

Overall View About Evolution of TGD

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Contents

1	Introduction	5
2	Evolution of classical TGD	8
2.1	Quantum classical correspondence and why classical TGD is so important?	9
2.2	Classical fields	10
2.2.1	Geometrization of classical fields and of quantum numbers	10
2.2.2	The new physics associated with classical gauge fields	12
2.3	Many-sheeted space-time concept	13
2.3.1	Join along boundaries contacts and join along boundaries condensate	15
2.3.2	Wormhole contacts	16
2.3.3	Topological field quantization	20
2.3.4	Negative energy space-time sheets and new view about energy	22
2.3.5	More precise view about topological condensate	23
2.4	Classical non-determinism of Kähler action	24
2.4.1	Matter-mind duality geometrically	24
2.4.2	Association sequence concept and a mind like space-time sheets	26
2.4.3	Vacuum degeneracy and spin glass analogy	26
2.4.4	Connection with catastrophe theory and Haken's theory of self-organization for spin glass	28
2.5	Quantum classical correspondence as an interpretational guide	29

2.5.1	The implications deriving from the topology of space-time surface and from the properties of induced gauge fields	29
3	Evolution of p-adic ideas	30
3.1	p-Adic numbers	30
3.2	Evolution of physical ideas	32
3.2.1	p-Adic length scale hypothesis	32
3.2.2	CP_2 type extremals and elementary particle black hole analogy	34
3.2.3	p-Adic thermodynamics and particle massivation . . .	35
3.2.4	p-Adic coupling constant evolution	36
3.2.5	Vacuum degeneracy of the Kähler action and spin glass analogy	36
3.3	Evolution of mathematical ideas	39
3.3.1	p-Adic physics as physics of cognition and intentionality and generalization of number concept	39
3.3.2	Algebraic continuation as a basic principle	42
3.4	Generalized Quantum Mechanics	44
3.4.1	Quantum mechanics in H_F as a algebraic continuation of quantum mechanics in H_Q	45
3.4.2	Could U_F describe dispersion from H_Q to the spaces H_F ?	46
3.5	Do state function reduction and state-preparation have number theoretical origin?	47
3.5.1	Negentropy Maximization Principle as variational principle of cognition	48
3.5.2	NMP and number theory	49
4	The boost from TGD inspired theory of consciousness	50
4.1	The anatomy of the quantum jump	51
4.1.1	The three non-determinisms	52
4.1.2	How psychological time and its arrow emerge?	53
4.1.3	Macro-temporal quantum coherence and spin glass degeneracy	55
4.2	Negentropy Maximization Principle and new information measures	57
4.2.1	p-Adic entropies	58
4.2.2	Number theoretic entropies and bound states	59

4.2.3	Number theoretic information measures at the space-time level	60
5	TGD as a generalized number theory	60
5.1	The painting is the landscape	61
5.2	p-Adic physics as physics of cognition	62
5.2.1	Real and p-adic regions of the space-time as geometric correlates of matter and mind	62
5.2.2	The generalization of the notion of number and p-adicization program	63
5.3	Space-time-surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?	64
5.3.1	Transition from string models to TGD as replacement of real/complex numbers with quaternions/octonions	65
5.3.2	Physics as a generalized algebraic number theory and Universe as algebraic hologram	67
5.4	Infinite primes and physics in TGD Universe	67
5.4.1	Infinite primes and infinite hierarchy of second quantizations	68
5.4.2	Infinite primes as a bridge between quantum and classical	68
5.4.3	Various equivalent characterizations of space-times as surfaces	69
5.4.4	Infinite primes and quantum gravitational holography	69
5.5	Infinite primes and more precise view about p-adic length scale hypothesis	70
5.5.1	How to define the notion of elementary particle?	70
5.5.2	What effective p-adic topology really means?	72
5.5.3	Do infinite primes code for q-adic effective space-time topologies?	73
5.5.4	Under what conditions space-time sheets can be connected by $\#_B$ contact?	75
5.5.5	What about the integer characterizing graviton?	76
5.6	Infinite primes, cognition and intentionality	77
5.6.1	Infinite primes very briefly	78
5.6.2	Precise space-time correlates of cognition and intention	79
5.7	Complete algebraic, topological, and dimensional democracy?	81

Abstract

This chapter provides a bird's eye view about TGD in its 27th birthday with the hope that this kind of summary might make it easier to follow the more technical representation provided by sub-sequent chapters. The geometrization of fundamental interactions assuming that space-times are representable as 4-surfaces of $H = M_+^4 \times CP_2$ is wherefrom everything began. The two manners to understand TGD is TGD as a Poincare invariant theory of gravitation obtained by fusing special and general relativities, and TGD as a generalization of string model obtained by replacing 1-dimensional strings with 3-surfaces. The fusion of these approaches leads to the notion of the many-sheeted space-time.

The evolution of quantum TGD involve five threads which have become more and more entangled with each other. The first great vision was the reduction of the entire quantum physics (apart from quantum jump) to the geometry of classical spinor fields of the infinite-dimensional space of 3-surfaces in H , the great idea being that infinite-dimensional Kähler geometric existence and thus physics is unique from the requirement that it is free of infinities. The outcome is geometrization and generalization of the known structures of the quantum field theory and of string models.

The second thread is p-adic physics. p-Adic physics was initiated by more or less accidental observations about reduction of basic mass scale ratios to the ratios of square roots of Mersenne primes and leading to the p-adic thermodynamics explaining elementary particle mass scales and masses with an unexpected success. p-Adic physics turned eventually to be the physics of cognition and intentionality. Consciousness theory based ideas have led to a generalization of the notion of number obtained by gluing real numbers and various p-adic number fields along common rationals to a more general structure and implies that many-sheeted space-time contains also p-adic space-time sheets serving as space-time correlates of cognition and intentionality. The hypothesis that real and p-adic physics can be regarded as algebraic continuation of rational number based physics provides extremely strong constraints on the general structure of quantum TGD.

TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory replacing the notion of observer as an outsider with the notion of self. The detailed analysis of what happens in quantum jump have brought considerable understanding about the basic structure of quantum TGD itself. It seems that even quantum jump itself could be seen as a number theoretical necessity in the sense that state function reduction and state preparation by self measurements are necessary in order to reduce the generalized quantum state which is a formal superposition over components in different number fields to a state which contains only rational

or finitely-extended rational entanglement identifiable as bound state entanglement. The number theoretical information measures generalizing Shannon entropy (always non-negative) are one of the important outcomes of consciousness theory combined with p-adic physics.

Physics as a generalized number theory is the fourth thread. The key idea is that the notion of divisibility could make sense also for literally infinite numbers and perhaps make them useful from the point of view of physicist. The great surprise was that the construction of infinite primes corresponds to the repeated quantization of a supersymmetric arithmetic quantum field theory. This led to the vision about physics as a generalized number theory involving infinite primes, integers, rationals and reals, as well as their quaternionic and octonionic counterparts. A further generalization is based on the generalization of the number concept already mentioned. Space-time surfaces could be regarded in this framework as concrete representations for infinite primes and integers, whereas the dimensions 8 and 4 for imbedding space and space-time surface could be seen as reflecting the dimensions of octonions and quaternions and their hyper counterparts obtained by multiplying imaginary units by $\sqrt{-1}$. Also the dimension 2 emerges naturally as the maximal dimension of commutative sub-number field and relates to the ordinary conformal invariance central also for string models.

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

This chapter represents a overall view about evolution of classical TGD and of p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, the vision about physics as a generalized number theory.

1 Introduction

Topological Geometroynamics was born for twenty five years ago as an attempt to construct a Poincare invariant theory of gravitation by assuming that physically allowed space-times are representable as surfaces in the

space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 is complex projective space having real dimension four (see the appendix of the book). Poincare group was identified as the isometry group of M^4 rather than of the space-time surface itself. The isometries of CP_2 were identified as color group and the geometrization of electro-weak gauge fields and elementary particle quantum numbers was achieved in terms of the spinor structure of CP_2 . Rather remarkably, for a quarter century after this discovery one can still say that CP_2 codes the known elementary particle quantum numbers and interactions in its geometry. The construction of quantum theory suggests the replacement of M^4 with M^4_+ , the interior of the future light cone of Minkowski space so that Poincare invariance is broken by the global geometry of the light cone but not locally.

It took almost half decade to develop the new view about space-time implied by the basic hypothesis: this is summarized in my PhD thesis [16]. The construction of a mathematical theory around these physically very attractive ideas became the basic challenge and I have devoted my professional life to the realization of this dream. The great idea was that quantum physics reduces to the construction of Kähler metric and spinor structure for the infinite-dimensional space CH of all possible 3-surfaces of H . Physical states correspond to classical spinor fields in this space and a natural geometrization of fermionic statistics in terms of gamma matrices emerges [B1, B2, B3].

p-Adic number fields R_p [18] (one number field for each prime obtained as a completion of the rational numbers) emerged for about ten years ago as a separate thread only loosely related to quantum TGD. What made them so attractive was that, with certain additional assumptions about physically favored p-adic primes, it became possible to understand the basic elementary particle mass scales number theoretically. This led to a successful calculation of the elementary particle masses using p-adic thermodynamics assuming that Super Virasoro algebra and related Kac Moody algebras, which are also basic algebraic structures of string models, act as symmetries of TGD [F2, F3, F4, F5]. The success of the mass calculations in turn forced the attempts to understand how Super Virasoro and related symmetries might emerge from basic TGD. Several trials led finally to the realization that these super algebras (or actually the proper generalizations of them) are the basic symmetries of quantum TGD. One of the most dramatic predictions is the uniqueness of the space H : quantum TGD exists mathematically (cancellation of various infinities occurs) only for the space $M^4_+ \times CP_2$, the choice which is forced also by the cosmological and symmetry considerations. One can say that infinite-dimensional Kähler geometric existence and thus

physics is unique.

A third thread to the development emerged when I started systematic development of TGD inspired theory of consciousness [TGDconsc]. This work has led to dramatic increase of understanding also at the level of basic quantum TGD and allowed to develop quantum measurement theory in which conscious observer is not anymore Cartesian outsider but an essential part of quantum physics. The need to understand the mechanism making bio-systems macroscopic quantum systems led to a dramatic progress in the understanding of the new physics implied by the notion of many-sheeted space-time. Dramatic change in views about the relation between subjectively experienced and geometric time of physicist emerges and leads to the solution of the basic paradoxes of quantum physics. It became also clear that p-adic numbers are indeed an absolutely essential element of the mathematical formulation of quantum TGD proper and that the general properties of quantum TGD force the introduction of the p-adic numbers. One can say that physics involves both real and p-adic number fields with real numbers describing the topology of the real world and various p-adic number fields serving as correlates of cognition with the prime p labelling the p-adic topology serving as kind of intelligence quotient.

A further thread into the development of ideas came from the realization that physics might be basically number theory in generalized sense. TGD more or less forces the notion of infinite primes [E3], and it turned out that their construction reduces to a repeated second quantization of arithmetic quantum field theory. Generalization of the concept of integer and real number emerges implying that the configuration space and state space of TGD could be imbedded into the field of generalized reals which is infinite-dimensional algebraic extension of ordinary reals. Physics could be basically theory of generalized reals! The dimensions of space-time *resp.* imbedding space correspond to the dimensions of quaternion *resp.* octonion fields as well as the dimensions of algebraic extensions of $p > 2$ - *resp.* 2-adics allowing square root of ordinary p-adic number. The discussions with Tony Smith suggested that one can endow space-time and imbedding space with what might be called local quaternion and octonion structures.

This stimulated a development, which led to the notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [E2]. What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multi-

plied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

Further important number theoretical ideas emerged from the attempt to construct a model for how intentions are transformed to actions. The process was interpreted as a quantum jump in which p-adic space-time sheet representing intention is transformed to a real one. This model led to a bundle of ideas and conjectures.

a) The core idea is the generalization of the notion of number obtained by gluing all number number fields together along rationals and algebraic numbers common to them. This means a generalization of the notion of manifold. In particular, imbedding space is obtained by gluing real and p-adic imbedding spaces together along rational points. This picture also justifies the decomposition of space-time surface to real and p-adic space-time sheets. Also finite-dimensional algebraic extensions, even extensions involving transcendentals like e are needed.

b) p-Adic space-time sheets are identified as correlates of intentionality and cognition. The differences between real and p-adic topologies (two rationals near to each other as p-adic numbers are very far in real sense) have deep implications concerning the understanding of cognitive consciousness. The evolution of cognition corresponds naturally to the increase of the p-adic prime and dimension of the extension of p-adic numbers.

c) Real physics and various p-adic physics are obtained from finitely extended rational physics by algebraic continuation to p-adic number fields and their extensions analogous to analytic continuation in complex analysis. This algebraic continuation is performed both at space-time level, state space level, and configuration space level. One can also generalize the notion of unitarity and the generalization poses extremely strong conditions on S-matrix.

This chapter represents a overall view of classical TGD, a discussion of the p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, and the vision about physics as generalized number theory.

2 Evolution of classical TGD

The TGD based space-time concept means a radical generalization of standard views already in the real context. Many-sheetedness means a hierarchy of space-time sheets of increasing size making possible to understand the emergence of structures in terms of the macroscopic space-time topology.

The non-determinism of the Kähler action forces the notion of the association sequence defined as a union of space-like 3-surfaces with time-like separations: association sequence provides a geometric correlate for thought as simulation of the classical history. Non-determinism forces also the notion of mind like space-time sheet defined as a space-time sheet having finite temporal duration, which is an attractive candidate for the geometric correlate of self. Topological field quantization means that space-time topology provides classical correlates for the basic notions of the quantum field theory. The decomposition of space-time surface into real and p-adic regions brings in besides the matter also cognitive representations of material world.

2.1 Quantum classical correspondence and why classical TGD is so important?

In standard quantum physics classical theory is seen as a result of some kind of approximation procedure, say stationary phase approximation. In TGD framework classical physics is an exact part of quantum physics, and even more of configuration space geometry since, apart from the complications caused by the classical non-determinism of the Kähler action, the definition of the Kähler geometry in terms of Kähler action assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$.

The evolution of TGD inspired theory of consciousness has gradually led to the notion of quantum classical correspondence which states that every quantum aspect of existence has space-time correlate. The correspondence is certainly not faithful but rather like the representation of contents of consciousness provided by spoken or written language. Space-time surface can be indeed seen as a symbolic representation, kind of written language. Not only the characteristics of quantum states, but also quantum jumps and their sequences defining the contents of conscious experience, have space-time correlates made possible by the classical determinism of the Kähler action, and the inherent p-adic non-determinism of p-adic counterparts of the field equations. In fact, there are reasons to believe that classical non-determinism of the Kähler action and a p-adic non-determinism have close relationship in the sense that the effective topology of the real space-time sheets is expected to correspond to p-adic topology in some length scale range.

2.2 Classical fields

In TGD framework the physics of classical fields are an essential part of the quantum theory and the study of classical fields has provided the easiest manner to get grasp about the physics of TGD Universe.

2.2.1 Geometrization of classical fields and of quantum numbers

The basic motivation for TGD was provided by the finding that known interactions at classical level and quantum number spectrum of known particles could be readily understood from the assumption that space-time is a 4-surface in $H = M^4 \times CP_2$.

The geometrization of classical gauge fields is based on the following identifications.

a) The classical gravitational field is identified as the induced metric. The still open question is whether the classical gravitational fields couple to matter with the gravitational constant $G \simeq kR^2$, $k \simeq 10^{-8}$, where R is CP_2 size (the length of CP_2 geodesic line). There is however an argument leading to a precise and correct prediction for k , and fixing the value of the Kähler coupling strength α_K at electron length scale to a value rather near to that of the fine structure constant.

b) The geometrization of electro-weak gauge fields reduces to the curvature of CP_2 just like the geometrization of gravitation reduces to the curvature of the space-time surface. Classical electro-weak fields are identified as components of CP_2 spinor connection projected to the space-time surface. The holonomy group of CP_2 spinor connection is $U(2)$ and naturally identifiable as electro-weak gauge group.

c) Color symmetries correspond to the isometries of CP_2 so that there is deep and unexpected connection between electro-weak and color interactions. Color gauge potentials are identified in the spirit of Kaluza-Klein theory as projections of the Killing vector fields of color isometries to the space-time surface. Color gauge fields are of form $F_{\alpha\beta}^A \propto H^A \times J_{\alpha\beta}$, where H^A is the Hamiltonian of the color isometry and J denotes the induced Kähler form. Therefore the vacuum extremals of Kähler action carry also non-vanishing color gauge fields.

Also elementary particle quantum numbers can be understood in terms of the induced spinor structure and simple 3-topology.

a) CP_2 does not allow ordinary spinor structure and it is necessary to couple CP_2 spinors to the Kähler potential of CP_2 . The couplings are different for different H -chiralities identifiable as leptonic and quark like spinors.

Baryon and lepton numbers are separately conserved for both the ordinary massless Dirac action and modified Dirac action. The modified Dirac action is fixed uniquely by requiring that it has the vacuum degeneracy of Kähler action. The modified Dirac action allows local super-symmetries generated by the right-handed neutrino.

b) At the fundamental level color quantum numbers are not spin like quantum numbers but can be said to correspond to the color partial waves in CP_2 center of mass degrees of freedom of the 3-surface representing the elementary particle. Ordinary Dirac equation for CP_2 predicts wrong correlations between electro-weak and color quantum numbers of the color partial waves associated with the spinor harmonics. This was a longstanding problem of TGD approach but the construction of physical states as representations of the Super Kac Moody algebra allows to obtain correct correlations and an interpretation in terms of electro-weak symmetry breaking coded already into the CP_2 geometry.

c) The first guess was that the genus of the two-dimensional boundary associated with the 3-surface representing particle explains family replication phenomenon. The identification of the super-conformal symmetries as symmetries associated with light like effectively 2-dimensional 3-surfaces X_l^3 acting as causal determinants suggests a more concrete identification.

Quaternion conformal invariance allows to assign to X_l^3 a highly unique 2-dimensional surface X^2 as a surface at which superconformal structure reduces to ordinary conformal structure and thus becomes Abelian. The genus of this surface telling whether the surface is sphere, torus, etc... determines the particle family. X_l^3 could correspond to either a boundary of 3-surface or to an elementary particle horizon. Elementary particle horizon would surround the wormhole contact connecting CP_2 extremal with an Euclidian signature of the induced metric to a larger space-time sheet with a Minkowskian signature of metric. The induced metric is degenerate at the elementary particle horizon so that this surface is indeed metrically 2-dimensional.

More concretely, sphere, torus, and sphere with two handles would correspond to (e, ν_e) , (μ, ν_μ) , (τ, ν_τ) in the leptonic sector and (u, d) , (c, s) , and (t, b) in the quark sector respectively. The experimental absence of heavier particle families would be most naturally due to the fact that they are extremely heavy. The 3 lowest particle families differ from the higher genera in the sense that 2-surfaces with genus $g < 3$ are always hyper-elliptic, that is they allow always Z_2 conformal symmetry, whereas higher genera generically do not allow any conformal symmetries. Hyper-ellipticity is an excellent candidate for an explanation of the lightness of $g < 3$ genera. The

construction of elementary particle functionals as functionals in the conformal equivalence classes of the 2-surface X^2 associated with X_l^3 allows to formulate this argument more precisely.

The explanation of Cabibbo mixing as being due to the mixing of boundary topologies, and number theoretic arguments (complex rationality of CKM matrix) lead to a highly unique CKM matrix for quarks and also leptonic mixings can be fixed highly uniquely. Also bosons are predicted to possess family replication phenomenon.

2.2.2 The new physics associated with classical gauge fields

Long range electro-weak, in particular Z^0 , vacuum gauge fields are unavoidable in TGD: this is a necessary outcome of the induced gauge field concept reducing the number of the primary bosonic field variables to four (CP_2 coordinates)! The interpretation of this puzzling prediction has been a long standing challenge of TGD. There are three alternative options to consider.

Option I: Classical gauge fields are space-time correlates for gauge bosons with mass scale determined by the p-adic length scale of the space-time sheet in question. The electro-weak charges of elementary particles are screened by vacuum gauge charges (possible in TGD) in a region of size L_W of order intermediate boson length scale. This option does not explain the presence of long range electro-weak gauge fields unavoidably present if the dimension of CP_2 projection of space-time sheet is higher than 2 nor classical color gauge fields present for non-vacuum extremals.

Option II: Electro-weak gauge charges are not screened in the length scale L_W and the gauge fluxes of elementary particles flow to larger space-time sheets via # throats within region of size L_W and elementary particles have the quantized values of em Z^0 charges. The problem for this option are anomalously large Rutherford cross sections in condensed matter and large parity breaking effects in hadronic, nuclear, and atomic length scales. Despite this I regarded this option as the most realistic one until the realization that the mysterious long ranged weak fields could be assigned to dark matter particles at various space-time sheets.

Option III: There is a hierarchy of color electro-weak physics such that weak bosons are massless below the p-adic length scale determining the mass scale of weak bosons. Classical long range gauge fields serve as space-time correlates for gauge bosons below the p-adic length scale in question.

The unavoidable long ranged electro-weak and color gauge fields are created by dark matter and dark particles can screen dark nuclear electro-weak charges below the weak scale above which vacuum screening occurs as

for ordinary weak interactions. Dark gauge bosons are massless below the appropriate p-adic length scale but massive above it and $U(2)_{ew}$ is broken only in the fermionic sector. For dark copies of ordinary fermions masses are essentially identical with those of ordinary fermions.

This option is consistent with the standard elementary particle physics for visible matter apart from predictions such as the possibility of p-adically scaled up versions of ordinary quarks predicted to appear already in ordinary low energy hadron physics. The most interesting implications are seen in longer length scales. Dark quarks and gluons and a scaled up copy of ordinary gluons emerge already in ordinary nuclear physics [F8] and explain some recently discovered anomalies such as neutron halos and tetra-neutron. The field bodies associated with are predicted to have sizes of order atom size. Also scaled down versions of weak bosons giving to interactions between exotic quarks with a range of order atomic length scale are predicted.

The new nuclear physics has deep implications for chemistry and condensed matter where color bonds between neighboring atoms might be part of the chemical bonding [F9]. Long ranged repulsive weak force behind exotic quarks compensated by color force would contribute to the repulsive force assumed in van der Waals equations of state for condensed matter. No strong isotopic dependence is predicted.

Classical long range weak and color forces become also key players at the level of molecular physics and biophysics. Chiral selection of bio-molecules can be seen as one direct signature of the long ranged weak force which suggests that non-broken $U(2)_{ew}$ symmetry and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. The central role of the long ranged weak forces in bio-systems and in pre-biotic evolution is discussed in [9, M3, 11].

Classical em fields and Z^0 fields are not invariant under color rotations acting as exact symmetries and are accompanied by classical color gauge fields. This implies new physics potentially important for TGD inspired theory of consciousness. For instance, in TGD Universe the original joke like term "quark color" inspired by certain algebraic similarities ceases to be a joke since it is possible to reduce the 3+3 primary colors in color vision to the 3+3 different increments of color quantum numbers induced by the absorption or emission of color octet gluon.

2.3 Many-sheeted space-time concept

The detailed study of TGD led to a further generalization of the space-time concept and the end result is what I have used to call topological

condensate or many-sheeted space-time. The 3-space is many-sheeted such that the sheets of 3-space have finite size and outer boundary. The physical interpretation of a given space-time sheet of a finite size is as a 'particle'. Depending on their size, these particles correspond to elementary particles, nucleons, atomic nuclei, atoms, molecules, cells, ourselves, stars, galaxies, etc. For instance, my skin corresponds to the outer boundary of a 3-surface glued to a larger 3-surface identifiable as the room in which I sit! I am a small Universe glued to a larger one, the 3-space associated with me literally ends on my skin just as string ends at its end! The surface of earth, the outer surfaces of trees, etc...: everywhere I can see nontrivial 3-topology.

Important new physics is associated with the extremely tiny wormholes contacts with size of order CP_2 length needed to perform the gluing operation. Join along boundaries bonds serving as space-time correlates for the bound state formation is second important notion. The larger sheets of the many-sheeted space-time are ideal for carrying various macroscopic quantum phases. Topological field quantization allows to define precisely the notions of coherence and de-coherence and also means that one can assign to a given material system what might be called field body or magnetic body.

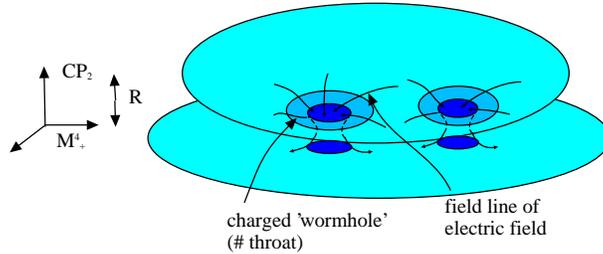


Figure 1: Charged wormholes feed the electromagnetic gauge flux to the 'lower' space-time sheet.

Obviously the outcome is a thorough-going generalization of the space-time concept and means that TGD has highly nontrivial consequences in all length scales rather than in particle physics only, as one might naively expect.

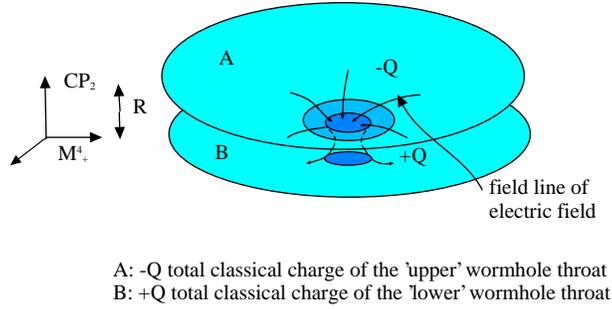


Figure 2: The two throats of wormhole behave as classical charges of opposite sign.

2.3.1 Join along boundaries contacts and join along boundaries condensate

The recipe for constructing TGD:eish 3-space is simple. Take 3-surfaces with boundaries, glue them by topological sum to larger 3-surfaces, glue these 3-surfaces in turn on even larger 3-surfaces, etc.. The smallest 3-surfaces correspond to CP_2 type extremals that is elementary particles and they are at the top of hierarchy. In this manner You get quarks, hadrons, nuclei, atoms, molecules,... cells, organs, ..., stars, ...,galaxies, etc...

Besides this, one can also glue different 3-surfaces together by tubes connecting their *boundaries*: this is just connected sum operation for boundaries. Take disks D^2 on the boundaries of two objects and connect these disks by cylinder $D^2 \times D^1$ having D^2 :s as its ends. Or more concretely: let the two 3-surfaces just touch each other.

Depending on the scale join along boundaries bonds are identified as color flux tubes connecting quarks, bonds giving rise to strong binding between nucleons inside nuclei, bonds connecting neutrons inside neutron star, chemical bonds between atoms and molecules, gap junctions connecting cells, the bond which is formed when You touch table with Your finger, etc.

One can construct from a group of nearby disjoint 3-surfaces so called join along boundaries condensate by allowing them to touch each other here and there.

The formation of join along boundaries condensates creates clearly strong

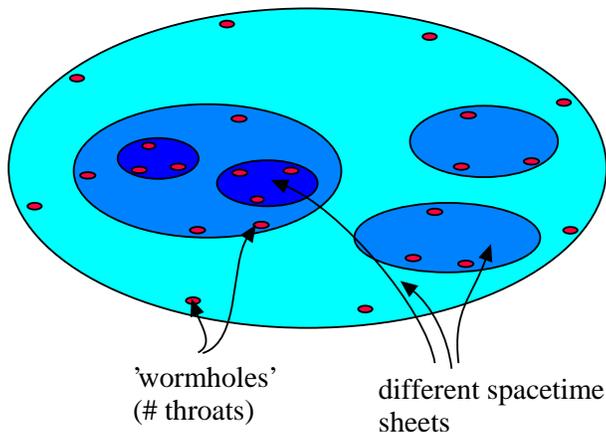


Figure 3: Many-sheeted space-time structure results from the requirement of gauge flux conservation.

correlation between two quantum systems and it is assumed that the formation of join along boundaries condensate is necessary prerequisite for the formation of *macroscopic quantum systems*. Crucially important examples in biology are gap junctions connecting cells and MAPs (micro-tubule associated proteins) connecting micro-tubules.

Quantum classical correspondence inspires the hypothesis that quite generally join along boundaries bonds are space-time correlates for the formation of the bound state entanglement. Since join along boundaries bonds between space-time sheets condensed on larger space-time sheets having no join along boundaries bonds between them is possible, one is forced to conclude that entanglement between sub-systems of un-entangled systems is possible in the many-sheeted space-time. The paradox disappears when entanglement is understood as a length scale dependent notion so that the bound state entanglement of sub-systems is not visible in the length and time scales of the systems.

2.3.2 Wormhole contacts

The gauge and gravitational fluxes at the boundary of a given space-time sheet must go somewhere by gauge flux conservation. This forces the existence of a larger space-time sheet and of tiny wormhole contacts connecting the two space-time sheets and feeding the gauge fluxes from the smaller sheet

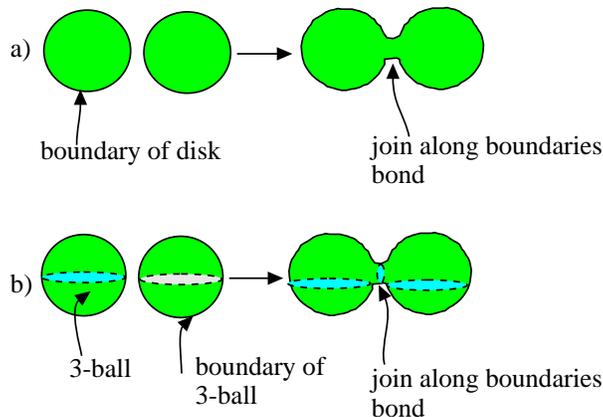


Figure 4: Join along boundaries bond a): in two dimensions and b): in 3-dimensions for solid balls.

to the larger one. Wormhole contacts ($\#$ contacts) are elementary particle like objects (actually deformed pieces of so called CP_2 type extremals) having size of order CP_2 size about 10^4 Planck lengths and, being sources and sinks of gauge field lines, wormhole throats effectively like classical charges, the charges of throats at the two space-time sheets being of opposite sign. Hence wormhole contacts look like dipoles and couple to the difference of the classical gauge potentials associated with the two space-time sheets. Also the coupling to the difference of the gauge potentials serving as order parameters for the coherent states of photons is possible.

The crucial experiment would be the one demonstrating the existence of the wormholes.

a) There are good reasons to expect that wormhole gauge flux is quantized. The reason for quantization would be the absolute minimization of Kähler action (or its variant discussed in [E2]), which is mathematically a condition very similar to the Bohr's quantization condition. In the usual initial value problem one would fix only the imbedding space coordinates of 4-surface for given value of time and allow their time derivatives be arbitrary. Now absolute minimization fixes the values of the time derivatives just like Bohr's quantization rules fix the momenta. The most aesthetic possibility is that the unit of wormhole em charge is the smallest possible elementary particle charge $1/3$ associated with d quarks but also integer charge could be considered.

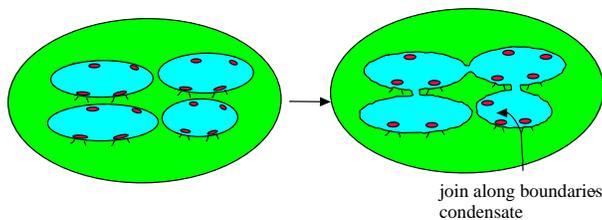


Figure 5: Join along boundaries condensate in 2 dimensions.

b) If wormhole charge is quantized then the gauge flux of an external em field running from a larger space-time sheet to a smaller one is quantized. The experimental arrangement should demonstrate that this flux indeed can change by a multiple of the elementary flux only. One could also try to detect wormhole currents. It must be emphasized that wormhole current is a pseudo current in the sense that two space-time sheets carry opposite classical currents. These currents are created, when magnetic field penetrates from space-time sheet to another. The detection of charge $1/3$ for the charge carriers of this current would be a triumph.

c) One cannot exclude the possibility that the recently found evidence for $1/3$ charge in condensed matter systems (quantum Hall effect) could be interpreted in terms of an em gauge flux quantized in this manner. Electron current flowing inside a planar layer like structure is studied. Strong magnetic field, which could lead to a generation of wormhole currents is present! Evidence for some quasi particles in current flow possessing this charge has been found. The anyon interpretation of quasi particles as bound states of

magnetic flux quanta and electrons explains the effect (McLaughlin wave function). The prediction is however that also fluxes of $m/5$, $m/7, \dots$, m integer, should be observed. Only $1/3$ has been detected hitherto and it is not understood why higher charges have not been observed. The question is whether the quasi particles are actually wormholes created by the penetration of magnetic field and flowing along the boundaries of the arrangement.

One application of the new space-time concept is a model of brain. The basic idea is that brain can be regarded as a macroscopic quantum system and that our experiences of free will correspond to quantum jumps which are unpredictable as also is the end result of a free choice. The idea that quantum theory might provide some light in the problem of consciousness has become popular during the last years and a serious building of quantum theories of consciousness has begun. The bottleneck problem is how the brain can be a macroscopic quantum system. Some kind of super conductivity looks a promising idea but standard physics does not provide promising candidates for a super conductor like system. Wormholes might provide one such system besides high T_c electronic and protonic superconductors and Bose-Einstein condensates of bosonic ions.

To see what is involved, consider in more precise manner how many sheeted 3-space is constructed. When one glues a sheet of 3-space to a larger sheet of 3-space one does it by constructing extremely tiny elementary particle sized wormholes connecting the two sheets of 3-space.

These wormholes serve important function. For instance, the flux of the electric field (usually it is unlucky space traveller) flows to this kind of wormhole on the smaller sheet of 3-space and comes back from it to the larger sheet of 3-space. Since the field lines of the electric field flow to the wormhole on the smaller sheet of 3-space, the wormhole looks like a charge since it acts as a sink of field lines. Same applies on the larger sheet of 3-space except that the sign of the charge is opposite. Hence, on both space-time sheets wormhole looks classically like a charged particle. Shortly, wormholes behave like particles and represent a new exotic form of matter. More generally, it seems that many-sheeted nature of the space-time is crucial for the understanding of a bio-system as a macroscopic quantum system.

The interaction between space-time sheets is mediated by these wormholes having size of order CP_2 radius R and located near the boundaries of the smaller space-time sheet. Wormholes feed various gauge fluxes from the smaller space-time sheet to the larger one (say from the atomic sheet to some molecular sheet). p -Adic considerations suggest that wormholes are light having mass of order $1/L_p$: this implies that they suffer Bose-Einstein

condensation on the ground state. One could even say that space-time sheets "perceive" the external world and act on it with the help of the charged wormhole BE condensates near their boundaries. Wormholes provide a very general mechanism making possible the transfer of classical electromagnetic fields and various quantum numbers such as energy, momentum and angular momentum, between different space-time sheets and bio-systems are especially promising as far as applications are considered.

2.3.3 Topological field quantization

Topological field quantization [D7] implies that various notions of quantum field theory have rather precise classical analogies. Topological field quantization is basically implied by the compactness of CP_2 , which typically implies that a given Maxwell field allows only a partial imbedding as a space-time surface in H . One can say that magnetic fields, electric fields and radiation fields decompose into field quanta.

The energies and other classical charges of the topological field quanta are quantized by the absolute minimization of the Kähler action making classical space-time surfaces the counterparts of the Bohr orbits. Feynman diagrams become classical space-time surfaces with lines thickened to 4-manifolds. For instance, "massless extremals" representing topologically quantized classical radiation fields are the classical counterparts of gravitons and photons. Topologically quantized non-radiative nearby fields give rise to various geometric structures such as magnetic and electric flux tubes.

Topological field quantization provides the correspondence between the abstract Fock space description of elementary particles and the description of the elementary particles as concrete geometric objects detected in the laboratory. In standard quantum field theory this kind of correspondence is lacking since classical fields are regarded as a phenomenological concept only.

Topological field quanta define coherence regions for the classical gauge fields and induced spinor fields and classical coherence is the prerequisite of the quantum coherence. Whether and how macroscopic and macro-temporal quantum coherence are possible in living matter is the basic question of quantum consciousness theories and quantum biology. In TGD this question is even more difficult since the first estimate for de-coherence time is CP_2 time which is about 10^4 Planck times. The length scale hierarchy of space-time sheets allows immediately to understand at the level of space-time correlates how macroscopic and macro-temporal quantum coherence are possible. A good order of magnitude guess for the zero point energy

of a particle at a space-time sheet of size L is given by $E = \pi^2/2mL^2$. $T \leq \pi^2/2mL^2$ gives an estimate for the temperature of the space-time sheet populated by particles of mass m : the larger the size of the space-time sheet, the lower the temperature. Superconductivity and various macroscopic phenomena become thus possible at larger space-time sheets. TGD based model of living matter is based on the hypothesis that large space-time sheets are responsible for quantum control.

The virtual particles of quantum field theory have also classical counterparts. In particular, the virtual particles of quantum field theory can have negative energies: this is true also for the TGD counterparts of the virtual particles. The fundamental difference between TGD and GRT is that in TGD the sign of energy depends on the time orientation of the space-time sheet: this is due to the fact that in TGD energy current is vector field rather than part of tensor field. Therefore space-time sheets with negative energies are possible.

One can criticize the notion of time orientation. A more precise definition of the time orientation requires the realization that configuration space of 3-surfaces, call it CH , can be understood as a union of corresponding configuration spaces associated with unions of arbitrary many light cones, both future and past light cones with positive/negative energies assignable to future/past lightcones. This brings in in a natural manner also the super-canonical symmetries associated with the boundaries of the lightcones.

Negative energies would have quite dramatic technological consequences: consider only the possibility of generating energy from vacuum and classical signalling backwards in time along negative energy space-time sheets [G1]. Also bio-systems might have invented negative energy space-time sheets: in fact, they define the basic mechanism for the realization of intentional action, long term memory, and metabolism [I5].

Quantum classical correspondence suggests that quantum entanglement has the formation of the join along boundaries bonds as its geometric correlate. The superposition of the topologically quantized space-time surfaces in the state $U\Psi$ could be regarded as a geometric correlate for quantum fields: creation/annihilation operators would correspond to positive/negative energy space-time sheets. This hypothesis, together with the expansion of the interacting quantum field in terms of creation and annihilation operators, would make it possible to make quantitative estimates about the fraction of energy density carried by the negative energy space-time sheets, in particular, about the energy density associated with the massless extremals.

In TGD Universe topological field quanta serve as templates for the formation of the bio-structures. Thus topologically quantized classical elec-

tromagnetic fields associated with the material objects, field bodies or more concretely, magnetic bodies, could be equally important for the functioning of the living systems as the structures formed by the visible bio-matter and the visible part of bio-system might represent only a dip of an ice berg. For instance, in [L1] the implications of the notion of field body for the understanding of bio-systems and pre-biotic evolution are discussed in detail.

2.3.4 Negative energy space-time sheets and new view about energy

Negative energy space-time sheets represents an important distinction between TGD and standard physics. They are possible because energy momentum tensor is replaced by a collection of conserved currents associated with various components of four momentum. This resolves the energy problem of general relativity but, since the sign of the conserved charged depends on the time orientation of the space-time sheet, the sign of energy is not positive definite anymore.

Quantum classical correspondence implies that also elementary particles can have negative energies and this means a new kind of physics. It seems that this physics has been already discovered: the strange properties of phase conjugate laser waves can be understood if they consist of negative energy photons.

Negative energy space-time sheets have far reaching implications for TGD inspired theory of consciousness. The so called time mirror mechanism involves the reflection of negative energy signals sent to the geometric past from population inverted lasers as amplified positive energy signals propagating to the geometric future. Time mirror mechanism provides the holy grail to the understanding of the mechanisms of brain functioning and also of the workings of the living matter. There are obvious implications for communication and energy technologies since negative energy signals could make possible instantaneous remote sensing and quantum control over arbitrarily long distances so that light velocity would cease to be a restriction forcing us to be habitants of 3-space instead of space-time.

If Kähler action were strictly deterministic, the only possible choice for H would be $M_+^4 \times CP_2$. Together with negative energies the classical non-determinism of the Kähler action it is possible to assume that imbedding space is $M^4 \times CP_2$ meaning exact Poincare invariance. The point is that generation of pairs of positive and negative energy space-time sheet at light-like 7-surfaces $X_l^3 \times CP_2$ means emergence of new kind of causal determinants generalizing the light cone boundary $\delta M_+^4 \times CP_2$ as a fundamental causal de-

terminant. All states of the Universe have vanishing net quantum numbers and everything in the Universe would have been pair-created from vacuum. Future light cones containing positive energy could also be created when negative energy radiation (in particular gravitons) is generated and propagates to the geometric past and leaks from the future light cone. This vision can be applied also to the second quantization of fermions by giving fermions and anti-fermions opposite energies. Depending on time orientation either fermions or anti-fermions have negative energy.

By crossing symmetry the assumption that the net quantum numbers of the Universe vanish is not in conflict with elementary particle physics. In macroscopic length scales the identification of the gravitational energy as the difference of inertial (Poincare) energies of positive and negative energy matter plus the possibility that negative and positive energy matter interact weakly allows to understand why western view about objective reality with conserved positive total energy is so good an approximation. The non-conservation of the gravitational energy can be understood, and vacuum extremals, of which Robertson-Walker cosmologies, are most important examples find interpretation. The non-determinism of Kähler action explains naturally the fact that Universe is to some extent an outcome of engineering. The notion of gravitational energy generalizes to that of gravitational quantum numbers and the inertial-gravitational dichotomy is a direct correlate for the geometric-subjective dichotomy for time discovered while developing TGD inspired theory of consciousness. Indeed, positive and negative energy space-time sheets correspond to initial and final states of quantum jump so that gravitational quantum numbers characterize changes.

This vision would resolve the unpleasant philosophical questions like "What is the total fermion number of the Universe". One could see entire universe as a result of intentional actions in which intentions represented by p-adic space-time sheets are transformed to actions represented by real space-time sheets. Everyone knows the anecdotes about yogis and gurus creating material objects from nothing and very few "scientifically thinking" westerner can take these stories really seriously. Whether or not these stories are true, they might however express a deep truth about reality.

2.3.5 More precise view about topological condensate

The challenge is to define precisely the concepts like classical gauge charge, gauge flux, wormhole contacts, join along boundaries bonds, topological condensation and evaporation, etc... Number theoretical vision allows to achieve this goal [F6, D2].

The crucial ingredients in the model are so called CP_2 type vacuum extremals. The realization that $\#$ contacts (topological sum contacts and $\#_B$ contacts (join along boundaries bonds) are accompanied by causal horizons which carry quantum numbers and allow identification as partons leads to a more detailed articulation of these notions.

The partons associated with topologically condensed CP_2 type extremals carry elementary particle vacuum numbers whereas the parton pairs associated with $\#$ contacts connecting two space-time sheets with Minkowskian signature of induced metric define parton pairs. These parton pairs do not correspond to ordinary elementary particles. Gauge fluxes through $\#$ contacts can be identified as gauge charges of the partons. Gauge fluxes between space-time sheets can be transferred through $\#$ and $\#_B$ contacts concentrated near the boundaries of the smaller space-time sheet. The dynamics of topological condensation and evaporation can be formulated in terms of gauge interactions of partons and splitting and fusion of CP_2 type extremals. This picture generalizes to the case of gravitational flux which need not be well-defined purely classically.

Number theoretical vision and p-adic length scale hypothesis allow to quantify this picture and lead to an overall view about interactions of particles in many-sheeted space-time. A far reaching generalization of standard physics results predicting an infinite hierarchy of dark matters besides ordinary elementary particles of standard model. In particular, the partons associated with $\#$ and $\#_B$ contacts represent dark matter.

2.4 Classical non-determinism of Kähler action

The classical non-determinism of Kähler action has been deep source of inspiration and challenges and guided the evolution of TGD inspired theory of consciousness and finally also of quantum TGD proper. In nut-shell, classical non-determinism makes possible quantum-classical correspondence in the sense that space-time surface becomes a symbolic representation for the quantum states and quantum jump sequences defining conscious experience.

2.4.1 Matter-mind duality geometrically

The non-determinism of Kähler action implies huge vacuum degeneracy: any 4-surface whose projection belongs to $M_+^4 \times Y^2$, where Y^2 is so called Lagrange manifold of CP_2 (has vanishing induced Kähler form), is a vacuum extremal. This suggests that one must radically generalize the concept of space-time. It seems that the correct picture is roughly like follows. Space-

time is many-sheeted. Each sheet can be regarded as a slightly deformed piece of M^4 in H containing smaller sheets glued to it and being itself glued to a larger space-time sheet. Gluing means the formation of topological sum contacts between the space-time sheets. There are reasons to believe that topological sum contacts, "wormhole contacts" are located near the boundaries of the smaller space-time sheet.

Material space-time sheets have infinitely long time duration if they possess non-vanishing energy (and provided that they do feed their energy to some other space-time sheets). "Mind like" space-time sheets can be regarded as obtained by gluing space-time sheets with finite time duration to material space-time sheets. The gluing operation implies that tiny amounts of energy and momenta and other conserved quantities flow to the mind like space-time sheet when it begins and back to the material space-time sheets when mind like space-time sheet ends. Mind like space-time sheets are space-time correlates for contents of consciousness. In particular, they form symbolic representations for material space-time sheets. For instances, the frequencies of various oscillatory processes are mapped also to frequencies of processes occurring in mind like space-time sheets. The possibility of mind like space-time sheets implies that the absolute minima of Kähler action (or more general preferred extremals defining analogs of Bohr orbits [E2]) are degenerate: one can glue mind like space-time sheets to given absolute minimum to get new absolute minima. This conforms with the fact that contents of consciousness are defined by a sequence of non-deterministic quantum jumps.

This picture must of course be taken with strong reservations, and one should actually state more precisely what "mind like" means. The interpretation of p-adic space-time sheets as correlates of intentions and cognitions gives some ideas about what aspects of consciousness mind like space-time sheets correlate with. The model for how intentions are realized as actions in quantum jump assumes that p-adic "topological light rays" representing intentions are transformed to real topological light rays with negative energy serving as correlates of desires, which in turn induce the action initiated in the geometric past. Thus it would seem that real "mind like" space-time sheets with negative energy would serve as correlates for desires.

The precise definition of p-adic space-time sheets is a separate question and requires a precise vision about how real and various p-adic physics integrate to a coherent whole. This requires a generalization of the number concept based on the fusion of real and p-adic number number fields to a larger book like structure along common rationals. The precise definition of p-adic space-time sheets is discussed in [E2]. The surprising outcome,

basically due to the difference between real and p-adic notions of distance, is that most points of p-adic space-time sheets can be said to reside at infinity of the real imbedding space and the projection to real space-time consists of a discrete set of rational points. Thus cognition can be said to look material cosmos from outside.

2.4.2 Association sequence concept and a mind like space-time sheets

The vacuum degeneracy of the Kähler action defining quantum TGD solves the difficulty. The vacuum degeneracy implies spin glass analogy and strongly suggests that the Bohr orbit like space-time surface defined as a preferred extremal of Kähler action and going through a given space-like 3-surface, cannot be unique in general. To achieve uniqueness one must generalize the concept of 3-surface to what might be called association sequence . In order to specify uniquely one of the degenerate absolute minimum space-times going through a given 3-surface one must fix some minimum number, say N , of 3-surfaces on a given preferred extremal. These sequences of disjoint 3-surfaces with time-like separations can be regarded as a simulations of the classical time development and hence as a geometric correlate of conscious experience localized temporally. It seems that in real case geometric correlates of sensory experiences are in question whereas in p-adic case correlates of thoughts are in question.

Association sequences are very probably not all that is needed to overcome the complications caused by the non-determinism of Kähler action. The enormous vacuum degeneracy of Kähler action suggests strongly that the classical non-determinism does not reduce to simple sequences of bifurcations. Hence it seems that must give up the idea of identifying space-like 3-surfaces given value of geometric time as causal determinants which are possibly degenerate because of the bifurcations.

2.4.3 Vacuum degeneracy and spin glass analogy

Kähler action determines configuration space geometry and is hence a cornerstone of quantum TGD. Kähler action can be regarded as a Maxwell action for the Kähler form of CP_2 induced to space-time surface and defining nonlinear Maxwell field. Kähler action possesses enormous vacuum degeneracy. Any space-time surface in $M_+^4 \times CP_2$, where Y^2 is so called Lagrange sub-manifold of CP_2 having by definition vanishing induced Kähler form, is vacuum extremal. In canonical coordinates (P_i, Q_i) for CP_2 Lagrange

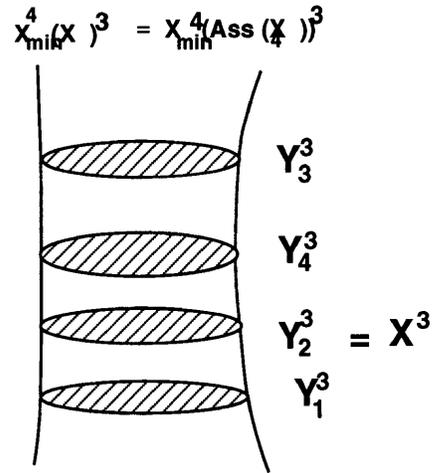


Figure 6: 'Association sequence': a geometric model for thought as a sequence of disjoint 3-surfaces with time-like separations.

sub-manifolds correspond to functions

$$P_i = \nabla_i f(Q_j) .$$

This means that there is infinite number of vacuum sectors since all 4-surfaces in any six-dimensional space $M_+^4 \times Y^2$ are vacua.

Also non-vacuum configurations are almost degenerate. Only the gravitational effects caused by the presence of the induced metric in the Maxwell action for the induced Kähler form of CP_2 on space-time surface breaks the canonical invariance of the Kähler action. Canonical transformations of CP_2 act as $U(1)$ gauge transformations and in the absence of gravitation one would have ordinary $U(1)$ gauge invariance. Gravitation however changes the situation. Various canonically related configurations are physically *non-equivalent*. This means a characteristic degeneracy analogous to the degeneracy of the states for spin glass rather than to the physically uninteresting gauge degeneracy. The effective breaking of $U(1)$ gauge invariance makes possible vacuum charge densities, scalar wave pulses propagating with light velocity and carrying longitudinal electric field parallel to the propagation

direction, and topological light rays carrying light like vacuum current and transversal electric and magnetic fields are predicted.

Contrary to the original beliefs, p-adic physics does not seem to follow from vacuum degeneracy alone. Rather, p-adic space-time topology is a genuine rather than only effective space-time topology and emerges independently from the vacuum degeneracy. p-Adic topology seems however to serve as effective topology for the real space-time sheets in the sense that the non-determinism implied by the vacuum degeneracy mimics the inherent non-determinism of p-adic field equations for some value of p so that one can indeed assign a definite p-adic prime to a given real space-time sheet. Vacuum degeneracy has a wide spectrum of implications. For instance, the spin glass degeneracy implied by it allows to understand at quantum level generation of macroscopic and macro-temporal quantum coherence. The same mechanism explains also color confinement.

The p-adic fractality of real space-time sheets is in turn implied by the fact that p-adic and real space-time sheets have common rational points which implies that the purely local p-adic physics sets constraints on the long ranged real physics because rational points close to each other p-adically are very distant in real sense.

2.4.4 Connection with catastrophe theory and Haken's theory of self-organization for spin glass

If the effects related to the induced metric (classical gravitation) are neglected, canonical transformations of CP_2 act as $U(1)$ gauge symmetries and all canonically related surfaces are physically equivalent. Classical gravitation however breaks this gauge invariance but due to the extreme weakness of the gravitational interaction one has good reasons to expect that the maxima of Kähler function for given values of the zero modes are highly degenerate. The hypothesis that single maximum of Kähler function with respect to fiber degrees of freedom is selected in quantum jump, means huge simplification of the mathematical theory.

Besides the degeneracy resulting from the non-determinism, there is also the spin glass degeneracy related to zero modes. The nonphysical $U(1)$ gauge degeneracy is transformed to physical spin glass degeneracy. The energies of various absolute minima differ only by the classical gravitational energy. Zero modes serve as coordinates for the "energy" landscape of quantum spin glass and the energy landscape of non-equilibrium thermodynamics is fractal containing valleys inside valleys...inside valleys.

One naturally ends up with a generalization of the catastrophe theory

[20] to the infinite-dimensional configuration space context. Zero modes play the role of the control parameters forming master slave-hierarchy and non-zero modes characterizing various degenerate absolute minima of Kähler action correspond to the state variables [I1]. There is natural connection with the non-equilibrium thermodynamics of Haken [23]. Since time development by quantum jumps means hopping in the zero modes characterizing the macroscopic space-time surfaces associated with the final states of the quantum jumps, Haken's classical theory applies almost as such. Asymptotically the self-organizing quantum jumping system (self) ends up to a fixed point, limiting cycle, strange attractor, etc. near the bottom of some valley of the energy landscape. The bottom of a valley corresponds to a maximum of the Kähler function rather than minimum of free energy as in thermodynamics since vacuum functional is exponent of Kähler function. Self-organization in spin glass energy landscape by quantum jumps is extremely powerful notion allowing to understand general features of living systems.

2.5 Quantum classical correspondence as an interpretational guide

The overall view about interpretation of TGD can be deduced from the general properties of space-time surfaces, the notion of induced gauge field, the general properties of Kähler action, and the known extremals using quantum classical correspondence. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence.

2.5.1 The implications deriving from the topology of space-time surface and from the properties of induced gauge fields

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales. It took relatively long time to realize that perhaps the only manner to understand this is a generalization of the quantum theory itself by allowing Planck constant to be dynamical and quantized. TGD leads indeed to a "prediction" for the spectrum of Planck constants and macroscopic quantum phases with large value of

Planck constant allow an identification as a dark matter hierarchy.

Also long ranged classical color and electro-weak fields are an unavoidable prediction and it took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labelled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy defines as $\hbar = \lambda^k \hbar_0$, $k_d = 0, 1, \dots$. λ depends logarithmically on p-adic length scale $L(k)$ and satisfies $\lambda \simeq 2^{11}$ in atomic length scale $L(k = 137)$. Dark space-time sheets are identifiable as space-time sheets defining locally λ^k -fold covering of M^4 factor of imbedding space.

The new view about energy and time means that the sign of inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric past. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

A further important fact is that the holonomy group of induced color gauge field is Abelian. Together with quantum classical correspondences this suggests a weak form of color confinement in the sense that only color neutral states of color multiplets are realized as physical states. This would mean a weak form of color confinement.

3 Evolution of p-adic ideas

It took quite a long time to end up with the recent picture how p-adic numbers emerge as a basic aspect of quantum TGD and what p-adicization of TGD might mean. Of course, recent picture need not be the final yet and there are several unsolved problems. In the following the basic properties of the p-adic numbers are described shortly and then it is demonstrated how p-adic numbers might emerge from TGD and how one should formulate p-adic version of quantum TGD formalism.

3.1 p-Adic numbers

Like real numbers, p-adic numbers can be regarded as completions of the rational numbers to a larger number field allowing the generalization of differential calculus. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x, y)$ (the counterpart of $|x - y|$ in the real context) satisfies the inequality

$$d(x, z) \leq \text{Max}\{d(x, y), d(y, z)\} ,$$

(Max(a,b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x, z) \leq d(x, y) + d(y, z) .$$

p-Adic numbers have expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n \geq 0} x_n p^n ,$$

and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x = \sum_{n \geq 0} x_n p^n) = p^{-n_0} ,$$

and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x, y) = N_p(x - y)$.

p-Adic numbers allow the generalization of the differential calculus and of the concept of analytic function $f(x) = \sum f_n x^n$. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which depend on a finite number of positive pinary digits of x only so that one has

$$f_N(x = \sum_n x_n p^n) = f(x_N = \sum_{n < N} x_n p^n) .$$

In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic.

An essential element is the map of the p-adic numbers to the positive real numbers by the so called canonical identification I :

$$I : \sum x_n p^n \in R_p \rightarrow \sum_n x_n p^{-n} \in R .$$

Canonical identification has inverse, which is single valued for the real numbers having infinite number of pinary digits but two-valued for real numbers having finite number of pinary digits (the reason is that real number with finite number or pinary digits has two equivalent pinary expansions:

($x = 1 = .999999\dots$ in case of decimal expansion and $x = 1 = 0yyyy\dots$, $y = p - 1$, in the case of pinary expansion).

Canonical identification in its basic form cannot map real space-time surface to p-adic ones or vice versa because it is not a general coordinate invariant notion. A variant of canonical identification, call it I_Q , maps defined only for rationals is given by $I(q = m/n) = I(m)/I(n)$, where $q = m/n$ is the unique representation of rational q in terms of integers [E1].

I_Q can be applied to map rational points of p-adic CP_2 to their real counterparts whereas the points of p-adic M^4 are mapped as such to real points as such [E1]. General coordinate invariance is not lost since the projection of p-adic space-time sheet to real imbedding space is discrete and genuinely p-adic points are at infinite real distance, "outside the real cosmos". This means a deep number theoretic difference between M^4 and CP_2 and gives one reason for the product decomposition of the imbedding space. O_Q makes it also possible to map the predictions of the p-adic probability theory and thermodynamics to real numbers so that probability is conserved.

3.2 Evolution of physical ideas

In the sequel the evolution of physical ideas related to p-adic numbers is summarized.

3.2.1 p-Adic length scale hypothesis

p-Adic length scale hypothesis [6] states that to a given p-adic prime p there corresponds a primary p-adic length scale $L_p = \sqrt{pl}$, $l \simeq 1.288 \times 10^4 \sqrt{G}$ (\sqrt{G} denotes Planck length) and that physically favored primes correspond to $p \simeq 2^k$, k power of prime. The corresponding p-adic time scale is obtained as $T_p = L_p/c$. The justification for the first part of the hypothesis comes from Uncertainty Principle and from the p-adic mass calculations [6] predicting that the mass of elementary particle, resulting from the mixing of massless states with $10^{-4} m_{Planck}$ mass states described by p-adic thermodynamics, is of order $1/L_p$ for the light states.

The first principle explanation for p-adic length scale hypothesis derives from the fusion of real and p-adic physics to a single larger framework. The fact that real and p-adic space-time sheets can have common points implies that local p-adic physics give rise to p-adic fractality of real physics. Also multi-p p-adic fractality is possible. $p \simeq 2^k$ would reflect the presence of 2-adic fractality besides $p > 2$ -adic fractality.

A heuristic justification for the preferred values of p comes from elementary particle black hole analogy [E5] generalizing the Bekenstein-Hawking area-entropy law to apply to the elementary particle horizon defined as the surface at which the Euclidian signature for the so called CP_2 type extremal describing elementary particle changes to the Minkowskian signature of the background space-time at which elementary particle has suffered topological condensation.

The hypothesis is especially interesting above the elementary particle length scales $p > M_{127}$ and has testable implications in nuclear physics, atomic physics and condensed matter length scales. The most convincing support for this hypothesis are provided by the elementary particle mass calculations: if one assumes that the p -adic primes associated with elementary particles are primes near prime powers of two, one can predict lepton and gauge boson masses with accuracy better than one per cent. Also quark masses can be predicted but the calculation of the hadron masses requires some modelling (CKM matrix, color force, etc...). The existing empirical information about neutrino mass squared differences suggests that the allowed values of k are indeed *powers* of prime rather than primes.

It is natural to postulate that space-time sheets form a hierarchy with respect to p in the sense that the lower bound for the size of the space-time sheets at level p is of order L_p and that $p_1 < p_2$ sheets condensed on p_2 sheets behave like particles on sheet p_2 .

The following table lists the p -adic length scales L_p , $p \simeq 2^k$, k power or prime, which might be interesting as far as condensed matter is considered (the notation $L(k)$ will be used instead of L_p). It must be emphasized that the definition of the length scale is bound to contain some unknown numerical factor K : the requirement that the thickness of cell membrane corresponds to $L(151)$ fixes the proportionality coefficient K to $K \simeq 1.1$.

k	127	131	137	139	149
$L_p/10^{-10}m$.025	.1	.8	1.6	50
k	151	157	163	167	169
$L_p/10^{-8}m$	1	8	64	256	512
k	173	179	181	191	193
$L_p/10^{-4}m$.2	1.6	3.2	100	200
k	197	199	211	223	227
L_p/m	.08	.16	10	640	2560

Table 1. Primary p -adic length scales $L_p = 2^{k-151}L_{151}$, $p \simeq 2^k$, k prime,

possibly relevant to bio physics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L(151)$ is take to be thickness of cell scale, which is 10^{-8} meters in good approximation.

The assumption that p-adic space-time regions provide cognitive representations of the real space-time regions forces to conclude that cognition is present in all length scales and that the properties of the p-adic space-time regions reflect those of the real space-time regions. p-adic-real phase transitions and identifiable as transformation of intentions to actions [H8] occurring even at elementary particle length scales would explain this elegantly.

Besides primary p-adic length scales also n-ary p-adic length scales defined as $L_p(n) = p^{(n-1)/2}L_p$ and corresponding time scales are possible and form a fractal hierarchy coming as powers of \sqrt{p} . Accepting these scales means that all length scales $L(n)$ coming as powers of $2^{n/2}$, n a positive integer, should have a preferred physical role. The TGD inspired model for living matter lends support for the hypothesis that biologically important length and time scales indeed appear as half octaves. A possible explanation for this is the existence of a hierarchy of cognitive codes associated with the time scales $T(n)$. Any prime power factor k^i in the decomposition of the integer n to a product of prime power factors defines a candidate for a cognitive code. The duration of code word would be $T(n)$ and the number of bits would be k^i . For prime values of n the information content of the code word is maximal so that one could understand why prime values of n are especially important.

3.2.2 CP_2 type extremals and elementary particle black hole analogy

CP_2 type extremals are vacuum extremals having a finite negative action so that one can lower the action of the ordinary vacuum extremals by gluing CP_2 type extremals to them. CP_2 type extremals have one-dimensional M_+^4 projection which is light like random curve. Light likeness condition leads to classical Virasoro algebra constraints. $M^4 \times SO(3, 1) \times SU(3) \times SU(2)_{ew}$ Super-Kac-Moody algebra acts as symmetries and the spectrum of elementary particles is precisely known. The obvious interpretation of the CP_2 type extremals is as a model of elementary particle.

CP_2 type extremals are much like black holes in the sense that they possess elementary particle horizon: this is the surface at which the Eu-

clidian signature of the metric of the CP_2 type extremal changes to the Minkowskian signature of the background space-time. One can indeed generalize Bekenstein-Hawking law to a statement saying that the real counterpart of the p-adic entropy predicted by the p-adic thermodynamics is proportional to the surface area of the elementary particle horizon. In particular, for primes $p \sim 2^k$, where k is power of prime, the radius of the elementary particle horizon is itself a p-adic length scale. This suggests a double p-adicization associated with p and k and an additional cognitive degeneracy due to the k-adic non-determinism, and hence also the dominance of the final states of quantum jump for which $p \simeq 2^k$ holds true: there would be simply very many physically equivalent physical states for these values of p .

3.2.3 p-Adic thermodynamics and particle massivation

The underlying idea of TGD based description of particle massivation is following. Due to the interaction of a topologically condensed 3-surface describing elementary particle with the background space-time, massless ground states are thermally mixed with the excitations with mass of order $m_0 \sim 1/R$ (R is CP_2 length scale, $1/R$ of order 10^{-4} Planck masses) created by the Super Virasoro generators. Instead of energy, the Virasoro generator L_0 (essentially mass squared) is thermalized. This guarantees Lorentz invariance automatically. p-Adic temperature is quantized by purely number theoretical constraints (the Boltzmann weight $\exp(-E/kT)$ is replaced with p^{L_0/T_p} , $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to \hbar so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0 representing scaling!)).

Optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The calculations support the existence of massless gluons and electro-weak quanta associated with so called massless extremals (MEs). One important prediction is that p-adic thermodynamics cannot explain the masses of the intermediate gauge bosons although the predictions for the fermion masses are excellent. This observation led to the identification of the TGD counterpart of Higgs field whose vacuum expectation provides the dominating contribution to the bosonic masses and only shifts bosonic masses [F2].

3.2.4 p-Adic coupling constant evolution

The original hypothesis was that Kähler coupling strength α_K is completely fixed by quantum criticality implying that α_K is analogous to critical temperature. p-Adic considerations led to the view that there is infinite number of critical values of α_K labelled by p-adic primes. In many-sheeted space-time one can indeed consider the possibility that α_K is not a universal constant. This would mean that space-time sheets joined only by wormhole contacts and surrounded by light like elementary particle horizons would be characterized by different values of Kähler coupling strength.

Since p-adic primes correspond to p-adic length scales this inspires the idea that the ordinary coupling constant evolution is replaced by a discrete coupling constant evolution. This view is also consistent with the criticality of the Kähler coupling constant. The assumption that gravitational constant is invariant under critical temperature-adic coupling constant evolution fixes highly unique the evolution of Kähler coupling strength. This picture makes sense if one can assign to a given 3-surface a unique p-adic prime and there are good reasons to believe that this is indeed the case.

The progress in the understanding of the spectrum of Planck constants predicted by TGD however forced to question the idea about p-adic evolution of the Kähler coupling strength and consider the possibility that the original vision is correct after all. Assume that gauge bosons and graviton correspond to Mersenne primes and that graviton, or more generally, the space-time sheets mediating gravitational interaction, corresponds to the largest Mersenne prime for which the p-adic length scale is non-super-astronomical. This Mersenne is M_{127} defining the p-adic length scale of electron. If only $p = M_{127}$ is experimentally relevant, one can tolerate the proportionality $G = \exp(S_K(CP_2))L_p^2$ following from simple dimensional considerations ($S_K(CP_2)$ denotes Kähler action for CP_2 type extremals representing elementary particles) and meaning a rapid increase of G as a function of L_p if α_K is RG invariant. This leads to a highly predictive scenario reproducing the basic features of electro-weak and color coupling constant evolution and also allowing to deduce the value of R^2/CP_2 with electro-weak coupling $\alpha_{U(1)}(M_{127})$.

3.2.5 Vacuum degeneracy of the Kähler action and spin glass analogy

The space of minima of free energy for spin glass is known to have ultrametric topology. p-Adic topology is also ultra-metric and this motivated

the hypothesis that quantum average space-time, 'topological condensate', defined as a maximum of Kähler function can be obtained by gluing together regions characterized by various values of the p-adic prime p . It must be emphasized that this hypothesis is just a guess and not even correct as such, and it seems that TGD as a generalized number theory vision gives the real justification for the p-adics. A good guess is however that the ultra-metric topology of the reduced configuration space consisting of the maxima of the Kähler function is induced from the p-adic norm and that there is a close connection between the two p-adicities. The following arguments tries to make this idea more precise.

The unique feature of the Kähler action is its enormous vacuum degeneracy: any space-time surface, whose CP_2 projection is a so called Lagrange manifold (having dimension $D \leq 2$) is vacuum extremal. This is expected to imply a large degeneracy of the absolute minimum space-times: for instance, several absolute minima with the same action are possible for single 3-surface (this forces to a generalization of space-time concept obtained by introducing 'association sequences'). The degeneracy means an obvious analogy with the spin glass phase characterized by 'frustration' implying a large number of degenerate ground states. In the construction of the configuration space geometry the analogy between quantum TGD and spin glass becomes precise.

Spin glass consists of magnetized regions such that the direction of the magnetization varies randomly in the spatial degrees of freedom but is frozen in time. What is peculiar that, although there are large gradients on the boundaries of the regions with a definite direction of magnetization, no large surface energies are generated. An obvious p-adic explanation suggests itself: p-adic magnetization could be pseudo constant and hence piecewise constant with a vanishing derivative on the boundaries of the magnetized regions so that no p-adic surface energy would be generated.

In the description of the spin glass phase also ultra-metricity, which is the basic property of the p-adic topology, emerges in a natural manner. The energy landscape describing the free energy of spin glass as a function of various parameters characterizing spin glass, is fractal like function and there are infinite number of energy minima. In this case there is a standard manner to endow the space of the free energy minima with an ultra-metric topology [24].

The counterpart of the energy landscape in TGD can be constructed as follows. The configuration space of TGD (the space of 3-surfaces in H) has fiber-space like structure deriving from the decomposition $CH = \cup_{zeromodes} G/H$. The fiber is the coset space G/H such that G is the group

of the canonical transformation of the light cone boundary. In particular, the canonical transformations of CP_2 act in the fiber as isometries. The base space is the infinite-dimensional space of the zero modes characterizing the size and shape as well as the classical Kähler field at the 3-surface.

To calculate S-matrix element, one must form Fock space inner product as a functional of 3-surface X^3 multiplied with the vacuum functional $exp(K)$ and integrate it over the entire configuration space:

$$S_{i \rightarrow f} = \int \langle \Psi_f, \Psi_i \rangle (X^3) exp(K(X^3)) \sqrt{GD} X^3 .$$

The integration over the fiber degrees of freedom reduces to a Gaussian integration around the maxima of the Kähler function with respect to the fiber coordinates. The equally poorly defined Gaussian and metric determinants cancel each other in this integration and one obtains a well defined end result. Canonical transformations are 'almost gauge symmetries' since only classical gravitational fields destroy canonical symmetries acting as $U(1)$ gauge transformations. This means that the action for several canonically related configurations can be degenerate and several maxima are expected for given values of the zero modes. This means that the subset CH_0 of the configuration space consisting of the maxima of the Kähler function has many sheets parameterized by the zero modes and that generalized catastrophe theory is obtained.

If a localization in the zero modes occurs in the quantum jump, one can circumvent the integration over the zero modes in practice. The exponent for the maximum of the Kähler action is expected to have maxima as a function of the zero modes too. The maxima of $exp(K_{max})$ as function of zero modes define the counterpart of the energy landscape and $exp(K_{max})$ is the counterpart of the energy serving as a height function of the energy landscape. It could quite well be that this height function can be induced from a p-adic norm. If so, the allowed values of p define a decomposition of the space of zero modes to sectors D_p . For 'full' CP_2 type extremals representing virtual gravitons the exponent is indeed proportional to $1/p$ if one takes seriously the argument determining the possible values of the Kähler coupling strength. Thus cognitive p-adicity and spin glass p-adicity would be related to each other. The connection with gravitons is especially interesting since also classical gravitation is closely related to the spin glass degeneracy.

3.3 Evolution of mathematical ideas

The evolution of mathematical ideas has been driven by the following frequently asked questions.

a) Is p-adicity realized at space-time level or only at the level of p-adic thermodynamics which was the first application of p-adic numbers? If p-adic space-time regions really make sense, what is their physical interpretation?

b) Physics seems to require correspondence between p-adic and real numbers. What is the role of canonical identification: does it only map p-adic probabilities to their real counterparts or could it be applied also at space-time level despite the obvious difficulties with general coordinate invariance? What about correspondence defined by rational numbers which can be regarded as numbers common to all number fields. Is it possible to assign to a real space-time surface a p-adic counterpart by procedure respecting general coordinate invariance?

c) Does the notion of p-adicization of real physics make sense? How one might achieve the p-adicization in general coordinate invariance manner? What should one p-adicize: only probability calculus and thermodynamics? Or should one include also Hilbert space level? What about p-adicization at space-time level and perhaps even configuration space-level?

d) What is the origin of p-adicity? What is the origin of p-adic length scale hypothesis? How it is possible to assign p-adic prime to a given real space-time sheet as required by the p-adic mass calculations?

e) There have been also technical problems. Besides differential calculus also integral calculus is basic element of classical physics since all variational principles involve integrals over space-time. Also the functional integral over configuration space is needed in order to define S-matrix elements. How one could circumvent the difficulties caused by the non-existence of a p-adic valued define integral based on Riemann sum.

3.3.1 p-Adic physics as physics of cognition and intentionality and generalization of number concept

The identification of p-adic physics as physics of cognition and intention suggests strongly connections between cognition, intentionality, and number theory. The new idea is that also real transcendental numbers can appear in the extensions of p-adic numbers which must be assumed to be finite-dimensional at least in the case of human cognition.

The basic ingredient is the new view about numbers: real and p-adic number fields are glued together like pages of a book along common rationals

representing the rim of the book. Also the rational multiples of algebraic numbers existing p-adically are shared in this manner so that the pages of the book can be stuck together along these lines. This generalizes to the extensions of p-adic number fields and the outcome is a complex fractal book like structure containing books within books. This holds true also for manifolds and one ends up to the view about many-sheeted space-time realized as 4-surface in 8-D generalized imbedding space and containing both real and p-adic space-time sheets. The transformation of intention to action corresponds to a quantum jump in which p-adic space-time sheet is replaced with a real one.

One implication is that the rationals having short distance p-adically are very far away in the real sense. This implies that p-adically short temporal and spatial distances correspond to long real distances and that the evolution of cognition proceeds from long to short temporal and spatial scales whereas material evolution proceeds from short to long scales. Together with p-adic non-determinism due the fact that the integration constants of p-adic differential equations are piecewise constant functions this explains the long range temporal correlations and apparent local randomness of intentional behavior. The failure of the real statistics and its replacement by p-adic fractal statistics for time series defined by varying number N of measurements performed during a fixed time interval T allows very general tests for whether the system is intentional and what is the p-adic prime p characterizing the "intelligence quotient" of the system. The replacement of $\log(p_n)$ in the formula $S = -\sum_n p_n \log(p_n)$ of Shannon entropy with the logarithm of the p-adic norm $|p_n|_p$ of the rational valued probability allows to define a hierarchy of number theoretic information measures which can have both negative and positive values.

Since p-adic numbers represent a highly number theoretical concept one might expect that there are deep connections between number theory and intentionality and cognition. The discussions with Uwe Kämpf in CASYS'2003 conference in Liege indeed stimulated a bundle of ideas allowing to develop a more detailed view about intention-to-action transformation and to disentangle these connections. These discussions made me aware of the fact that my recent views about the role of extensions of p-adic numbers are perhaps too limited. To see this consider the following arguments.

a) Pure p-adic numbers predict only p-adic length scales proportional to $p^{n/2}l$, l CP_2 length scale about 10^4 Planck lengths, $p \simeq 2^k$, k prime or power of prime. As a matter fact, all positive integer values of k are possible. This is however not enough to explain all known scale hierarchies. Fibonacci numbers $F_n : F_{n+1} = F_n + F_{n-1}$ behave asymptotically like $F_n = kF_{n-1}$,

k solution of the equation $k^2 = k + 1$ given by $k = \Phi = (1 + \sqrt{5})/2 \simeq 1.6$. Living systems and self-organizing systems represent a lot of examples about scale hierarchies coming in powers of the Golden Mean $\Phi = (1 + \sqrt{5})/2$.

By allowing the extensions of p-adics by algebraic numbers one ends up to the idea that also the length scales coming as powers of x , where x is a unit of algebraic extension analogous to imaginary unit, are possible. One would however expect that the generalization of the p-adic length scale hypothesis alone would predict only the powers $\sqrt{x}p^{n/2}$ rather than $x^k p^{n/2}$, $k = 1, 2, \dots$. Perhaps the purely kinematical explanation of these scales is not possible and genuine dynamics is needed. For sinusoidal logarithmic plane waves the harmonics correspond to the scalings of the argument by powers of some scaling factor x . Thus the powers of Golden Mean might be associated with logarithmic sinusoidal plane waves.

b) Physicist Hartmuth Mueller has developed what he calls Global Scaling Theory [25] based on the observation that powers of e (Neper number) define preferred length scales. These powers associate naturally with the nodes of logarithmic sinusoidal plane waves and correspond to various harmonics (matter tends to concentrate on the nodes of waves since force vanishes at the nodes). Mueller talks about physics of number line and there is great temptation to assume that deep number theory is indeed involved. What is troubling from TGD point of view that Neper number e is not algebraic. Perhaps a more general approach allowing also transcendentals must be adopted.

c) Classical mathematics, such as the theory of elementary functions, involves few crucially important transcendentals such as e and π . This might reflect the evolution of cognition: these numbers should be cognitively and number theoretically very special. The numbers e and π appear also repeatedly in the basic formulas of physics. They however look p-adically very troublesome since it has been very difficult to imagine a physically acceptable generalization of such simple concepts as exponent function, trigonometric functions, and logarithm resembling its real counterpart by allowing only the extensions of p-adic numbers based on algebraic numbers.

These considerations stimulate the question whether, besides the extensions of p-adics by algebraic numbers, also the extensions of p-adic numbers involving π and e and other transcendentals might be needed. The intuitive expectation motivated by the finiteness of human intelligence is that these extensions should have finite algebraic dimensions, and it indeed turns out that this is possible under some conditions which can be formulated as very general number theoretical conjectures. Since e^p exists p-adically, the powers e, \dots, e^{p-1} define a p-dimensional extension as do also the roots of polynomi-

als with coefficients which are in an extension of rationals containing e and its powers. Contrary to the original conjecture, π however cannot belong to a finite-dimensional extension of p-adics. It is an open question whether one should allow infinite-dimensional extension of p-adic numbers containing π . In any case, the special role of π however becomes an extremely strong constraint for the p-adicization of quantum TGD by algebraic continuation from the realm of rationals to real and p-adic number fields.

Second question is whether there might be some dynamical mechanism allowing to understand the hierarchy of scalings coming in powers of some preferred transcendentals and algebraic numbers like Golden Mean. Conformal invariance implying that the system is characterized by a universal spectrum of scaling momenta for the logarithmic counterparts of plane waves seems to provide this mechanism. This spectrum is determined by the requirement that it exists for both reals and all p-adic number fields assuming that finite-dimensional extensions are allowed in the latter case. The spectrum corresponds to the zeros of the Riemann Zeta if Zeta is required to exist for all number fields in the proposed sense, and a lot of new understanding related to Riemann hypothesis emerges and allows to develop further the previous TGD inspired ideas about how to prove Riemann hypothesis [17, 53].

3.3.2 Algebraic continuation as a basic principle

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function. Logarithm is also such a function provided that the above mentioned number theoretic conjecture holds true.

The definition of a definite integral for p-adic numbers has been the key challenge in attempts to construct p-adic physics and algebraic continuations seems to solve this problem. The first problem is that p-adic numbers are not well ordered and one cannot define what ordered integration interval $[a, b]$ means p-adically. The second problem is that Riemann sum gives identically vanishing p-adic integral if coordinate increments approach zero

at the limit. One can however define the definite integral in terms of the integral function:

$$\int_a^b f(x)dx = F(b) - F(a) ; f(x) = \frac{dF(x)}{dx} .$$

Integral function $F(x)$ is obtained using the inverse of the derivation just as in the real context. If integration limits are restricted to be rational numbers or finitely extended rational numbers, they can be ordered using the ordering of real numbers. This would essentially mean that p-adic integration measure is an algebraic continuation of the real integration measure.

Also residy calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residy formula. One can also imagine of extending residy calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". This could mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residy integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Gaussian integration as a purely algebraic process gives hopes to define p-adic variants of configuration space integrals but only in the case that the integral over the configuration space reduces effectively to the Gaussian integral of a free quantum field theory. If configuration space is indeed a union of symmetric spaces, there are good hopes for achieving this (Duistermaat-Hecke theorem).

p-Adic integration is not necessarily needed to define the p-adic counterpart for the field equations associated with Kähler action but the continuation of the physics from real configuration space to the p-adic variants of the configuration spaces requires the existence of the p-adic valued Kähler action. If it is possible to assign to a given real space-time surface a p-adic counterpart uniquely in a given resolution for rational numbers, one can define the p-adic Kähler action as the real action interpreted as p-adic number in case that the real action belongs to a finite extension of rationals. This would also take care of the absolute minimization of the p-adic Kähler action which does not make sense as a genuinely p-adic concept.

3.4 Generalized Quantum Mechanics

One can consider two generalizations of quantum mechanics to a fusion of p-adic and real quantum mechanics.

a) For the first generalization the guiding principle for the generalization of quantum mechanics is that quantum mechanics in a given number field is obtained as an algebraic continuation of the quantum mechanics in the field of rational numbers common to all number fields or in finite-dimensional extensions of rational numbers. This means that U -matrices U_F for transitions from H_Q to H_F , where F refers to various completions of rationals, are obtained as algebraic continuations of the unitary U -matrix U_Q for H_Q . The generalization means enormously strong algebraic constraints on the form of the U -matrix.

b) A more radical option is that transitions from rational Hilbert space H_Q to the Hilbert spaces H_F associated with different number fields occur. This requires that U -process is followed by a process analogous to a state function reduction and preparation takes care that the resulting states become states in H_Q : this is what makes this generalization of a special interest. In this case one can speak about total scattering probability from H_Q to H_F . The U -matrices U_F are not anymore mere analytic continuations of U_Q . A possible interpretation of the unitary process $H_Q \rightarrow H_F$ is as generation of intention whereas the reduction and preparation means the transformation of the intention to action.

The assumption that H_Q allows an algebraic continuation to the spaces H_F is probably too strong an idealization in p-adic and even in the real case. For instance, one cannot allow all rational valued momenta in p-adic case for the simple reason that the continuation to the p-adic case involves always some momentum cutoff if the extension of p-adics remains finite. Even in the real case the summation over all rational momenta in the unitarity conditions of U -matrix fails to make sense and cutoff is needed. A hierarchy of cutoffs suggests itself and has a natural interpretation as number theoretical hierarchy of extensions of p-adics.

In order to avoid un-necessary complications the following formal discussion however uses H_Q as a universal Hilbert space contained by the various state spaces H_F .

3.4.1 Quantum mechanics in H_F as an algebraic continuation of quantum mechanics in H_Q

The rational Hilbert space H_Q is representable as the set of sequences of real or complex rationals of which only finite number are non-vanishing. Real and p-adic Hilbert spaces are obtained as the numbers in the sequences to become real or p-adic numbers and no limitations are posed to the number of non-vanishing elements. All these Hilbert spaces have rational Hilbert space H_Q as a common sub-space. Also momenta and other continuous quantum numbers are replaced by a discrete value set. Superposition principle holds true only in a restricted sense, and state function reduction and preparation leads always to a final state which corresponds to a state in H_Q . This picture differs from the earlier one in which p-adic and real Hilbert spaces were assumed to form a direct sum.

The notion of unitarity generalizes. Contrary to the earlier beliefs, U -matrix does not possess matrix elements between different number fields but between rational Hilbert space and Hilbert spaces associated with various completions of rationals. This makes sense since the final state of the quantum jump (and thus the initial state of the unitary process, is always in H_Q .

The U -matrix is a collection of matrices U_F having matrix elements in the number field F . U_F maps H_Q to H_F . Each of these U -matrices is unitary. Also U_Q is unitary and U_F is obtained by algebraic continuation in the quantum numbers labelling the states of U_Q to U_F .

Hermitian conjugation makes sense since the defining condition

$$\langle \alpha_F | U n_Q \rangle = \langle U^\dagger \alpha_F | n_Q \rangle . \quad (1)$$

allows to interpret $|n_Q\rangle$ also as an element of H_F . If U would map different completed number fields to each other, hermiticity conditions would not make sense.

The hermitian conjugate of U -matrix maps H_F to H_Q so that UU^\dagger resp. $U^\dagger U$ maps H_F resp. H_Q to itself. This means that there are two independent unitarity conditions

$$\begin{aligned} U_F U_F^\dagger &= Id_F , \\ U_F^\dagger U_F &= Id_Q . \end{aligned} \quad (2)$$

One can write $U = P_Q + T_F$ and $U^\dagger = P_Q + T_F^\dagger$, where P_Q refers to the projection operator to H_Q .

This gives

$$\begin{aligned} T_F + T_F^\dagger &= -T_F T_F^\dagger , \\ P_Q T_F + T_F^\dagger P_Q &= -T_F^\dagger T_F . \end{aligned} \quad (3)$$

It is convenient to introduce the notations $T_Q = P_Q T_F$ and $T_Q^\dagger = T_F^\dagger P_Q$ with analogous notations for U and U^\dagger . The first condition, when multiplied from both sides by P_Q , gives together with the second equation unitarity conditions for T_Q

$$\begin{aligned} T_Q + T_Q^\dagger &= -T_Q T_Q^\dagger , \\ T_Q + T_Q^\dagger &= -T_F^\dagger T_F . \end{aligned} \quad (4)$$

This means that the restriction of the U-matrix to H_Q is unitary.

The difference between the right hand sides of the equation should vanish. The understanding of how this happens requires more delicate considerations. For instance, in the case of $F = C$ continuous sum over indices appears at the right hand side coming from four-momenta labelling the states. The restrictions of quantum numbers to Q and its subsets could be a process analogous to the momentum cutoff of quantum field theories. The continuation from discrete integer valued labels of, say discrete momenta, to continuous values is performed routinely in various physical models routinely, and it would seem that this process has cognitive and physical counterparts. This picture conforms with the vision that the rational (or extended rational) U-matrix U_Q gives the U-matrices U_F by an algebraic continuation in the quantum numbers labelling the states (say 4-momenta).

3.4.2 Could U_F describe dispersion from H_Q to the spaces H_F ?

One can also consider a more general situation in which the states in H_Q can be said to disperse to the sectors H_F . In this case one can write

$$T = \text{''} \sum_F \text{''} T_F . \quad (5)$$

Here the sum has only a symbolic meaning since different number fields are in question and an actual summation is not possible. The T -matrix T_Q is

the sum of the restrictions of T_F to H_Q and is the sum of rational valued T -matrices: $T_Q = \sum_F P_Q T_F$.

The T -matrices T_F are not anymore obtainable by algebraic continuation from same T -matrix T_Q . The unitarity conditions

$$\sum_F (P_Q T_F + T_F^\dagger P_Q) = - \sum_F T_F^\dagger T_F \quad (6)$$

make sense only if they are satisfied separately for each T_F , exactly as in the previous case. T

The diagonal elements

$$T_F^{mm} + \bar{T}_F^{mm} = \sum_\alpha T_F^{m\alpha} \bar{T}_F^{m\alpha} = \sum_r T_F^{mr} \bar{T}_F^{mr}$$

give essentially total scattering probabilities from the state $|m\rangle$ of H_Q to the sector H_F , and must be rational (or extended rational) numbers. One can therefore say that each U -process leads with a definite probability to a particular sector of the state space.

The fact that states which are superpositions of states in different spaces H_F does not make sense mathematically, forces the occurrence of a process, which might be regarded as a number theoretical counterpart of state function reduction and preparation. First a sector H_F is selected with probability p_F . Then F -valued (in particular complex valued) entanglement in H_F is reduced by state reduction and preparation type processes to a rational or extended rational entanglement having interpretation as bound state entanglement. It would be natural to assume that Negentropy Maximization Principle governs this process. Obviously the possibility to reduce state function reduction to number theory forces to consider quite seriously the proposed option.

3.5 Do state function reduction and state-preparation have number theoretical origin?

The foregoing considerations support the view that state function reduction and state preparation are number theoretical necessities so that there would be a deep connection between number theory and free will. One could even say that free will is a number theoretic necessity. The resulting more unified view provides the reason why for state function reduction, and preparation and allows to generalize previous views developed gradually by physics and consciousness inspired educated guess work.

3.5.1 Negentropy Maximization Principle as variational principle of cognition

It is useful to discuss the original view about Negentropy Maximization Principle (NMP) before considering the possible generalization of NMP inspired by the number theoretic vision.

NMP was originally motivated by the need to construct a TGD based quantum measurement theory. Gradually it however became clear that standard quantum measurement theory more or less follows from the assumption that the world of conscious experience is classical: this meant that NMP became a principle governing only state preparation.

State function reduction is achieved if a localization in zero modes occurs in each quantum jump, and if U matrix in zero modes corresponds to a flow in some orthogonal basis for the configuration space spinor fields in the quantum fluctuating fiber degrees of freedom of the configuration space. The requirement that U -matrix induces effectively a flow in zero modes is consistent with the effective classicality of the zero modes requiring that quantum evolution causes no dispersion. The one-one correlation between preferred quantum state basis in quantum fluctuating degrees of freedom and zero modes implies nothing but a one-one correspondence between quantum states and classical variables crucial for the interpretation of quantum theory. It seems that number theoretical vision forces to generalize this view, and to raise NMP to a completely general principle applying also to the state function reduction as the original proposal indeed was.

In its original form NMP governs the dynamics of self measurements and thus applies to the quantum jumps reducing the entanglement between quantum fluctuating degrees of freedom for given values of zero modes. Self measurements reduce the entanglement only between sub-systems in quantum fluctuating degrees of freedom since they occur after the localization in the zero modes. Self measurement is repeated again and again for the unentangled sub-systems resulting in each self measurement. This cascade of self measurements leads to a state possessing only extended rational entanglement identifiable as bound state entanglement and having negative number theoretic entanglement entropy. This process should be equivalent with the state preparation process assumed to be performed by a conscious observer in standard quantum measurement theory.

NMP states that the self measurement can be regarded as a quantum measurement of the sub-system's density matrix reducing the counterpart of the entanglement entropy of some sub-system to a smaller value, and that this occurs for the sub-system for which the reduction of the entanglement

entropy is largest among all sub-systems of the p-adic self. Inside each self NMP fixes some sub-system which is quantum measured in the quantum jump. One could perhaps say that self measurements make possible quantum level self repair since they allow the system in self state to fight against thermalization which results from the generation of unbound entanglement between sub-system-complement pairs.

3.5.2 NMP and number theory

The requirement the universe of conscious experience is classical is one manner to justify the notion of quantum jump. This hypothesis could be replaced by a postulate that state function reduction and preparation project quantum states to a definite number field and that only extended rational entanglement identifiable as bound state entanglement is stable. This is consistent with NMP since it is possible to assign to an extended rational entanglement a non-negative number theoretic negentropy as the maximum over entropies defined by various p-adic entropies $S_p = -\sum p_k \log(|p_k|_p)$.

The unitary process U would thus start from a product of bound states for which entanglement coefficients are extended rationals, and would lead to a formal superposition of states belonging to different number fields. Both state function reduction and state preparation would begin with a localization to a definite number field. This localization would be followed by a self measurement cascade reducing the entanglement to extended rational entanglement.

This vision forces to challenge the earlier views about state function reduction.

a) There is no good reason for why NMP could not be applied to both state function reduction and preparation.

b) If the entanglement between zero modes and quantum fluctuating degrees of freedom involves only discrete values of zero modes, the problems caused by the fact that no well-defined functional integral measure over zero modes exists, find an automatic resolution. Since extended rational entanglement possesses negative entanglement entropy, it is stable also against reduction if NMP applies completely generally. A discrete entanglement involving transcendentals not contained to any *finite* extension of any p-adic number field is unstable and reduced.

c) The quantum measurement lasts for a time determined by the lifetime of the bound state entanglement between zero modes and quantum fluctuating degrees of freedom. Physical considerations of course support the view that it takes more than single quantum jump (10^{-39} seconds of

psychological time) for the state function reduction to take place. The notion of zero mode-zero mode bound state entanglement seems however to be self-contradictory. If join along boundaries bonds are space-time correlates for the bound state entanglement, their formation should transform roughly half of the zero modes associated with the two space-time sheets to quantum fluctuating degrees of freedom.

d) If p-adic length scale hierarchy has as its counterpart a hierarchy of state function reduction and preparation cascades, one must accept the quantum parallel occurrence of state function reduction and preparation processes in the parallel quantum universes corresponding to different p-adic length scales. This picture provides a justification for the modelling of hadron as a quantum system in long length and time scales and as a dissipative system consisting of quarks and gluons in shorter length and time scales. The bound state entanglement between sub-systems of entangled systems having as a space-time correlate join along boundaries bonds connecting sub-system space-time sheets, is a second important implication of the new sub-system concept, and plays a central role in TGD inspired theory of consciousness.

4 The boost from TGD inspired theory of consciousness

Quite generally, TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory. The identification of quantum jump as a moment of consciousness is analogous to the identification of elementary particles as basic building blocks of matter. The observer is an outsider in standard quantum measurement theory and is replaced by the notion of self in TGD inspired theory of consciousness. Selves identified as systems able to avoid bound state entanglement and identifiable as ensembles of quantum jumps, are analogous to many-particle states. The sensory and other qualia of self are determined as statistical averages over quantum number and zero mode increments for the increasing sequence of quantum jumps defining self. Especially important are selves, which are in a state of macro-temporal quantum coherence since for these selves the entropy of the ensemble defined by the quantum jumps does not increase and the qualia stay sharp. These selves are analogous to bound states of elementary particles and their formation actually corresponds to the generation of bound state entanglement.

4.1 The anatomy of the quantum jump

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories rather than to a non-deterministic behavior of a single quantum history. Therefore U -matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of U -matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

a) The classical time development determined by the absolute minimization of Kähler action or some more general principle selecting preferred extremals as generalized Bohr orbits [E2].

b) The unitary "time development" defined by U associated with each quantum jump

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f ,$$

and defining U -matrix. One cannot however assign to the U -matrix an interpretation as a unitary time-translation operator and this means that one must leave open the identification of U -matrix with S-matrix.

c) The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness. A crucial role is played by the classical non-determinism of Kähler action implying that the non-determinism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration since there is *no* real time evolution or translation involved. It is not clear whether one should regard U -matrix and S-matrix as two different things or not: U -matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix

understood in the spirit of superstring models is however something very different and could correspond to U -matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. Physically it seems obvious that U matrix should decompose to a cosmological U -matrix representing dispersion in configuration space and U -matrix representing local dynamics: this indeed occurs thanks to the classical non-determinism of the Kähler action. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation. An important exception are the zero modes characterizing center of mass degrees of freedom of 3-surface which correspond to the isometries of $M_+^4 \times CP_2$. In these degrees of freedom localization does not occur. At the limit when 3-surfaces are regarded as pointlike objects theory should obviously reduce to quantum field theory.

4.1.1 The three non-determinisms

Besides the non-determinism of quantum jump, TGD allows two other kinds of non-determinisms: the classical non-determinism basically due the vacuum degeneracy of the Kähler action and p-adic non-determinism of p-adic differential equations due to the fact that functions with vanishing p-adic derivative correspond to piecewise constant functions.

To achieve classical determinism in a generalized sense, one must generalize the definition of the 3-surfaces Y^3 (belonging to light cone boundary) by allowing also "association sequences", that is 3-surfaces which have, besides the component belonging to the light cone boundary, also disjoint components which do not belong to the light cone boundary and have mutual *time-like separations*. This means the introduction of additional, one might hope typically discrete, degrees of freedom (consider non-determinism based on bifurcations as an example). It is even possible to have quantum entanglement between the states corresponding to different values of time.

Without the classical and p-adic non-determinisms general coordinate invariance would reduce the theory to the light cone boundary and this would mean essentially the loss of time which occurs also in the quantization of gen-

eral relativity as a consequence of general coordinate invariance. Classical and p-adic non-determinisms imply that one can have quantum jumps with non-determinism (in conventional sense) located to a finite time interval. If quantum jumps correspond to moments of consciousness, and if the contents of consciousness are determined by the locus of the non-determinism, then these quantum jumps must give rise to a conscious experience with contents located in a finite time interval.

Also p-adic space-time sheets obey their own quantum physics and are identifiable as seats of cognitive representations. p-Adic non-determinism is the basic prerequisite for imagination and simulation. The notion of cognitive space-time sheet as a space-time sheet having finite time duration is one aspect of the p-adic non-determinism and allows to understand how the notion of psychological time emerges. Cognitive space-time sheets simply drift quantum jump by quantum to the direction of geometric future since there is much more room there in the light cone cosmology.

The classical non-determinism is maximal for CP_2 type extremals for which the M_+^4 projection of the space-time surface is random lightlike curve. In this case, basic objects are essentially four- rather than 3-dimensional. The basic implication of the classical non-determinism is that quantum theory does not reduce to the light cone boundary. Secondly, U -matrix reduces to a tensor product of a cosmological U -matrix and local U -matrices relevant for particle physics. As a matter fact, an entire hierarchy of U -matrices defined in various p-adic time scales is expected to appear in the hierarchy. Thirdly, the classical non-determinism of CP_2 type extremals allows a topologization of the Feynman diagrammatics of quantum field theories and string models. Although localization in zero modes characterizing zitterbewegung orbit occur in quantum jump, there is integral over the positions of vertices which correspond to cm degrees of freedom for imbedding space, and this gives rise to a sum over various Feynman diagrams.

4.1.2 How psychological time and its arrow emerge?

How psychological time and its arrow emerge is the basic challenge for the hypothesis that quantum jumps occur between quantum histories and are identifiable as moments of consciousness. Mind like space-time sheets provide a geometric model of unconscious mind in TGD framework and make it possible to solve the puzzle of psychological time. The first argument is following.

Mind like space-time sheets have well center of mass time coordinate and this coordinate is zero mode identifiable as psychological time. Localization

in zero modes means that final states of quantum jumps correspond to quantum superpositions of space-time surfaces having same number of mind like space-time sheets such that given mind like space-time sheet possesses same value of psychological time for all space-time surfaces appearing in the superposition. The arrow of psychological time follows from the gradual drift of the mind like space-time sheets in future direction occurring quantum jump by quantum jump and is implied by the geometry of future light cone (there is more volume in the future of a given light cone point than in its future). The simplest assumption is that the average increment of psychological time in single quantum jump is of order CP_2 time, which is about 10^4 Planck times.

Besides classical non-determinism there is also p-adic non-determinism and one should keep mind open in the attempts to identify the roles of these two non-determinisms. The interpretation taken as a working hypothesis in the recent version of TGD inspired theory of consciousness is that p-adic space-time regions provide cognitive representations of the real regions and serve as correlates for intentions. Real regions are in turn symbolic representations for the material world in TGD sense of the word. This means that besides ordinary matter also higher level physical states associated with the real space-time sheets of a finite duration and having vanishing net energy are possible. The zero energy states representing pairs of incoming and outgoing states could make possible self-referential real physics representing the laws of physics in the structure of the higher level physical states. Real space-time sheets of finite temporal duration might be interpreted also as correlates of pure sensory experience as opposed to p-adic space-time sheets which can be identified as correlates of thoughts. Also volition could be assigned to the quantum jumps involving selection between various branches of multifurcations implied by the classical non-determinism.

A more refined argument explaining the arrow of psychological time is based on the idea that psychological time correspond to the moment of geometric time which gives the dominant contribution to the conscious experience, and that it is the transformation of intentions to actions which provides this contribution. The transformation of intentions to actions corresponds to the transformation of p-adic space-time sheets to real ones, and one can identify psychological time as characterizing the position of the intention-to action phase transition front. In order to have consistency with the basic facts about everyday conscious experience one must assume that the geometric past remains unable to express intentions for a period of time longer than the life cycle since otherwise the decisions made in say my geometric youth subjectively now could induce dramatic changes in my recent

life. This dead time would be analogous to the recovery time of neuron after the generation of nerve pulse.

4.1.3 Macro-temporal quantum coherence and spin glass degeneracy

At the space-time level the generation of macroscopic quantum coherence is easy to understand if one accepts the identification of the space-time sheets as coherence regions. Quantum criticality and the closely related spin glass degeneracy are essential for the fractal hierarchy of space-time sheets. The problem of understanding macro-temporal and macroscopic quantum coherence at the level of configuration space (of 3-surfaces) is a more tricky challenge although quantum-classical correspondence strongly suggests that this is possible.

Concerning macro-temporal quantum coherence, the situation in quantum TGD seems at the first glance to be even worse than in standard physics. The problem is that simplest estimate for the increment in psychological time in single quantum jump is about 10^{-39} seconds derived from the idea that single quantum jump represent a kind of elementary particle of consciousness and thus corresponds to CP_2 time of about 10^{-39} seconds. If this time interval defines coherence time one ends up to a definite contradiction with the standard physics. Of course, the average increment of the geometric time during single quantum jump could vary and correspond to the de-coherence time. The idea of quantum jump as an elementary particle of consciousness does not support this assumption.

To understand how this naive conclusion is wrong, one must look more precisely the anatomy of quantum jump. The unitary process $\Psi_i \rightarrow U\Psi_i$, where Ψ_i is a prepared maximally unentangled state, corresponds to the quantum computation producing maximally entangled multi-verse state. Then follows the state function reduction and after this the state preparation involving a sequence of self measurements and given rise to a new maximally unentangled state Ψ_f .

a) What happens in the state function reduction is a localization in zero modes, which do not contribute to the line element of the configuration space metric. They are non-quantum fluctuating degrees of freedom and TGD counterparts of the macroscopic, classical degrees of freedom. There are however also quantum-fluctuating degrees of freedom and the assumption that zero modes and quantum fluctuating degrees of freedom are correlated like the direction of a pointer of a measurement apparatus and quantum numbers of the quantum system, implies standard quantum measurement

theory.

b) Bound state entanglement is assumed to be stable against state function reduction and preparation. Bound state formation has as a geometric correlate formation of join along boundaries bonds between space-time sheets representing free systems. Thus the members of a pair of disjoint space-time sheets are joined to single space-time sheet. Half of the zero modes is transformed to quantum fluctuating degrees of freedom and only overall center of mass zero modes remain zero modes. These new quantum fluctuating degrees of freedom represent macroscopic quantum fluctuating degrees of freedom. In these degrees of freedom localization does not occur since bound states are in question.

Both state function reduction and state preparation stages leave this bound state entanglement intact, and in these degrees of freedom the system behaves effectively as a quantum coherent system. One can say that a sequence of quantum jumps binds to form a single long-lasting quantum jump effectively. This is in complete accordance with the fractality of consciousness. Quantum jumps represent moments of consciousness which are "elementary particles of consciousness" and in macro-temporal quantum coherent state these elementary particles bind to form atoms, molecules, etc. of consciousness.

c) The properties of the bound state plus its interaction with the environment allow to estimate the typical duration of the bound state. This time takes the role of coherence time. This suggests a connection with the standard approach to quantum computation. An essential element is spin glass degeneracy. The generation of join along boundaries bonds connecting the space-time sheets of the composite systems is the space-time correlate for the formation of the bound states. Spin glass degeneracy is much higher for the bound states because of the presence of the join along boundaries bonds. This together with the fact that these degenerate states are almost identical so that transition amplitudes between them are also almost identical, implies that the life-time of the majority of bound states is much longer than one might expect otherwise. The detailed argument is carried out in [C1] and can be applied to show that spin glass degeneracy for the color flux tubes explains color confinement [D2].

e) The number theoretic notion of information relies on Shannon entropy in which the logarithms of probabilities are replaced by logarithms of their p -adic norms. This requires that the probabilities are rational or belong to a finite-dimensional extension of rationals. What is so important is that this entropy can have also negative values. If one assumes that bound states form a hierarchy such that the entanglement coefficients belong always to

a finite-dimensional extension of rationals, one can define the entanglement entropy as a number theoretic entropy associated with some prime p . In p-adic context the prime is unique whereas in the real context the value of the prime can be selected in such a manner that the entropy is maximally negative. This prime would be naturally a maximal prime factor of the integer N defining the number of strictly deterministic regions of the space-time sheet in question. If this assumption is made, NMP alone implies the stability of bound states against state preparation by self measurements. This generalization of the information concept has far reaching implications in TGD inspired theory consciousness.

4.2 Negentropy Maximization Principle and new information measures

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = - \sum_n p_n \log(p_n) . \quad (7)$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

4.2.1 p-Adic entropies

The key observation is that in the p-adic context the logarithm function $\log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $\log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $\log(x)$ with the logarithm $\log_p(|x|_p)$ of the p-adic norm of x , where \log_p denotes p-based logarithm. This logarithm is integer valued ($\log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) , \\ k(p_n) &= -\log_p(|p_n|) . \end{aligned} \quad (8)$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $\log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $\log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = - \sum_n p_n \log(p_n) = - \sum_n p_n \left[-k(p_n) \log(p) + p^{k_n} \log(p_n/p^{k_n}) \right] . \quad (9)$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times \log(p) , \\ \left(\sum_n x_n p^n \right)_R &= \sum_n x_n p^{-n} . \end{aligned} \quad (10)$$

The real counterpart of the p-adic entropy is non-negative.

4.2.2 Number theoretic entropies and bound states

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p . The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = - \sum_n p_n \log_p(|p_n|) \log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $\log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv \text{Max}\{-S_p, p \text{ prime}\} . \quad (11)$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP, and has a natural interpretation as bound state entanglement. The prediction would be that the bound states of real systems form a number theoretical hierarchy according to the prime p and dimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes projecting the physical states from either real or p-adic sectors of the state space to their intersection. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the U -matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

4.2.3 Number theoretic information measures at the space-time level

Quantum classical correspondence suggests that the notion of entropy should have also space-time counterpart. Entropy requires ensemble and both the p-adic non-determinism and the non-determinism of Kähler action allow to define the required ensemble as the ensemble of strictly deterministic regions of the space-time sheet. One can measure various observables at these space-time regions, and the frequencies for the outcomes are rational numbers of form $p_k = n(k)/N$, where N is the number of strictly deterministic regions of the space-time sheet. The number theoretic entropies are well defined and negative if p divides the integer N . Maximum is expected to result for the largest prime power factor of N . This would mean the possibility to assign a unique prime to a given real space-time sheet and thus solve the basic problem created already by p-adic mass calculations.

The classical non-determinism resembles p-adic non-determinism in the sense that the space-time sheet obeys effective p-adic topology in some length and time scale range is consistent with this idea since p-adic fractality suggests that N is power of p .

5 TGD as a generalized number theory

The vision about a number theoretic formulation of quantum TGD is based on the gradual accumulation of wisdom coming from different sources. The attempts to find a formulation allowing to understand real and p-adic physics as aspects of some more general scenario have been an important stimulus and generated a lot of, not necessarily mutually consistent ideas, some of which might serve as building blocks of the final formulation. The original chapter representing the number theoretic vision as a consistent narrative grew so massive that I decided to divide it to three parts.

The first part is devoted to the p-adicization program attempting to construct physics in various number fields as an algebraic continuation of

physics in the field of rationals (or appropriate extension of rationals). The program involves in essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals. Highly non-trivial number theoretic conjectures are an outcome of the program.

Second part focuses on the idea that the tangent spaces of space-time and imbedding space can be regarded as 4- *resp.* 8-dimensional algebras such that space-time tangent space defines sub-algebra of imbedding space. The basic candidates for the pair of algebras are hyper-quaternions and hyper-octonions. The problems are caused by the Euclidian signature of the Euclidian norm.

The great idea is that space-time surfaces X^4 correspond to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of $HO = M^8$. The possibility to assign to X^4 a surface in $M^4 \times CP_2$ means a number theoretic analog for spontaneous compactification. Of course, nothing dynamical is involved: a dual relation between totally different descriptions of the physical world are in question. In the spirit of generalized algebraic geometry one can ask whether hyper-quaternionic space-time surfaces and their duals could be somehow assigned to hyper-octonion analytic maps $HO \rightarrow HO$, and there are good arguments suggesting that this is the case.

The third part is devoted to infinite primes. Infinite primes are in one-one correspondence with the states of super-symmetric arithmetic quantum field theories. The infinite-primes associated with hyper-quaternionic and hyper-octonionic numbers are the most natural ones physically because of the underlying Lorentz invariance, and the possibility to interpret them as momenta with mass squared equal to prime. Most importantly, the polynomials associated with hyper-octonionic infinite primes have automatically space-time surfaces as representatives so that space-time geometry becomes a representative for the quantum states.

5.1 The painting is the landscape

The work with TGD inspired theory of consciousness has led to a vision about the relationship of mathematics and physics. Physics is not in this view a model of reality but objective reality itself: painting is the landscape. One can also equate mathematics and physics in a well defined sense and the often implicitly assumed Cartesian theory-world division disappears. Physical realities are mathematical ideas represented by configuration space spinor fields (quantum histories) and quantum jumps between quantum histories give rise to consciousness and to the subjective existence of mathematician.

The concrete realization for the notion algebraic hologram based on the notion of infinite prime is a second new element. The notion of infinite rationals leads to the generalization of also the notion of finite number since infinite-dimensional space of real units obtained from finite rational valued ratios q of infinite integers divided by q . These units are not units in p-adic sense. The generalization to the (hyper-)quaternionic and (hyper-)octonionic context means that ordinary space-time points become infinitely structured and space-time point is able to represent even the quantum physical state of the Universe in its algebraic structure. Single space-time point becomes the Platonia not visible at the level of real physics but essential for mathematical cognition.

In this view evolution becomes also evolution of mathematical structures, which become more and more self-conscious quantum jump by quantum jump. The notion of p-adic evolution is indeed a basic prediction of quantum TGD but even this vision might be generalized by allowing rational-adic topologies for which topology is defined by a ring with unit rather than number field.

5.2 p-Adic physics as physics of cognition

5.2.1 Real and p-adic regions of the space-time as geometric correlates of matter and mind

The solutions of the equations determining space-time surfaces are restricted by the requirement that imbedding space-coordinates are real. When this is not the case, one might apply instead of a real completion with some rational-adic or p-adic completion: this is how rational-adic p-adic physics could emerge from the basic equations of the theory. One could interpret the resulting rational-adic or p-adic regions as geometrical correlates for 'mind stuff'.

p-Adic non-determinism implies extreme flexibility and therefore makes the identification of the p-adic regions as seats of cognitive representations very natural. Unlike real completion, p-adic completions preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with 'mind like' regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of 'self' and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real

elementary particle regions rather than elementary particles themselves!

5.2.2 The generalization of the notion of number and p-adicization program

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

For instance, residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to vari-

ous number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition and intentionality. The basic stumbling block of this program is integration and algebraic continuation should allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. Two types of cutoffs are predicted: p-adic length scale cutoff and a cutoff due to phase resolution. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and Beraha numbers $B_n = 4\cos^2(\pi/n)$, $n \geq 3$ related closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra [O4]. This cutoff hierarchy seems to relate closely to the hierarchy of cutoffs defined by the hierarchy of subalgebras of the super-canonical algebra defined by the hierarchy of sets (z_1, \dots, z_n) , where z_i are the first n non-trivial zeros of Riemann Zeta [C5]. Hence there are good hopes that the p-adicization program might unify apparently unrelated branches of mathematics.

5.3 Space-time-surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?

Second thread in the development of ideas has been present for only few years ideas inspired by the possibility that quaternions and octonions might allow a deeper understanding of TGD. This thread emerged from the discussions with Tony Smith which stimulated very general ideas about space-time surface as associative, quaternionic sub-manifold of octonionic 8-space. Also the observation that quaternionic and octonionic primes have norm squared equal to prime in complete accordance with p-adic length scale hypothesis, led to suspect that the notion of primeness for quaternions, and perhaps even for octonions, might be fundamental for the formulation of quantum TGD [E2]. It turned out that, much in spirit with transition from Riemannian to pseudo-Riemannian geometry, hyper-quaternions and hyper-octonions are forced by physical considerations.

5.3.1 Transition from string models to TGD as replacement of real/complex numbers with quaternions/octonions

One can fairly say, that quantum TGD results from string model with the pair of real and complex numbers replaced with the pair of hyper-quaternions and hyper-octonions. Hyper is necessary in order to take into the Minkowskian signature of the metric.

Space-time identified as a hyper-quaternionic sub-manifold of the hyper-octonionic space in the sense that the tangent space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point. One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered.

Some delicacies are caused by the question whether the induced algebra at X^4 is just the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If the normal part of the product is projected out, the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2 . This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and

appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH - -M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteeing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2 . This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter family co-hyper-quaternionic space-time surfaces.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that the absolute minima of Kähler action in $H = M^4 \times CP_2$ correspond to the hyper-quaternion analytic surfaces in HO . The map $f : HO \rightarrow S^6$ would probably satisfy some constraints posed by the requirement that the resulting surfaces define solutions of field equations in $M^4 \times CP_2$ picture. This conjecture has several variants. It could be that only the asymptotic behavior corresponds to hyper-quaternion analytic function but that hyper-quaternionicity is a general property of absolute minima. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates coding for the local selection of the plane of non-physical polarizations, appear naturally also in the construction of general solutions of field equations [D1].

5.3.2 Physics as a generalized algebraic number theory and Universe as algebraic hologram

The third stimulus encouraging to think that TGD might be reduced to algebraic number theory and algebraic geometry in some generalized sense, came from the work with Riemann hypothesis [E8]. One can assign to Riemann Zeta a super-conformal quantum field theory and identify Zeta as a Hermitian form in the state space possibly defining a Hilbert space metric. The proposed form of the Riemann hypothesis implies that the zeros of ζ code for infinite primes which in turn have interpretation as Fock states of a super-symmetric quantum field theory if the proposed vision is correct.

A further stimulus came from the realization that algebraic extensions of rationals, which make possible a generalization of the notion of prime, could provide enormous representative and information storage power in arithmetic quantum field theory. Algebraic symmetries defined as transformations preserving the algebraic norm represent new kind of symmetries commuting with ordinary quantum numbers. Fractal scalings and discrete symmetries are in question so that the notion of fractality emerges to the fundamental physics in this manner.

The basic observation, completely consistent with fractality, is that these symmetries make possible what might be called *algebraic hologram*. The algebraic quantum numbers associated with elementary particle depend on the environment of the particle. The only possible conclusion seems to be that these fractal quantum numbers provide some kind of 'cognitive representation' about external world. This kind of an algebraic hologram would be in complete accordance with fractality and would provide first principle realization for fractality observed everywhere in Nature but not properly understood in standard physics framework. A further basic idea which emerged was the principle of *algebraic democracy*: all possible algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions are possible and emerge dynamically as properties of physical systems in algebraic physics.

5.4 Infinite primes and physics in TGD Universe

The notion of infinite primes emerged originally from TGD inspired theory of consciousness [15] but it soon turned out that the notion could be used to build a number theoretic interpretation of quantum TGD and relate quantum to classical. Also the notion of infinite-P p-adicity emerges naturally and could replace real topology with something more refined and appropriate for description of the space-time correlates of cognition.

5.4.1 Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes was one important step in the development suggesting strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes so that it applies even in the case of hyper-quaternionic and hyper-octonionic primes. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

What is remarkable is that one has quite realistic possibilities to understand the quantum numbers of physical particles in terms of hyper-octonionic infinite primes. Also the TGD inspired model for $1/f$ noise [I5] based on thermal arithmetic quantum field theory encouraged also to consider the idea about hyper-quaternionic or hyper-octonionic arithmetic quantum field theory as an essential element of quantum TGD.

5.4.2 Infinite primes as a bridge between quantum and classical

The final stimulus came from the observation stimulated by algebraic number theory [40]. Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes. Infinite primes allow nice interpretation as Fock states of a second quantized super-symmetric quantum field theory. Also bound states are included.

This in turn led to the observation that one can represent infinite primes (integers) geometrically as surfaces related to the polynomials associated with infinite primes (integers). Thus infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite numbers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

The original mapping to 4-surfaces inspired by algebraic geometry was essentially as zeros of polynomials. It however turned out that the mapping is more delicate and based on the idea that space-time surfaces correspond

to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of imbedding space with hyper-octonionic structure. Also the attribute maximally associative or co-associate could be used. The assignment of a space-time surface to an infinite prime boils down to an assignment of a hyper-octonion analytic polynomial to infinite prime, which in turn defines a foliation of $M^4 \times CP_2$ by hyper-quaternionic space-time surfaces. The procedure generalizes also to the higher levels of the hierarchy and the natural interpretation is in terms of the hierarchical structure of the many-sheeted space-time.

The connection with the basic ideas of algebraic geometry from the possibility to order space-time surfaces according to the complexity of the polynomial involved (at higher levels rational coefficients of the polynomial are replaced with rational polynomials). In particular, the notions of degree and genus make sense for space-time surface.

5.4.3 Various equivalent characterizations of space-times as surfaces

The idea about space-times as associative, hyper-quaternionic surfaces of a hyper-octonionic imbedding space M^8 and the notion of infinite prime serving as a bridge between classical and quantum are the two basic tenets of the algebraic approach. This vision leads to an equivalence of quite different views about space-time: space-time as an associative/hyper-quaternionic or co-associative/co-hyperquaternionic surface of an hyper-octonionic imbedding space $HO = M^8$; space-time as a surface in $H = M^4 \times CP_2$; space-time as a geometric counterpart of an infinite prime representing also Fock state identifiable as a particular ground state of super-canonical representation; and finally, space-time surface as an absolute minimum of the Kähler action. The great challenge is to prove that the last characterization is equivalent with the others.

5.4.4 Infinite primes and quantum gravitational holography

Infinite primes emerge naturally in the realization of the quantum gravitational holography in terms of the modified Dirac operator and provide a deeper understanding of the basic aspects of the configuration space geometry.

a) Two types of infinite primes are predicted corresponding to the two types of fermionic vacua $X \pm 1$, where X is the product of all finite primes. The physical interpretation for the two types of infinite primes $X \pm 1$ is in terms of two quantizations for which creation and oscillator operators

change role and which correspond to the two signs of inertial energy in TGD Universe. In particular, phase conjugate photons would be negative energy photons erratically believed to reduce to standard physics.

b) The new view about gravitational and inertial masses forced by TGD leads also the view that positive and negative energy space-time sheets are created pairwise at space-like 3-surfaces located at 7-D light-like causal determinants $X_{\pm}^7 = \delta M_{\pm}^4 \times CP_2$. The conjecture is that the ratio of Dirac determinants associated with the positive and negative energy space-time sheets, which is finite, equals to the exponent of Kähler function which would be thus determined completely by the data at 3-dimensional causal determinants and realizing quantum gravitational holography.

c) The spectra associated with the space-time sheets X_{+}^4 and X_{-}^4 meeting at X^3 would correspond to the infinite primes built from the vacua corresponding to the infinite primes $X \pm 1$. The close analogy of the product of all finite hyper-octonionic primes with Dirac determinant suggest that the ratio of the determinants corresponds to the ratio of infinite primes defining X_{+}^4 and X_{-}^4 . The theory predicts the dependence of the eigenvalues of the modified Dirac operator on the value of the Kähler action. Both Kähler coupling strength and gravitational coupling strength are expressible in terms of the finite primes characterizing the ratio of the infinite primes and this ratio depends on the p-adic prime characterizing X_{+}^4 and X_{-}^4 .

d) Some modes of the spectrum of the modified Dirac operator at X_{\pm}^4 become zero modes, and by the resulting spectral asymmetry the ratio of the determinants differs from unity. Thus the spectral asymmetry or the infinite primes defining the space-time sheets X_{+}^4 and X_{-}^4 is all that would be needed to deduce the value of the vacuum functional once causal determinants are known.

5.5 Infinite primes and more precise view about p-adic length scale hypothesis

Number theoretical considerations allow to develop more quantitative vision about the how p-adic length scale hypothesis relates to the ideas just described.

5.5.1 How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass

squared. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles. This might apply also to photons. Infinite range might mean that the join along boundaries bonds mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [C2, C5, E3].

a) If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$ -adic topology is homeomorphic with p-adic topology).

b) One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation

of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89} , if this p-adic prime does not somehow characterize also the particle itself.

c) The formulation of quantum TGD based on the identification of light-like 3-surfaces as fundamental dynamical objects (supported by 4-D general coordinate invariance) suggests that light-like 3 surface identifiable as orbits of partons are characterized by p-adic primes and one can even characterize what this means at the level of the modified Dirac operator characterizing quantum dynamics at parton level [C1]. Space-time sheet itself would be characterized by a collection of p-adic primes so that multi-p-p-adicity would emerge naturally. Even q-adicity might make sense. In the lowest order approximation only partonic boundary components with same prime would interact. The hierarchy of space-time sheets would give rise to a hierarchy of infinite primes. This view leads also to a nice interpretation of infinite primes and fermion-boson dichotomy in terms of cognition and intentionality.

5.5.2 What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

a) The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

b) A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [E3].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

5.5.3 Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [E3], hierarchy of Jones inclusions [O5], hierarchy of dark matters with increasing values of \hbar [F9, J6], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [E3]. Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. One can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets char-

acterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q -adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p -adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n .

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

a) The first interpretation is that there is single q -adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p -adicity ($1/p$ -adicity) for the field modes describing the states.

b) Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p -adic topologies consistent with m - and n -adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by $\#$ contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of $\#$ contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q -adic effective topology,

which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n . This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [E3] coding information too subtle to be caught by ordinary physical measurements [O4].

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

5.5.4 Under what conditions space-time sheets can be connected by $\#_B$ contact?

Assume that particles are characterized by a p-adic prime determining its mass scale plus p-adic primes characterizing the gauge bosons to which they couple and assume that $\#_B$ contacts mediate gauge interactions. The question is what kind of space-time sheets can be connected by $\#_B$ contacts.

a) The first working hypothesis that comes in mind is that the p-adic primes associated with the two space-time sheets connected by $\#_B$ contact must be identical. This would require that particle is many-sheeted structure with no other than gravitational interactions between various sheets. The problem of the multi-sheeted option is that the characterization of events like electron-positron annihilation to a weak boson looks rather clumsy.

b) If the notion of multi-p p-adicity is accepted, space-time sheets are characterized by integers and the largest prime dividing the integer might characterize the mass of the particle. In this case a common prime factor p for the integers characterizing the two space-time sheets could be enough for the possibility of $\#_B$ contact and this contact would be characterized by this prime. If no common prime factors exist, only $\#$ contacts could connect the space-time sheets. This option conforms with the number theoretical vision. This option would predict that the transition to large \hbar phase occurs simultaneously for all interactions.

5.5.5 What about the integer characterizing graviton?

If one accepts the hypothesis that graviton couples to both visible and dark matter, graviton should be characterized by an integer dividing the integers characterizing all particles. This leaves two options.

Option I: gravitational constant characterizes graviton number theoretically

The argument leading to an expression for gravitational constant in terms of CP_2 length scale led to the proposal that the product of primes $p \leq 23$ are common to all particles and one interpretation was in terms of multi-fractality. If so, graviton would be characterized by a product of some or all primes $p \leq 23$ and would thus correspond to a very small p-adic length scale. This might be also the case for photon although it would seem that photon cannot couple to dark matter always. $p = 23$ might characterize the transversal size of the massless extremal associated with the space-time sheet of graviton.

Option II: graviton behaves as a unit with respect to multiplication

One can also argue that if the largest prime assignable to a particle characterizes the size of the particle space-time sheet it does not make sense to assign any finite prime to a massless particle like graviton. Perhaps graviton corresponds to simplest possible infinite prime $P = X \pm 1$, X the product of all primes.

As found, one can assign to any infinite prime, integer, and rational a rational number $q = m/n$ to which one can assign a q -adic topology as effective space-time topology and as a special case effective p-adic topologies corresponding to prime factors of m and n .

In the case of $P = X \pm 1$ the rational number would be equal to ± 1 . Graviton could thus correspond to $p = 1$ -adic effective topology. The "prime" $p = 1$ indeed appears as a factor of any integer so that graviton would couple to any particle. Formally the 1-adic norm of any number would be 1 or 0 which would suggest that a discrete topology is in question.

The following observations help in attempts to interpret this.

a) CP_2 type extremals having interpretation as gravitational instantons are non-deterministic in the sense that M^4 projection is random light-like curve. This condition implies Virasoro conditions which suggests interpretation in terms topological quantum theory limit of gravitation involving vanishing four-momenta but non-vanishing color charges. This theory would represent gravitation at the ultimate CP_2 length scale limit without the

effects of topological condensation. In longer length scales a hierarchy of effective theories of gravitation corresponds to the coupling of space-time sheets by join along boundaries bonds would emerge and could give rise to "strong gravities" with strong gravitational constant proportional to L_p^2 . It is quite possible that the M-theory based vision about duality between gravitation and gauge interactions applies to electro-weak interactions and in these "strong gravities".

b) p-Adic length scale hypothesis $p \simeq 2^k$, k integer, implies that $L_k \propto \sqrt{k}$ corresponds to the size scale of causal horizon associated with $\#$ contact. For $p = 1$ k would be zero and the causal horizon would contract to a point which would leave only generalized Feynman diagrams consisting of CP_2 type vacuum extremals moving along random light-like orbits and obeying Virasoro conditions so that interpretation as a kind of topological gravity suggests itself.

c) $p = 1$ effective topology can make marginally sense for vacuum extremals with vanishing Kähler form and carrying only gravitational charges. The induced Kähler form vanishes identically by the mere assumption that X^4 , be it continuous or discontinuous, belongs to $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2 .

Why topological graviton, or whatever the particle represented by CP_2 type vacuum extremals should be called, should correspond to the weakest possible notion of continuity? The most plausible answer is that discrete topology is *consistent* with any other topology, in particular with any p-adic topology. This would express the fact that CP_2 type extremals can couple to any p-adic prime. The vacuum property of CP_2 type extremals implies that the splitting off of CP_2 type extremal leaves the physical state invariant and means effectively multiplying integer by $p = 1$.

It seems that Option I suggested by the deduction of the value of gravitational constant looks more plausible as far as the interpretation of gravitation is considered. This does not however mean that CP_2 type vacuum extremals carrying color quantum numbers could not describe gravitational interactions in CP_2 length scale.

5.6 Infinite primes, cognition and intentionality

Somehow it is obvious that infinite primes must have some very deep role to play in quantum TGD and TGD inspired theory of consciousness. What this role precisely is has remained an enigma although I have considered several detailed interpretations, one of them above.

In the following an interpretation allowing to unify the views about

fermionic Fock states as a representation of Boolean cognition and p-adic space-time sheets as correlates of cognition is discussed. Very briefly, real and p-adic partonic 3-surfaces serve as space-time correlates for the bosonic super algebra generators, and pairs of real partonic 3-surfaces and their algebraically continued p-adic variants as space-time correlates for the fermionic super generators. Intentions/actions are represented by p-adic/real bosonic partons and cognitions by pairs of real partons and their p-adic variants and the geometric form of Fermi statistics guarantees the stability of cognitions against intentional action. It must be emphasized that this interpretation is not identical with the one discussed above since it introduces different identification of the space-time correlates of infinite primes.

5.6.1 Infinite primes very briefly

Infinite primes have a decomposition to infinite and finite parts allowing an interpretation as a many-particle state of a super-symmetric arithmetic quantum field theory for which fermions and bosons are labelled by primes. There is actually an infinite hierarchy for which infinite primes of a given level define the building blocks of the infinite primes of the next level. One can map infinite primes to polynomials and these polynomials in turn could define space-time surfaces or at least light-like partonic 3-surfaces appearing as solutions of Chern-Simons action so that the classical dynamics would not pose too strong constraints.

The simplest infinite primes at the lowest level are of form $m_B X / s_F + n_B s_F$, $X = \prod_i p_i$ (product of all finite primes). The simplest interpretation is that X represents Dirac sea with all states filled and $X / s_F + s_F$ represents a state obtained by creating holes in the Dirac sea. m_B , n_B , and s_F are defined as $m_B = \prod_i p_i^{m_i}$, $n_B = \prod_i q_i^{n_i}$, and $s_F = \prod_i q_i$, m_B and n_B have no common prime factors. The integers m_B and n_B characterize the occupation numbers of bosons in modes labelled by p_i and q_i and $s_F = \prod_i q_i$ characterizes the non-vanishing occupation numbers of fermions.

The simplest infinite primes at all levels of the hierarchy have this form. The notion of infinite prime generalizes to hyper-quaternionic and even hyper-octonionic context and one can consider the possibility that the quaternionic components represent some quantum numbers at least in the sense that one can map these quantum numbers to the quaternionic primes.

The obvious question is whether configuration space degrees of freedom and configuration space spinor (Fock state) of the quantum state could somehow correspond to the bosonic and fermionic parts of the hyper-quaternionic generalization of the infinite prime. That hyper-quaternionic (or possi-

bly hyper-octonionic) primes would define as such the quantum numbers of fermionic super generators does not make sense. It is however possible to have a map from the quantum numbers labelling super-generators to the finite primes. One must also remember that the infinite primes considered are only the simplest ones at the given level of the hierarchy and that the number of levels is infinite.

5.6.2 Precise space-time correlates of cognition and intention

The best manner to end up with the proposal about how p-adic cognitive representations relate bosonic representations of intentions and actions and to fermionic cognitive representations is through the following arguments.

a) In TGD inspired theory of consciousness Boolean cognition is assigned with fermionic states. Cognition is also assigned with p-adic space-time sheets. Hence quantum classical correspondence suggests that the decomposition of the space-time into p-adic and real space-time sheets should relate to the decomposition of the infinite prime to bosonic and fermionic parts in turn relating to the above mention decomposition of physical states to bosonic and fermionic parts.

If infinite prime defines an association of real and p-adic space-time sheets and this association could serve as a space-time correlate for the Fock state defined by configuration space spinor for given 3-surface. Also spinor field as a map from real partonic 3-surface would have as a space-time correlate a cognitive representation mapping real partonic 3-surfaces to p-adic 3-surfaces obtained by algebraic continuation.

b) Consider first the concrete interpretation of integers m_B and n_B . The most natural guess is that the primes dividing $m_B = \prod_i p^{m_i}$ characterize the effective p-adicities possible for the real 3-surface. m_i could define the numbers of disjoint partonic 3-surfaces with effective p_i -adic topology and associated with with the same real space-time sheet. These boundary conditions would force the corresponding real 4-surface to have all these effective p-adicities implying multi-p-adic fractality so that particle and wave pictures about multi-p-adic fractality would be mutually consistent. It seems natural to assume that also the integer n_i appearing in $m_B = \prod_i q_i^{n_i}$ code for the number of real partonic 3-surfaces with effective q_i -adic topology.

c) Fermionic statistics allows only single genuinely q_i -adic 3-surface possibly forming a pair with its real counterpart from which it is obtained by algebraic continuation. Pairing would conform with the fact that n_F appears both in the finite and infinite parts of the infinite prime (something absolutely essential concerning the consistency of interpretation!).

The interpretation could be as follows.

i) Cognitive representations must be stable against intentional action and fermionic statistics guarantees this. At space-time level this means that fermionic generators correspond to pairs of real effectively q_i -adic 3-surface and its algebraically continued q_i -adic counterpart. The quantum jump in which q_i -adic 3-surface is transformed to a real 3-surface is impossible since one would obtain two identical real 3-surfaces lying on top of each other, something very singular and not allowed by geometric exclusion principle for surfaces. The pairs of boson and fermion surfaces would thus form cognitive representations stable against intentional action.

ii) Physical states are created by products of super algebra generators. Bosonic generators can have both real or p-adic partonic 3-surfaces as space-time correlates depending on whether they correspond to intention or action. More precisely, m_B and n_B code for collections of real and p-adic partonic 3-surfaces. What remains to be interpreted is why m_B and n_B cannot have common prime factors (this is possible if one allows also infinite integers obtained as products of finite integer and infinite primes).

iii) Fermionic generators to the pairs of a real partonic 3-surface and its p-adic counterpart obtained by algebraic continuation and the pictorial interpretation is as fermion hole pair. Unrestricted quantum super-position of Boolean statements requires that many-fermion state is accompanied by a corresponding many-antifermion state. This is achieved very naturally if real and corresponding p-adic fermion have opposite fermion numbers so that the kicking of negative energy fermion from Dirac sea could be interpreted as creation of real-p-adic fermion pairs from vacuum.

If p-adic space-time sheets obey same algebraic expressions as real sheets (rational functions with algebraic coefficients), the Chern-Simons Noether charges associated with real partons defined as integrals can be assigned also with the corresponding p-adic partons if they are rational or algebraic numbers. This would allow to circumvent the problems related to the p-adic integration. Therefore one can consider also the possibility that p-adic partons carry Noether charges opposite to those of corresponding real partons sheet and that pairs of real and p-adic fermions can be created from vacuum. This makes sense also for the classical charges associated with Kähler action in space-time interior if the real space-time sheet obeying multi-p p-adic effective topology has algebraic representation allowing interpretation also as p-adic surface for all primes involved.

iv) This picture makes sense if the partonic 3-surfaces containing a state created by a product of super algebra generators are unstable against decay to this kind of 3-surfaces so that one could regard partonic 3-surfaces as a

space-time representations for a configuration space spinor field.

e) Are alternative interpretations possible? For instance, could $q = m_B/n_B$ code for the effective q -adic topology assignable to the space-time sheet. That q -adic numbers form a ring but not a number field casts however doubts on this interpretation as does also the general physical picture.

5.7 Complete algebraic, topological, and dimensional democracy?

Without the notion of Platonia allowing realization of all imaginable algebraic structures cognitively but leaving no trace on the physics of matter, the idea about dimensional democracy would look almost compelling despite the fact that it might well be in conflict with the special role of the dimensions associated with the classical number fields. One can imagine several realizations of this idea.

a) The most (if not the only) plausible realization for the dimensional hierarchy would be following. Both fractal cosmology, non-determinism of Kähler action, and Poincare invariance favor the option in which configuration space is a union of sectors characterized by unions of future and past light cones $M_{\pm}^4(a)$ where a characterizes the position a of the dip of the light-cone in M^4 . Future/past dichotomy would correspond to positive/negative energy dichotomy and to the two kinds of infinite primes constructed from $X \pm 1$, X the product of all finite primes. Hence the cm degrees of freedom for the sectors of the configuration space would correspond to the union of the spaces $(M^4)^m \times (M^4)^n$ of dimension $D = 4(m+n)$, and the dimensional democracy would conform with the 8-dimensionality of the imbedding space.

b) The most plausible identification consistent with the p -adic length scale hierarchy is as unions of n disjoint 4-surfaces of H . This correspondence is completely analogous to that involved when the configuration space of n point-like particles is identified as $(E^3)^n$ in wave mechanics.

c) One might also consider of assigning with hyper-octonionic infinite primes of level n $4n$ -dimensional surfaces in $8n$ -dimensional space $H^n = (M_+^4 \times CP_2)^n$. This would suggest a dimensional hierarchy of space-time surfaces and a complete dimensional and algebraic democracy: quite a considerable generalization of quantum TGD from its original formulation. This option does not however look physically plausible since it is not consistent with the hierarchical "abstractions about abstractions" structure of infinite primes and corresponding space-time representations.

Since quantum field theories are based on the notion of point like particles, the hierarchy of arithmetic quantum field theories associated with in-

finite primes cannot code entire quantum TGD but only the ground states of the super-canonical representations. This might however be the crucial element needed to understand the construction S-matrix of quantum TGD at the general level.

One can imagine also a topological democracy and an evolution of algebraic topological structures. At the lowest, primordial level there are just algebraic surfaces allowing no completion to smooth ...-adic or real surfaces, and defined only in algebraic extensions of rationals by algebraic field equations. At higher levels rational-adic, p-adic and even infinite-P p-adic completions of infinite primes could appear and provide natural completions of function spaces. Of course, all these generalizations might make sense only as cognitive structures in Platonia and it is comforting to know that there is room in just a single point of TGD Universe for all this richness of imaginable structures!

The reader not familiar with the basic algebra of quaternions and octonions is encouraged to study some background material: the homepage of Tony Smith provides among other things an excellent introduction to quaternions and octonions [35]. String model builders are beginning to grasp the potential importance of octonions and quaternions and the articles about possible applications of octonions [36, 37, 50] provide an introduction to octonions using the language of physicist.

Personally I found quite frustrating to realize that I had neglected totally learning of the basic ideas of algebraic geometry, despite its obvious potential importance for TGD and its applications in string models. This kind of losses are the price one must pay for working outside the scientific community. It is not easy for a physicist to find readable texts about algebraic geometry and algebraic number theory from the bookshelves of mathematical libraries. The book "Algebraic Geometry for Scientists and Engineers" by Abhyankar [39], which is not so elementary as the name would suggest, introduces in enjoyable manner the basic concepts of algebraic geometry and binds the basic ideas with the more recent developments in the field. "Problems in Algebraic Number Theory" by Esmonde and Murty [40] in turn teaches algebraic number theory through exercises which concretize the abstract ideas. The book "Invitation to Algebraic Geometry" by K. E. Smith, L. Kahanpää, P. Kekäläinen and W. Traves is perhaps the easiest and most enjoyable introduction to the topic for a novice. It also contains references to the latest physics inspired work in the field.

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