 1 & q-q^{-1} & \\ & & 1 & \\ & & & q \end{pmatrix}$$

follows by a delicate limiting process from a unitary factorizing S-matrix at the limit $\eta_{12} \rightarrow \pm\infty$ as shown by Jimbo [40]. Could the Kac-Moody R-matrix follow by a limiting procedure from a unitary R-matrix by allowing an additional continuous parameter analogous to rapidity to approach some limit?

2. The invariance under isometries requires that central extension must vanish in center of mass degrees of freedom so that only finite-dimensional representations are possible.

3. Only a finite number of degrees of freedom are observable in the sense that they appear in the S-matrix between zero energy states and this requires $\mathcal{M} \rightarrow \mathcal{M}/\mathcal{N}$ reduction for Kac-Moody algebra leading to finite-dimensional Kac-Moody/quantum group representations.

9.2.5 What the reduction to braid group representations means physically?

One could choose also $M^1 \times S^2$ decomposition instead of $M^2 \times E^2$. $M^1 \times S^2$ option gives ordinary braid group representations as the limit $\eta_{ij} = 0$ meaning that the tips of light cones are at rest relative to each other. There is no convincing argument forbidding the braid group representations and they would be absolutely essential for topological quantum computation utilizing braiding S-matrices [E9].

For CP_2 the two options correspond to (S^1, S^2) and (S^2, S^1) decompositions and are equivalent and $SU(2)_L \times U(1)_R \subset SU(3)_L \times SU(3)_R$ reduction. A reduction to braid group representation occurs always in $U(1)_R$ factor and is accompanied by a similar reduction in electro-weak degrees of freedom.

The geodesic circle $S^1 \subset S^2$ with $\theta = \pi/2$ implies $(I_3+Y)/\sqrt{I(I+1)+Y^2} = 0$ meaning the absence of $I_3 + Y$ color rotation. The second color quantum number $I_3 - Y$ is represented by a geodesic circle $S^1_\perp$ orthogonal to S^2 and should vanish by the same argument. Quantum classical correspondence predicts that physical states correspond to $(I_3, Y) = (0, 0)$ states of color multiplets: the interpretation is as a weak form of color confinement. The vanishing of I_3 and Y implied by the weak form of color confinement means a reduction to

$U(1)_L \times U(1)_R \subset SU(3)_L \times SU(3)_R$ so that S-matrix reduces to a braiding S-matrix in both $S^1_\perp$ and S^2 factors and also for electro-weak sector.

9.2.6 The relationship with Jones inclusions

The factorization of S-matrix to four factorizing tensor factors suggest similar structure for Jones inclusions.

1. The four basic basic types of Jones inclusions

Four kinds of Jones inclusions can be assigned with the pairs (M^2, E^2) and (S^2, S^1). Same applies in case of (M^1, S^2) and (S^1, S^1) in TGD framework.

1. In $M^2 \times E^2$ case the discrete subgroups of $O(1,1)$ and $O(2)$ would characterize Jones inclusions. For E^2 only $G = A_n$ or D_{2n} are possible. For M^2 the subgroups generated by powers of Lorentz boost and reflection are possible. The infinite order for these groups strongly suggests $\beta = 4$. The quantum phase $q = exp(i\pi/n)$ would emerge naturally if the action of Lorentz boosts on configuration space spinor fields is unitary and reduces to a cyclic action represented by A_{n-1}. This would be very much analogous to the reduction of the quantum group representations to finite-dimensional ones as q becomes a root of unity.

2. $M^1 \times S^2$ option allows also $G = E_6, E_8$ (tetrahedral and icosahedral groups) and $SU(2)$.

3. For CP_2 all groups $G \subset SU(2)_L$ and $A_n \subset U(1)_R$ could define Jones inclusions. For color confined states only $G_L = A_{n_L}$ and $G_R = A_{n_R}$ are possible.

2. The type of braiding correlates with the type of Jones inclusion

Jones inclusions come in two very different types corresponding to $\beta < 4$ defined by the subgroups $G \subset SU(2)$ and $\beta = 4$ defined by $G = SU(2)$ or infinite subgroups of $SU(2)$. The two kinds of S-matrices could correspond to the two types of Jones inclusions as following arguments suggest.

1. Constant Yang-Baxter matrices defining braid group representations emerge as intertwiners of quantum versions of Lie algebras whereas more general Yang-Baxter matrices emerge as intertwiners for the representations of quantum versions of Kac Moody algebras [40]. Thus M^2 *resp.* S^2 would correspond to a representation of quantum Kac-Moody algebra whereas M^1 *resp.* S^1 would represent a degeneration to a purely topological braid group representation in the case of SU(2).

2. According to the arguments of [A9] $\beta < 4$ corresponds to quantum group representations characterized by finite sub-groups $G \subset SU(2)$ whereas $\beta = 4$ representations corresponds to Kac Moody representations with monodromies of n-point functions characterized by the quantum phase q.

It would seem that an equivalent characterization is as representations of quantum Kac Moody algebras.

3. Consistency with the TGD based explanation for McKay correspondence

These observations relate also interestingly to the proposal that TGD physics is universal in the sense of being able to mimic almost any physics obeying Kac Moody symmetry [A9].

1. McKay correspondence states that the finite subgroups $G \subset SU(2)$ characterizing $\beta < 4$ inclusions are labelled by ADE diagrams (A_n, D_{2n}, E_6 and E_8 are allowed). A concrete proposal was made for constructing the representations of the corresponding Kac-Moody algebras from these data by utilizing the new discrete degrees of freedom implied by the fact that space-time sheets define $n(G)$-fold coverings of M^4 (of CP_2 for $SU(2) \subset SL(2,C)$). The group algebra of G associated with multiple coverings of M^4 or CP_2 gave the multiplets.

   The degeneration of the S-matrix to braiding S-matrix does not kill this conjecture. The point is that $n \geq 3$ condition for quantum phase excludes the Jones inclusion corresponding to A_2 (two-element subgroup of SU(2)). It would be just the representation of SU(2) realized in terms of quantum spinors which would degenerate to the braid group representation whereas other representations for which spin like degrees of freedom are represented in terms of group algebra of G are not lost.

2. For $\beta = 4$ one obtains all extended ADE diagrams as characterizers of Jones inclusions, and an analogous construction of corresponding Kac Moody representations was proposed with quantum phase assigned with a non-trivial monodromy for n-point functions in S^2/G, S^2 a non-trivial geodesic sphere of CP_2. The natural identification would be as representations of quantum Kac-Moody algebra. All extended ADE diagrams are allowed which conforms with the fact that now SU(2) can be realized using quantum spinors. The representations of D_{2n+1} and E_7 should involve both quantum spinors and the $n(G)$-fold covering of S^2/G defining the monodromy.

3. These proposals do not seem so speculative when one realizes that the finite dimensional representations of quantum groups can be regarded also as representations of quantum Kac-Moody algebras [40]. As found, the generators in defining representations appear also as conserved charges in the quantum field theory models giving rise to factorizing S-matrices.

4. According to the construction for $\beta < 4$ the dimension of CP_2 projection of the partonic 2-surface can be smaller than two: this excludes homological non-triviality. For $\beta = 4$ CP_2 projection would be homologically non-trivial geodesic sphere. This is in harmony with the assumption that

geodesic circle S^1 and homologically non-trivial geodesic sphere S^2 characterize the sub-manifold of CP_2 to which the arguments of n-point functions belong for these representations.

9.2.7 Is factorizable QFT in M^2 associated with quantum criticality?

2-D QFT:s in M^2 are almost trivial and generalize topological QFT:s associated with braids. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2_{II} \subset M^4 \times CP_2$, where S^2_{II} is the homologically trivial geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2_{II}$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane.

This leads to the generalization of the imbedding space discussed in detail in [A9]. The generalized imbedding space is obtained by gluing together along $M^2 \times S^2_{II}$ the infinite collections of spaces consisting of $\hat{M}^4/G_a \times \hat{CP}_2/G_b$, $(\hat{M}^4 \hat{\times} G_a) \times (\hat{CP}_2 \hat{\times} G_b)$, $(\hat{M}^4/G_a) \times (\hat{CP}_2 \hat{\times} G_b)$, $(\hat{M}^4 \hat{\times} G_a) \times \hat{CP}_2/G_b$. Here $X \hat{\times} G$ *resp.* X/G mean singular G-covering *resp.* G-factor space of X, and G is discrete subgroup of $SO(3)$. For groups G_a with genuinely three-dimensional G_a-action (tedrahedral, octahedral, and icosahedral groups) M^2 is replaced with its image under G_a in the case of factor spaces. Hence the previous framework generalizes considerably by the allowance of both coset spaces and covering spaces.

Planck constant depends on the sector of generalized imbedding space and is ill-defined in $M^2 \times S^2_{II}$ which thus represents quantum critical sub-manifold and must be a vacuum extremal. TGD should reduce to a pure topological QFT for partons moving in this sub-manifold. Since partons are 2-dimensional, one would have essentially light-like geodesics as allowed solutions of field equations and thus classical theory of free massless particles. Hence factorizing QFT would be a natural description for the quantum critical dynamics at quantum criticality. This conforms also with the idea that intentional action takes place at quantum criticality.

This picture raises a question concerning the precise identification of the intersection points of number theoretical braids with $\delta M^4_\pm \times CP_2$. Should the points correspond to the intersections of the 2-D CP_2 projection of the partonic 2-surface in $\delta M^4_\pm \times CP_2$ with S^2_{II}? In the generic case the intersection would consist of discrete points and for non-vacuum extremals this would certainly be the case. Similar intersection makes sense also for $r_M = constant$ sphere of light-cone boundary. The intersection should consist of algebraic points: the condition that CP_2 projection is an algebraic surface is a necessary condition for this.

9.3 Are unitarity and Lorentz invariance consistent for the U-matrix constructed from factorizing S-matrices?

Factorizable M^2 S-matrices do not allow particle creation and the sets of initial and final state momenta are identical. The possibility to exchange internal quantum numbers possible in equal mass case could make possible momentum exchange in a very limited sense.

The extension to TGD framework brings in additional problems since the decomposition $M^4 = M^2 \times E^2$ breaks manifest Lorentz invariance. Also color invariance is broken. The question is how to achieve unitarity and Lorentz invariance simultaneously. The loss of these symmetries in case of U-matrix which characterizes universe rather than quantum state would be a catastrophe. This problem can be however circumvented.

9.3.1 Lorentz and color invariance are consistent with unitarity

U-matrix constructible using the proposed decomposition $M^2, E^2, S^2, S^1_{\perp}$ or its variant $(M^1, S^2), (S^1, S^1_{\perp})$ should be unitary. Unitarity is trivial to achieve if one just restricts to a given decomposition. Since Jones inclusions have a concrete effect on imbedding space geometry and topology, one could argue that this decomposition indeed reduces Lorentz symmetry to $SO(1,1) \times SO(2)$ and color symmetry to $U(2)$ or $U(1)$.

There is a way out of the problem. One can extend the U-matrix by introducing a complete orthogonal basis of wave functions in the projective sphere P^2 labelling the choices $M^4 = M^2 \times E2$ and in the space of geodesic spheres $S^2 \subset CP_2$. The extended U-matric is obtained by convoluting the factorizing S-matrix with this function basis. Completeness and orthonormalization of the basis reduce unitarity conditions for those of U-matrix for a fixed choice of (M^2, E^2) and $(S^2, S^1_{\perp})$ pairs.

9.3.2 Is the momentum exchange in transversal degrees of freedom possible?

The basic objection against the factorizing S-matrices as basic building blocks of S-matrix (as opposed to U-matrix) is that scattering leaves the set of M^2 and E^2 projections of 4-momenta invariant. Also the conservation for the number of light cones is questionable aspect. In the case of U-matrix these properties are well-come. The natural question is whether it is possible to circumvent this objection in 4-D case at least partially. The following arguments suggest that this is not the case.

1. The basic aspect of 2-D scattering matrices is that in equal mass case the exchange of the internal quantum numbers is possible in the pass-by process [40]. This applies to both M^2 and E^2 scattering. The exchange can occur if the longitudinal *resp.* transversal masses of the particles are same. Transversal (longitudinal) momenta are internal quantum numbers from the point of view of M^2 (E^2) scattering so that their exchange is possible.

2. The problem is that elementary particles should have same mass and identical longitudinal (transversal) momenta meaning that this kind of

situation is non-generic so that this mechanism cannot contribute to the scattering.

3. The particles are in the recent case typically particle groups associated with light cones so that the mass squared for them is continuous. By performing a Lorentz boost for M^2 in a suitable direction it is possible to achieve a situation in which the longitudinal masses of given two particles are same since their transversal masses can be different. For this choice of M^2 the exchange of transverse momenta is possible in the pass-by process for a given particle pair. Also same value of transverse masses for a given particle pair can be achieved by a suitable Lorentz boost for M_2 so that exchange of longitudinal momenta becomes possible.

4. The problem with this argument is that it applies to a single particle pair only and one would like to have all possible exchanges of transversal momenta. Physical intuition also suggests that the exchanged momentum must be distributed between the particles inside the light-cone and it is not clear whether U-matrix in configuration space degrees of freedom can characterize this distribution.

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