An Overview about the Evolution of Quantum TGD

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Abstract

This chapter provides a bird's eye view about TGD in its 25^{th} birthday with the hope that this kind of summary might make it easier to follow the more technical representation provided by sub-sequent chapters. The geometrization of fundamental interactions assuming that space-times are representable as 4-surfaces of $H = M^4_+ \times CP_2$ is wherefrom everything began. The two manners to understand TGD is TGD as a Poincare invariant theory of gravitation obtained by fusing special and general relativities, and TGD as a generalization of string model obtained my replacing 1-dimensional strings with 3-surfaces. The fusion of these approaches leads to the notion of the many-sheeted space-time.

The evolution of quantum TGD involves four threads which have become more and more entangled with each other. The first great vision was the reduction of the entire quantum physics (apart from quantum jump) to the geometry of classical spinor fields of the infinite-dimensional space of 3-surfaces in H, the great idea being that infinite-dimensional Kähler geometric existence and thus physics is unique from the requirement that it is free of infinities. The outcome is geometrization and generalization of the known structures of the quantum field theory and of string models.

The second thread is p-adic physics. p-Adic physics was initiated by more or less accidental observations about reduction of basic mass scale ratios to the ratios of square roots of Mersenne primes and leading to the p-adic thermodynamics explaining elementary particle mass scales and masses with an unexpected success. p-Adic physics turned eventually to be the physics of cognition and intentionality. Consciousness theory based ideas have led to a generalization of the notion of number obtained by gluing real numbers and various p-adic number fields along common rationals to a more general structure and implies that many-sheeted space-time contains also p-adic space-time sheets serving as space-time correlates of cognition and intentionality. The hypothesis that real and p-adic physics can be regarded as algebraic continuation of rational number based physics provides extremely strong constraints on the general structure of quantum TGD.

TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory replacing the notion of observer as an outsider with the notion of self. The detailed analysis of what happens in quantum jump have brought considerable understanding about the basic structure of quantum TGD itself. It seems that even quantum jump itself could be seen as a number theoretical necessity in the sense that state function reduction and state preparation by self measurements are necessary in order to reduce the generalized quantum state which is a formal superposition over components in different number fields to a state which contains only rational

or finitely-extended rational entanglement identifiable as bound state entanglement. The number theoretical information measures generalizing Shannon entropy (always non-negative) are one of the important outcomes of consciousness theory combined with p-adic physics.

Physics as a generalized number theory is the fourth thread. The key idea is that the notion of divisibility could make sense also for literally infinite numbers and perhaps make them useful from the point of view of physicist. The great surprise was that the construction of infinite primes corresponds to the repeated quantization of a supersymmetric arithmetic quantum field theory. This led to the vision about physics as a generalized number theory involving infinite primes, integers, rationals and reals, as well as their quaternionic and octonionic counterparts. A further generalization is based on the generalization of the number concept already mentioned. Space-time surfaces could be regarded in this framework as concrete representations for infinite primes and integers, whereas the dimensions 8 and 4 for imbedding space and space-time surface could be seen as reflecting the dimensions of octonions and quaternions and their hyper counterparts obtained by multiplying imaginary units by $\sqrt{-1}$. Also the dimension 2 emerges naturally as the maximal dimension of commutative sub-number field and relates to the ordinary conformal invariance central also for string models.

This chapter represents a overall view of classical TGD, a discussion of the p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, and the vision about physics as a generalized number theory. Also the construction of configuration space geometry and spinor structure, and of S-matrix are also described at the level of general principles.

1 Introduction

Topological Geometrodynamics was born for twenty five years ago as an attempt to construct a Poincare invariant theory of gravitation by assuming that physically allowed space-times are representable as surfaces in space $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 is complex projective space having real dimension four (see the appendix of the book). Poincare group was identified as the isometry group of M^4 rather than of the space-time surface itself. The isometries of CP_2 were identified as color group and the geometrization of electro-weak gauge fields and elementary particle quantum numbers was achieved in terms of the spinor structure of CP_2. Rather remarkably, for a quarter century after this discovery one can still say that CP_2 codes the known elementary particle quantum numbers and interactions in its geometry. The construction of quantum theory suggests the replacement of M^4 with M^4_+, the interior of the future light cone of Minkowski space so that Poincare invariance is broken by the global geometry of the light cone but not locally.

It took almost half decade to develop the new view about space-time implied by the basic hypothesis: this is summmarized in my PhD thesis [1]. The construction of a mathematical theory around these physically very attractive ideas became the basic challenge and I have devoted my professional life to the realization of this dream. The great idea was that quantum physics reduces to the construction of Kähler metric and spinor structure for the infinite-dimensional space CH of all possible 3-surfaces of H. Physical states correspond to classical spinor fields in this space and a natural geometrization of fermionic statistics in terms of gamma matrices emerges [B1, B2, B3].

p-Adic number fields R_p [2] (one number field for each prime obtained as a completion of the rational numbers) emerged for about ten years ago as a separate thread only loosely related to quantum TGD. What made them so attractive was that, with certain additional assumptions about physically favoured p-adic primes, it became possible to understand the basic elementary particle mass scales number theoretically. This led to a successful calculation of the elementary particle masses using p-adic thermodynamics assuming that Super Virasoro algebra and related Kac Moody algebras, which are also basic algebraic structures of string models, act as symmetries of TGD [F2, F3, F4, F5]. The success of the mass calculations in turn forced the attempts to understand how Super Virasoro and related symmetries might emerge from basic TGD. Several trials led finally to the realization that these super algebras (or actually the proper generalizations of them) are the basic symmetries of quantum TGD. One of the most dramatic predictions is the uniqueness of the space H: quantum TGD exists mathematically (cancellation of various infinities occurs) only for the space $M^4_+ \times CP_2$, the choice which is forced also by the cosmological and symmetry considerations. One can say that infinite-dimensional Kähler geometric existence and thus physics is unique.

A third thread to the development emerged when I started systematic development of TGD inspired theory of consciousness [**?**]. This work has led to dramatic increase of understanding also at the level of basic quantum TGD and allowed to develop quantum measurement theory in which conscious observer is not anymore Cartesian outsider but an essential part of quantum physics. The need to understand the mechanism making biosystems macroscopic quantum systems led to a dramatic progress in the understanding of the new physics implied by the notion of many-sheeted space-time. Dramatic change in views about the relation between subjectively experienced and geometric time of physicist emerges and leads to the solution of the basic paradoxes of quantum physics. It became also clear that p-adic numbers are indeed an absolutely essential element of the mathematical formulation of quantum TGD proper and that the general properties of quantum TGD force the introduction of the p-adic numbers. One can say that physics involves both real and p-adic number fields with real numbers describing the topology of the real world and various p-adic number fields serving as correlates of cognition with the prime p labelling the p-adic topology serving as kind of intelligence quotient.

A further thread into the development of ideas came from the realization that physics might be basically number theory in generalized sense. TGD more or less forces the notion of infinite primes [E3], and it turned out that their construction reduces to a repeated second quantization of arithmetic quantum field theory. Generalization of the concept of integer and real number emerges implying that the configuration space and state space of TGD could be imbedded into the field of generalized reals which is infinite-dimensional algebraic extension of ordinary reals. Physics could be basically theory of generalized reals! The dimensions of space-time *resp.* imbedding space correspond to the dimensions of quaternion *resp.* octonion fields as well as the dimensions of algebraic extensions of $p > 2$- *resp.* 2-adics allowing square root of ordinary p-adic number. The discussions with Tony Smith suggested that one can endow space-time and imbedding space with what might be called local quaternion and octonion structures.

This stimulated a development, which led to the notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [E2]. What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

Further important number theoretical ideas emerged from the attempt to construct a model for how intentions are transformed to actions. The process was interpreted as a quantum jump in which p-adic space-time sheet representing intention is transformed to a real one. This model led to a bundle of ideas and conjectures.

a) The core idea is the generalization of the notion of number obtained by gluing all number number fields together along rationals and algebraic numbers common to them. This means a generalization of the notion of manifold. In particular, imbedding space is obtained by gluing real and p-adic imbedding spaces together along rational points. This picture also justifies the decomposition of space-time surface to real and p-adic space-time sheets. Also finite-dimensional algebraic extensions, even extensions involving transcendentals like e are needed.

b) p-Adic space-time sheets are identified as correlates of intentionality and cognition. The differences between real and p-adic topologies (two rationals near to each other as p-adic numbers are very far in real sense) have deep implications concerning the understanding of cognitive consciousness. The evolution of cognition corresponds naturally to the increase of the p-adic prime and dimension of the extension of p-adic numbers.

c) Real physics and various p-adic physics are obtained from finitely extended rational physics by algebraic continuation to p-adic number fields and their extensions analogous to analytic continuation in complex analysis. This algebraic continuation is performed both at space-time level, state space level, and configuration space level. One can also generalize the notion of unitarity and the generalization poses extremely strong conditions on S-matrix.

This chapter represents a overall view of classical TGD, a discussion of the p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, and the vision about physics as generalized number theory. Also the construction of configuration space geometry and spinor structure, and of S-matrix are also described at the level of general principles.

2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model (or of super string model).

2.1 Energy problem of GRT and TGD as a Poincare invariant theory of gravitation

The two great achievements of Einstein were Special Relativity and General Relativity and TGD can in well defined sense be regarded as a synthesis of these two theories. The need for a synthesisis became obvious to me when I was considering the so called energy problem of General Relativity.

The space-time of the Special Relativity is so called Minkowski space (M^4). The basic hypothesis of General Relativity is that the presence of matter makes space-time curved. This idea however leads to difficulties with the well tested deep ideas about symmetries. Already in classical mechanics every symmetry of a system gives rise to conservation law as discoved by mathematician Emmy Noether. For instance, translational symmetry with respect to the spatial degrees of freedom gives rise to momentum conservation and translational symmetry with respect to time gives rise to energy conservation. The content of the translational symmetries is roughly that the physical laws are same everywhere and for all times.

For empty Minkowksi space the presence of translational symmetries is obvious. The problem is that the presence of matter makes space-time curved and curved space-time does not possess translational symmetry anymore. This is obvious in the 3-dimensional visualization. Planar surface remains as such in translation. For a deformed surface different points are not anymore related by a translational symmetry: for instance, gravitational fields are different in different points. The question is 'How can one define conserved energy and momentum when the symmetries are lost?'. Usually this is not regarded as a very serious problem since gravitational interaction is extremely weak after all.

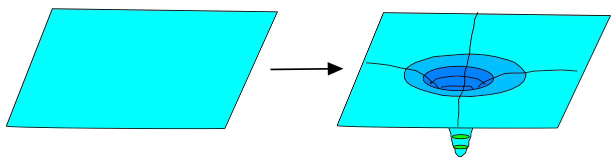

Figure 1: Matter makes space-time curved and spoils translational invariance. Two-dimensional illustration.

There is a manner to circumvent the energy problem. Assume that space-time is representable as a 4-dimensional surface of some higher-dimensional space time obtained by replacing every point of an empty Minkowski space-time with an N-dimensional space S with a very small size (of order Planck length which corresponds to about 10^{-35} meters). This D=4+N-dimensional space is denoted by $H = M^4xS$. This space possesses the symmetries of the empty Minkowski space plus some additional symmetries, namely those of S. This suggests the existence of a theory for which space-time is a 4-dimensional surface of H determined by field equations which allow the symmetries of H as symmetries. This would mean a solution of the energy problem since energy would now correspond to time translations of H rather than of 4-dimensional space-time as in General Relativity.

As a by product one obtains also additional symmetries: namely those of S and identifying these symmetries as color symmetries characteristic for quarks and gluons, one can identify S uniquely as so call ed complex projective space $S = CP_2$ having dimension four so that the space H is 8-dimensional and theory becomes unique In fact, it turns out that the interior the future light cone of M^4, to be denoted by M^4_+, must be chosen instead of M^4 for both mathematical and cosmological reasons. This choice preserves Poincare group as the local symmetries of the theory and gives rise to classical conservation laws.

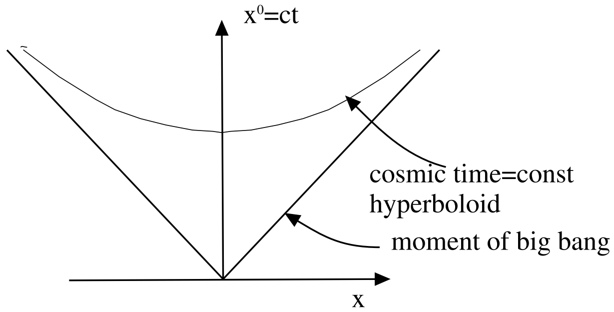

Figure 2: Future light cone of Minkowski space.

2.2 Geometrization of fields and quantum numbers

The geometrization of fields was the great dream of Einstein. General Relativity provided the geometrization of the gravitational field but not much success was achieved in the geometrization of the electromagnetic field. Later also other fields related to the weak interactions were discovered. The pleasant surprise was that the identification of space-time as a 4-surface solves this problem.

The first observation is that the metric of H mathematisizing the concept of the length measurement defines also metric on space-time surface. Physically this means that the distances on the space-time surfaces are measured using same meter stick as used to measure distances in H. Since space-time metric corresponds to gravitational field, this means a geometrization of the

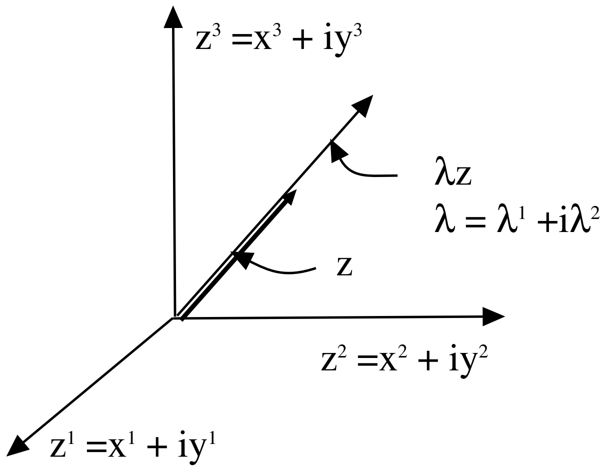

Figure 3: CP_2 is obtained by identifying all points of C^3, space having 3 complex dimensions, which differ by a complex scaling Lambda: z is identified with $\Lambda \times z$.

gravitational field in the same manner as it is achieved in GRT. Mathematically this means what is called the induction of the metric of H to space-time surface.

Parallel translation is a second basic concept of the Riemannian geometry besides length measurement. In Euclidian space the concept of parallel translation is obvious: move a vector so that its direction and length remain unchanged. In a curved space-time parallel translation is defined to be a process, which preserves the length of the vector and in case of the parallel translation along a geodesic (counterpart of a straight line) also the angle formed by the vector and the geodesic. For curved spaces the parallel translation along a closed curve in general brings back the original vector as a rotated one.

The mathematical realization of the parallel translation is in terms of Riemann connection playing key role in General Relatitivy. Riemann connection generalizes to a so called spinor connection (a great triumph of Special Relativity was the geometrization of the electron spin in terms of spinor concept). The concept of connection is actually extremely general and the identification of the bosons as quanta of the gauge potentials regarded as components of a gauge connection was a great triumph of the

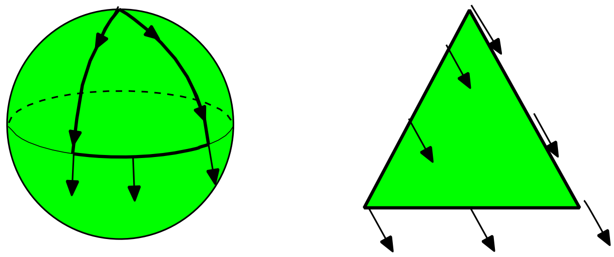

Figure 4: Parallel translation on sphere and on plane.

standard model. What remained however lacking was the direct connection with the geometry of space-time and in TGD this connection is achieved.

What happens is following. The parallel translation along space-time surface can be performed by regarding the curve in question as a curve of the imbedding space. Mathematically this means that the spinor connection of the space H is *induced* to the space-time surface. This induced connection can be interpreted as a gauge potential. For $S = CP_2$ the identification of induced spinor connection in terms of electromagnetic field and classical fields corresponding to the so called intermediate gauge bosons, is possible. Also the classical color field can be identified as a purely geometric field. As a byproduct the geometrization of the electro-weak and color quantum numbers of the elementary particles is achieved. The separate conservation of baryon and lepton numbers follows as a particular consequence. Hence for $H = M^4_+ \times CP_2$ one indeed achieves an elegant realization of Einstein's dream.

The construction of QFT limit of TGD can be achieved very elegantly by generalizing the induction procedure to the quantum fields of H representing light particles in the point particle approximation. This description is in nice accordance with the particle massivation described by p-adic thermodynamics. TGD allows also Higgs particles but it seems that the coupling to Higgs particle explain only the masses of electro-weak gauge bosons whereas fermionic masses receive only minor contributions from the coupling to the Higgs. The weaker couplings of the Higgs a la TGD to fermions could explain the continual failure to detect Higgs particle.

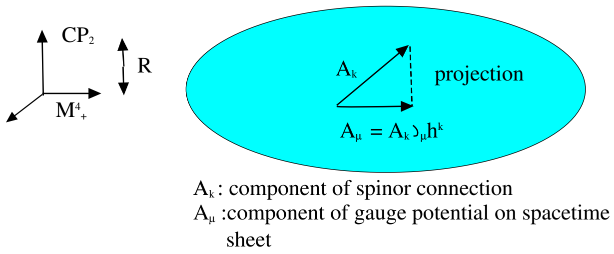

Figure 5: Classical electro-weak gauge potentials at space-time surface are obtained as projections of the components of CP_2 spinor connection.

2.3 TGD as a generalization of string model

The second manner to end up with TGD is to start from the old fashioned string model, which also served as a starting point of super string models, which have been in fashion during the last ten years. Mesons are strongly interacting particles and string model description was in terms of a string with quark and antiquark attached to the ends of the string. A problem was encountered in an attempt to generalize this description to apply to baryons which consist of three quarks. One cannot put 3 quarks to the ends of the string since it has only two ends and this led to the ugly looking attempts to construct baryons using strings (see the figure).

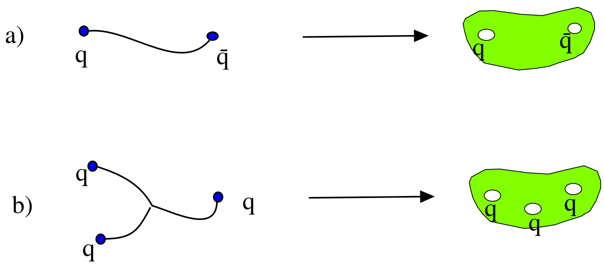

Figure 6: The transition from hadronic string model to TGD.

The solution of the problem is simple. Replace one-dimensional strings with small 3-dimensional surfaces. Since the ends of the string correspond to the boundaries of a one-dimensional manifold they correspond in 3-dimensional case boundaries of small holes drilled in 3-dimensional space. Put quarks on these boundaries. In 3-dimensional case one can drill arbitrary number of these holes so that also baryons can be described in this kind of model.

A very nice side product is an explanation for the so called family replication phenomenon. Both leptons and quarks seem to exist in at least three essentially identical copies. There are three leptons electron, muon and tau and corresponding neutrinos. Also there are three quarks u, c, t with charge 2/3 and d,s,b with charge -1/3. The explanation is in terms of the topology of the boundary component. The hole drilled in 3-space can have topology of a sphere, torus, sphere with two handles, etc. corresponding to e,mu, tau and possible new particle families not yet detected. It turns out that entire elementary particle spectrum can be understood in TGD framework if this identification is made. One could interpret TGD also as a generalization of the super string model and this interpretation is more appropriate since the original TGD:eish picture of hadron makes sense only in very rough topological sense.

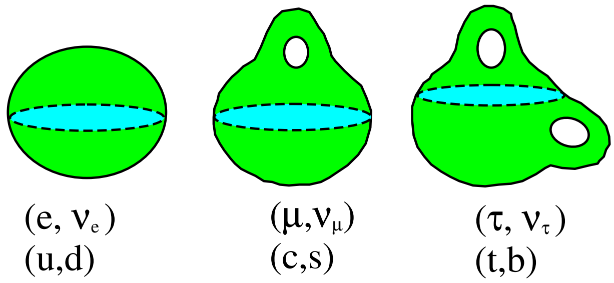

Figure 7: Various fermion families correspond to genera for the boundary component of CP_2 type extremal describing elementary particle.

2.4 Fusion of the two approaches and the notions of topological condensate and many-sheeted space-time

The only manner to unify the two TGD:s given by the Poincare invariant gravitation on one hand, and by the generalization of the string model on the other hand, is provided a generalization of the space-time concept. The macroscopic space-time with matter is identified as a surface to which smaller 3-surfaces representing various physical objects are glued by topological sum operation by connecting different space-time sheets by very tiny wormholes. In particular, elementary particles correspond to so called CP_2 type extremals, which have Euclidian metric and negative finite action and have very much the same role in TGD as black holes in GRT. Also 'vapour phase', i.e. small particle like surface residing (at least part of the time) outside the macroscopic space-time surface are possible, and are the counterpart of the Baby Universies of GRT.

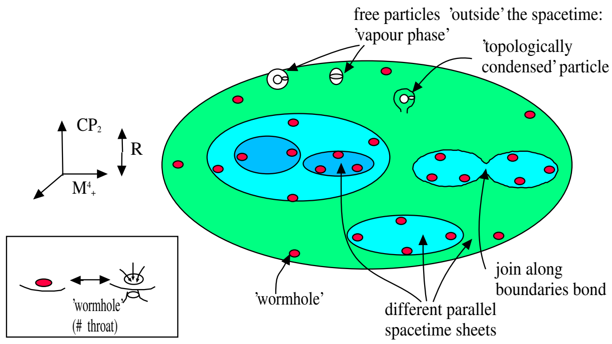

Figure 8: This is what TGD:eish 3-space would look if it were a 2-dimensional surface in 3-dimensional slab of thickness of order 10^4 Planck lengths.

A further generalization of the space-time concept is forced by the classical non-determinism of the Kähler action leading to the notion of mind like space-time sheets having finite time duration. The resulting many-sheeted space-time implies a lot of new physics since the larger than space-time sheet, the lower the temperature so that various macroscopic quantum phases become possible. The identification of the space-time sheets as coherence regions of classical fields and space-time spinor fields provides a connection with quantum mechanics and provides a long sought solution to the problem of defining the notions of classical and quantum coherence rigorously. Topological field quantization implies that classical fields decompose into space-time quanta such as magnetic flux tubes, electric flux quanta, and topological light rays: the fact that one can assign to a given material system field body has deep implications in TGD based model of living matter.

p-Adicity forces a further generalization so that space-time surface consists of real representing real physics and p-adic space-time sheets providing cognitive representations for the real space-time regions. The vision about TGD as a generalized number theory allows a rigorous mathematical formulation of the notion of topological condensate in terms of generalized number concept resulting when real numbers and various p-adic number fields are glued together along common rational numbers.

3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [3, 4, 5]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B+C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces?

Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in [**?**] and leads to a quantitative understanding of the relationship between sensory qualia and EEG [K3, M4, M5]. The basic elements of the theory are following.

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \ ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m - M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape. Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. In case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric representation of thought. When non-determinism has long last-

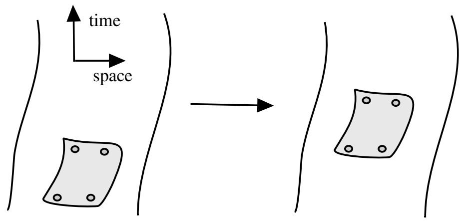

Figure 9: The mechanism giving rise to the arrow of psychological time

ing and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4$ CP_2 times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1].

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obeys the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself.

3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [E2].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encourage by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

3.5.1 Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [D6].

I have proposed already earlier the possibility that Planck constant is quantized and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes.

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Shrödinger equation holds true only inside flux quanta for the dark matter.

a) During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).

b) In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.

c) The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n \ mod \ 5 = 0$ for some reason.

d) A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

3.5.2 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

3.5.3 Dark matter hierarchy and consciousness

The emergence of the vision about dark matter hierarchy has meant a revolution in TGD inspired theory of consciousness. Dark matter hierarchy means also a hierarchy of long term memories with the span of the memory identifiable as a typical geometric duration of moment of consciousness at the highest level of dark matter hierarchy associated with given self so that even human life cycle represents at this highest level single moment of consciousness.

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [O5]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [O5].

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

Higher levels of dark matter hierarchy provide neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k = 4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

4 Evolution of classical TGD

The TGD based space-time concept means a radical generalization of standard views already in the real context. Many-sheetedness means a hierarchy of space-time sheets of increasing size making possible to understand the emergence of structures in terms of the macroscopic space-time topology. The non-determinism of the Kähler action forces the notion of the association sequence defined as a union of space-like 3-surfaces with time-like separations: association sequence provides a geometric correlate for thought as simulation of the classical history. Non-determinism forces also the notion of mind like space-time sheet defined as a space-time sheet having finite temporal duration, which is an attractive candidate for the geometric correlate of self. Topological field quantization means that space-time topology provides classical correlates for the basic notions of the quantum field theory. The decomposition of space-time surface into real and p-adic regions brings in besides the matter also cognitive representations of material world.

4.1 Quantum classical correspondence and why classical TGD is so important?

In standard quantum physics classical theory is seen as a result of some kind of approximation procedure, say stationary phase approximation. In TGD framework classical physics is exact part of quantum physics, and even more of configuration space geometry since, apart from the complications caused by the classical non-determinism of the Kähler action, the definition of the Kähler geometry in terms of Kähler action assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$.

The evolution of TGD inspired theory of consciousness has gradually led to the notion of quantum classical correspondence which states that every quantum aspect of existence has space-time correlate. The correspondence is certainly not faithful but rather like the representation of contents of consciousness provided by spoken or written language. Space-time surface can be indeed seen as a symbolic representation, kind of written language. Not only the characteristics of quantum states, but also quantum jumps and their sequences defining the contents of conscious experience, have space-time correlates made possible by the classical determinism of the Kähler action, and the inherent p-adic non-determinism of p-adic counterparts of the field equations. In fact, there are reasons to believe that classical non-determinism of the Kähler action and a p-adic non-determinism have close relationship in the sense that the effective topology of the real space-time sheets is expected to correspond to p-adic topology in some length scale range.

4.2 Classical fields

In TGD framework the physics of classical fields are an essential part of the quantum theory and the study of classical fields has provided the easiest manner to get grasp about the physics of TGD Universe.

4.2.1 Geometrization of classical fields and of quantum numbers

The basic motivation for TGD was provided by the finding that known interactions at classical level and quantum number spectrum of known particles could be readily understood from the assumption that space-time is a 4-surface in $H = M^4 \times CP_2$.

The geometrization of classical gauge fields is based on the following identifications.

a) The classical gravitational field is identified as the induced metric. The still open question is whether the classical gravitational fields couple to matter with the gravitational constant $G \simeq kR^2$, $k \simeq 10^{-8}$, where R is CP_2 size (the length of CP_2 geodesic line). There is however an argument leading to a precise and correct prediction for k, and fixing the value of the Kähler coupling strength α_K at electron length scale to a value very near to that of the fine structure constant.

b) The geometrization of electro-weak gauge fields reduces to the curvature of CP_2 just like the geometrization of gravitation reduces to the curvature of the space-time surface. Classical electro-weak fields are identified as components of CP_2 spinor connection projected to the space-time surface. The holonomy group of CP_2 spinor connection is $U(2)$ and naturally identifiable as electro-weak gauge group.

c) Color symmetries correspond to the isometries of CP_2 so that there is deep and unexpected connection between electro-weak and color interactions. Color gauge potentials are identified in the spirit of Kaluza-Klein theory as projections of the Killing vector fields of color isometries to the space-time surface. Color gauge fields are of form $F^A_{\alpha\beta} \propto H^A \times J_{\alpha\beta}$, where H^A is the Hamiltonian of the color isometry and J denotes the induced Kähler form. Therefore the vacuum extremals of Kähler action carry also non-vanishing color gauge fields.

Also elementary particle quantum numbers can be understood in terms of the induced spinor structure and simple 3-topology.

a) CP_2 does not allow ordinary spinor structure and it is necessary to couple CP_2 spinors to the Kähler potential of CP_2. The couplings are different for different H-chiralities identifiable as leptonic and quark like spinors. Baryon and lepton numbers are separately conserved for both the ordinary massless Dirac action and modified Dirac action. The modified Dirac action is fixed uniquely by requiring that it has the vacuum degeneracy of Kähler action. The modified Dirac action allows local super-symmetries generated by the right-handed neutrino.

b) At the fundamental level color quantum numbers are not spin like quantum numbers but can be said to correspond to the color partial waves in CP_2 center of mass degrees of freedom of the 3-surface representing the elementary particle. Ordinary Dirac equation for CP_2 predicts wrong correlations between electro-weak and color quantum numbers of the color partial waves associated with the spinor harmonics. This was a longstanding problem of TGD approach but the construction of physical states as representations of the Super Kac Moody algebra allows to obtain correct correlations and an interpretation in terms of electro-weak symmetry breaking coded already into the CP_2 geometry.

c) The first guess was that the genus of the two-dimensional boundary associated with the 3-surface representing particle explains family replication phenomenon. The identification of the super-conformal symmetries as symmetries associated with light like effectively 2-dimensional 3-surfaces X^3_l

acting as causal determinants suggests a more concrete identification.

Quaternion conformal invariance allows to assign to X_l^3 a highly unique 2-dimensional surface X^2 as a surface at which superconformal structure reduces to ordinary conformal structure and thus becomes Abelian. The genus of this surface telling whether the surface is sphere, torus, etc... determines the particle family. X_l^3 could correspond to either a boundary of 3-surface or to an elementary particle horizon. Elementary particle horizon would surround the wormhole contact connecting CP_2 extremal with an Euclidian signature of the induced metric to a larger space-time sheet with a Minkowskian signature of metric. The induced metric is degenerate at the elementary particle horizon so that this surface is indeed metrically 2-dimensional.

More concretely, sphere, torus, and sphere with two handles would correspond to (e, ν_e), (μ, ν_μ), (τ, ν_τ) in the leptonic sector and and (u, d), (c, s), and (t, b) in the quark sector respectively. The experimental absence of heavier particle families would be most naturally due to the fact that they are extremely heavy. The 3 lowest particle families differ from the higher genera in the sense that 2-surfaces with genus $g < 3$ are always hyper-elliptic, that is they allow always Z_2 conformal symmetry, whereas higher genera generically do not allow any conformal symmetries. Hyper-ellipticity is an excellent candidate for an explanation of the lightness of $g < 3$ genera. The construction of elementary particle functionals as functionals in the conformal equivalence classes of the 2-surface X^2 associated with X_l^3 allows to formulate this argument more precisely.

The explanation of Cabibbo mixing as being due to the mixing of boundary topologies, and number theoretic arguments (complex rationality of CKM matrix) lead to a highly unique CKM matrix for quarks and also leptonic mixings can be fixed highly uniquely. Also bosons are predicted to possess family replication phenomenon.

4.2.2 The new physics associated with classical gauge fields

Long range electro-weak, in particular Z^0, vacuum gauge fields are unavoidable in TGD: this is a necessary outcome of the induced gauge field concept reducing the number of the primary bosonic field variables to four (CP_2 coordinates)! The interpretation of this puzzling prediction has been a long standing challenge of TGD. There are three alternative options to consider.

Option I: Classical gauge fields are space-time correlates for gauge bosons with mass scale determined by the p-adic length scale of the space-time sheet in question. The electro-weak charges of elementary particles are screened by vacuum gauge charges (possible in TGD) in a region of size L_W of order intermediate boson length scale. This option does not explain the presence of long range electro-weak gauge fields unavoidably present if the dimension of CP_2 projection of space-time sheet is higher than 2 nor classical color gauge fields present for non-vacuum extremals.

Option II: Electro-weak gauge charges are not screened in the length scale L_W and the gauge fluxes of elementary particles flow to larger space-time sheets via # throats within region of size L_W and elementary particles have the quantized values of em Z^0 charges. The problem for this option are anomalously large Rutherford cross sections in condensed matter and large parity breaking effects in hadronic, nuclear, and atomic length scales. Despite this I regarded this option as the most realistic one until the realization that the mysterious long ranged weak fields could be assigned to dark matter particles at various space-time sheets.

Option III: There is a hierarchy of color electro-weak physics such that weak bosons are massless below the p-adic length scale determining the mass scale of weak bosons. Classical long range gauge fields serve as space-time correlates for gauge bosons below the p-adic length scale in question.

The unavoidable long ranged electro-weak and color gauge fields are created by dark matter and dark particles can screen dark nuclear electro-weak charges below the weak scale above which vacuum screening occurs as for ordinary weak interactions. Dark gauge bosons are massless below the appropriate p-adic length scale but massive above it and $U(2)_{ew}$ is broken only in the fermionic sector. For dark copies of ordinary fermions masses are essentially identical with those of ordinary fermions.

This option is consistent with the standard elementary particle physics for visible matter apart from predictions such as the possibility of p-adically scaled up versions of ordinary quarks predicted to appear already in ordinary low energy hadron physics. The most interesting implications are seen in longer length scales. Dark quarks and gluons and a scaled up copy of ordinary gluons emerge already in ordinary nuclear physics [F8] and explain some recently discovered anomalies such as neutron halos and tetraneutron. The field bodies associated with are predicted to have sizes of order atom size. Also scaled down versions of weak bosons giving to interactions between exotic quarks with a range of order atomic length scale are predicted.

The new nuclear physics has deep implications for chemistry and condensed matter where color bonds between neighboring atoms might be part of the chemical bonding [F9]. Long ranged repulsive weak force behind exotic quarks compensated by color force would contribute to the repulsive force assumed in van der Waals equations of state for condensed matter. No strong isotopic dependence is predicted.

Classical long range weak and color forces become also key players at the level of molecular physics and biophysics. Chiral selection of bio-molecules can be seen as one direct signature of the long ranged weak force which suggests that non-broken $U(2)_{ew}$ symmetry and and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. The central role of the long ranged weak forces in bio-systems and in pre-biotic evolution is discussed in [TGDware, M3, TGDgeme].

Classical em fields and Z^0 fields are not invariant under color rotations acting as exact symmetries and are accompanied by classical color gauge fields. This implies new physics potentially important for TGD inspired theory of consciousness. For instance, in TGD Universe the original joke like term "quark color" inspired by certain algebraic similarities ceases to be a joke since it is possible to reduce the 3+3 primary colors in color vision to the 3+3 different increments of color quantum numbers induced by the absorption or emission of color octet gluon.

4.3 Many-sheeted space-time concept

The detailed study of TGD led to a further generalization of the space-time concept and the end result is what I have used to call topological condensate or many-sheeted space-time. The 3-space is many-sheeted such that the sheets of 3-space have finite size and outer boundary. The physical interpretation of a given space-time sheet of a finite size is as a 'particle'. Depending on their size, these particles correspond to elementary particles, nucleons, atomic nuclei, atoms, molecules, cells, ourselves, stars, galaxies, etc. For instance, my skin corresponds to the outer boundary of a 3-surface glued to a larger 3-surface identifiable as the room in which I sit! I am a small Universe glued to a larger one, the 3-space associated with me literally ends on my skin just as string ends at its end! The surface of earth, the outer surfaces of trees, etc...: everywhere I can see nontrivial 3-topology.

Important new physics is associated with the extremely tiny wormholes contacts with size of order CP_2 length needed to perform the gluing operation. Join along boundaries bonds serving as space-time correlates for the bound state formation is second important notion. The larger sheets of the many-sheeted space-time are ideal for carrying various macroscopic quantum phases. Topological field quantization allows to define precisely the notions of coherence and de-coherence and also means that one can assign to a given material system what might be called field body or magnetic body.

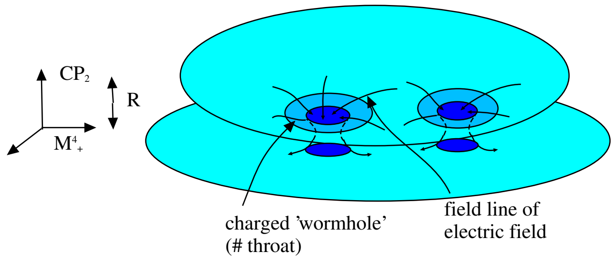

Figure 10: Charged wormholes feed the electromagnetic gauge flux to the 'lower' space-time sheet.

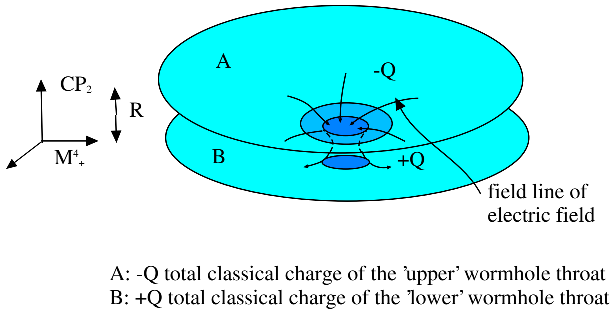

Figure 11: The two throats of wormhole behave as classical charges of opposite sign.

Obviously the outcome is a thorough-going generalization of the space-time concept and means that TGD has highly nontrivial consequences in all length scales rather than in particle physics only, as one might naively expect.

4.3.1 Join along boundaries contacts and join along boundaries condensate

The recipe for constructing TGD:eish 3-space is simple. Take 3-surfaces with boundaries, glue them by topological sum to larger 3-surfaces, glue these 3-surfaces in turn on even larger 3-surfaces, etc.. The smallest 3-surfaces correspond to CP_2 type extremals that is elementary particles and

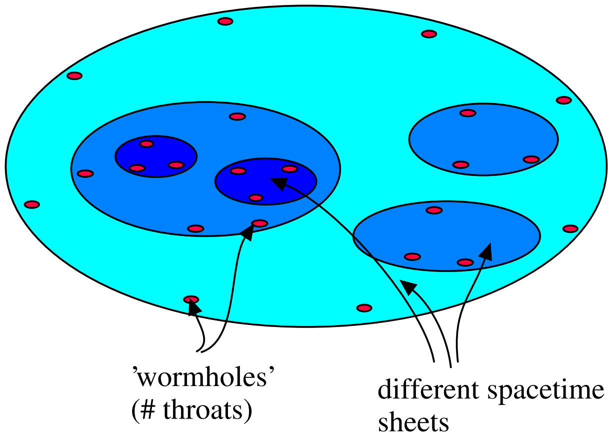

Figure 12: Many-sheeted space-time structure results from the requirement of gauge flux conservation.

they are at the top of hierarchy. In this manner You get quarks, hadrons, nuclei, atoms, molecules,... cells, organs, ..., stars, ..,galaxies, etc...

Besides this, one can also glue different 3-surfaces together by tubes connecting their *boundaries*: this is just connected sum operation for boundaries. Take disks D^2 on the boundaries of two objects and connect these disks by cylinder D^2xD1 having D^2:s as its ends. Or more concretely: let the two 3-surfaces just touch each other.

Depending on the scale join along boundaries bonds are identified as color flux tubes connecting quarks, bonds giving rise to strong binding between nucleons inside nuclei, bonds connecting neutrons inside neutron star, chemical bonds between atoms and molecules, gap junctions connecting cells, the bond which is formed when You touch table with Your finger, etc.

One can construct from a group of nearby disjoint 3-surfaces so called join along boundaries condensate by allowing them to touch each other here and there.

The formation of join along boundaries condensates creates clearly strong correlation between two quantum systems and it is assumed that the formation of join along boundaries condensate is necessary prerequisite for the formation of *macroscopic quantum systems*. Crucially important examples

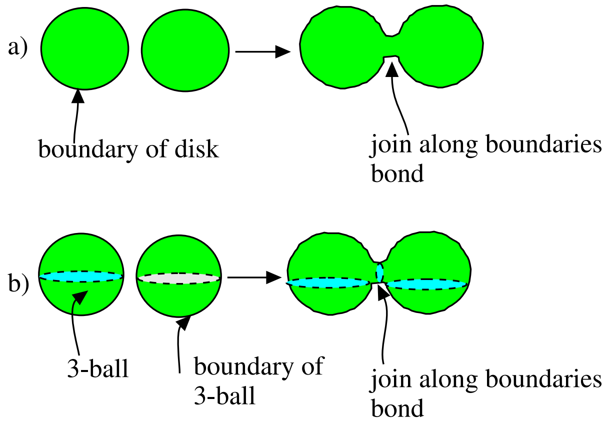

Figure 13: Join along boundaries bond a): in two dimensions and b): in 3-dimensions for solid balls.

in biology are gap junctions connecting cells and MAPs (micro-tubule associated proteins) connecting micro-tubules.

Quantum classical correspondence inspires the hypothesis that quite generally join along boundaries bonds are space-time correlates for the formation of the bound state entanglement. Since join along boundaries bonds between space-time sheets condensed on larger space-time sheets having no join along boundaries bonds between them is possible, one is forced to conclude that entanglement between subsystems of un-entangled systems is possible in the many-sheeted space-time. The paradox disappears when entanglement is understood as a length scale dependent notion so that the bound state entanglement of sub-systems is not visible in the length and time scales of the systems.

4.3.2 Wormhole contacts

The gauge and gravitational fluxes at the boundary of a given space-time sheet must go somewhere by gauge flux conservation. This forces the existence of a larger space-time sheet and of tiny wormhole contacts connecting the two space-time sheets and feeding the gauge fluxes from the smaller sheet to the larger one. Wormhole contacts (# contacts) are elementary particle like objects (actually deformed pieces of so called CP_2 type extremals) having size of order CP_2 size about 10^4 Planck lengths and, being sources and

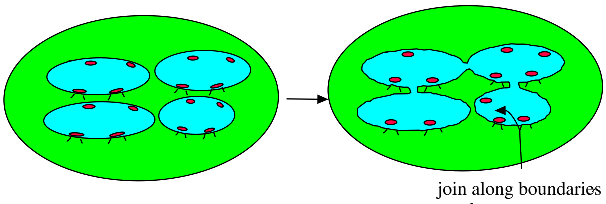

Figure 14: Join along boundaries condensate in 2 dimensions.

sinks of gauge field lines, wormhole throats effectively like classical charges, the charges of throats at the two space-time sheets being of opposite sign. Hence wormhole contacts look like dipoles and couple to the difference of the classical gauge potentials associated with the two space-time sheets. Also the coupling to the difference of the gauge potentials serving as order parameters for the coherent states of photons is possible.

The crucial experiment would be the one demonstrating the existence of the wormholes.

a) There are good reasons to expect that wormhole gauge flux is quantized. The reason for quantization would be the absolute minimization of Kähler action, which is mathematically a condition very similar to the Bohr's quantization condition. In the usual initial value problem one would fix only the imbedding space coordinates of 4-surface for given value of time and allow their time derivatives be arbitrary. Now absolute minimization fixes the values of the time derivatives just like Bohr's quantization rules fix the momenta. The most aesthetic possibility is that the unit of wormhole em charge is the smallest possible elementary particle charge 1/3 associated with d quarks but also integer charge could be considered.

b) If wormhole charge is quantized then the gauge flux of an external em field running from a larger space-time sheet to a smaller one is quantized. The experimental arrangement should demonstrate that this flux indeed can change by a multiple of the elementary flux only. One could also try to detect wormhole currents. It must be emphasized that wormhole current is a pseudo current in the sense that two space-time sheets carry opposite classical currents. These currents are created, when magnetic field penetrates from space-time sheet to another. The detection of charge 1/3 for the charge carriers of this current would be a triumph.

c) One cannot exclude the possibility that the recently found evidence for 1/3 charge in condensed matter systems (quantum Hall effect) could be interpreted in terms of an em gauge flux quantized in this manner. Electron current flowing inside a planar layer like structure is studied. Strong magnetic field, which could lead to a generation of wormhole currents is present! Evidence for some quasi particles in current flow possessing this charge has been found. The anyon interpretation of quasi particles as bound states of magnetic flux quanta and electrons explains the effect (McLaughlin wave function). The prediction is however that also fluxes of m/5, m/7,... , m integer, should be observed. Only 1/3 has been detected hitherto and it is not understood why higher charges have not been observed. The question is whether the quasi particles are actually wormholes created by the penetration of magnetic field and flowing along the boundaries of the arrangement.

One application of the new space-time concept is a model of brain. The basic idea is that brain can be regarded as a macroscopic quantum system and that our experiences of free will correspond to quantum jumps which are unpredictable as also is the end result of a free choice. The idea that quantum theory might provide some light in the problem of consciousness has become popular during the last years and a serious building of quantum theories of consciousness has begun. The bottleneck problem is how the brain can be a macroscopic quantum system. Some kind of super conductivity looks a promising idea but standard physics does not provide promising candidates for a super conductor type system. In TGD the situation is however different.

To see what is involved, consider in more precise manner how many sheeted 3-space is constructed. When one glues a sheet of 3-space to a larger sheet of 3-space one does it by constructing extremely tiny wormholes connecting the two sheets of 3-space.

These wormholes serve important function. For instance, the flux of the electric field (usually it is unlucky space traveller) flows to this kind of wormhole on the smaller sheet of 3-space and and comes back from it to the larger sheet of 3-space. Since the field lines of the electric field flow to the wormhole on the smaller sheet of 3-space, the wormhole looks like a charge since it acts as a sink of field lines. Same applies on the larger sheet of 3-space except that the sign of the charge is opposite. Hence, on both space-time sheets wormhole looks classically like a charged particle. Shortly, wormholes behave like particles and represent a new exotic form of matter. More generally, it seems that many-sheeted nature of the space-time is crucial for the understanding of a bio-system as a macroscopic quantum system.

The interaction between space-time sheets is mediated by the the extremely tiny "wormholes" having size of order CP_2 radius R and located near the boundaries of the smaller space-time sheet. Wormholes feed various gauge fluxes from the smaller space-time sheet to the larger one (say from the atomic sheet to some molecular sheet). p-Adic considerations suggest that wormholes are light having mass of order $1/L_p$: this implies that they suffer Bose-Einstein condensation on the ground state. One could even say that space-time sheets "perceive" the external world and act on it with the help of the charged wormhole BE condensates near their boundaries. Wormholes provide a very general mechanism making possible the transfer of classical electromagnetic fields and various quantum numbers such as energy, momentum and angular momentum, between different space-time sheets and bio-systems are especially promising as far as applications are considered.

4.3.3 Topological field quantization

Topological field quantization [D7] implies that various notions of quantum field theory have rather precise classical analogies. Topological field quantization is basically implied by the compactness of CP_2, which typically implies that a given Maxwell field allows only a partial imbedding as a space-time surface in H. One can say that magnetic fields, electric fields and radiation fields decompose into field quanta.

The energies and other classical charges of the topological field quanta are quantized by the absolute minimization of the Kähler action making classical space-time surfaces the counterparts of the Bohr orbits. Feynman diagrams become classical space-time surfaces with lines thickened to 4-manifolds. For instance, "massless extremals" representing topologically quantized classical radiation fields are the classical counterparts of gravitinos and photons. Topologically quantized non-radiative nearby fields give rise to various geometric structures such as magnetic and electric flux tubes.

Topological field quantization provides the correspondence between the abstract Fock space description of elementary particles and the description of the elementary particles as concrete geometric objects detected in the laboratory. In standard quantum field theory this kind of correspondence is lacking since classical fields are regarded as a phenomenological concept only.

Topological field quanta define regions of coherence for the classical gauge fields and induced spinor fields and classical coherence is the prerequisite of the quantum coherence. Whether and how macroscopic and macro-temporal quantum coherence are possible in living matter is the basic question of quantum consciousness theories and quantum biology. In TGD this question is even more difficult since the first estimate for de-coherence time is CP_2 time which is about 10^4 Planck times. The length scale hierarchy of space-time sheets allows immediately to understand at the level of space-time correlates how macroscopic and macro-temporal quantum coherence are possible. A good order of magnitude guess for the zero point energy of a particle at a space-time sheet of size L is given by $E = \pi^2/2mL^2$. $T \leq \pi^2/2mL^2$ gives an estimate for the temperature of the space-time sheet populated by particles of mass m: the larger the size of the space-time sheet, the lower the temperature. Superconductivity and various macroscopic phenomena become thus possible at larger space-time sheets. TGD based model of living matter is based on the hypothesis that large space-time sheets are responsible for quantum control.

The virtual particles of quantum field theory have also classical counterparts. In particular, the virtual particles of quantum field theory can have negative energies: this is true also for the TGD counterparts of the virtual particles. The fundamental difference between TGD and GRT is that in TGD the sign of energy depends on the time orientation of the space-time sheet: this is due to the fact that in TGD energy current is vector field rather than part of tensor field. Therefore space-time sheets with negative energies are possible. This could have quite dramatic technological consequences: consider only the possibility of generating energy from vacuum and classical signalling backwards in time along negative energy space-time sheets [G1]. Also bio-systems might have invented negative energy space-time sheets: in fact, so called "massless extremals" provide an ideal manner to generate coherent motions as recoil effects caused by the creation of negative energy massless extremals [I5].

Quantum classical correspondence suggests that quantum entanglement has the formation of the join along boundaries bonds as its geometric correlate. The superposition of the topologically quantized space-time surfaces in the state $U\Psi$ could be regarded as a geometric correlate for quantum fields: creation/annihilation operators would correspond to positive/negative energy space-time sheets. This hypothesis, together with the expansion of the interacting quantum field in terms of creation and annihilation operators, would make it possible to make quantitative estimates about the fraction of energy density carried by the negative energy space-time sheets, in particular, about the energy density associated with the massless extremals.

In TGD Universe topological field quanta serve as templates for the formation of the bio-structures. Thus topologically quantized classical electromagnetic fields associated with the material objects, field bodies or more concretely, magnetic bodies, could be equally important for the functioning of the living systems as the structures formed by the visible bio-matter and the visible part of bio-system might represent only a dip of an ice berg. In the book "Genes, Memes, Qualia, and Semitrance" the implications of the notion of field body for the understanding of bio-systems and pre-biotic evolution are discussed in detail.

4.3.4 Negative energy space-time sheets and new view about energy

Negative energy space-time sheets represents an important distinction between TGD and standard physics. They are possible because energy momentum tensor is replaced by a collection of conserved currents associated with various components of four momentum. This resolves the energy problem of general relativity but, since the sign of the conserved charged depends on the time orientation of the space-time sheet, the sign of energy is not positive definite anymore.

Quantum classical correspondence implies that also elementary particles can have negative energies and this means a new kind of physics. It seems that this physics has been already discovered: the strange properties of phase conjugate laser waves can be understood if they consist of negative energy photons.

Negative energy space-time sheets have far reaching implications for TGD inspired theory of consciousness. The so called time mirror mechanism involves the reflection of negative energy signals sent to the geometric past from population inverted lasers as amplified positive energy signals propagating to the geometric future. Time mirror mechanism provides the holy grail to the understanding of the mechanisms of brain functioning and also of the workings of the living matter. There are obvious implications for communication and energy technologies since negative energy signals could make possible instantaneous remote sensing and quantum control over arbitrarily long distances so that light velocity would cease to be a restriction forcing us to be habitants of 3-space instead of space-time.

If Kähler action were strictly deterministic, the only possible choice for H would be $M^4_+ \times CP_2$. Together with negative energies the classical non-determinism of the Kähler action however means that one cannot exclude the possibility that imbedding space is $M^4 \times CP_2$ meaning exact Poincare invariance. The point is that generation of pairs of positive and negative energy space-time sheet at light-like 7-surfaces $X^3_l \times CP_2$ means emergence of new kind of causal determinants generalizing the light cone boundary $\delta M^4_+ \times CP_2$ as a fundamental causal determinant. All states of the Universe have vanishing net quantum numbers and everything in the Universe would have been pair-created from vacuum. Future light cones containing positive energy could also be created when negative energy radiation (in particular gravitons) is generated and propagates to the geometric past and leaks from the future light cone. This vision can be applied also to the second quantization of fermions by giving fermions and anti-fermions opposite energies. Depending on time orientation either fermions or anti-fermions have negative energy.

By crossing symmetry the assumption that the net quantum numbers of the Universe vanish is not in conflict with elementary particle physics. In macroscopic length scales the identification of the gravitational energy as the difference of inertial (Poincare) energies of positive and negative energy matter plus the possibility that negative and positive energy matter interact weakly allows to understand why western view about objective reality with conserved positive total energy is so good an approximation. The non-conservation of the gravitational energy can be understood, and vacuum extremals, of which Robertson-Walker cosmologies, are most important examples find interpretation. The non-determinism of Kähler action explains naturally the fact that Universe is to some extended product of engineering. The notion of gravitational energy generalizes to that of gravitational quantum numbers and the inertial-gravitational dichotomy is a direct correlate for the geometric-subjective dichotomy for time discovered while developing TGD inspired theory of consciousness. Indeed, positive and negative energy space-time sheets correspond to initial and final states of quantum jump so that gravitational quantum numbers characterize changes.

This vision would resolve the unpleasant philosophical questions like "What is the total fermion number of the Universe". One could see entire universe as a result of intentional actions in which intentions represented by p-adic space-time sheets are transformed to actions represented by real space-time sheets. Everyone knows the anecdotes about yogis and gurus creating material objects from nothing and very few "scientifically thinking" westerner can take these stories really seriously. Whether or not these stories are true, they might however express a deep truth about reality.

4.3.5 More precise view about topological condensate

The challenge is to define precisely the concepts like classical gauge charge, gauge flux, wormhole contacts, join along boundaries bonds, topological condensation and evaporation, etc... Number theoretical vision allows to achieve this goal [F6].

The crucial ingredients in the model are so called CP_2 type extremals. The realization that # contacts (topological sum contacts and $\#_B$ contacts (join along boundaries bonds) are accompanied by causal horizons which carry quantum numbers and allow identification as partons leads to a more detailed articulation of these notions.

The partons associated with topologically condensed CP_2 type extremals carry elementary particle vacuum numbers whereas the parton pairs associated with # contacts connecting two space-time sheets with Minkowskian signature of induced metric define parton pairs. These parton pairs do not correspond to ordinary elementary particles. Gauge fluxes through # contacts can be identified as gauge charges of the partons. Gauge fluxes between space-time sheets can be transferred through # and $\#_B$ contacts concentrated near the boundaries of the smaller space-time sheet. The dynamics of topological condensation and evaporation can be formulated in terms of gauge interactions of partons and splitting and fusion of CP_2 type extremals. This picture generalizes to the case of gravitational flux which need not be well-defined purely classically.

Number theoretical vision and p-adic length scale hypothesis allow to quantify this picture and lead to an overall view about interactions of particles in many-sheeted space-time. A far reaching generalization of standard physics results predicting an infinite hierarchy of dark matters besides ordinary elementary particles of standard model. In particular, the partons associated with # and $\#_B$ contacts represent dark matter.

4.4 Classical non-determinism of Kähler action

The classical non-determinism of Kähler action has been deep source of inspiration and challenges and guided the evolution of TGD inspired theory of consciousness and finally also of quantum TGD proper. In nut-shell, classical non-determinism makes possible quantum-classical correspondence in the sense that space-time surface becomes a symbolic representation for the quantum states and quantum jump sequences defining conscious experience.

4.4.1 Matter-mind duality geometrically

The non-determinism of Kähler action implies huge vacuum degeneracy: any 4-surface whose projection belongs to $M^4_+ \times Y^2$, where Y^2 is so called Legendre manifold, is vacuum extremal. This suggests that one must radically generalize the concept of space-time. It seems that the correct picture is roughly like follows. Space-time is many-sheeted. Each sheet can be regarded as a slightly deformed piece of M^4 in H containing smaller sheets glued to it and being itself glued to a larger space-time sheet. Gluing means the formation of topological sum contacts between the space-time sheets. There are reasons to believe that topological sum contacts, "wormholes" are located near the boundaries of the smaller space-time sheet.

Material space-time sheets have infinitely long time duration if they possess non-vanishing energy (and provided that they do feed their energy to some other space-time sheets). "Mind like" space-time sheets can be regarded as obtained by gluing space-time sheets with finite time duration to material space-time sheets. The gluing operation implies that tiny amounts of energy and momenta and other conserved quantities flow to the mind like space-time sheet when it begins and back to the material space-time sheets when mind like space-time sheet ends. Mind like space-time sheets are space-time correlates for contents of consciousness. In particular, they form symbolic representations for material space-time sheets. For instances, the frequencies of various oscillatory processes are mapped also to frequencies of processes occurring in mind like space-time sheets. The possibility of mind like space-time sheets implies that the absolute minima of Kähler action are degenerate: one can glue mind like space-time sheets to given absolute minimum to get new absolute minima. This conforms with the fact that contents of consciousness are defined by a sequence of non-deterministic quantum jumps.

This picture must of course be taken with strong reservations, and one should actually state more precisely what "mind like" means. The interpretation of p-adic space-time sheets as correlates of intentions and cognitions gives some ideas about what aspects of consciousness mind like space-time sheets correlate with. The model for how intentions are realized as actions in quantum jump assumes that p-adic "topological light rays" representing intentions are transformed to real topological light rays with negative energy serving as correlates of desires, which in turn induce the action initiated in the geometric past. Thus it would seem that real "mind like" space-time sheets with negative energy would serve as correlates for desires.

4.4.2 Association sequence concept and a mind like space-time sheets

The vacuum degeneracy of the Kähler action defining quantum TGD solves the difficulty. The vacuum degeneracy implies spin glass analogy and strongly suggests that the effective space-time surface defined as the absolute minimum of the effective action and going through a given space-like 3-surface, cannot be unique in general. To achieve uniqueness one must generalize the concept of 3-surface. In order to specify uniquely one of the degenerate absolute minimum space-times going through a given 3-surface one must fix some minimum number, say N, of 3-surfaces on a given absolute minima. These sequences of disjoint 3-surfaces with time-like separations can be regarded as a simulations of the classical time development and hence as a geometric correlate of conscious experience localized temporally. It seems that in real case geometric correlates of sensory experiences are in question whereas in p-adic case correlates of thoughts are in question.

Association sequences are ver probably not all that is needed to overcome the complications caused by the non-determinism of Kähler action. The enormous vacuum degeneracy of Kähler action suggests strongly that the classical non-determinism does not reduce to simple sequences of bifurcations. Hence it seems that must give up the idea of identifying space-like 3-surfaces given value of geometric time as causal determinants which are possibly degenerate because of the bifurcations.

4.4.3 Vacuum degeneracy and spin glass analogy

Kähler action determines configuration space geometry and is hence a cornerstone of quantum TGD. Kähler action can be regarded as a Maxwell action for the Kähler form of CP_2 induced to space-time surface and defining nonlinear Maxwell field. Kähler action possesses enormous vacuum degeneracy. Any space-time surface in $M^4_+ \times CP_2$, where Y^2 is so called Legendre

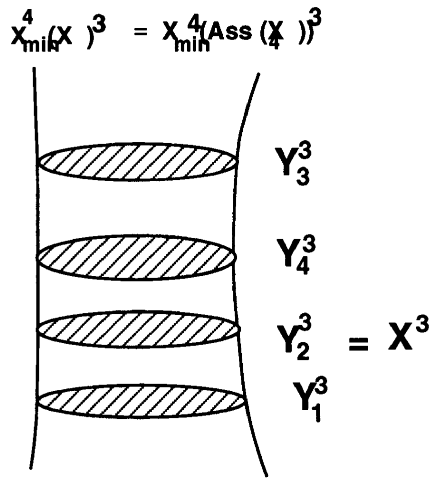

Figure 15: 'Association sequence': a geometric model for thought as a sequence of disjoint 3-surfaces with time-like separations.

sub-manifold of CP_2 having by definition vanishing induced Kähler form, is vacuum extremal. In canonical coordinates (P_i, Q_i) for CP_2 Legendre sub-manifolds correspond to functions

$$P_i = \nabla_i f(Q_j) \ .$$

This means that there is infinite number of vacuum sectors since all 4-surfaces in any six-dimensional space $M^4_+ \times Y^2$ are vacua.

Also non-vacuum configurations are almost degenerate. Only the gravitational effects caused by the presence of the induced metric in the Maxwell action for the induced Kähler form of CP_2 on space-time surface breaks the canonical invariance of the Kähler action. Canonical transformations of CP_2 act as $U(1)$ gauge transformations and in the absence of gravitation one would have ordinary $U(1)$ gauge invariance. Gravitation however changes the situation. Various canonically related configurations are physically *non-equivalent*. This means a characteristic degeneracy analogous to the degeneracy of the states for spin glass rather than to the physically uninteresting gauge degeneracy. The effective breaking of $U(1)$ gauge invariance makes possible vacuum charge densities, scalar wave pulses propagating with light velocity and carrying longitudinal electric field parallel to the propagation direction, and topological light rays carrying light like vacuum current and transversal electric and magnetic fields are predicted.

Contrary to the original beliefs, p-adic physics does not seem to follow from vacuum degeneracy alone. Rather, p-adic space-time topology is a genuine rather than effective space-time topology and emerges independently from the vacuum degeneracy. p-Adic topology seems however to serve as effective topology for the real space-time sheets in the sense that the non-determinism implied by the vacuum degeneracy mimics the inherent non-determinism of p-adic field equations for some value of p so that one can indeed assign a definite p-adic prime to a given real space-time sheet. Vacuum degeneracy has a wide spectrum of implications. For instance, the spin glass degeneracy implied by it allows to understand at quantum level generation of macroscopic and macro-temporal quantum coherence. The same mechanism explains also color confinement.

4.4.4 Connection with catastrophe theory and Haken's theory of self-organization for spin glass

If the effects related to the induced metric (classical gravitation) are neglected, canonical transformations of CP_2 act as $U(1)$ gauge symmetries and all canonically related surfaces are physically equivalent. Classical gravitation however breaks this gauge invariance but due to the extreme weakness of the gravitational interaction one has good reasons to expect that the maxima of Kähler function for given values of the zero modes are highly degenerate. The hypothesis that single maximum of Kähler function with respect to fiber degrees of freedom is selected in quantum jump, means huge simplification of the mathematical theory.

Besides the degeneracy resulting from the non-determinism, there is also the spin glass degeneracy related to zero modes. The nonphysical $U(1)$ gauge degeneracy is transformed to physical spin glass degeneracy. The energies of various absolute minima differ only by the classical gravitational energy. Zero modes serve as coordinates for the "energy" landscape of quantum spin glass and the energy landscape of nonequilibrium thermodynamics is fractal containing valleys inside valleys...inside valleys.

One naturally ends up with a generalization of the catastrophe theory [4] to the infinite-dimensional configuration space context. Zero modes play the role of the control parameters forming master slave-hierarchy and non-zero modes characterizing various degenerate absolute minima of Kähler action correspond to the state variables [I1]. There is natural connection with the non-equilibrium thermodynamics of Haken [8]. Since time development by quantum jumps means hopping in the zero modes characterizing the macroscopic space-time surfaces associated with the final states of the quantum jumps, Haken's classical theory applies almost as such. Asymptotically the self-organizing quantum jumping system (self) ends up to a fixed point, limiting cycle, strange attractor, etc. near the bottom of some valley of the energy landscape. The bottom of a valley corresponds to a maximum of the Kähler function rather than minimum of free energy as in thermodynamics since vacuum functional is exponent of Kähler function. Self-organization in spin glass energy landscape by quantum jumps is extremely powerful notion allowing to understand general features of living systems.

5 Evolution of p-adic ideas

It took quite a long time to end up with the recent picture how p-adic numbers emerge as a basic aspect of quantum TGD and what p-adicization of TGD might mean. Of course, recent picture need not be the final yet and there are several unsolved problems. In the following the basic properties of the p-adic numbers are described shortly and then it is demonstrated how p-adic numbers might emerge from TGD and how one should formulate p-adic version of quantum TGD formalism.

5.1 p-Adic numbers

Like real numbers, p-adic numbers can be regarded as completions of the rational numbers to a larger number field allowing the generalization of differential calculus. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. This means that the distance function $d(x,y)$ (the counterpart of $|x-y|$ in the real context) satisfies the inequality

$$d(x,z) \leq Max\{d(x,y), d(y,z)\} \ ,$$

(Max(a,b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x,z) \leq d(x,y) + d(y,z) \ .$$

p-Adic numbers have expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n \geq 0} x_n p^n \ ,$$

and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x = \sum_{n \geq 0} x_n p^n) = p^{-n_0} \ ,$$

and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.

p-Adic numbers allow the generalization of the differential calculus and of the concept of analytic function $f(x) = \sum f_n x^n$. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which depend on a finite number of positive pinary digits of x only so that one has

$$f_N(x = \sum_n x_n p^n) = f(x_N = \sum_{n<N} x_n p^n) \ .$$

In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic.

An essential element is the map of the p-adic numbers to the positive real numbers by the so called canonical identification I:

$$I : \sum x_n p^n \in R_p \rightarrow \sum_n x_n p^{-n} \in R \ .$$

Canonical identification makes it possible to map the predictions of the p-adic probability theory and thermodynamics to real numbers. Canonical identification cannot map real space-time surface to p-adic ones or vice versa because it is not a general coordinate invariant notion. Canonical identification has inverse, which is single valued for the real numbers having infinite number of pinary digits but two-valued for real numbers having finite number of pinary digits (the reason is that real number with finite number or pinary digits has two equivalent pinary expansions: ($x = 1 = .999999...$ in case of decimal expansion and $x = 1 = 0yyyy...$, $y = p-1$, in the case of pinary expansion).

5.2 Evolution of physical ideas

In the sequel the evolution of physical ideas related to p-adic numbers is summarized.

5.2.1 p-Adic length scale hypothesis

p-Adic length scale hypothesis [TGDpad] states that to a given p-adic prime p there corresponds a primary p-adic length scale $L_p = \sqrt{p}l$, $l \simeq 1.288 \times 10^4\sqrt{G}$ ($\sqrt{G}$ denotes Planck length) and that physically favored primes correspond to $p \simeq 2^k$, k power of prime. The corresponding p-adic time scale is obtained as $T_p = L_p/c$. The justification for the first part of the hypothesis comes from Uncertainty Principle and from the p-adic mass calculations predicting that the mass of elementary particle, resulting from the mixing of massless states with $10^{-4}m_{Planck}$ mass states described by p-adic thermodynamics, is of order $1/L_p$ for the light states.

The justification for the preferred values of p comes from elementary particle black hole analogy [E5] generalizing the Bekenstein-Hawking area-entropy law to apply to the elementary particle horizon defined as the surface at which the Euclidian signature for the so called CP_2 type extremal describing elementary particle changes to the Minkowskian signature of the background space-time at which elementary particle has suffered topological condensation.

The hypothesis is especially interesting above the elementary particle length scales $p > M_{127}$ and has testable implications in nuclear physics, atomic physics and condensed matter length scales. The most convincing support for this hypothesis are provided by the elementary particle mass calculations: if one assumes that the p-adic primes associated with elementary particles are primes near prime powers of two, one can predict lepton and gauge boson masses with accuracy better than one per cent. Also quark masses can be predicted but the calculation of the hadron masses requires some modelling (CKM matrix, color force, etc...). The existing empirical information about neutrino mass squared differences suggests that the allowed values of k are indeed *powers* of prime rather than primes.

It is natural to postulate that space-time sheets form a hierarchy with respect to p in the sense that the lower bound for the size of the space-time sheets at level p is of order L_p and that $p_1 < p_2$ sheets condensed on p_2 sheets behave like particles on sheet p_2.

The following table lists the p-adic length scales L_p, $p \simeq 2^k$, k power or prime, which might be interesting as far as condensed matter is considered (the notation $L(k)$ will be used instead of L_p). It must be emphasized that the definition of the length scale is bound to contain some unknown numerical factor K: the requirement that the thickness of cell membrane corresponds to $L(151)$ fixes the proportionality coefficient K to $K \simeq 1.1$.

k	127	131	137	139	149
$L_p/10^{-10}m$	.025	.1	.8	1.6	50
k	151	157	163	167	169
$L_p/10^{-8}m$	1	8	64	256	512
k	173	179	181	191	193
$L_p/10^{-4}m$	.2	1.6	3.2	100	200
k	197	199	211	223	227
L_p/m	.08	.16	10	640	2560

Table 1. Primary p-adic length scales $L_p = 2^{k-151}L_{151}$, $p \simeq 2^k$, k prime, possibly relevant to bio physics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L(151)$ is take to be thickness of cell scale, which is 10^{-8} meters in good approximation.

The assumption that p-adic space-time regions provide cognitive representations of the real space-time regions forces to conclude that cognition is present in all length scales and that the properties of the p-adic space-time regions reflect those of the real space-time regions. p-adic–real phase transitions occurring even at elementary particle length scales would explain this elegantly.

Besides primary p-adic length scales also n-ary p-adic length scales defined as $L_p(n) = p^{(n-1)/2}L_p$ and corresponding time scales are possible and form a fractal hierarchy coming as powers of $\sqrt{p}$. Accepting these scales means that all length scales $L(n)$ coming as powers of $2^{n/2}$, n a positive integer, should have a preferred physical role. The TGD inspired model for living matter lends support for the hypothesis that biologically important length and time scales indeed appear as half octaves. A possible explanation for this is the existence of a hierarchy of cognitive codes associated with the time scales $T(n)$. Any prime power factor k^i in the decomposition of the integer n to a product of prime power factors defines a candidate for a cognitive code. The duration of code word would be $T(n)$ and the number of bits would be k^i. For prime values of n the information content of the code word is maximal so that one could understand why prime values of n

are especially important.

5.2.2 CP_2 type extremals and elementary particle black hole analogy

CP_2 type extremals are vacuum extremals having a finite negative action so that one can lower the action of the ordinary vacuum extremals by gluing CP_2 type extremals to them. CP_2 type extremals have one-dimensional M^4_+ projection which is light like random curve. Light likeness condition leads to classical Virasoro algebra constraints. $M^4 \times SO(3,1) \times SU(3) \times SU(2)_{ew}$ Super-Kac-Moody algebra acts as symmetries and the spectrum of elementary particles is precisely known. The obvious interpretation of the CP_2 type extremals is as a model of elementary particle.

CP_2 extremals are much like black holes in the sense that they possess elementary particle horizon: this is the surface at which the Euclidian signature of the metric of the CP_2 type extremal changes to the Minkowskian signature of the bacground space-time. One can indeed generalize Bekenstein-Hawking law to a statement saying that the real counterpart of the p-adic entropy predicted by the p-adic thermodynamics is proportional to the surface area of the elementary particle horizon. In particular, for primes $p \sim 2^k$, where k is power of prime, the radius of the elementary particle horizon is itself a p-adic length scale. This suggests a double p-adicization associated with p and k and an additional cognitive degeneracy due to the k-adic non-determinism, and hence also the dominance of the final states of quantum jump for which $p \simeq 2^k$ holds true: there would be simply very many physically equivalent physical states for these values of p.

5.2.3 p-Adic thermodynamics and particle massivation

The underlying idea of TGD based description of particle massivation is following. Due to the interaction of a topologically condensed 3-surface describing elementary particle with the background space-time, massless ground states are thermally mixed with the excitations with mass of order $m_0 \sim 1/R$ (R is CP_2 length scale, $1/R$ of order 10^{-4} Planck masses) created by the Super Virasoro generators. Instead of energy, the Virasoro generator L_0 (essentially mass squared) is thermalized. This guarantees Lorentz invariance automatically. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

Optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The calculations support the existence of massless gluons and electro-weak quanta associated with so called massless extremals (MEs). One important prediction is that p-adic thermodynamics cannot explain the masses of the intermediate gauge bosons although the predictions for the fermion masses are excellent. This observation led to the identification of the TGD counterpart of Higgs field whose vacuum expectation provides the dominating contribution to the bosonic masses and only shifts bosonic masses.

5.2.4 p-Adic coupling constant evolution

The original hypothesis was that Kähler coupling strength α_K is completely fixed by quantum criticality implying that α_K is analogous to critical temperature. p-Adic considerations led to the view that there is infinite number of critical values of α_K labelled by p-adic primes. In many-sheeted space-time one can indeed consider the possibility that α_K is not a universal constant. This would mean that space-time sheets joined only by wormhole contacts and surrounded by light like elementary particle horizons would be characterized by different values of Kähler coupling strength.

Since p-adic primes correspond to p-adic length scales this inspires the idea that the ordinary coupling constant evolution is replaced by a discrete coupling constant evolution. This view is also consistent with the criticality of the Kähler coupling constant. The assumption that gravitational constant is invariant under p-adic coupling constant evolution fixes highly unique the evolution of Kähler coupling strength. This picture makes sense if one can assign to a given 3-surface a unique p-adic prime and there are good reasons to believe that this is indeed the case.

5.2.5 Vacuum degeneracy of the Kähler action and spin glass analogy

The space of minima of free energy for spin glass is known to have ultra-metric topology. p-Adic topology is also ultra-metric and this motivated the hypothesis that quantum average space-time, 'topological condensate', defined as a maximum of Kähler function can be obtained by gluing together regions characterized by various values of the p-adic prime p. It must be emphasized that this hypothesis is just a guess and not even correct as such, and it seems that TGD as a generalized number theory vision gives the real justification for the p-adics. A good guess is however that the ultra-metric topology of the reduced configuration space consisting of the maxima of the Kähler function is induced from the p-adic norm and that there is a close connection between the two p-adicities. The following arguments tries to make this idea more precise.

The unique feature of the Kähler action is its enormous vacuum degeneracy: any space-time surface, whose CP_2 projection is a so called Legendre manifold (having dimension $D \leq 2$) is vacuum extremal. This is expected to imply a large degeneracy of the absolute minimum space-times: for instance, several absolute minima with the same action are possible for single 3-surface (this forces to a generalization of space-time concept obtained by introducing 'association sequences'). The degeneracy means an obvious analogy with the spin glass phase characterized by 'frustration' implying a large number of degenerate ground states. In the construction of the configuration space geometry the analogy between quantum TGD and spin glass becomes precise.

Spin glass consists of magnetized regions such that the direction of the magnetization varies randomly in the spatial degrees of freedom but is frozen in time. What is peculiar that, although there are large gradients on the boundaries of the regions with a definite direction of magnetization, no large surface energies are generated. An obvious p-adic explanation suggests itself: p-adic magnetization could be pseudo constant and hence piecewise constant with a vanishing derivative on the boundaries of the magnetized regions so that no p-adic surface energy would be generated.

In the description of the spin glass phase also ultra-metricity, which is the basic property of the p-adic topology, emerges in a natural manner. The energy landscape describing the free energy of spin glass as a function of various parameters characterizing spin glass, is fractal like function and there are infinite number of energy minima. In this case there is a standard manner to endow the space of the free energy minima with an ultra-metric topology [9].

The counterpart of the energy landscape in TGD can be constructed as follows. The configuration space of TGD (the space of 3-surfaces in H) has fiber-space like structure deriving from the decomposition $CH = \cup_{zeromodes} G/H$. The fiber is the coset space G/H such that G is the group of the canonical transformation of the light cone boundary. In particular, the canonical transformations of CP_2 act in the fiber as isometries. The base space is the infinite-dimensional space of the zero modes characterizing the size and shape as well as the classical Kähler field at the 3-surface.

To calculate S-matrix element, one must form Fock space inner product as a functional of 3-surface X^3 multiplied with the vacuum functional $exp(K)$ and integrate it over the entire configuration space:

$$S_{i\to f} = \int \langle \Psi_f, \Psi_i \rangle (X^3) exp(K(X^3)\sqrt{G} DX^3 \ .$$

The integration over the fiber degrees of freedom reduces to a Gaussian integration around the maxima of the Kähler function with respect to the fiber coordinates. The equally poorly defined Gaussian and metric determinants cancel each other in this integration and one obtains a well defined end result. Canonical transformations are 'almost gauge symmetries' since only classical gravitational fields destroy canonical symmetries acting as $U(1)$ gauge transformations. This means that the action for several canonically related configurations can be degenerate and several maxima are expected for given values of the zero modes. This means that the subset CH_0 of the configuration space consisting of the maxima of the Kähler function has many sheets parameterized by the zero modes and that generalized catastrophe theory is obtained.

If a localization in the zero modes occurs in the quantum jump, one can circumvent the integration over the zero modes in practice. The exponent for the maximum of the Kähler action is expected to have maxima as a function of the zero modes too. The maxima of $exp(K_{max})$ as function of zero modes define the counterpart of the energy landscape and $exp(K_{max})$ is the counterpart of the energy serving as a height function of the energy landscape. It could quite well be that this height function can be induced from a p-adic norm. If so, the allowed values of p define a decomposition of the space of zero modes to sectors D_p. For 'full' CP_2 type extremals representing virtual gravitons the exponent is indeed proportional to $1/p$ if one takes seriously the argument determining the possible values of the Kähler coupling strength. Thus cognitive p-adicity and spin glass p-adicity would be related to each other. The connection with gravitons is especially interesting since also classical gravitation is closely related to the spin glass degeneracy.

5.3 Evolution of mathematical ideas

The evolution of mathematical ideas has been driven by the following frequently asked questions.

a) Is p-adicity realized at space-time level or only at the level of p-adic thermodynamics which was the first application of p-adic numbers? If p-adic space-time regions really make sense, what is their physical interpretation?

b) Physics seems to require correspondence between p-adic and real numbers. What is the role of canonical identification: does it only map p-adic probabilities to their real counterparts or could it be applied also at space-time level despite the obvious difficulties with general coordinate invariance? What about correspondence defined by rational numbers which can be regarded as numbers common to all number fields. Is it possible to assign to a real space-time surface a p-adic counterpart by procedure respecting general coordinate invariance?

c) Does the notion of p-adicization of real physics make sense? How one might achieve the p-adicization in general coordinate invariance manner? What should one p-adicize: only probability calculus and thermodynamics? Or should one include also Hilbert space level? What about p-adicization at space-time level and perhaps even configuration space-level?

d) What is the origin of p-adicity? What is the origin of p-adic length scale hypothesis? How it is possible to assign p-adic prime to a given real space-time sheet as required by the p-adic mass calculations?

e) There have been also technical problems. Besides differential calculus also integral calculus is basic element of classical physics since all variational principles involve integrals over space-time. Also the functional integral over configuration space is needed in order to define S-matrix elements. How one could circumvent the difficulties caused by the non-existence of a p-adic valued define integral based on Riemann sum.

5.3.1 p-Adic physics as physics of cognition and intentionality and generalization of number concept

The identification of p-adic physics as physics of cognition and intention suggests strongly connections between cognition, intentionality, and number theory. The new idea is that also real transcendental numbers can appear in the extensions of p-adic numbers which must be assumed to be finite-dimensional at least in the case of human cognition.

The basic ingredient is the new view about numbers: real and p-adic number fields are glued together like pages of a book along common rationals representing the rim of the book. Also the rational multiples of algebraic numbers existing p-adically are shared in this manner so that the pages of the book can be stuck together along these lines. This generalizes to the extensions of p-adic number fields and the outcome is a complex fractal book like structure containing books within books. This holds true also for manifolds and one ends up to the view about many-sheeted space-time realized as 4-surface in 8-D generalized imbedding space and containing both real and p-adic space-time sheets. The transformation of intention to action corresponds to a quantum jump in which p-adic space-time sheet is replaced with a real one.

One implication is that the rationals having short distance p-adically are very far away in the real sense. This implies that p-adically short temporal and spatial distances correspond to long real distances and that the evolution of cognition proceeds from long to short temporal and spatial scales whereas material evolution proceeds from short to long scales. Together with p-adic non-determinism due the fact that the integration constants of p-adic differential equations are piecewise constant functions this explains the long range temporal correlations and apparent local randomness of intentional behavior. The failure of the real statistics and its replacement by p-adic fractal statistics for time series defined by varying number N of measurements performed during a fixed time interval T allows very general tests for whether the system is intentional and what is the p-adic prime p characterizing the "intelligence quotient" of the system. The replacement of $log(p_n)$ in the formula $S = -\sum_n p_n log(p_n)$ of Shannon entropy with the logarithm of the p-adic norm $|p_n|_p$ of the rational valued probability allows to define a hierarchy of number theoretic information measures which can have both negative and positive values.

Since p-adic numbers represent a highly number theoretical concept one might expect that there are deep connections between number theory and intentionality and cognition. The discussions with Uwe Kämpf in CASYS$'$2003 conference in Liege indeed stimulated a bundle of ideas allowing to develop a more detailed view about intention-to-action transformation and to disentangle these connections. These discussions made me aware of the fact that my recent views about the role of extensions of p-adic numbers are perhaps too limited. To see this consider the following arguments.

a) Pure p-adic numbers predict only p-adic length scales proportional to $p^{n/2}l$, l CP_2 length scale about 10^4 Planck lengths, $p \simeq 2^k$, k prime or power of prime. As a matter fact, all positive integer values of k are possible. This is however not enough to explain all known scale hierarchies. Fibonacci numbers $F_n : F_n + 1 = F_n + F_{n-1}$ behave asymptotically like $F_n = kF_{n-1}$, k solution of the equation $k^2 = k + 1$ given by $k = \Phi = (1 + \sqrt{5})/2 \simeq 1.6$. Living systems and self-organizing systems represent a lot of examples about scale hierarchies coming in powers of the Golden Mean $\Phi = (1 + \sqrt{5})/2$.

By allowing the extensions of p-adics by algebraic numbers one ends up to the idea that also the length scales coming as powers of x, where x is a unit of algebraic extension analogous to imaginary unit, are possible. One would however expect that the generalization of the p-adic length scale hypothesis alone would predict only the powers $\sqrt{x}p^{n/2}$ rather than $x^k p^{n/2}$, $k = 1, 2, ...$ Perhaps the purely kinematical explanation of these scales is not possible and genuine dynamics is needed. For sinusoidal logarithmic plane waves the harmonics correspond to the scalings of the argument by powers of some scaling factor x. Thus the powers of Golden Mean might be associated with logarithmic sinusoidal plane waves.

b) Physicist Hartmuth Mueller has developed what he calls Global Scaling Theory [10] based on the observation that powers of e (Neper number) define preferred length scales. These powers associate naturally with the nodes of logarithmic sinusoidal plane waves and correspond to various harmonics (matter tends to concentrate on the nodes of waves since force vanishes at the nodes). Mueller talks about physics of number line and there is great temptation to assume that deep number theory is indeed involved. What is troubling from TGD point of view that Neper number e is not algebraic. Perhaps a more general approach allowing also transcendentals must be adopted.

c) Classical mathematics, such as the theory of elementary functions, involves few crucially important transcendentals such as e and π. This might reflect the evolution of cognition: these numbers should be cognitively and number theoretically very special. The numbers e and π appear also repeatedly in the basic formulas of physics. They however look p-adically very troublesome since it has been very difficult to imagine a physically acceptable generalization of such simple concepts as exponent function, trigonometric functions, and logarithm resembling its real counterpart by allowing only the extensions of p-adic numbers based on algebraic numbers.

These considerations stimulate the question whether, besides the extensions of p-adics by algebraic numbers, also the extensions of p-adic numbers involving π and e and other transcendentals might be needed. The intuitive expectation motivated by the finiteness of human intelligence is that these extensions should have finite algebraic dimensions, and it indeed turns out that this is possible under some conditions which can be formulated as very general number theoretical conjectures. Since e^p exists p-adically, the powers $e, ..., e^{p-1}$ define a p-dimensional extension as do also the roots of polynomials with coefficients which are in an extension of rationals containing e and its powers. Contrary to the original conjecture, π however cannot belong to a finite-dimensional extension of p-adics. It is an open question whether one should allow infinite-dimensional extension of p-adic numbers containing π.

In any case, the hypothesis that $log(p) \times \pi$ for p prime, and $log(\Phi) \times \pi$ are rational numbers, guarantees the minimum amount of transcendentality required by classical physics and basic calculus using only a finite-dimensional extensions of p-adic numbers. The special role of π however becomes an extremely strong constraint for the p-adicization of quantum TGD by algebraic continuation from the realm of rationals to real and p-adic number fields.

Second question is whether there might be some dynamical mechanism allowing to understand the hierarchy of scalings coming in powers of some preferred transcendentals and algebraic numbers like Golden Mean. Conformal invariance implying that the system is characterized by a universal spectrum of scaling momenta for the logarithmic counterparts of plane waves seems to provide this mechanism. This spectrum is determined by the requirement that it exists for both reals and all p-adic number fields assuming that finite-dimensional extensions are allowed in the latter case. The spectrum corresponds to the zeros of the Riemann Zeta if Zeta is required to exist for all number fields in the proposed sense, and a lot of new understanding related to Riemann hypothesis emerges and allows to develop further the previous TGD inspired ideas about how to prove Riemann hypothesis [11, 6].

5.3.2 Algebraic continuation as a basic principle

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function. Logarithm is also such a function provided that the above mentioned number theoretic conjecture holds true.

The definition of a definite integral for p-adic numbers has been the key challenge in attempts to construct p-adic physics and algebraic continuations seems to solve this problem. The first problem is that p-adic numbers are not well ordered and one cannot define what ordered integration interval $[a, b]$ means p-adically. The second problem is that Riemann sum gives identically vanishing p-adic integral if coordinate increments approach zero at the limit. One can however define the definite integral in terms of the integral function:

$$\int_a^b f(x)dx = F(b) - F(a) \ ; \ f(x) = \frac{dF(x)}{dx} \ .$$

Integral function $F(x)$ is obtained using the inverse of the derivation just as in the real context. If integration limits are restricted to be rational numbers or finitely extended rational numbers, they can be ordered using the ordering of real numbers. This would essentially mean that p-adic integration measure is an algebraic continuation of the real integration measure.

Also residy calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residy formula. One can also imagine of extending residy calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". This could mean that the integral could be calculated at any page having the pole common . In particular, could a p-adic residy integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Gaussian integration gives hopes to define p-adic variants of configuration space integrals but only in the case that the integral over the configuration space reduces effectively to the Gaussian integral of a free quantum field theory. If configuration space is indeed a union of symmetric spaces, there are good hopes for achieving this (Duistermaat-Hecke theorem).

p-Adic integration is not necessarily needed to define the p-adic counterpart for the field equations associated with Kähler action but the continuation of the physics from real configuration space to the p-adic variants of the configuration spaces requires the existence of the p-adic valued Kähler action. If it is possible to assign to a given real space-time surface a p-adic counterpart uniquely in a given resolution for rational numbers, one can define the p-adic Kähler action as the real action interpreted as p-adic number in case that the real action belongs to a finite extension of rationals. This would also take care of the absolute minimization of the p-adic Kähler action which does not make sense as a genuinely p-adic concept.

5.4 Generalized Quantum Mechanics

One can consider two generalizations of quantum mechanics to a fusion of p-adic and real quantum mechanics.

a) For the first generalization the guiding principle for the generalization of quantum mechanics is that quantum mechanics in a given number field is obtained as an algebraic continuation of the quantum mechanics in the field of rational numbers common to all number fields or in finite-dimensional extensions of rational numbers. This means that U-matrices U_F for transitions from H_Q to H_F, where F refers to various completions of rationals, are obtained as algebraic continuations of the unitary U-matrix U_Q for H_Q. The generalization means enormously strong algebraic constraints on the form of the U-matrix.

b) A more radical option is that transitions from rational Hilbert space H_Q to the Hilbert spaces H_F associated with different number fields occur. This requires that U-process is followed by a process analogous to a state function reduction and preparation takes care that the resulting states become states in H_Q: this is what makes this generalization of a special interest. In this case one can speak about total scattering probability from H_Q to H_F. The U-matrices U_F are not anymore mere analytic continuations of U_Q. A possible interpretation of the unitary process $H_Q \rightarrow H_F$ is as generation of intention whereas the reduction and preparation means the transformation of the intention to action.

The assumption that H_Q allows an algebraic continuation to the spaces H_F is probably too strong an idealization in p-adic and even in the real case. For instance, one cannot allow all rational valued momenta in p-adic case for the simple reason that the continuation to the p-adic case involves always some momentum cutoff if the extension of p-adics remains finite. Even in the real case the summation over all rational momenta in the unitarity conditions of U-matrix fails to make sense and cutoff is needed. A hierarchy of cutoffs suggests itself and has a natural interpretation as number theoretical hierarchy of extensions of p-adics.

In order to avoid un-necessary complications the following formal discussion however uses H_Q as a universal Hilbert space contained by the various state spaces H_F.

5.4.1 Quantum mechanics in H_F as a algebraic continuation of quantum mechanics in H_Q

The rational Hilbert space H_Q is representable as the set of sequences of real or complex rationals of which only finite number are non-vanishing. Real and p-adic Hilbert spaces are obtained as the numbers in the sequences to become real or p-adic numbers and no limitations are posed to the number of non-vanishing elements. All these Hilbert spaces have rational Hilbert space H_Q as a common sub-space. Also momenta and other continuous quantum numbers are replaced by a discrete value set. Superposition principle holds true only in a restricted sense, and state function reduction and preparation leads always to a final state which corresponds to a state in H_Q. This picture differs from the earlier one in which p-adic and real Hilbert spaces were assumed to form a direct sum.

The notion of unitarity generalizes. Contrary to the earlier beliefs, U-matrix does not possess matrix elements between different number fields but between rational Hilbert space and Hilbert spaces associated with various completions of rationals. This makes sense since the final state of the quantum jump (and thus the initial state of the unitary process, is always in H_Q.

The U-matrix is a collection of matrices U_F having matrix elements in the number field F. U_F maps H_Q to H_F. Each of these U-matrices is unitary. Also U_Q is unitary and U_F is obtained by algebraic continuation in the quantum numbers labelling the states of U_Q to U_F.

Hermitian conjugation makes sense since the defining condition

$$\langle \alpha_F | U n_Q \rangle = \langle U^{\dagger} \alpha_F | n_Q \rangle \ . \tag{1}$$

allows to interpret $|n_Q\rangle$ also as an element of H_F. If U would map different completed number fields to each other, hermiticity conditions would not make sense.

The hermitian conjugate of U-matrix maps H_F to H_Q so that $UU^{\dagger}$ *resp.* $U^{\dagger}U$ maps H_F *resp.* H_Q to itself. This means that there are two independent unitarity conditions

$$\begin{aligned} U_F U_F^{\dagger} &= Id_F \ , \\ U_F^{\dagger} U_F &= Id_Q \ . \end{aligned} \tag{2}$$

One can write $U = P_Q + T_F$ and $U^{\dagger} = P_Q + T_F^{\dagger}$, where P_Q refers to the projection operator to H_Q.

This gives

$$\begin{aligned} T_F + T_F^\dagger &= -T_F T_F^\dagger \ , \\ P_Q T_F + T_F^\dagger P_Q &= -T_F^\dagger T_F \ . \end{aligned} \tag{3}$$

It is convenient to introduce the notations $T_Q = P_Q T_F$ and $T_Q^\dagger = T_F^\dagger P_Q$ with analogous notations for U and $U^\dagger$. The first condition, when multiplied from both sides by P_Q, gives together with the second equation unitarity conditions for T_Q

$$\begin{aligned} T_Q + T_Q^\dagger &= -T_Q T_Q^\dagger \ , \\ T_Q + T_Q^\dagger &= -T_F^\dagger T_F \ . \end{aligned} \tag{4}$$

This means that the restriction of the U-matrix to H_Q is unitary.

The difference between the right hand sides of the equation should vanish. The understanding of how this happens requires more delicate considerations. For instance, in the case of $F = C$ continuous sum over indices appears at the right hand side coming from four-momenta labelling the states. The restrictions of quantum numbers to Q and its subsets could be a process analogous to the momentum cutoff of quantum field theories. The continuation from discrete integer valued labels of, say discrete momenta, to continuous values is performed routinely in various physical models routinely, and it would seem that this process has cognitive and physical counterparts. This picture conforms with the vision that the rational (or extended rational) U-matrix U_Q gives the U-matrices U_F by an algebraic continuation in the quantum numbers labelling the states (say 4-momenta).

5.4.2 Could U_F describe dispersion from H_Q to the spaces H_F?

One can also consider a more general situation in which the states in H_Q can be said to disperse to the sectors H_F. In this case one can write

$$T = \text{"}\sum_F\text{"} T_F \ . \tag{5}$$

Here the sum has only a symbolic meaning since different number fields are in question and an actual summation is not possible. The T-matrix T_Q is the sum of the restrictions of T_F to H_Q and is the sum of rational valued T-matrices: $T_Q = \sum_F P_Q T_F$.

The T-matrices T_F are not anymore obtainable by algebraic continuation from same T-matrix T_Q. The unitarity conditions

$$\sum_F (P_Q T_F + T_F^{\dagger} P_Q) \quad = \quad -\sum_F T_F^{\dagger} T_F \tag{6}$$

make sense only if they are satisfied separately for each T_F, exactly as in the previous case. T

The diagonal elements

$$T_F^{mm} + \overline{T}_F^{mm} = \sum_{\alpha} T_F^{m\alpha} \overline{T}_F^{m\alpha} = \sum_r T_F^{mr} \overline{T}_F^{mr}$$

give essentially total scattering probabilities from the state $|m\rangle$ of H_Q to the sector H_F, and must be rational (or extended rational) numbers. One can therefore say that each U-process leads with a definite probability to a particular sector of the state space.

The fact that states which are superpositions of states in different spaces H_F does not make sense mathematically, forces the occurrence of a process, which might be regarded as a number theoretical counterpart of state function reduction and preparation. First a sector H_F is selected with probability p_F. Then F-valued (in particular complex valued) entanglement in H_F is reduced by state reduction and preparation type processes to a rational or extended rational entanglement having interpretation as bound state entanglement. It would be natural to assume that Negentropy Maximization Principle governs this process. Obviously the possibility to reduce state function reduction to number theory forces to consider quite seriously the proposed option.

5.5 Do state function reduction and state-preparation have number theoretical origin?

The foregoing considerations support the view that state function reduction and state preparation are number theoretical necessities so that there would be a deep connection between number theory and free will. One could even say that free will is a number theoretic necessity. The resulting more unified view provides the reason why for state function reduction, and preparation and allows to generalize previous views developed gradually by physics and consciousness inspired educated guess work.

5.5.1 Negentropy Maximization Principle as variational principle of cognition

It is useful to discuss the original view about Negentropy Maximization Principle (NMP) before considering the possible generalization of NMP inspired by the number theoretic vision.

NMP was originally motivated by the need to construct a TGD based quantum measurement theory. Gradually it however became clear that standard quantum measurement theory more or less follows from the assumption that the world of conscious experience is classical: this meant that NMP became a principle governing only state preparation.

State function reduction is achieved if a localization in zero modes occurs in each quantum jump, and if U matrix in zero modes corresponds to a flow in some orthogonal basis for the configuration space spinor fields in the quantum fluctuating fiber degrees of freedom of the configuration space. The requirement that U-matrix induces effectively a flow in zero modes is consistent with the effective classicality of the zero modes requiring that quantum evolution causes no dispersion. The one-one correlation between preferred quantum state basis in quantum fluctuating degrees of freedom and zero modes implies nothing but a one-one correspondence between quantum states and classical variables crucial for the interpretation of quantum theory. It seems that number theoretical vision forces to generalize this view, and to raise NMP to a completely general principle applying also to the state function reduction as the original proposal indeed was.

In its original form NMP governs the dynamics of self measurements and thus applies to the quantum jumps reducing the entanglement between quantum fluctuating degrees of freedom for given values of zero modes. Self measurements reduce the entanglement only between subsystems in quantum fluctuating degrees of freedom since they occur after the localization in the zero modes. Self measurement is repeated again and again for the unentangled subsystems resulting in each self measurement. This cascade of self measurements leads to a state possessing only extended rational entanglement identifiable as bound state entanglement and having negative number theoretic entanglement entropy. This process should be equivalent with the state preparation process assumed to be performed by a conscious observer in standard quantum measurement theory.

NMP states that the self measurement can be regarded as a quantum measurement of the subsystem's density matrix reducing the counterpart of the entanglement entropy of some subsystem to a smaller value, and that this occurs for the subsystem for which the reduction of the entanglement entropy is largest among all subsystems of the p-adic self. Inside each self NMP fixes some subsystem which is quantum measured in the quantum jump. One could perhaps say that self measurements make possible quantum level self repair since they allow the system in self state to fight against thermalization which results from the generation of unbound entanglement between subsystem-complement pairs.

5.5.2 NMP and number theory

The requirement the universe of conscious experience is classical is one manner to justify quantum jump. This hypothesis could be replaced by a postulate that state function reduction and preparation project quantum states to a definite number field and that only extended rational entanglement identifiable as bound state entanglement is stable. This is consistent with NMP since it is possible to assign to an extended rational entanglement a non-negative number theoretic negentropy as the maximum over entropies defined by various p-adic entropies $S_p = -\sum p_k log(|p_k|_p)$.

The unitary process U would thus start from a product ofbound states for which entanglement coefficient are extended rationals, and would lead to a formal superposition of states belonging to different number fields. Both state function reduction and state preparation would begin with a localization to a definite number field. This localization would be followed by a self measurement cascade reducing the entanglement to extended rational entanglement.

This vision forces to challenge the earlier views about state function reduction.

a) There is no good reason for why NMP could not be applied to both state function reduction and preparation.

b) If the entanglement between zero modes and quantum fluctuating degrees of freedom involves only discrete values of zero modes, the problems caused by the fact that no well-defined functional integral measure over zero modes exists, find an automatic resolution. Since extended rational entanglement possesses negative entanglement entropy, it is stable also against reduction if NMP applies completely generally. A discrete entanglement involving transcendentals not contained to any *finite* extension of any p-adic number field is unstable and reduced.

c) The quantum measurement lasts for a time determined by the lifetime of the bound state entanglement between zero modes and quantum fluctuating degrees of freedom. Physical considerations of course support the view that it takes more than single quantum jump (10^{-39} seconds of psychological time) for the state function reduction to take place. The notion of zero mode-zero mode bound state entanglement seems however to be self-contradictory. If join along boundaries bonds are space-time correlates for the bound state entanglement, their formation should transform roughly half of the zero modes associated with the two space-time sheets to quantum fluctuating degrees of freedom.

d) If p-adic length scale hierarchy has as its counterpart a hierarchy of state function reduction and preparation cascades, one must accept the quantum parallel occurrence of state function reduction and preparation processes in the parallel quantum universes corresponding to different p-adic length scales. This picture provides a justification for the modelling of hadron as a quantum system in long length and time scales and as a dissipative system consisting of quarks and gluons in shorter length and time scales. The bound state entanglement between subsystems of entangled systems having as a space-time correlate join along boundaries bonds connecting subsystem space-time sheets, is a second important implication of the new sub-system concept, and plays a central role in TGD inspired theory of consciousness.

6 The boost from TGD inspired theory of consciousness

Quite generally, TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory. The identification of quantum jump as a moment of consciousness is analogous to the identification of elementary particles as basic building blocks of matter. The observer is an outsider in standard quantum measurement theory and is replaced by the notion of self in TGD inspired theory of consciousness. Selves identified as systems able to avoid bound state entanglement and identifiable as ensembles of quantum jumps, are analogous to many-particle states. The sensory and other qualia of self are determined as statistical averages over quantum number and zero mode increments for the increasing sequence of quantum jumps defining self. Especially important are selves, which are in a state of macro-temporal quantum coherence since for these selves the entropy of the ensemble defined by the quantum jumps does not increase and the qualia stay sharp. These selves are analogous to bound states of elementary particles and their formation actually corresponds to the generation of bound state entanglement.

6.1 The anatomy of the quantum jump

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories rather than to a non-deterministic behaviour of a single quantum history. Therefore U-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of U-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

a) The classical time development determined by the absolute minimization of Kähler action.

b) The unitary "time development" defined by U associated with each quantum jump

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \ ,$$

and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator and this means that one must leave open the identification of U-matrix with S-matrix.

c) The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness. A crucial role is played by the classical non-determinism of Kähler action implying that the non-determinism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration since there is *no* real time evolution or translation involved. It is not clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. Physically it seems obvious that U matrix should decompose to a cosmological U-matrix representing dispersion in configuration space and U-matrix representing local dynamics: this indeed occurs thanks to the classical non-determinism of the Kähler action. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation. An important exception are the zero modes characterizing center of mass degrees of freedom of 3-surface which correspond to the isometries of $M^4_+ \times CP_2$. In these degrees of freedom localization does not occur. At the limit when 3-surfaces are regarded as pointlike objects theory should obviously reduce to quantum field theory.

6.1.1 The three non-determinisms

Besides the non-determinism of quantum jump, TGD allows two other kinds of non-determinisms: the classical non-determinism basically due the vacuum degeneracy of the Kähler action and p-adic non-determinism of p-adic differential equations due to the fact that functions with vanishing p-adic derivative correspond to piecewise constant functions.

To achieve classical determinism in a generalized sense, one must generalize the definition of the 3-surfaces Y^3 (belonging to light cone boundary) by allowing also "association sequences", that is 3-surfaces which have, besides the component belonging to the light cone boundary, also disjoint components which do not belong to the light cone boundary and have mutual *time-like separations*. This means the introduction of additional, one might hope typically discrete, degrees of freedom (consider non-determinism based on bifurcations as an example). It is even possible to have quantum entanglement between the states corresponding to different values of time.

Without the classical and p-adic non-determinisms general coordinate invariance would reduce the theory to the light cone boundary and this would mean essentially the loss of time which occurs also in the quantization of general relativity as a consequence of general coordinate invariance. Classical and p-adic non-determinisms imply that one can have quantum jumps with non-determinism (in conventional sense) located to a finite time interval. If quantum jumps correspond to moments of consciousness, and if the contents of consciousness are determined by the locus of the non-determinism, then these quantum jumps must give rise to a conscious experience with contents located in a finite time interval.

Also p-adic space-time sheets obey their own quantum physics and are identifiable as seats of cognitive representations. p-Adic non-determinism is the basic prerequisite for imagination and simulation. The notion of cognitive space-time sheet as a space-time sheet having finite time duration is one aspect of the p-adic non-determinism and allows to understand how the notion of psychological time emerges. Cognitive space-time sheets simply drift quantum jump by quantum to the direction of geometric future since there is much more room there in the light cone cosmology.

The classical non-determinism is maximal for CP_2 type extremals for which the M^4_+ projection of the space-time surface is random lightlike curve. In this case, basic objects are essentially four- rather than 3-dimensional. The basic implication of the classical non-determinism is that quantum theory does not reduce to the light cone boundary. Secondly, U-matrix reduces to a tensor product of a cosmologcal U-matrix and local U-matrices relevant for particle physics. As a matter fact, an entire hierarchy of U-matrices defined in various p-adic time scales is expected to appear in the hierarchy. Thirdly, the classical non-determinism of CP_2 type extremals allows a topologization of the Feynman diagrammatics of quantum field theories and string models. Although localization in zero modes characterizing zitterbewegung orbit occur in quantum jump, there is integral over the positions of vertices which correspond to cm degrees of freedom for imbedding space, and this gives rise to a sum over various Feynman diagrams.

6.1.2 How psychological time and its arrow emerge?

How psychological time and its arrow emerge is the basic challenge for the hypothesis that quantum jumps occur between quantum histories and are identifiable as moments of consciousness. Mind like space-time sheets provide a geometric model of unconscious mind in TGD framework and make it possible to solve the puzzle of psychological time. The first argument is following.

Mind like space-time sheets have well center of mass time coordinate and this coordinate is zero mode identifiable as psychological time. Localization in zero modes means that final states of quantum jumps correspond to quantum superpositions of space-time surfaces having same number of mind like space-time sheets such that given mind like space-time sheet possesses same value of psychological time for all space-time surfaces appearing in the superposition. The arrow of psychological time follows from the gradual drift of the mind like space-time sheets in future direction occurring quantum jump by quantum jump and is implied by the geometry of future light cone (there is more volume in the future of a given light cone point than in its future). The simplest assumption is that the average increment of psychological time in single quantum jump is of order CP_2 time, which is about 10^4 Planck times.

Besides classical non-determinism there is also p-adic non-determinism and one should keep mind open in the attempts to identify the roles of these two non-determinisms. The interpretation taken as a working hypothesis in the recent version of TGD inspired theory of consciousness is that p-adic space-time regions provide cognitive representations of the real regions and serve as correlates for intentions. Real regions are in turn symbolic representations for the material world in TGD sense of the word. This means that besides ordinary matter also higher level physical states associated with the real space-time sheets of a finite duration and having vanishing net energy are possible. The zero energy states representing pairs of incoming and outgoing states could make possible self-referential real physics representing the laws of physics in the structure of the higher level physical states. Real space-time sheets of finite temporal duration might be interpreted also as correlates of pure sensory experience as opposed to p-adic space-time sheets which can be identified as correlates of thoughts. Also volition could be assigned to the quantum jumps involving selection between various branches of multifurcations implied by the classical non-determinism.

A more refined argument explaining the arrow of psychological time is based on the idea that psychological time correspond to the moment of geometric time which gives the dominant contribution to the conscious experience, and that it is the transformation of intentions to actions which provides this contribution. The transformation of intentions to actions corresponds to the transformation of p-adic space-time sheets to real ones, and one can identify psychological time as characterizing the position of the intention-to action phase transition front. In order to have consistency with the basic facts about everyday conscious experience one must assume that the geometric past remains unable to express intentions for a period of time longer than the life cycle since otherwise the decisions made in say my geometric youth subjectively now could induce dramatic changes in my recent life. This dead time would be analogous to the recovery time of neuron after the generation of nerve pulse.

6.1.3 Macro-temporal quantum coherence and spin glass degeneracy

At the space-time level the generation of macroscopic quantum coherence is easy to understand if one accepts the identification of the space-time sheets as coherence regions. Quantum criticality and the closely related spin glass degeneracy are essential for the fractal hierarchy of space-time sheets. The problem of understanding macro-temporal and macroscopic quantum coherence at the level of configuration space (of 3-surfaces) is a more tricky challenge although quantum-classical correspondence strongly suggests that this is possible.

Concerning macro-temporal quantum coherence, the situation in quantum TGD seems at the first glance to be even worse than in standard physics. The problem is that simplest estimate for the increment in psychological time in single quantum jump is about 10^{-39} seconds derived from the idea that single quantum jump represent a kind of elementary particle of consciousness and thus corresponds to CP_2 time of about 10^{-39} seconds. If this time interval defines coherence time one ends up to a definite contradiction with the standard physics. Of course, the average increment of the geometric time during single quantum jump could vary and correspond to the de-coherence time. The idea of quantum jump as an elementary particle of consciousness does not support this assumption.

To understand how this naive conclusion is wrong, one must look more precisely the anatomy of quantum jump. The unitary process $\Psi_i \rightarrow U\Psi_i$, where Ψ_i is a prepared maximally unentangled state, corresponds to the quantum computation producing maximally entangled multi-verse state. Then follows the state function reduction and after this the state preparation involving a sequence of self measurements and given rise to a new maximally unentangled state Ψ_f.

a) What happens in the state function reduction is a localization in zero modes, which do not contribute to the line element of the configuration space metric. They are non-quantum fluctuating degrees of freedom and TGD counterparts of the macroscopic, classical degrees of freedom. There are however also quantum-fluctuating degrees of freedom and the assumption that zero modes and quantum fluctuating degrees of freedom are correlated like the direction of a pointer of a measurement apparatus and quantum numbers of the quantum system, implies standard quantum measurement theory.

b) Bound state entanglement is assumed to be stable against state function reduction and preparation. Bound state formation has as a geometric correlate formation of join along boundaries bonds between space-time sheets representing free systems. Thus the members of a pair of disjoint space-time sheets are joined to single space-time sheet. Half of the zero modes is transformed to quantum fluctuating degrees of freedom and only overall center of mass zero modes remain zero modes. These new quantum fluctuating degrees of freedom represent macroscopic quantum fluctuating degrees of freedom. In these degrees of freedom localization does not occur since bound states are in question.

Both state function reduction and state preparation stages leave this bound state entanglement intact, and in these degrees of freedom the system behaves effectively as a quantum coherent system. One can say that a sequence of quantum jumps binds to form a single long-lasting quantum jump effectively. This is in complete accordance with the fractality of consciousness. Quantum jumps represent moments of consciousness which are "elementary particles of consciousness" and in macro-temporal quantum coherent state these elementary particles bind to form atoms, molecules, etc. of consciousness.

c) The properties of the bound state plus its interaction with the environment allow to estimate the typical duration of the bound state. This time takes the role of coherence time. This suggests a connection with the standard approach to quantum computation. An essential element is spin glass degeneracy. The generation of join along boundaries bonds connecting the space-time sheets of the composite systems is the space-time correlate for the formation of the bound states. Spin glass degeneracy is much higher for the bound states because of the presence of the join along boundaries bonds. This together with the fact that these degenerate states are almost identical so that transition amplitudes between them are also almost identical, implies that the life-time of the majority of bound states is much longer than one might expect otherwise. The detailed argument is carried out in [C1] and can be applied to show that spin glass degeneracy for the color flux tubes explains color confinement [D2].

e) The number theoretic notion of information relies on Shannon entropy in which the logarithms of probabilities are replaced by logarithms of their p-adic norms. This requires that the probabilities are rational or belong to an finite-dimensional extension of rationals. What is so important is that this entropy can have also negative values. If one assumes that bound states form a hierarchy such that the entanglement coefficients belong always to a finite-dimensional extension of rationals, one can define the entanglement entropy as a number theoretic entropy associated with some prime p. In p-adic context the prime is unique whereas in the real context the value of the prime can be selected in such a manner that the entropy is maximally negative. This prime would be naturally a maximal prime factor of the integer N defining the number of strictly deterministic regions of the space-time sheet in question. If this assumption is made, NMP alone implies the stability of bound states against state preparation by self measurements. This generalization of the information concept has far reaching implications in TGD inspired theory consciousness.

6.2 Negentropy Maximization Principle and new information measures

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = -\sum_n p_n log(p_n) \ . \tag{7}$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

6.2.1 p-Adic entropies

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of x, where log_p denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) \ , \\ k(p_n) &= -log_p(|p_n|) \ . \end{aligned} \tag{8}$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n)log(p) + p^{k_n} log(p_n/p^{k_n} \right] \ . \quad (9)$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times log(p)) \ , \\ (\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ . \end{aligned} \quad (10)$$

The real counterpart of the p-adic entropy is non-negative.

6.2.2 Number theoretic entropies and bound states

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|) log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv Max\{-S_p, \ p \text{ prime}\} \ . \quad (11)$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP, and has a natural interpretation as bound state entanglement. The prediction would be that the bound states of real systems form a number theoretical hierarchy according to the prime p and and dimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes projecting the physical states from either real or p-adic sectors of the state space to their intersection. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the U-matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

6.2.3 Number theoretic information measures at the space-time level

Quantum classical correspondence suggests that the notion of entropy should have also space-time counterpart. Entropy requires ensemble and both the p-adic non-determinism and the non-determinism of Kähler action allow to define the required ensemble as the ensemble of strictly deterministic regions of the space-time sheet. One can measure various observables at these space-time regions, and the frequencies for the outcomes are rational numbers of form $p_k = n(k)/N$, where N is the number of strictly deterministic regions of the space-time sheet. The number theoretic entropies are well defined and negative if p divides the integer N. Maximum is expected to result for the largest prime power factor of N. This would mean the possibility to assign a unique prime to a given real space-time sheet and thus the solve the basic problem created already by p-adic mass calculations.

The classical non-determinism resembles p-adic non-determinism in the sense that the space-time sheet obeys effective p-adic topology in some length and time scale range is consistent with this idea since p-adic fractality suggests that N is power of p.

7 TGD as a generalized number theory

The vision about a number theoretic formulation of quantum TGD is based on the gradual accumulation of wisdom coming from different sources. The attempts to find a formulation allowing to understand real and p-adic physics as aspects of some more general scenario have been an important stimulus and generated a lot of, not necessarily mutually consistent ideas, some of which might serve as building blocks of the final formulation. The original chapter representing the number theoretic vision as a consistent narrative grew so massive that I decided to divide it to three parts.

The first part is devoted to the p-adicization program attempting to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals. Highly non-trivial number theoretic conjectures are an i outcome of the program.

Second part focuses on the idea that the tangent spaces of space-time and imbedding space can be regarded as 4- *resp.* 8-dimensional algebras such that space-time tangent space defines sub-algebra of imbedding space. The basic candidates for the pair of algebras are hyper-quaternions and hyper-octonions. The problems are caused by the Euclidian signature of the Euclidian norm.

The great idea is that space-time surfaces X^4 correspond to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of $HO = M^8$. The possibility to assign to X^4 a surface in $M^4 \times CP_2$ means a number theoretic analog for spontaneous compactification. Of course, nothing dynamical is involved: a dual relation between totally different descriptions of the physical world are in question. In the spirit of generalized algebraic geometry one can ask whether hyper-quaternionic space-time surfaces and their duals could be somehow assigned to hyper-octonion analytic maps $HO \rightarrow HO$, and there are good arguments suggesting that this is the case.

The third part is devoted to infinite primes. Infinite primes are in one-one correspondence with the states of super-symmetric arithmetic quantum field theories. The infinite-primes associated with hyper-quaternionic and hyper-octonionic numbers are the most natural ones physically because of the underlying Lorentz invariance, and the possibility to interpret them as momenta with mass squared equal to prime. Most importantly, the polynomials associated with hyper-octonionic infinite primes have automatically space-time surfaces as representatives so that space-time geometry becomes a representative for the quantum states.

7.1 The painting is the landscape

The work with TGD inspired theory of consciousness has led to a vision about the relationship of mathematics and physics. Physics is not in this view a model of reality but objective reality itself: painting is the landscape. One can also equate mathematics and physics in a well defined sense and the often implicitly assumed Cartesian theory-world division disappears. Physical realities are mathematical ideas represented by configuration space spinor fields (quantum histories) and quantum jumps between quantum histories give rise to consciousness and to the subjective existence of mathematician.

The concrete realization for the notion algebraic hologram based on the notion of infinite prime is a second new element. The notion of infinite rationals leads to the generalization of also the notion of finite number since infinite-dimensional space of real units obtained from finite rational valued ratios q of infinite integers divided by q. These units are not units in p-adic sense. The generalization to the (hyper-)quaternionic and (hyper-)octonionic context means that ordinary space-time points become infinitely structured and space-time point is able to represent even the quantum physical state of the Universe in its algebraic structure. Single space-time point becomes the Platonia not visible at the level of real physics but essential for mathematical cognition.

In this view evolution becomes also evolution of mathematical structures, which become more and more self-conscious quantum jump by quantum jump. The notion of p-adic evolution is indeed a basic prediction of quantum TGD but even this vision might be generalized by allowing rational-adic topologies for which topology is defined by a ring with unit rather than number field.

7.2 p-Adic physics as physics of cognition

7.2.1 Real and p-adic regions of the space-time as geometric correlates of matter and mind

The solutions of the equations determining space-time surfaces are restricted by the requirement that imbedding space-coordinates are real. When this is not the case, one might apply instead of a real completion with some rational-adic or p-adic completion: this is how rational-adic p-adic physics could emerge from the basic equations of the theory. One could interpret the resulting rational-adic or p-adic regions as geometrical correlates for 'mind stuff'.

p-Adic non-determinism implies extreme flexibility and therefore makes the identification of the p-adic regions as seats of cognitive representations very natural. Unlike real completion, p-adic completions preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with 'mind like' regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of 'self' and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves!

7.2.2 The generalization of the notion of number and p-adicization program

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

For instance, residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition and intentionality. The basic stumbling block of this program is integration and algebraic continuation should allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. Two types of cutoffs are predicted: p-adic length scale cutoff and a cutoff due to phase resolution. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$ related closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra [O4]. This cutoff hierarchy seems to relate closely to the hierarchy of cutoffs defined by the hierarchy of subalgebras of the super-canonical algebra defined by the hierarchy of sets $(z_1, ...z_n)$, where z_i are the first n non-trivial zeros of Riemann Zeta [C5]. Hence there are good hopes that the p-adicization program might unify apparently unrelated branches of mathematics.

7.3 Space-time-surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?

Second thread in the development of ideas has been present for only few years ideas inspired by the possibility that quaternions and octonions might allow a deeper understanding of TGD. This thread emerged from the discussions with Tony Smith which stimulated very general ideas about space-time surface as associative, quaternionic sub-manifold of octonionic 8-space. Also the observation that quaternionic and octonionic primes have norm squared equal to prime in complete accordance with p-adic length scale hypothesis, led to suspect that the notion of primeness for quaternions, and perhaps even for octonions, might be fundamental for the formulation of quantum TGD [E2]. It turned out that, much in spirit with transition from Riemannian to pseudo-Riemannian geometry, hyper-quaternins and hyper-octonions are forced by physical considerations.

7.3.1 Transition from string models to TGD as replacement of real/complex numbers with quaternions/octonions

One can fairly say, that quantum TGD results from string model with the pair of real and complex numbers replaced with the pair of hyper-quaternions and hyper-octonions. Hyper is necessary in order to take into the Minkowskian signature of the metric.

Space-time identified as a hyper-quaternionic sub-manifold of the hyper-octonionic space in the sense that the tangent space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point. One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered.

Some delicacies are caused by the question whether the induced algebra at X^4 is just the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If the normal part of the product is projected out, the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2. This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH - -M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2. This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter familty co-hyper-quaternionic space-time surfaces.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that the absolute minima of Kähler action in $H = M^4 \times CP_2$ correspond to the hyper-quaternion analytic surfaces in HO. The map $f : HO \rightarrow S^6$ would probably satisfy some constraints posed by the requirement that the resulting surfaces define solutions of field equations in $M^4 \times CP_2$ picture. This conjecture has several variants. It could be that only the asymptotic behavior corresponds to hyper-quaternion analytic function but that hyper-quaternionicity is a general property of absolute minima. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates coding for the local selection of the plane of non-physical polarizations, appear naturally also in the construction of general solutions of field equations [D1].

7.3.2 Physics as a generalized algebraic number theory and Universe as algebraic hologram

The third stimulus encouraging to think that TGD might be reduced to algebraic number theory and algebraic geometry in some generalized sense, came from the work with Riemann hypothesis [E8]. One can assign to Riemann Zeta a super-conformal quantum field theory and identify Zeta as a Hermitian form in the state space possibly defining a Hilbert space metric. The proposed form of the Riemann hypothesis implies that the zeros of ζ code for infinite primes which in turn have interpretation as Fock states of a super-symmetric quantum field theory if the proposed vision is correct.

A further stimulus came from the realization that algebraic extensions of rationals, which make possible a generalization of the notion of prime, could provide enormous representative and information storage power in arithmetic quantum field theory. Algebraic symmetries defined as transformations preserving the algebraic norm represent new kind of symmetries commuting with ordinary quantum numbers. Fractal scalings and discrete symmetries are in question so that the notion of fractality emerges to the fundamental physics in this manner.

The basic observation, completely consistent with fractality, is that these symmetries make possible what might be called *algebraic hologram*. The algebraic quantum numbers associated with elementary particle depend on the environment of the particle. The only possible conclusion seems to be that these fractal quantum numbers provide some kind of 'cognitive representation' about external world. This kind of an algebraic hologram would be in complete accordance with fractality and would provide first principle realization for fractality observed everywhere in Nature but not properly understood in standard physics framework. A further basic idea which emerged was the principle of *algebraic democracy*: all possible algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions are possible and emerge dynamically as properties of physical systems in algebraic physics.

7.4 Infinite primes and physics in TGD Universe

The notion of infinite primes emerged originally from TGD inspired theory of consciousness [?] but it soon turned out that the notion could be used to build a number theoretic interpretation of quantum TGD and relate quantum to classical. Also the notion of infinite-P p-adicity emerges naturally and could replaces real topology with something more refined and appropriate for description of the space-time correlates of cognition.

7.4.1 Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes was one important step in the development suggesting strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes so that it applies even in the case of hyper-quaternionic and hyper-octonionic primes. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.

What is remarkable is that one has quite realistic possibilities to understand the quantum numbers of physical particles in terms of hyper-octonionic infinite primes. Also the TGD inspired model for $1/f$ noise [I5] based on thermal arithmetic quantum field theory encouraged also to consider the idea about hyper-quaternionic or hyper-octonionic arithmetic quantum field theory as an essential element of quantum TGD.

7.4.2 Infinite primes as a bridge between quantum and classical

The final stimulus came from the observation stimulated by algebraic number theory [26]. Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes. Infinite primes allow nice interpretation as Fock states of a second quantized super-symmetric quantum field theory. Also bound states are included.

This in turn led to the observation that one can represent infinite primes (integers) geometrically as surfaces related to the polynomials associated with infinite primes (integers). Thus infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite numbers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

The original mapping to 4-surfaces inspired by algebraic geometry was essentially as zeros of polynomials. It however turned out that the mapping is more delicate and based on the idea that space-time surfaces correspond to hyper-quaternionic or co-hyper-quaternionic sub-manifolds of imbedding space with hyper-octonionic structure. Also the attribute maximally associative or co-associate could be used. The assignment of a space-time surface to an infinite prime boils down to an assignment of a hyper-octonion analytic polynomial to infinite prime, which in turn defines a foliation of $M^4 \times CP_2$ by hyper-quaternionic space-time surfaces. The procedure generalizes also to the higher levels of the hierarchy and the natural interpretation is in terms of the hierarchical structure of the many-sheeted space-time.

The connection with the basic ideas of algebraic geometry from the possibility to order space-time surfaces according to the complexity of the polynomial involved (at higher levels rational coefficients of the polynomial are replaced with rational polynomials). In particular, the notions of degree and genus make sense for space-time surface.

7.4.3 Various equivalent characterizations of space-times as surfaces

The idea about space-times as associative, hyper-quaternionic surfaces of a hyper-octonionic imbedding space M^8 and the notion of infinite prime serving as a bridge between classical and quantum are the two basic tenets of the algebraic approach. This vision leads to an equivalence of quite different views about space-time: space-time as an associative/hyper-quaternionic or co-associative/co-hyperquaterionic surface of an hyper-octonionic imbedding space $HO = M^8$; space-time as a surface in $H = M^4 \times CP_2$; space-time as a geometric counterpart of an infinite prime representing also Fock state identifiable as a particular ground state of super-canonical representation; and finally, space-time surface as an absolute minimum of the Kähler action. The great challenge is to prove that the last characterization is equivalent with the others.

7.4.4 Infinite primes and quantum gravitational holography

Infinite primes emerge naturally in the realization of the quantum gravitational holography in terms of the modified Dirac operator and provide a deeper understanding of the basic aspects of the configuration space geometry.

a) Two types of infinite primes are predicted corresponding to the two types of fermionic vacua $X \pm 1$, where X is the product of all finite primes. The physical interpretation for the two types of infinite primes $X \pm 1$ is in terms of two quantizations for which creation and oscillator operators change role and which correspond to the two signs of inertial energy in TGD Universe. In particular, phase conjugate photons would be negative energy photons erratically believed to reduce to standard physics.

b) The new view about gravitational and inertial masses forced by TGD leads also the view that positive and negative energy space-time sheets are created pairwise at space-like 3-surfaces located at 7-D light-like causal determinants $X^7_\pm = \delta M^4_\pm \times CP_2$. The conjecture is that the ratio of Dirac determinants associated with the positive and negative energy space-time sheets, which is finite, equals to the exponent of Kähler function which would be thus determined completely by the data at 3-dimensional causal determinants and realizing quantum gravitational holography.

c) The spectra associated with the space-time sheets X^4_+ and X^4_- meeting at X^3 would correspond to the infinite primes built from the vacua corresponding to the infinite primes $X \pm 1$. The close analogy of the product of all finite hyper-octonionic primes with Dirac determinant suggest that the ratio of the determinants corresponds to the ratio of infinite primes defining X^4_+ and X^4_-. The theory predicts the dependence of the eigenvalues of the modified Dirac operator on the value of the Kähler action. Both Kähler coupling strength and gravitational coupling strength are expressible in terms of the finite primes characterizing the ratio of the infinite primes and this ratio depends on the p-adic prime characterizing X^4_+ and X^4_-.

d) Some modes of the spectrum of the modified Dirac operator at $X^4_\pm$ become zero modes, and by the resulting spectral asymmetry the ratio of the determinants differs from unity. Thus the spectral asymmetry or the infinite primes defining the space-time sheets X^4_+ and X^4_- is all that would be needed to deduce the value of the vacuum functional once causal determinants are known.

7.5 Infinite primes and more precise view about p-adic length scale hypothesis

Number theoretical considerations allow to develop more quantitative vision about the how p-adic length scale hypothesis relates to the ideas just described.

7.5.1 How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass squared. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles. This might apply also to photons. Infinite range might mean that the join along boundaries bonds mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [C2, C5, E3].

a) If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$-adic topology is homeomorphic with p-adic topology).

b) One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89}, if this p-adic prime does not somehow characterize also the particle itself.

7.5.2 What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

a) The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

b) A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [E3].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

7.5.3 Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [E3], hierarchy of Jones inclusions [O5], hierarchy of dark matters with increasing values of $\hbar$ [F9, J6], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [E3]. Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

a) The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

b) Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [E3] coding information too subtle to be catched by ordinary physical measurements [O4].

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

7.5.4 Under what conditions space-time sheets can be connected by $\#_B$ contact?

Assume that particles are characterized by a p-adic prime determining it mass scale plus p-adic primes characterizing the gauge bosons to which they couple and assume that $\#_B$ contacts mediate gauge interactions. The question is what kind of space-time sheets can be connected by $\#_B$ contacts.

a) The first working hypothesis that comes in mind is that the p-adic primes associated with the two space-time sheets connected by $\#_B$ contact must be identical. This would require that particle is many-sheeted structure with no other than gravitational interactions between various sheets. The problem of the multi-sheeted option is that the characterization of events like electron-positron annihilation to a weak boson looks rather clumsy.

b) If the notion of multi-p p-adicity is accepted, space-time sheets are characterized by integers and the largest prime dividing the integer might characterize the mass of the particle. In this case a common prime factor p for the integers characterizing the two space-time sheets could be enough for the possibility of $\#_B$ contact and this contact would be characterized by this prime. If no common prime factors exist, only # contacts could connect the space-time sheets. This option conforms with the number theoretical vision. This option would predict that the transition to large $\hbar$ phase occurs simultaneously for all interactions.

7.5.5 What about the integer characterizing graviton?

If one accepts the hypothesis that graviton couples to both visible and dark matter, graviton should be characterized by an integer dividing the integers characterizing all particles. This leaves two options.

Option I: gravitational constant characterizes graviton number theoretically

The argument leading to an expression for gravitational constant in terms of CP_2 length scale led to the proposal that the product of primes $p \leq 23$ are common to all particles and one interpretation was in terms of multi-fractality. If so, graviton would be characterized by a product of some or all primes $p \leq 23$ and would thus correspond to a very small p-adic length scale. This might be also the case for photon although it would seem that photon cannot couple to dark matter always. $p = 23$ might characterize the transversal size of the massless extremal associated with the space-time sheet of graviton.

Option II: graviton behaves as a unit with respect to multiplication

One can also argue that if the largest prime assignable to a particle characterizes the size of the particle space-time sheet it does not make sense to assign any finite prime to a massless particle like graviton. Perhaps graviton corresponds to simplest possible infinite prime $P = X \pm 1$, X the product of all primes.

As found, one can assign to any infinite prime, integer, and rational a rational number $q = m/n$ to which one can assign a q-adic topology as effective space-time topology and as a special case effective p-adic topologies corresponding to prime factors of m and n.

In the case of $P = X \pm 1$ the rational number would be equal to ± 1. Graviton could thus correspond to $p = 1$-adic effective topology. The "prime" $p = 1$ indeed appears as a factor of any integer so that graviton would couple to any particle. Formally the 1-adic norm of any number would be 1 or 0 which would suggest that a discrete topology is in question.

The following observations help in attempts to interpret this.

a) CP_2 type extremals having interpretation as gravitational instantons are non-deterministic in the sense that M^4 projection is random light-like curve. This condition implies Virasoro conditions which suggests interpretation in terms topological quantum theory limit of gravitation involving vanishing four-momenta but non-vanishing color charges. This theory would represent gravitation at the ultimate CP_2 length scale limit without the effects of topological condensation. In longer length scales a hierarchy of effective theories of gravitation corresponds to the coupling of space-time sheets by join along boundaries bonds would emerge and could give rise to "strong gravities" with strong gravitational constant proportional to L_p^2. It is quite possible that the M-theory based vision about duality between gravitation and gauge interactions applies to electro-weak interactions and in these "strong gravities".

b) p-Adic length scale hypothesis $p \simeq 2^k$, k integer, implies that $L_k \propto \sqrt{k}$ corresponds to the size scale of causal horizon associated with # contact. For $p = 1$ k would be zero and the causal horizon would contract to a point which would leave only generalized Feynman diagrams consisting of CP_2 type vacuum extremals moving along random light-like orbits and obeying Virasoro conditions so that interpretation as a kind of topological gravity suggests itself.

c) $p = 1$ effective topology can make marginally sense for vacuum extremals with vanishing Kähler form and carrying only gravitational charges. The induced Kähler form vanishes identically by the mere assumption that X^4, be it continuous or discontinuous, belongs to $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2.

Why topological graviton, or whatever the particle represented by CP_2 type vacuum extremals should be called, should correspond to the weakest possible notion of continuity? The most plausible answer is that discrete topology is *consistent* with any other topology, in particular with any p-adic topology. This would express the fact that CP_2 type extremals can couple to any p-adic prime. The vacuum property of CP_2 type extremals implies that the splitting off of CP_2 type extremal leaves the physical state invariant and means effectively multiplying integer by $p = 1$.

It seems that Option I suggested by the deduction of the value of gravitational constant looks more plausible as far as the interpretation of gravitation is considered. This does not however mean that CP_2 type vacuum extremals carrying color quantum numbers could not describe gravitational interactions in CP_2 length scale.

7.6 Complete algebraic, topological, and dimensional democracy?

Without the notion of Platonia allowing realization of all imaginable algebraic structures cognitively but leaving no trace on the physics of matter, the idea about dimensional democracy would look almost compelling despite the fact that it might well be in conflict with the special role of the dimensions associated with the classical number fields. One can imagine several realizations of this idea.

a) The most (if not the only) plausible realization for the dimensional hierarchy would be following. Both fractal cosmology, non-determinism of Kähler action, and Poincare invariance favor the option in which configuration space is a union of sectors characterized by unions of future and past light cones $M^4_\pm(a)$ where a characterizes the position a of the dip of the light-cone in M^4. Future/past dichotomy would correspond to positive/negative energy dichotomy and to the two kinds of infinite primes constructed from $X \pm 1$, X the product of all finite primes. Hence the cm degrees of freedom for the sectors of the configuration space would correspond to the union of the spaces $(M^4)^m \times (M^4)^n$ of dimension $D = 4(m+n)$, and the dimensional democracy would conform with the 8-dimensionality of the imbedding space.

b) The most plausible identification consistent with the p-adic length scale hierarchy is as unions of n disjoint 4-surfaces of H. This correspondence is completely analogous to that involved when the configuration space of n point-like particles is identified as $(E^3)^n$ in wave mechanics.

c) One might also consider of assigning with hyper-octonionic infinite primes of level n $4n$-dimensional surfaces in $8n$-dimensional space $H^n = (M^4_+ \times CP_2)^n$. This would suggests a dimensional hierarchy of space-time surfaces and a complete dimensional and algebraic democracy: quite a considerable generalization of quantum TGD from its original formulation. This option does not however look physically plausible since it is not consistent with the hierarchical "abstractions about abstractions" structure of infinite primes and corresponding space-time representations.

Since quantum field theories are based on the notion of point like particles, the hierarchy of arithmetic quantum field theories associated with infinite primes cannot code entire quantum TGD but only the ground states of the super-canonical representations. This might however be the crucial element needed to understand the construction S-matrix of quantum TGD at the general level.

One can imagine also a topological democracy and an evolution of algebraic topological structures. At the lowest, primordial level there are just algebraic surfaces allowing no completion to smooth ...-adic or real surfaces, and defined only in algebraic extensions of rationals by algebraic field equations. At higher levels rational-adic, p-adic and even infinite-P p-adic completions of infinite primes could appear and provide natural completions of function spaces. Of course, all these generalizations might make sense only as cognitive structures in Platonia and it is comforting to know that there is room in just a single point of TGD Universe for all this richness of imaginable structures!

The reader not familiar with the basic algebra of quaternions and octonions is encourated to study some background material: the homepage of Tony Smith provides among other things an excellent introduction to quaternions and octonions [21]. String model builders are beginning to grasp the potential importance of octonions and quaternions and the articles about possible applications of octonions [22, 23, 24] provide an introduction to octonions using the language of physicist.

Personally I found quite frustrating to realize that I had neglected totally learning of the basic ideas of algebraic geometry, despite its obvious potential importance for TGD and its applications in string models. This kind of losses are the price one must pay for working outside the scientific community. It is not easy for a physicist to find readable texts about algebraic geometry and algebraic number theory from the bookshelves of mathematical libraries. The book "Algebraic Geometry for Scientists and Engineers" by Abhyankar

[25], which is not so elementary as the name would suggest, introduces in enjoyable manner the basic concepts of algebraic geometry and binds the basic ideas with the more recent developments in the field. "Problems in Algebraic Number Theory" by Esmonde and Murty [26] in turn teaches algebraic number theory through exercises which concretize the abstract ideas. The book "Invitation to Algebraic Geometry" by K. E. Smith. L. Kahanpää, P. Kekäläinen and W. Traves is perhaps the easiest and most enjoyable introduction to the topic for a novice. It also contains references to the latest physics inspired work in the field.

8 Dualities and conformal symmetries in TGD framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the modified Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of configuration space geometry. Also the duality between 7-D and 3-D CDs relating closely to quantum gravitational holography and YM-gravity duality and representing basically field-particle duality suggests itself. In this case strict duality seems however too strong an assumption.

8.1 Electric-magnetic duality

Electric-magnetic duality for the induced Kähler induced field is present also in TGD (CP_2 Kähler form is self-dual). My original belief was that it corresponds to a self duality leaving Kähler coupling constant invariant as an analog of critical temperature: $\alpha_K \rightarrow \alpha_K$ in this transformation [B2, B3]. This duality would allow to construct configuration space Kähler metric in terms of Kähler electric or magnetic fluxes.

It is however possible to imagine a second variant of electric-magnetic duality, not in fact a genuine duality, could correspond to $\alpha_K \rightarrow -\alpha_K$ [B2, B3, E3], which would be a logarithmic version of $g \rightarrow 1/g$ duality and makes sense in TGD framework since g_K does not appear as a coupling constant. This "duality" would have nothing to do with whether the configuration space Hamiltonians are defined in terms of Kähler magnetic or electric fluxes. The two directions of geometric and subjective time and two possible signs of inertial energy would correspond to the two signs (phase conjugate photons would provide example of negative energy particles propagating in the direction of geometric past). Electric *resp.* magnetic flux tubes would be favored in the two phases and a beautiful consistency with TGD inspired cosmology results [D5]. It is questionable whether one can speak of duality now since the two phases seem to be physically different so that alternative descriptions would not be in question. In any case, if a genuine duality is in question it is enough to use either future directed 7-D CDs or past directed CDs. If not, both are required.

This duality relates in an interesting manner to the idea that space-time surfaces can be regarded either hyper-quaternionic sub-manifolds of M^8 endowed with hyper-octonionic tangent space or as 4-surfaces in $M^4 \times CP_2$ [E2]. The point is that one can consider also the dual definition for which the 4-D normal space defines 4-D subalgebra of 8-D algebra at each point of the space-time surface. Future-past duality could basically reduce to this purely geometric duality and would basically reflect bra-ket duality.

Unfortunately, the construction of S-matrix [C2] suggests strongly that the change of sign of α_K breaks unitarity (this is not related to the naive expectation that g_K becomes imaginary since the absolute minima would be different and tend to be dominated by Kähler magnetic fields). In pure quantum context the cosmological argument does not actually bite since one can simply regard the moment of "big bang" as a creation of pairs of positive and negative energy cosmic strings. Whether the time development is dissipative in ordinary or reversed time direction for negative energy cosmic strings is not relevant. Negative energy space-time sheets could be final points of dissipative evolution from the beginning somewhere in the geometric future. It is also possible that the dissipative time evolution occurs in the reversed time direction only in the fermionic degrees of freedom.

8.2 Duality of 3-D and 7-D causal determinants as particle-field duality

As already described, TGD predicts two kinds of super-conformal symmetries corresponding to 7-D and 3-D causal determinants and that their duality would generalize the age-old field-particle particle duality so that quantum gravitational holography and YM-gravitational duality could be seen as particular aspects of field particle duality. The two dual super symmetry algebras defined by super-canonical and Super Kac-Moody algebras at configuration space level define spectrum generating algebras whereas at space-time level they define pure super gauge symmetry algebras eliminating half of the helicities of the induced Dirac spinor fields at each point.

1. The conformal symmetries associated with 7-D CDs and space-time interior

The super-canonical conformal invariance is associated with 7-dimensional light like CDs $\delta M^4_{\pm}(a) \times CP_2$ and their unions at the level of the imbedding space. The GRT counterpart is the moment of local "big bang". The string model counterpart is a Kaluza-Klein type representation of quantum numbers used in superstring models relying on closed strings. Obviously this corresponds to the field aspect of the duality.

At space-time level these dynamical super-conformal symmetries have gauge super-symmetries as their counterpart. Hyper-quaternion analytic maps of the space-time surface regarded as a surface in hyper-octonionic M^8 induce deformations of boundaries of the space-time surface and could represent conformal transformations becoming dynamical at boundaries and causal determinants. These solutions would correspond to pure gauge degrees of freedom in the interior. Also the construction of space-time surfaces in terms of hyper-octonion real-analytic maps of M^8 could be interpreted in terms of a dynamical hyper-octonionic conformal symmetry.

The interior symmetry could be equivalent with $N = 4$ local gauge supersymmetry ($N = 4$ super-symmetric YM theory has been proposed to be closely linked with string models). There would be no global super symmetry. Since the hyper-quaternion conformal gauge symmetry can make sense only in interior, and since only the induced spinor field in the interior of space-time surface contributes to the configuration space super charges, hyper-quaternion conformal symmetry would indeed correspond to the field aspect of the field-particle duality.

2. Conformal symmetries associated with 3-D light like CDs and quantum gravitational holography

The Super-Kac Moody symmetry at the 3-D light like causal determinants and super-canonical symmetry at 7-D causal determinants define configuration space super-symmetries. These super-symmetries are dynamical and contrary to the original beliefs do not imply the existence of sparticles.

The conformal symmetry associated with the 3-dimensional light like CDs reduces to a generalization of the ordinary super-conformal symmetry. The derivative of the normal coordinate disappears from the modified Dirac operator and solutions are 3-dimensional spinorial shock waves having a very natural interpretation as representations of elementary particles. The GRT analog is black hole horizon. $N = 4$ superconformal symmetry in an almost ordinary sense is in question.

The induced spinor fields carry electro-weak quantum numbers as YM type quantum numbers, Poincare and color isometry charges, but not color as spin like degrees of freedom. Hence color degrees of freedom are analogous to rigid body rotational degrees of freedom of the 3-D causal determinant and genuine configuration space degrees of freedom having no counterpart at space-time level although conserved classical color charges make sense: obviously Kaluza-Klein type quantum numbers are in question.

7–3 duality however suggests that it is possible to code the information about configuration space color partial wave to the induced spinor field at X^3_l (for 7-D CDs this is not necessary) as a functional of X^3_l. The guess is that the shock wave solutions of the modified Dirac equation at 3-D CDs can be constructed by taking imbedding space spinor harmonics, operating on them by appropriate color Kac Moody generators to get a correct correlation between electro-weak and color quantum numbers, and applying the modified Dirac operator D to get a spinor basis $D\Psi_m$. If the spinor basis obtained in this manner satisfies $D\Psi_m = c_{mn}o\Psi_n$, where o is the contraction of the light like normal vector of CD with the induced gamma matrices appearing in the eigenvalue equation $D\Psi = \lambda o\Psi$ and defining boundary states for the induced spinor fields and Dirac determinant, the construction works. The fact that quark color does not have a direct space-time counterpart (imbedding space spinors allow color partial waves but induced spinors do not) might correlate with color confinement and with the impossibility to detect free quarks.

3. 7–3 duality and effective 2-dimensionality of 3-surfaces

Whether super-canonical and Kac-Moody algebras are dual is not at all obvious. The assumption that the situation reduces to the intersections X^2_i of the 3-D CDs X^3_l with 7-D CDs defining 2-sub-manifolds of X^3 concretizes the idea about duality. Duality would imply effective 2-dimensionality of 3-surfaces and the task is to understand what this could mean.

a) By duality both X^2_i-local H-isometries and the Hamiltonians of $\delta M^4_+ \times CP_2$ restricted to X^2_i span the tangent space of CH. A highly non-trivial implication would be a dramatic simplification of the construction of the configuration space Hamiltonians, Kähler metric, and gamma matrices since one could just sum only over the flux integrals over the sub-manifolds X^2_i. The best that one might hope is that it is possible to fix both 3-D light like CDs and their sub-manifolds X^2_i and 7-D CDs freely. There would be good hopes about achieving the p-adicization of the basic definitions.

One could assign unique modular degrees of freedom to X_i^2: this would be crucial for the unique definition of the elementary particle vacuum functionals [F1]. This would give rather good hopes of achieving a better understanding of why particle families corresponding to genera $g > 2$ are effectively absent from the spectrum. Elementary particle vacuum functionals vanish when $g > 2$ is hyper-elliptic, that is allows Z_2 conformal symmetry. The requirement that the tangent space 2-surface defines at each point a commutative sub-space of the octonionic tangent space might force $g > 2$ surfaces to be hyper-elliptic.

b) The reduction to dimension 2 could be understood in terms of the impossibility to choose X^3 freely once light like 3-D CDs are fixed but this does not remove the air of paradox. The resolution of the paradox comes from the following observation. The light likeness condition for 3-D CD can be written in the coordinates for which the induced metric is diagonal as a vanishing of one of the diagonal components of the induced metric, say g_{11}:

$$g_{1i}h_{kl}\partial_1 h^k \partial_i h^l \quad = \quad 0 \ , \ \ i = 1,2,3 \ . \tag{12}$$

The condition $g_{11} = 0$ is exactly like the light likeness condition for the otherwise random M^4 projection of CP_2 type extremals [D1]. When written in terms of the Fourier expansion this condition gives nothing but classical Virasoro conditions. This analog of the conformal invariance is different from the conformal invariance associated with transversal degrees of freedom and and from hyper-quaternion and -octonionic conformal symmetries encountered at the level of M^8. This symmetry conforms nicely with the duality idea since also the boundary of the light cone allows conformal invariance in both light like direction and transversal degrees of freedom.

One can consider two interpretations of this symmetry.

i) The degrees of freedom generating different light like 3-D CDs X_l^3 with a given intersection X^2 with 7-D CD correspond to zero modes. Physically this would mean that in each quantum jump a complete localization occurs in these degrees of freedom so that particles behave effectively classically. With this interpretation these degrees of freedom could perhaps be seen as dual for the zero mode degrees of freedom associated with the space-like 3-surfaces X^3 at 7-D CDs: deformation of X_l^3 would induce deformation of X^3.

ii) Gauge degrees of freedom could be in question so that one can make a gauge choice fixing the orbits within certain limits. The two symmetries could correspond to two different choices of gauge reflected as a choice of different space-time sheets. This would mean additional flexibility in the interpretation of this symmetry at the level of solutions of the modified Dirac equation.

At the level of configuration space geometry the result would mean that one can indeed code all data using only two-dimensional surfaces X_i^2 of X^3. This brings in mind a number theoretic realization for the quantum measurement theory. That only mutually commuting observables can be measured simultaneously would correspond to the assumption that all data about configuration space geometry and quantum physics must be given at 2-dimensional surfaces of H for which the tangent space at each point corresponds to an Abelian sub-algebra of octonions. Quantum TGD would reduce to something having very high resemblance with WZW model. One cannot deny the resemblance with M-theories with M interpreted as a membrane.

4. 7–3 duality and the equivalence of loop diagrams with tree diagrams

The 3-D light like CDs are expected to define analogs of Feynman diagrams. In the simplest case there would be past of future and past directed 7-D CDs $X^7_\pm = \delta M^4_\pm \times CP_2$, and the lines of the generalized Feynman diagram would begin from X^7_+ and terminate to X^7_-. In [C5] the generalization of duality symmetry of string models stating that generalized Feynman diagrams with loops are equivalent with tree diagrams is discussed. By quantum-classical correspondence this would mean that the conformal equivalence for Feynman diagrams defined by 3-D light like CDs generalizes to a topological equivalence. This is indeed as it should be since it is the intersections X_i^2 with $X^7_\pm$ which should code for physics and these intersections do not contain information about loops.

Interesting questions relate to the interpretation of the negative energy branches of the space-time surface. It would seem that also the surfaces X_i^2 are accompanied by negative energy branch. The branching brings in mind a space-time correlate for bra-ket dichotomy. The two branches would represent Feynman diagrams which are equivalent but correspond to different sign of Kähler coupling strength if the generalization of electric-magnetic duality is accepted.

5. 7-3 duality and quantum measurement theory

The action of Super Kac-Moody generators on configuration space Hamiltonians is well defined and one might hope that as a functional of 2-surface it could give rise to a unique superposition of super-canonical Hamiltonians. Same should apply to the action of super-canonical algebra on Kac Moody algebra. At the level of gamma matrices the question is whether the configuration space metric can be defined equivalently in terms of anti-commutators of super-canonical and Super Kac-Moody generators. If the answer is affirmative, then 7–3 duality would be nothing but a transformation between two preferred coordinates of the configuration space.

TGD inspired quantum measurement theory suggests however that the two super-conformal algebras correspond to each other like classical and quantal degrees of freedom. Super Kac-Moody algebra and super conformal algebra would act as transformations preserving the conformal equivalence class of the partonic 2-surfaces X^2 associated with the maxima of the Kähler function whereas super-canonical algebra in general changes conformal moduli and induces a conformal anomaly in this manner. Hence Kac-Moody algebra seems to act in the zero modes of the configuration space metric. In TGD inspired quantum measurement zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement.

Duality would be also realized in a well-defined sense at the level of configuration space conformal symmetries. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-canonical algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-canonical local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-canonical generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and thus mass.

8.3 Number-theoretical spontaneous compactification

The dimensions of space-time and imbedding space suggest that quaternions and octonions should play important role in the formulation of TGD. These ideas have now developed to what might be called number theoretical spontaneous compactification. This approach even suggests that TGD allows a dual formulation as 8-dimensional string theory.

8.3.1 Hyper-quaternions and -octonions

The original idea was that space-time surfaces could be regarded as four-surfaces in 8-D imbedding space with the property that the tangent spaces of these spaces can be locally regarded as 4- *resp.* 8-dimensional number fields of quaternions and octonions.

The difficulties caused by the Euclidian metric signature of the number theoretical norm have however forced to give up the original idea as such, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart of the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity.

The problem is that $H = M^4 \times CP_2$ cannot be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces are assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO. Since the hyper-quaternionic sub-spaces of HO with fixed complex structure are labelled by CP_2, each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

8.3.2 Space-time surface as a hyper-quaternionic sub-manifold of hyper-octonionic imbedding space?

Space-time identified as a hyper-quaternionic sub-manifold of the hyper-octonionic space in the sense that the tangent space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point. One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered. Some delicacies are caused by the question whether the induced algebra at X^4 is just the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If normal part of the product is projected out the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2. This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH - -M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2. This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter famility co-hyper-quaternionic space-time surfaces.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that the absolute minima of Kähler action correspond to the hyper-quaternion analytic surfaces. This conjecture has several variants. It could be that only asymptotic behavior corresponds to hyper-quaternion analytic function but that that hyper-quaternionicity is general property of absolute minima. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates appear naturally also in the construction of general solutions of field equations.

8.3.3 The notion of Kähler calibration

Calibration is a closed p-form, whose value for a given p-plane is not larger than its volume in the induced metric. What is important that if it is maximum for tangent planes of p-sub-manifold, minimal surface with smallest volume in its homology equivalence class results.

The idea of Kähler calibration is based on a simple observation. The octonionic spinor field defines a map $M^8 \rightarrow H = M^4 \times CP_2$ allowing to induce metric and Kähler form of H to M^8. Also Kähler action is well defined for the local hyper-quaternion plane.

The idea is that the non-closed 4-form associated the wedge product of unit tangent vectors of hyper-quaternionic plane in M^8 and saturating to volume for it becomes closed by multiplication with Kähler action density L_K. If L_K is minimal for hyper-quaternion plane, hyper-quaternionic manifolds define extremals of Kähler action for which the magnitudes of positive and negative contributions to the action are separately minimized.

This variational principle is not equivalent with the absolute minimization of Kähler action. Rather, Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant (they carry non-vanishing density gravitational energy). The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself. The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable construction of Kähler function.

8.3.4 Generalizing the notion of $HO-H$ duality to quantum level

The obvious question is how the $HO-H$ duality could generalize to quantum level. Number theoretical considerations combined with the general vision about generalized Feynman diagrams as a generalization of braid diagrams lead to general formulas for vertices in HO picture.

Simple arguments lead to the conclusion that strict duality can make sense only if the octonionic spinor field is second quantized such that the real Laurent coefficients correspond to a complete set of mutually commuting Hermitian operators having interpretation as observables. Space-time concept is well defined only for the eigen states of these operators and physical states are mapped to space-time surfaces. The Hermitian operators would naturally correspond to the state space spanned by super Kac-Moody and super-canonical algebras, and quantum states would have precise space-time counterparts in accordance with quantum-classical correspondence.

The regions inside which the power series representing HO analytic function and matrix elements of G_2 rotation converge are identified as counterparts of maximal deterministic regions of the space-time surface. The Hermitian operators replacing Laurent coefficients are assumed to commute inside these regions identifiable also as coherence regions for the generalized Schrödinger amplitude represented by the HO spinor field.

By quantum classical correspondence these regions are correlates for the final states of quantum jumps. The space-like 3-D causal determinants X^3 bounding adjacent regions of this kind represent quantum jumps. The octonionic part of the inner of the octonionic spinor fields at the two sides of the discontinuity defined as an integral over X^3 gives (or its sub-manifold) a number identifiable as complex number when imaginary unit is identified appropriately. The inner product is identified as a representation of S-matrix element for an internal transition of particle represented by 3-surface, that is 2-vertex.

For the generalized Feynman diagrams n-vertex corresponds to a fusion of n 4-surfaces along their ends at X^3. 3-vertex can be represented number theoretically as a triality of three hyper-octonion spinors integrated over the 3-surface in question. Higher vertices can be defined as composite functions of triality with a map $(h_1, h_2) \rightarrow \overline{h}_3$ defined by octonionic triality and by duality given by the inner product. More concretely, $m + n$ vertex corresponds in HO picture to the inner product for the local hyper-octonionic products of m outgoing and n incoming hyper-octonionic spinor fields integrated over the 3-surface defining the vertex. Both 2-vertices representing internal transitions and $n > 2$ vertices are completely fixed. This should give some idea about the power of the number theoretical vision.

8.3.5 Does TGD reduce to 8-D WZW string model?

Conservation laws suggests that in the case of non-vacuum extremals the dynamics of the local G_2 automorphism is dictated by field equations of some kind. The experience with WZW model suggests that in the case of non-vacuum extremals G_2 element could be written as a product $g = g_L(h)g_R^{-1}(\overline{h})$ of hyper-octonion analytic and anti-analytic complexified G_2 elements. g would be determined by the data at hyper-complex 2-surface for which the tangent space at a given point is spanned by real unit and preferred hyper-octonionic unit. Also Dirac action would be naturally restricted to this surface. The theory would reduce in HO picture to 8-D WZW string model both classically and quantally since vertices would reduce to integrals over 1-D curves. The interpretation of generalized Feynman diagrams in terms of generalized braid/ribbon diagrams and the unique properties of G_2 provide further support for this picture. In particular, G_2 is the lowest-dimensional Lie group allowing to realize full-powered topological quantum computation based on generalized braid diagrams and using the lowest level k=1 Kac Moody representation. Even if this reduction would occur only in special cases, such as asymptotic solutions for which Lorentz Kähler force vanishes or maxima of Kähler function, it would mean enormous simplification of the theory.

8.3.6 Spin crisis of proton as evidence for HO-H duality

The $SU(3)$ decomposition of hyper-octonionic spinors suggest strongly the interpretation in terms of leptons and quarks. The obvious objection is that these spinors do not possess electro-weak isospin and spin. The hint leading to the solution of puzzle is the observation that similar problem is encountered also in H picture. H spinors do not carry color as a spin like quantum number. Color resides at configuration space level or equivalently in color-rotational degrees of freedom of 3-surface. HO-H duality at configuration space level means that the spin-like quantum numbers in HO picture correspond to configuration space orbital quantum numbers in H picture and vice versa. The duality can be also seen as a kind of super-symmetry permuting orbital and spin degrees of freedom at the level of configuration space.

At the level of hadron physics $HO - H$ symmetry has a very concrete interpretation consistent with some, almost paradoxical, aspects of hadron physics. In particular, current quark-constituent quark duality corresponds to $HO-H$ duality, which among other things predicts correctly SO(4) chiral symmetry of low energy hadron physics. The puzzling finding that current quarks do not seem to contribute to proton spin [37] where as constituent quark model explains nicely the proton spin in terms of quark spins, known as proton spin crisis is in a nice accordance with the prediction that quarks in HO picture carry spin and electro-weak isospin as "orbital" quantum numbers in HO configuration space degrees of freedom.

Quantum classical correspondence suggests that the quantum numbers represented at configuration space degrees of freedom should have some kind of representation also at the space-time level. A representation in terms of anyons might be possible since both space-like partonic surfaces and the dual string orbit like surfacse are 2-dimensional. This representation could also allow to understand how states described by hyper-octonionic and ordinary H-spinors relate physically to each other. A detailed discussion of ew-color duality and the bundle of dualities related to HO-H duality can be found in [E2].

8.4 Quantum gravitational holography

The so called AdS/CFT of Maldacena [35] correspondence relates to quantum-gravitational holography states roughly that the gravitational theory in 10-dimensional $AdS_{10-n} \times S^n$ manifold is equivalent with the conformal field theory at the boundary of AdS_D factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD quantum gravitational holography is realized in terms of the modified Dirac action at light like 3-D CDs, which by their metric 2-dimensionality allow superconformal invariance and are very much like world sheets of closed super string or the ends of an open string.

It is possible to deduce the values of Kähler action at maximally deterministic regions of space-time sheets from the Dirac determinants at the CDs [E3] so that the enormously difficult solution of the absolute minimization of Kähler action would be reduced to local data stating that CDs are light like 3-surfaces which are also minimal surfaces in the case that Kähler action density is non-vanishing at them. This reduction has enormous importance for the calculability of the theory. Also the values of Kähler coupling strength and gravitational constant are predicted [E3].

Here a word of warning is in order: I do not know how to prove that the minimal surface property of the CDs implies the absolute minimization of Kähler action. One can even consider the possibility that absolute minimization is replaced with the minimal surface property in the case that the action density does not vanish at the CD.

Perhaps the most practical form of the quantum gravitational holography would be that the correlation functions of $N = 4$ super-conformal field theory at the light like 3-D CDs allow to construct the vertices needed to construct S-matrix of quantum TGD. Computationally TGD would reduce to almost string model since light like CD:s are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand. There is also an analogy with the Wess-Zumino-Witten model: light like CDs would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

Quantum gravitational holography could also mean that light like CDs define what might be called fundamental central nervous systems able to represent and process conscious information about the interior of the space-time surface in terms of its own quantum states which have interpretation either as a time evolution or state (duality again!). Topological quantum computation might be one of the activities associated with the light like CDs as proposed in [O3].

8.5 Super-symmetry at the space-time level

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X^3_l. Gauge symmetries are in question when the intersections of X^3_l with 7-D CDs X^7 are not changed. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced configuration space consisting of the light like 3-D CDs. The conformal symmetries in question imply a further degeneracy of the configuration space metric and effective metric 2-dimensionality of 3-surfaces as a consequence. These conformal symmetries are accompanied by $N = 4$ local super conformal symmetries defined by the solutions of the induced spinor fields.

Contrary to the original beliefs, these conformal symmetries do not seem to be continuable to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate. The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional.

A resolution of the interpretational problems came with the realization that it is hyper-quaternionic and -octonionic conformal symmetries, which are in question and that these symmetries are naturally associated with the description of the space-time surface as a 4-surface in hyper-quaternionic $HO = M^8$ rather than in H. These symmetries are realized also at the level of H. Note that hyper-quaternionic symmetries act trivially in the interior of X^4 but induce deformations of boundaries of X^4.

The solutions of the modified Dirac equation $D\Psi = 0$, define the modes which do not contribute to the Dirac determinant of the modified Dirac operator in terms of which the vacuum functional assumed to correspond to the exponent of the Kähler action is defined. Thus they define gauge super-symmetries. Usually D selects the physical helicities by the requirement that it annihilates physical states: now the situation is just the opposite. D^2 annihilates the generalized eigen states both at space-like and light like 3-surfaces. Hence the roles of the physical and non-physical helicities are switched. It is the generalized eigen modes of D with non-vanishing eigenvalues λ, which code for the physics whereas the solutions of the modified Dirac equation define super gauge symmetries.

At the space-like 3-surfaces associated with 7-D CDs the spinor harmonics of the configuration space satisfy the $M^4 \times CP_2$ counterpart of the massless Dirac equation so that non-physical helicities are eliminated in the standard sense at the imbedding space level. The righthanded neutrino does not generate an $N = 1$ space-time super-symmetry contrary to the long held belief.

8.6 Super-symmetry at the level of configuration space

The gamma matrices of the configuration space are defined as matrix elements of properly chosen operators between right-handed neutrino and second quantized induced spinor field at space-like boundaries X^3. These generators define the fermionic generators of what I call super-canonical algebra. The right handed neutrino can be replaced with any spinor harmonic of the imbedding space to obtain an extended super-algebra, which can be used to construct the physical states.

The requirement that super-generators vanish for the vacuum extremals requires that the modified Dirac operator D_+ or the inverse of D_- appearing in the matrix element of the "Hermitian conjugate" $S^- = (S^+)^\dagger$ of the super charge S^+. Here $\pm$ refers to the negative and positive energy space-time sheets meeting at X^3 or to the two maximally deterministic space-time regions separated by the causal determinant. The operators D_+ and D_-^{-1} are restricted to the spinor modes not annihilated by $D_\pm$. The super-generator generated by the covariantly constant right handed neutrino vanishes identically: a more rigorous argument showing that $N = 1$ global super symmetry is indeed absent.

If the configuration space decomposes into a union of sectors labelled by unions of light cones having dips at arbitrary points of M^4, the spinor harmonics can be assumed to define plane waves in M^4 and even possess well-defined four-momenta and mass squared values. Same applies to the super-canonical generators defined by their commutators. This means that the generators of the super-canonical algebra generated in this manner would possess well defined four-momenta and thus their action would change the mass of the state. Space-time super-symmetries would be absent. Similar argument applies to the Kac Moody algebras associated with the light like 3-D CDs if super-canonical Super Kac-Moody algebras provide dual representations of quantum states.

If the gist of these admittedly heuristic arguments is correct, they force to modify drastically the existing view about space-time super-symmetries. The problem how to break super-symmetry disappears since there is no space-time symmetry be broken down. Super-symmetries are realized as a spectrum generating algebra rather than symmetries in the standard sense.

I hasten to admit that I have myself believed that right handed neutrino defines a global super-symmetry and proposed that the topological condensation of sparticles and particles at space-time sheets with different p-adic primes would provide an elegant model for super-symmetry breaking using same general mass formulas but only a different mass scale. Giving up this assumption causes however only a sigh of relief. The predicted spectrum of massless states is reduced dramatically [F3]. p-Adic mass calculations based on p-adic thermodynamics and representations of super-conformal algebra are not affected since the global $N = 1$ super-symmetry implies only an additional vacuum degeneracy. Most predictions of TGD remain intact. One can however consider the possibility of light colored sneutrinos obtained by applying to a neutrino state a colored and thus non-vanishing super-canonical generator defined by right handed antineutrino.

It deserves to be noticed that the notion super-symmetry in configuration space sense was discovered with the advent of super string models and generalized to a space-time super-symmetry when gauge theories made their breakthrough. The notion of spontaneous compactification (we meet our friend again and again!) inspired then the hypothesis that this super-symmetry has a space-time counterpart and everyone believed. There is now an entire industry making similar purely formal out of context applications and generalizations of quantum groups, which originally emerged naturally in knot and braid theory and in the theory of von Neumann algebras [O4, O3].

9 Physics as geometry of configuration space spinor fields

The construction of the configuration space geometry has proceeded rather slowly. The experimentation with various ideas has however led to the identification of the basic constraints on the configuration space geometry.

9.1 Reduction of quantum physics to the Kähler geometry and spinor structure of configuration space of 3-surfaces

The basic philosophical motivation for the hypothesis that quantum physics could reduce to the construction of configuration space Kähler metric and spinor structure, is that infinite-dimensional Kähler geometric existence could be unique not only in the sense that the geometry of the space of 3-surfaces could be unique but that also the dimension of the space-time is fixed to $D = 4$ by this requirement and $M^4_+ \times CP_2$ is the only possible choice of imbedding space. This optimistic vision derives from the work of Dan Freed with loops spaces demonstrating that they possess unique Kähler geometry and from the fact that in $D > 1$ case the existence of Riemann connection, finiteness of Ricci tensor, and general coordinate invariance poses even stronger constraints.

9.2 Constraints on configuration space geometry

The detailed considerations of the constraints on configuration space geometry suggests that it should possess at least the following properties.

a) Metric should be Kähler metric. This property is necessary if one wants to geometrize the oscillator algebra used in the construction of the physical states and to obtain a well defined divergence free functional integration in the configuration space.

b) Metric should allow Riemann connection, which, together with the Kähler property, very probably implies the existence of an infinite dimensional isometry group as the construction of Kähler geometry for the loop spaces demonstrates [13].

c) The so called symmetric spaces classified by Cartant [14] are Cartesian products of the coset spaces G/H with maximal isometry group G. Symmetric spaces possess G invariant metric and curvature tensor is constant so that all points of the symmetric space are metrically equivalent. Symmetric space structure means that the Lie-algebra of G decomposes as

$$\begin{aligned} &g = h \oplus t \ , \\ &[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ , \end{aligned}$$

where g and h denote the Lie-algebras of G and H respectively and t denotes the complement of h in g. The existence of the $g = t + h$ decomposition poses an extremely strong constraint on the symmetry group G.

In the infinite-dimensional context symmetric space property would mean a drastic calculational simplification. The most one can hope is that configuration space is expressible as a union $\cup_i (G/H)_i$ of symmetric spaces. Reduction to a union of G/H is the best one can hope since 3-surface of Planck size cannot be metrically equivalent with a 3-surface having the size of galaxy! The coordinates labelling the symmetric spaces in this union do not appear as differentials in the line element of configuration space and are thus zero modes. They correspond to non-quantum fluctuating degrees of freedom in a well defined sense and are identifiable as classical variables of quantum measurement theory.

d) Metric should be Diff^4 (not only $Diff^3$!) invariant and degenerate and the definition of the metric should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 to act on. This requirement is absolutely crucial for all developments.

e) Divergence cancellation requirement for the functional integral over the configuration space requires that the metric is Ricci flat and thus satisfies vacuum Einstein equations.

9.3 Configuration space as a union of symmetric spaces

In the finite-dimensional context, globally symmetric spaces are of form G/H and connection and curvature are independent of the metric, provided it is left invariant under G. Good guess is that same holds true in the infinite-dimensional context. The task is to identify the infinite-dimensional groups G and H. Only quite recently, more than seven years after the discovery of the candidate for Kähler function defining the metric, it became finally clear that these identifications follow quite nicely from Diff^4 invariance and Diff^4 degeneracy.

The crux of the matter is Diff^4 degeneracy: all 3-surfaces on the orbit of 3-surface X^3 must be physically equivalent so that one can effectively replace all 3-surfaces Z^3 on the orbit of X^3 with a suitably chosen surface Y^3 on the orbit of X^3. The Lorentz and Diff^4 invariant choice of Y^3 is as the intersection of the 4-surface with the set $\delta M^4_+ \times CP_2$, where δM^4_+ denotes the boundary of the light cone: effectively the imbedding space can be replaced with the product $\delta M^4_+ \times CP_2$ as far as vibrational degrees of freedom are considered. More precisely: configuration space has a fiber structure: the 3-surfaces $Y^3 \subset \delta M^4_+ \times CP_2$ correspond to the base space and the 3-surfaces on the orbit of given Y^3 and diffeomorphic with Y^3 correspond to the fiber and are separated by a zero distance from each other in the configuration space metric.

These observations lead to the identification of the isometry group as some subgroup G of the group of the diffeomorphisms of $\delta H = \delta M^4_+ \times CP_2$. These diffeomorphisms indeed act in a natural manner in δCH, the space of the 3-surfaces in δH. Therefore one can identify the configuration space as the union of the coset spaces G/H, where H corresponds to the subgroup of G acting as diffeomorphisms for a given X^3. H depends on the topology of X^3 and since G does not change the topology of the 3-surface, each 3-topology defines a separate orbit of G. Therefore, the union involves the sum over all topologies of X^3 plus possibly other 'zero modes'.

The task is to identify correctly G as a sub-algebra of the diffeomorphisms of δH. The only possibility seems to be that the canonical transformations of δH generated by the function algebra of δH act as isometries of the configuration space. The canonical transformations act nontrivially also in δM^4_+ since δM^4_+ allows Kähler structure and thus also symplectic structure.

9.3.1 The magic properties of the light like 3-surfaces

In case of the Kähler metric, G- and H Lie-algebras must allow a complexification so that the isometries can act as holomorphic transformations. The unique feature of δM^4_+, realized already seven years ago, is its metric degeneracy: the boundary of the light cone is metrically 2-dimensional sphere although it is topologically 3-dimensional! This implies that light cone boundary allows an infinite-dimensional group of conformal symmetries generated by an algebra, which is a generalization of the ordinary Virasoro algebra! There is actually also an infinite-dimensional group of isometries (!) isomorphic with the group of the conformal transformations! Even more, in case of δH the groups of the conformal symmetries and isometries are local with respect to CP_2. Furthermore, light cone boundary allows infinite dimensional group of canonical transformations as the symmetries of the symplectic structure automatically associated with the Kähler structure. Therefore 4-dimensional Minkowski space is in a unique position in TGD approach. δM^4_+ allows also complexification and Kähler structure unlike the boundaries of the higher-dimensional light cones so that it becomes possible to define a complexification in the tangent space of the configuration space, too.

The space of the vector fields on $\delta H = \delta M^4_+ \times CP_2$ inherits the complex structure of the light cone boundary and CP_2. The complexification can be induced from the complex conjugation for the functions depending on the radial coordinate of the light cone boundary playing the same role as the time coordinate associated with string space-time sheet. In M^4_+ degrees of freedom complexification works only provided that the radial vector fields posses zero norm as configuration space vector fields (they have also zero norm as vector fields).

The effective two-dimensionality of the light cone boundary allows also to circumvent the no-go theorems associated with the higher-dimensional Abelian extensions. First, in the dimensions $D > 2$ Abelian extensions of the gauge algebra are extensions by an infinite dimensional Abelian group rather than central extensions by the group $U(1)$. In the present case the extension is a symplectic extension analogous to the extension defined by the Poisson bracket $\{p, q\} = 1$ rather than the standard central extension but is indeed 1- dimensional and well defined provided that the configuration space metric is Kähler. Secondly, $D > 2$ extensions possess no unitary faithfull representations (satisfying certain well motivated physical constraints) [15]. The point is that light cone boundary is metrically and conformally 2-sphere and therefore the gauge algebra is effectively the algebra associated with the 2-sphere and, as a consequence, also the configuration space metric is Kähler.

There is counter argument against complexifixation. The Kähler structure of the light cone boundary is not unique: various complex structures are parameterized by $SO(3,1)/SO(3)$ (Lobatchewski space). The definition of the Kähler function as absolute minimum of Kähler action however makes it possible to assign unique space-time surface $X^4(Y^3)$ to any Y^3 on the light cone boundary and the requirement that the group $SO(3)$ specifying the Kähler structure is isotropy group of the classical four-momentum associated with $X^4(Y^3)$, fixes the complex structure uniquely as a function of Y^3. Thus it seems that Kähler action is necessary ingredient of the group theoretical approach.

9.3.2 Symmetric space property reduces to conformal and canonical invariance

The idea about symmetric space is extremely beautiful but it millenium had to change before I was ripe to identify the precise form of the Cartan decomposition. The solution of the puzzle turned out to be amazingly simple.

The inspiration came from the finding that quantum TGD leads naturally to an extension of Super Algebras by combining Ramond and Neveu-Schwartz algebras into single algebra. This led to the introduction Virasoro generators and generators of canonical algebra of CP_2 localized with respect to the light cone boundary and carrying conformal weights with a half integer valued real part. Soon came the realization that the conformal weights $h = -1/2 - i\sum_i y_i$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta, are excellent candidates for the conformal weights. It took some time to answer affirmatively the question whether also the negatives of the trivial zeros $z = -2n$, $n > 0$ should be included. Thus the conjecture inspired by the work with Riemann hypothesis stating that the zeros of Riemann Zeta appear at the level of basic quantum TGD turned out be correct.

The generators whose commutators define the basis of the entire algebra have conformal weights given by the negatives of the zeros of Rieman Zeta. The algebra is a direct sum $g = g_1 \oplus g_2$ such that g_1 has $h = n$ as conformal weights and g_2 $h = n-1/2+iy$, where y is sum over imaginary parts y_i of non-trivial zeros of Zeta. Only $h = 2n$, $n > 1$, and $h = -1/2-iy+n$, such that n is even (odd) if y is sum of odd (even) number of y_i correspond to the weights labelling the generators of t in the Cartan decomposition $g = h + t$. The resulting super-canonical algebra would quite well be christened as Riemann algebra.

The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of configuration space, which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over configuration space by generalizing the Gaussian integration of free quantum field theories.

The light cone conformal invariance differs in many respects from the conformal invariance of string theories. Finite-dimensional Kac-Moody group is replaced by infinite-dimensional canonical group. Conformal weights correspond to zeros of Riemann zeta and suitable superpositions of them in case of trivial zeros, and physical states can have non-vanishing conformal weights just as the representations of color group in CP_2 can have non-vanishing color isospin and hyper charge. The conformal weights have also interpretation as quantum numbers associated with unitary representations of Lorentz group: thus there is no conflict between conformal invariance and Lorentz invariance in TGD framework.

9.4 An educated guess for the Kähler function

The turning point in the attempts to construct configuration space geometry was the realization that four-dimensional $Diff$ invariance (not only 3-dimensional $Diff$ invariance!) of General Relativity must have a counterpart in TGD. In order to realize this symmetry in the space of 3-surfaces, the definition of the configuration space metric should somehow associate to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$ for Diff^4 to act on. Physical considerations require that the metric should be, not only Diff^4 invariant, but also Diff^4 degenerate so that infinitesimal Diff^4 transformations should correspond to zero norm vector fields of the configuration space.

Since Kähler function determines Kähler geometry, the definition of the Kähler function should associate a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3. The natural physical interpretation for this space-time surface is as the classical space-time associated with X^3 so that in TGD classical physics ($X^4(X^3)$) becomes a part of the configuration space geometry and of the quantum theory.

One could try to construct the configuration space geometry by finding the metric for a single representative 3-surface at each orbit of G and extending it by left translations to the entire orbit of G. The metric for this representative should be $Diff^3$ invariant and somehow it should associate a unique space-time surface to the 3-surface in question. The original attempt was however more indirect and based on the realization that the construction of the Kähler geometry reduces to that of finding Kähler function $K(X^3)$ with the property that it associates a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3 and possesses mathematically and physically acceptable properties. The guess for the Kähler function is the following one.

By Diff^4 invariance one can restrict the consideration on the set of 3-surfaces Y^3 on the 'light cone boundary' $\delta H = \delta M^4_+ \times CP_2$ since one can define the space-time surface associated with $X^3 \subset X^4(Y^3)$ to be $X^4(X^3) = X^4(Y^3)$ in case that the initial value problem for X^3 has $X^4(Y^3)$ as its solution. This implies $K(X^3) = K(Y^3)$.

The value of the Kähler function K for a given 3-surface Y^3 on light cone boundary is obtained in the following manner.
a) Consider all possible 4-surfaces $X^4 \subset M^4_+ \times CP_2$ having Y^3 as its sub-

manifold: $Y^3 \subset X^4$. If Y^3 has boundary then it belongs to the boundary of X^4: $\delta Y^3 \subset \delta X^4$.
b) Associate to each four surface Kähler action as the Maxwell action for the Abelian gauge field defined by the projection of the CP_2 Kähler form to the four-surface. For a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density whereas for an Euclidian signature the action density is always non-positive.
c) Define the value of the Kähler function K for Y^3 as the absolute minimum of the Kähler action S_K over all possible four-surfaces having Y^3 as its submanifold: $K(Y^3) = Min\{S_K(X^4)|X^4 \supset Y^3\}$.

This definition of the Kähler function has several physically appealing features.

a) Kähler geometry associates with each X^3 a unique four-surface, which will be interpreted as the classical space-time associated with X^3. This means that the so called classical space time (and physics!) in TGD approach is not defined via some approximation procedure (stationary phase approximation of the functional integral) but is an essential part of not only quantum theory, but also of the configuration space geometry, which in turn might be determined by a mere mathematical consistency! Since quantum states are superpositions over these classical space-times, it is clear that the observed classical space-time is some kind of effective, quantum average space-time, presumably defined as an absolute minimum for the effective action of the theory.

b) The space-time surface associated with a given 3-surface is analogous to a Bohr orbit of the old fashioned quantum theory. The point is that the initial value problem in question differs from the ordinary initial value problem in that although the values of the H coordinates h^k as functions $h^k(x)$ of X^3 coordinates can be chosen arbitrarily, the time derivatives $\partial_t h^k(x)$ at X^3 are uniquely fixed by the absolute minimization requirement unlike in the ordinary variational problems encountered in the classical physics. This implies something closely analogous to the quantization of the canonical momenta so that the space-time surface can be regarded as a generalized Bohr orbit. The classical quantization of electric charge and mass are possible consequences of the Bohr orbit property.

c) Kähler function is Diff^4 invariant in the sense that the value of the Kähler function is same for all 3-surfaces belonging to the orbit of a given 3-surface. As a consequence, configuration space metric is Diff^4 degenerate. The implications of the Diff^4 invariance have turned out to be decisive, not only for the geometrization of the configuration space, but also for the construction of the quantum theory. For instance, time like vibrational modes tangential to the 4-surface imply tachyonic mass spectrum unless they correspond to the zero modes of the configuration space metric. Diff^4 invariance however guarantees the required kind of degeneracy of the metric.

d) The non-determinism of Kähler action means that the complete reduction to the light cone boundary is not possible. This means a mathematical challenge but is physically a highly desirable feature since otherwise time would be lost as it is lost in the canonically quantized general relativity.

The most general expectation is that configuration space can be regarded as a union of coset spaces: $C(H) = \cup_i G/H(i)$. Index i labels 3-topology and zero modes. The group G, which can depend on 3-surface, can be identified as a subgroup of diffeomorphisms of $\delta M^4_+ \times CP_2$ and H must contain as its subgroup a group, whose action reduces to $Diff(X^3)$ so that these transformations leave 3-surface invariant.

The task is to identify plausible candidate for G and to show that the tangent space of the configuration space allows Kähler structure, in other words that the Lie-algebras of G and $H(i)$ allow complexification. One must also identify the zero modes and construct integration measure for the functional integral in these degrees of freedom. Besides this one must deduce information about the explicit form of configuration space metric from symmetry considerations combined with the hypothesis that Kähler function is determined as absolute minimum of Kähler action.

It will be found that in the case of $M^4_+ \times CP_2$ Kähler geometry, or strictly speaking contact Kähler geometry, characterized by a degenerate Kähler form (Diff^4 degeneracy and plus possible other degeneracies) seems possible. Although it seems that this construction must be generalized by allowing all light like 7-surfaces $X^3_l \times CP_2$, at least those for which X^3_l is boundary of light-cone inside M^4_+ or M^4, with the physical interpretation differing dramatically from the original one, the original construction discussed in the sequel involves the most essential aspects of the problem.

9.5 An alternative for the absolute minimization of Kähler action

One can criticize the assumption that extremals correspond to absolute minima, and the number theoretical vision discussed in [E2] indeed favors the separate minimization of magnitudes of positive and negative contributions to the Kähler action.

For this option Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant: note that they would be only inertial vacua and carry non-vanishing density gravitational energy. The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself.

The 3-surfaces for which CP_2 projection is at least 2-dimensional and not a Lagrange manifold would correspond to non-vacua since conservation laws do not leave any other option. The variational principle would favor equally magnetic and electric configurations whereas absolute minimization of action based on S_K would favor electric configurations. The positive and negative contributions would be minimized for 4-surfaces in relative homology class since the boundary of X^4 defined by the intersections with 7-D light-like causal determinants would be fixed. Without this constraint only vacuum bubbles would result.

The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable solution to the construction of Kähler function.

Since the considerations of this chapter relate only to the extremals of Kähler action, I have not bothered to replace "absolute minimization" with "extremization" in the text. The number theoretic approach based on the properties of quaternions and octonions discussed in the chapter [E2] leads to a proposal for the general solution of field equations based on the generalization of the notion of calibration [36] providing absolute minima of volume to that of Kähler calibration. This approach will not be discussed in this chapter.

9.6 The construction of the configuration space geometry from symmetry principles

The gigantic size of the isometry group suggests that it might be possible to deduce very detailed information about the metric of the configuration space by group theoretical arguments. This turns out to be the case. In order to have a Kähler structure, one must define a complexification of the configuration space. Also one should identify the Lie algebra of the isometry group and try to derive explicit form of the Kähler metric using this information. One can indeed construct the metric in this manner but a rigorous proof that the corresponding Kähler function is the one defined by Kähler action does not exist yet although both approaches predict the same general qualitative properties for the metric. The argument stating the equivalence of the two approaches reduces to the hypothesis stating electric-magnetic duality of the theory. For the absolute minima of Kähler action magnetic configuration space Hamiltonians derivable from group theoretical approach are essentially identical with electric configuration space Hamiltonians derivable from Kähler action.

9.6.1 General Coordinate Invariance and generalized quantum gravitational holography

The basic motivation for the construction of configuration space geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional configuration space of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that configuration space possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as absolute minimum of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome.

This picture is however too simple.

a) The degeneracy of the absolute minima caused by the classical nondeterminism of Kähler action however brings in additional delicacies, and it seems that the reduction to the light cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving more general light like 7-surfaces $X^3_l \times CP_2$.

b) It has also become obvious that the gigantic symmetries associated with $\delta M^4_+ \times CP_2$ manifest themselves as the properties of propagators and vertices, and that M^4 is favored over M^4_+. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of the configuration space to a union of configuration spaces associated with various 7-D causal determinants. The minimum assumption is that all possible unions of future and past light cone boundaries $\delta M^4_\pm \times CP_2 \subset M^4 \times CP_2$ label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^3_+ \times CP_2$ generalize almost trivially.

This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers. One cannot exclude the possibility that even more general light like surfaces $X_l^3 \times CP_2$ of M^4 are important as causal determinants.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3 and also a preferred light like 7-surface $X_l^3 \times CP_2$.

This is indeed possible. The possibility of negative Poincare energies inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action in the set of 4-surfaces going through X^3. These space-time sheets should also define uniquely the light like 7-surface $X_l^3 \times CP_2$, most naturally as the "earliest" surface of this kind. Note that this means that it become possible to assign a unique value of geometric time to the space-time sheet.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X_l^3 of M^4 implies generalized conformal and canonical invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

9.6.2 Light like 3-D causal determinants, 7-3 duality, and effective 2-dimensionality

Thanks to the non-determinism of Kähler action, also light like 3-surfaces X_l^3 of space-time surface appear as causal determinants (CDs). Examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D CD. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to configuration space geometry and the interaction between the two conformal symmetries.

The possibility of spinorial shock waves at X_l^3 leads to the hypothesis that they correspond to particle aspect of field particle duality whereas the physics in the interior of space-time corresponds to field aspect. More generally, field particle duality in TGD framework states that 3-D light like CDs and 7-D CDs are dual to each other. In particular, super-canonical and Super Kac Moody symmetries are also dually related.

The underlying reason for 7–3 duality be understood from a simple geometric picture in which 3-D light like CDs X_l^3 intersect 7-D CDs X^7 along 2-D surfaces X^2 and thus form 2-sub-manifolds of the space-like 3-surface $X^3 \subset X^7$. One can regard either canonical deformations of X^7 or Kac-Moody deformations of X^2 as defining the tangent space of configuration space so that 7–3 duality would relate two different coordinate choices for CH.

The assumption that the data at either X^3 or X_l^3 are enough to determine configuration space geometry implies that the relevant data is contained to their intersection X^2. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One cannot over-emphasize the importance of this conclusion. It indeed stream lines dramatically the earlier formulas for configuration space metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. Most importantly, no data about absolute minima of Kähler are needed to construct the configuration space metric so that the construction is also practical.

The reduction of data to that associated with 2-D surfaces conforms with the number theoretic vision about imbedding space as having hyper-octonionic structure [E2]: the commutative sub-manifolds of $OH = M^8$ have dimension not larger than two and for them tangent space is complex sub-space of hyper-octonion tangent space. Number theoretic counterpart of quantum measurement theory forces the reduction of relevant data to 2-D commutative sub-manifolds of X^3. These points are discussed in more detail in the next chapter whereas in this chapter the consideration will be restricted to $X_l^3 = \delta M_+^4$ case which involves all essential aspects of the problem.

9.6.3 Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structures of the configuration space. There is almost unique identification for the symplectic structure. Configuration space counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signed and and unsigned Kähler magnetic fluxes

$$Q_m(H_A, X^2) = Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ ,$$

$$Q_m^+(H_A, r_M) = Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ ,$$

$$J \equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on canonical invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q_m^+.

$$Q_m^{\alpha,\beta}(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q_m^+(H_A, X^2) \ .$$

Thus it seems that symmetry arguments fix the form of the configuration space metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

The notion of 7–3-duality described in the introduction implies that the relevant data about configuration space geometry is contained by 2-D surfaces X^2 at the intersections of 3-D light like CDS and 7-D CDs such as $M^4_+ \times CP_2$. In this case the entire Hamiltonian could be defined as the sum of magnetic fluxes over surfaces $X^2_i \subset X^3$. The maximally optimistic guess would be that it is possible to fix both X^2_i and 7-D CDs freely with X^2_i possibly identified as commutative sub-manifold of octonionic H.

9.6.4 Electric Hamiltonians and electric-magnetic duality

Absolute minimization of Kähler action in turn suggests that one can identify configuration space Hamiltonians as classical charges $Q_e(H_A)$ associated with the Hamiltonians of the canonical transformations of the light cone boundary, that is as variational derivatives of the Kähler action with respect to the infinitesimal deformations induced by $\delta M_+^4 \times CP_2$ Hamiltonians. Alternatively, one might simply replace Kähler magnetic field J with Kähler electric field defined by space-time dual $*J$ in the formulas of previous section. These Hamiltonians are analogous to Kähler electric charge and the hypothesis motivated by the experience with the instantons of the Euclidian Yang Mills theories and 'Yin-Yang' principle, as well as by the duality of CP_2 geometry, is that for the absolute minima of the Kähler action these Hamiltonians are affinely related:

$$Q_e(H_A) = Z\left[Q_m(H_A) + q_e(H_A)\right] \quad .$$

Here Z and q_e are constants depending on canonical invariants only. Thus the equivalence of the two approaches to the construction of configuration space geometry boils down to the hypothesis of a physically well motivated electric-magnetic duality.

The crucial technical idea is to regard configuration space metric as a quadratic form in the entire Lie-algebra of the isometry group G such that the matrix elements of the metric vanish in the sub-algebra H of G acting as $Diff^3(X^3)$. The Lie-algebra of G with degenerate metric in the sense that H vector fields possess zero norm, can be regarded as a tangent space basis for the configuration space at point X^3 at which H acts as an isotropy group: at other points of the configuration space H is different. For given values of zero modes the maximum of Kähler function is the best candidate for X^3. This picture applies also in symplectic degrees of freedom.

9.6.5 Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in canonical degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states angular momentum (and possibly also of Lorentz boost), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to 'positive' frequencies and which to 'negative frequencies' and which to zero frequencies that is to decompose the generators of the canonical algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

a) It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.

b) If $k_2 = 0$ is possible one could have

$$
\begin{aligned}
Can_+ &= \{H^a_{m,n,k=k1+ik_2}, k_2 > 0\} \ , \\
Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\
Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ .
\end{aligned} \tag{13}
$$

c) If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$
\begin{aligned}
Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\
Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\
Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ .
\end{aligned} \tag{14}
$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to use the "half Poisson bracket"

$$
\begin{aligned}
J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\
G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ .
\end{aligned} \tag{15}
$$

Here the subscript $+$ and $-$ refer to complex isometry current and its complex conjugate in terms of which the "half Poisson bracket" can be expressed.

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

9.7 Configuration space spinor structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

a) The classical bosonic physics is coded into the definition of the configuration space metric; therefore the classical physics associated with the spinors of the imbedding space should be coded into the definition of the configuration space spinor structure. This means that the generalized massless Dirac equation for the induced spinor fields on $X^4(X^3)$ should be closely related to the definition of the configuration space gamma matrices.

b) Complex probability amplitudes (scalar fields) in the configuration space correspond to the second quantized boson fields in X^4. Hence the spinor fields of the configuration space should correspond to the second quantized, free, induced spinor fields on X^4. The space of the configuration space spinors should be just the Fock space of the second quantized fermions on X^4!

c) Canonical algebra might generalize to a super canonical algebra and that super generators should be linearly related to the gamma matrices of the configuration space. If this indeed is the case then the construction of the configuration space spinor structure becomes a purely group theoretical problem.

The realization of these ideas is simple in principle. Perform a second quantization for the free induced spinor field in X^4. Express configuration space gamma matrices and canonical super generators as superpositions of the fermionic oscillator operators. This means that configuration space gamma matrices are analogous to spin 3/2 fields and can be regarded as a superpartner of the gravitational field of the configuration space. Deduce the anti-commutation relations of the spinor fields from the requirement of super canonical invariance. Generalize the flux representation for the configuration space Hamiltonians to a spinorial flux representation for their super partners.

9.7.1 Configuration space gamma matrices as super algebra generators

The basic idea is that the space of the configuration space spinors must correspond to the Fock space for the second quantized induced spinor fields. In accordance with this the gamma matrices of the configuration space must be expressible as superpositions of the fermionic oscillator operators for the second quantized induced free spinor fields in X^4 so that they are analogous to spin 3/2 fields. The Dirac equation is fixed from the requirement of super symmetry and has same vacuum degeneracy as Kähler action. A further assumption is that the contractions of the gamma matrices with isometry currents correspond to super charges of the group of isometries of the configuration space so that the construction reduces to group theory. Also the super Kac Moody algebra associated with light like 3-D CDs defines candidates for gamma matrices defining the components of the metric as anti-commutators and the question is whether the two definitions are mutually consistent.

9.7.2 7–3 duality

The failure of the classical non-determinism forces to introduce two kinds of causal determinants (CDs). 7-D light like CDs are unions of the boundaries of future and past directed light cones in M^4 at arbitrary positions (also more general light like surfaces $X^7 = X^3_l \times CP_2$ might be considered). CH is a union of sectors associated with these 7-D CDs playing in a very rough sense the roles of big bangs and big crunches. The creation of pairs of positive and negative energy space-time sheets occurs at $X^3 \subset X^7$ in the sense that negative and positive energy space-time sheet meet at X^3. Negative and positive energy space-time sheets are space-time correlates for bras and kets and the meeting of negative and positive energy space-time sheets is the space-time correlate for their scalar product.

Also 3-D light like causal determinants $X^3_l \subset X^4$ must be introduced: elementary particle horizons provide a basic example of this kind of CDs. The deformations of the 2-surfaces defining X^3_l define Kac Moody type conformal symmetries.

7–3 duality states that the two kind of CDs play a dual role in the construction of the theory and implies that 3-surfaces are effectively two-dimensional with respect to the CH metric in the sense that all relevant data about CH geometry is contained by the two-dimensional intersections $X^2 = X^3_l \cap X^7$ defining 2-sub-manifolds of $X^3 \subset X^7$.

9.7.3 Modified Dirac equation and gamma matrices

The modified Dirac equation is deduced from Kähler action by requiring it to have the same vacuum degeneracy as Kähler action itself. The interpretation of the solutions of the modified Dirac equation is as super gauge symmetry generators whereas physical degrees of freedom corresponds to generalized eigen modes at X_l^3 and at space-like 3-surfaces $X^3 \subset X^7$.

The decisive property of the modified Dirac equation is that it allows shock wave solutions restricted to X_l^3: in terms of field-particle duality these shock waves correspond to the click caused by a particle in a detector. This allows to realize quantum gravitational holography and 7–3 duality in the sense that the induced second quantized spinor fields at the intersections $X^2 = X_l^3 \cap X^7$ determine the super-generators super-canonical and super Kac Moody algebras invariant under the super gauge symmetries generated by the solutions of the modified Dirac equation.

Both the function algebra and Poisson algebra of X^7 allow super-symmetrization and both N-S and Ramond type representations are possible. For Ramond type representation the modified Dirac operators D_+ and D_-^{-1} associated with the positive and negative energy space-time sheets $X^4_\pm$ meeting at X^3 are present in the expressions of the super generators. NS-type representations correspond to the replacement of these operators with projection operators to the space of spinor modes with non-vanishing eigenvalues of $D_\pm$. Both representations are necessary and correspond to leptonic and quark like representations of configuration space gamma matrices. Similar statements apply to super Kac-Moody representations. These two kinds of representations correspond to super and kappa symmetries of super-string models.

9.7.4 Expressing Kähler function in terms of Dirac determinants

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The ratio of Dirac determinants of the modified Dirac operators D_+ and D_- associated with the space-time regions separated by a 3-dimensional causal determinant is an excellent candidate in this respect and the guess is that it is expressible as an exponent for the difference of Kähler actions in the adjacent regions. This allows to deduce the exponent of Kähler function and one could identify it as a renormalized Dirac determinant.

If the operators D_+ and D_- commute and if spectrum of the operator $D_+D_-^{-1}$ is invariant under $\lambda \to 1/\lambda$ apart from a finite number of eigenvalues for which the ratio equals to the ratio of exponents of respective Kähler actions, the Dirac determinant in question is well-defined even without zeta function regularization, and allows to deduce Kähler function.

The modified Dirac operator indeed allows to realize quantum gravitational holography since it reduces to an effectively 3-dimensional Dirac operator by boundary conditions but depends on the normal derivatives of the imbedding space coordinates at the causal determinants, which are most naturally light like 3-surfaces. Hermiticity of $D_\pm$ poses conditions on normal derivatives of imbedding space coordinates unless Kähler action density vanishes at X^3_l but does not fully eliminate the effects of the classical non-determinism. Hence there are good hopes of evaluating the exponent of Kähler function as a Dirac determinant without solving the field equations.

9.7.5 The relationship between super-canonical and super Kac-Moody algebras

The conformal weights of the generating elements of super-canonical representations correspond to the zeros of Riemann Zeta and one can identify the Cartan decomposition of the super-canonical algebra crucial for defining configuration space of 3-surfaces as a union of symmetric Kähler manifolds labelled by zero modes. Super-canonical algebra differs dramatically from super Kac Moody algebra. 7-3 duality however allows to see super-canonical and super Kac-Moody algebras as associated with two different tangent space basis for CH and giving rise to different coordinate systems. Hence both super algebras could give rise to a gamma matrix algebra of CH.

7–3 duality allows to generalize the Olive-Goddard-Kent coset construction. By 7-3 duality the differences of the commuting Virasoro generators of super-canonical and super Kac-Moody algebras must annihilate the physical states. For the same reason the central charges of the two Virasoro algebras must be identical so that the net central charge vanishes. This condition leads to a generalization of stringy mass formula involving besides super Kac-Moody algebra also the super-canonical algebra and allowing continuum mass spectrum for many particle states.

The $N = 4$ super symmetries generated by the solutions of the modified Dirac equation are pure super gauge transformations. All CP_2 spinor harmonics except the covariantly constant right handed neutrino spinor carry color quantum numbers and thus a non-vanishing vacuum conformal weight:

hence only an $N = 1$ global super symmetry is in principle possible. Since the Ramond type super-generator corresponding to the covariantly constant neutrino vanishes identically even $N = 1$ global super-symmetry is absent and no sparticles are predicted. This means a decisive difference in comparison with super string models and M-theory.

Physical states satisfy both N-S and Ramond type Super Virasoro conditions separately: note that in super-canonical degrees of freedom Ramond/NS representations super generators involve carry quark/lepton number. The most obvious application of the mass formula would be to hadron physics. The effective 2-dimensionality allows to identify partons as 2-dimensional surfaces X_i^2, and the more than decade old notion of elementary particle vacuum functional finds a first principle justification as a functional in the modular degrees of freedom of X^2.

By quantum classical correspondence is that Virasoro algebra associated with super Kac-Moody algebra acts on the conformal weights of the super-canonical representations as conformal transformations and the generators of the super-canonical algebra can be regarded as conformal fields. This dictates the matrix elements of the algebra to a high degree as function of conformal weights. A connection with braid and quantum groups and II_1 sub-factors of type von Neumann algebras associated with the Clifford algebra of the configuration space emerges.

9.8 What about infinities?

The construction of a divergence free and unitary inner product for the configuration space spinor fields is one of the major challenges. In the sequel constraints on the geometry of the configuration space posed by the finiteness of the inner product are analyzed.

9.8.1 Inner product from divergence cancellation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('light cone boundary') using the exponent $exp(K)$ as a weight factor:

$$\langle \Psi_1 | \Psi_2 \rangle = \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3)exp(K)\sqrt{G}dY^3 \ ,$$

$$\overline{\Psi_1}(Y^3)\Psi_2(Y^3) \equiv \langle\Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \quad (16)$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [19]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \ . \quad (17)$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancellation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \rightarrow y,\beta) = \sum_{r,s} |S(r,\alpha \rightarrow s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

a) Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

b) α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

c) Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems stating that semiclassical approximation is exact for certain systems (for example for integrable systems [20]). Symmetric space property suggests that Kähler function might possess the properties guaranteeing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving configuration space spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that configuration space integrals are continuable to p-adic number fields requires this kind of reduction.

9.8.2 Divergence cancellation, Ricci flatness, and symmetric space and Hyper Kähler properties

In the case of the loop spaces left invariance implies that Ricci tensor is a multiple of the metric tensor so that Ricci scalar has an infinite value. Mathematical consistency (essentially the absence of the divergences in the integration over the configuration space) forces the geometry to be Ricci flat: in other words, vacuum Einstein's equations are satisfied. It can be shown that Hyper Kähler property guarantees Ricci flatness. The reason is that the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(\infty)$ generators instead of $U(\infty)$ generators as in case of loop spaces, so that the traces vanish.

Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper-Kähler property means the possibility to perform complexification in S^2-fold manners. An interesting possibility raised by the notion of number theoretical compactification [E2] is that hyper Kähler structure could be replaced with what might be called "hyper-hyper-Kähler structure" resulting when quaternionic tangent space is replaced with its hyper-quaternionic variant. This would conform with the Minkowski signature of the space-time surface. In this framework also hyper-octonionic structure might be considered. An interesting question not yet even touched, is whether the conjectured $M^8 - -M^4 \times CP_2$ duality is realized also at the level of the configuration space of 3-surfaces.

Consider now the arguments in favor of Ricci flatness of the configuration space.

a) The canonical algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the canonical generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

b) The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-canonical generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

c) In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property. In the following argument reader can well consider replacing the attribute "quaternionic" with "hyper-quaternionic".

a) The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of configuration space in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

b) S^2-fold degeneracy is indeed associated with the definition of the complex structure of the configuration space. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at $X^2_+ \times CP_2$ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

c) One can see the super-canonical conformal weights as points in a particular complex plane of the quaternionic space and the choice of this plane corresponds to a selection of one configuration space Kähler structure which are parameterized by S^2. The necessity to restrict the conformal weights to a complex plane brings in mind the commutativity constraint on simultaneously measurable quantum observables.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and canonical symmetries would also imply Hyper Kähler property of the configuration space and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

10 Particle massivation from the first principles

It took a decade to understand how p-adic thermodynamics description of the particle massivation emerges from the fundamental TGD. The understanding of 7–3 duality [B4, C2, F2] played a key role in the development of ideas leading to the understanding of what is involved. The thermodynamics for Virasoro generator L_0 makes sense also at the real context, and the requirement that there exists an algebraic continuation to the p-adic context implies the crucial quantization of the temperature parameter implying the emergence of the p-adic mass scales.

Basically configuration space Dirac equation corresponds to Super Virasoro conditions giving rise to Virasoro conditions determining the mass spectrum of partons. There are two kinds causal determinants (CDs): 3-dimensional light like surfaces $X_l^3 \subset X^4$ representing generalized Feynman diagrams and 7-D CDs of form $X_l^3 \times CP_2$, where X_l^3 in the general case is a union of boundaries of future and past directed light cones. X_l^4 have ends at future and past directed 7-D CDs somewhat like the ends of string connecting branes.

The two kinds of causal determinants give rise to super-canonical and Super Kac-Moody Dirac operators. In both cases quark-lepton degeneracy gives rise to two different Dirac operators corresponding to N-S (quarks) and Ramond (leptons) representations of Super Virasoro algebras. Dirac operators are identifiable as super-generators G of the light Super Virasoro algebra and Dirac equation states that Super Virasoro algebra acts as gauge transformations.

a) Supercanonical Dirac operators $D(SC)$ are associated with 7-D CDs $X^7 = X_l^3 \times CP_2$, X_l^3 light like surface. The interpretation is in terms of creation of pairs of space-time sheets branching from X^3 and with vanishing total classical energy. In the "vibrational" part of the $L_0(SC)$ appears the quadratic Casimir operator of Lorentz group, which can have complex values in accordance with the fact that super-canonical conformal weights are in general complex. The corresponding excitations have thus spin, color, electro-weak quantum numbers, and fermion numbers.

b) The light like 3-surfaces $Y_l^3 \subset X^4$ defining light like CDs event horizons and light like boundaries of space-time sheets define Super-Kac Moody Dirac operators. 7–3 duality reduces these operators as well as super-canonical Dirac operators to the intersections X_i^2 of Y_l^3 and X^3. These operators correspond to $P \times SU(3) \times U(2)_{ew}$ (P denotes Poincare group) localized with respect to Y_l^3 in the general case whereas the restriction of X_l^3 to surfaces $\delta M^4_\pm$ would reduce P to M^4. The counterpart of the stringy mass formula would result from the vanishing of $L_0(SKM)$.

The Dirac equations associated with these Dirac equations two kinds of Dirac operators can be regarded as independent, and the correlation comes from the assumption that super-conformal algebra generates super-canonical spectral flows acting as local gauge symmetries and braiding operations at global level. 7–3 duality indeed realizes this correlation concretely. In the sequel super symmetry based arguments are used to guess the explicit form of these operators and a general solution of configuration space Dirac equation is proposed by exploiting the analog with the Dirac equation in Minkowski space.

10.1 The analog of coset construction for super generators

The analog of Olive-Goddard-Kent coset construction for super Kac-Moody and super-canonical algebras provides a highly attractive manner to obtain a Virasoro algebra with a vanishing central extension term. A generalization of the coset construction is in question in the sense that Virasoro generators are differences of those for Super Kac-Moody algebra and super-canonical algebra. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-canonical local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. What is done is to construct first a state with a non-positive conformal weight using super-canonical generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and thus mass.

The coset construction giving $L_n = L_n(SKM) - L_n(SC)$ means at the level of super generators G_n the expression

$$\begin{aligned} G_n^+ &= G_n^+(SKM) - G_n^+(SC) \ , \\ G_n^- &= G_n^-(SKM) - G_n^-(SC) \ . \end{aligned} \tag{18}$$

Here $\pm$ refers to the positive and negative energy space-time sheets meeting at X^2 since "hermitian conjugation" (ket $\leftrightarrow$ bra) corresponds geometrically to a replacement of the space-time sheet with a positive time orientation with that having a negative time orientation. Note also that the super generators possess fermion number so that creation operators are replaced by annihilation operators in the conjugation.

If the conditions

$$\{G^+(SKM), G^-(SC)\} = \{G^+(SKM), G^-(SKM)\} \quad (19)$$

hold true, the anti-commutators give just the differences $L(SKM) - L(SC)$. If 7–3 duality corresponds to a coordinate change in CH, the central extension charges for SKM and SC Super Virasoro algebras are same and resulting Super Virasoro algebra has vanishing central extension terms.

This construction works perfectly for quark sector. Leptonic super-canonical generators defined assuming the presence of operators D_+ and D_-^{-1} however anti-commute to products of configuration space Hamiltonians rather than components of Kähler form. Hence the formula for super generators $G(SC)$ does not work. The conclusion can be avoided if there exists also a leptonic representation CH gamma matrices anti-commuting to Kähler form. As already found, this kind of representation is obtained by replacing the operators D^+ and D_1^{-1} in the definitions of the super generators by projectors to the space of spinor modes with non-vanishing generalized eigen value of $D_\pm$. Also quark like super generators anti-commuting to a function algebra exist with this assumption.

The Ramond-NS degeneracy is a further important factor which must be taken into account. Possible sparticles are associated with Ramond type representation and generated by the super generators defined by the covariantly constant right handed neutrino. The helicity for neutrino is fixed by requiring that the neutrino spinor is proportional to the operator $n^k\gamma_k$, where n^k corresponds to the radial light like vector for $M^4_\pm$. If the super generators containing D and D^{-1} correspond to a Ramond type representation then covariantly right handed neutrino spinor defines an identically vanishing super generator and spartners with degenerate masses are absent. In NS sector the conformal weights of the generators generating ground states are $\pm 1/2$ so that there is no problem with the ground state degeneracy. The conclusion would be that quark/lepton like super generators G correspond to Ramond/N-S type representation. Note that leptonic super generators $G(SC)$ must carry half odd integer conformal weight.

10.2 General mass formula

The vanishing condition for $L_0(SKM) - L_0(SC)$ leads to a general mass formula in quark and lepton sectors separately. One of the blessings of 7–3 duality is that one can treat different 2-surfaces X_i^2 as almost independent degrees of freedom. In the case of translations this does not seem to be true since independent translations lead the surfaces X^2 outside $X_l^3 \times CP_2$. Therefore one must consider two options.

a) If one neglects the correlation between the translations and assigns to each X_i^2 independent translational degrees of freedom a separate mass formula for each X_i^2 would result:

$$M_i^2 = -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \ . \quad (20)$$

Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known.

b) Perhaps the only internally consistent option is based on the assignment of the mass squared with the total cm. This would give

$$M^2 = (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i \cdot p_j = -\sum_i L_{0i}(SKM) + \sum_i L_{0i}(SC) \ . \quad (21)$$

Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. $L_0(SC)$ term contains only leptonic or quark oscillator operators unless one allows both the lepto-quark type gamma matrices involving both D^+ and D_-^{-1} and leptonic gamma matrices involving instead of $D_\pm^{\pm 1}$ the projector P to the spinor modes with a non-vanishing eigenvalue of D.

The decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses. It might be that there is no unique assignment of momenta to individual partons and that this non-uniqueness is part of the gauge symmetry implied by 7–3 duality.

10.2.1 Mass formula before massivation

If interactions can be neglected one could assume that the free particle mass formulas

$$M_i^2 = L_{0i}(SKM) + h_i(vac) \ , \quad (22)$$

where $h_i(vac)$ corresponds to vacuum conformal weight are satisfied and what remains is the condition

$$2\sum_{i\neq j} p_i \cdot p_j = \sum_i L_{0i}(SC) - \sum_i h_i(vac) \quad = \quad 0 \ . \tag{23}$$

In the case of single Kac Moody algebra this condition would be replaced with

$$2\sum_{i\neq j} p_i \cdot p_j \quad = \quad 0 \ . \tag{24}$$

The presence of the super-canonical algebra obviously allows much more flexibility but it seems that continuum mass spectrum is not possible.

The conformal weights of the super-canonical algebra related closely to the zeros of Riemann Zeta are complex and for the physical states the sum of the imaginary parts of the conformal weights must vanish: this is satisfied if the zeros and their conjugates appear as conformal weights of generators creating the state. The presence of super-canonical half integer contribution from super-canonical sector could explain the negative values of the ground state conformal weights forced by p-adic mass calculations.

The above formula does not contain all that is needed: also a contribution from modular degrees of freedom associated with the complex structures of X^2_i is necessary in order to understand the dependence of the mass of fermion families on the genus of X^2_i.

Hadron physics offers the most obvious application for the mass formula. The often used metaphor for the hadronic collision as a mini big bang would have a precise meaning in TGD framework, and the effective 2-dimensionality would provide a precise realization for the parton model of hadrons. The presence of both algebras could be essential for understanding the relationship between hadron and quark masses, and the presence of super-canonical spin could allow insights to the problem of proton spin.

10.2.2 Particle massivation

There is an objection against the proposed mass formula. The individual contributions to the mass squared eigenvalues are not fixed uniquely. The proposed identification of parton masses is only the simplest possibility, and one could add to the individual parton conformal weights contributions compensated by the super-canonical conformal weights. This might however be a blessing rather than a curse since it suggests a manner to understand particle massivation at the fundamental level.

a) A natural requirement is that parton masses are consistent with the poles of the S-matrix elements [C2]. This assumption is quite general and certainly makes sense if the S-matrix elements allow a decomposition to vertices and propagators for a tree diagram.

b) The time evolution for the quantum states of individual partons, which are always on mass shell (that is generalized eigen states of the modified Dirac operator) is a unitary process, and corresponds to a braiding for an N-puncture system defined by the product of N local operators creating the parton state. The basic requirement is that the flow contains information about the presence of other partons and thus about the normal derivatives of the imbedding space coordinates at X^3_l. Hence the S-matrix indeed contains information about the interior of the space-time surface and the effective two-dimensionality is indeed only effective. The condition that the S-matrix elements remain unchanged in conformal transformations obviously poses explicit conditions on the normal derivatives and can be regarded as conditions stating the vanishing of the corresponding beta function.

The best candidate for the flow is as the hydrodynamic flow defined by the discontinuity of the energy momentum tensor associated with the Kähler action at X^3_l representing what might be regarded as a hydrodynamical shock wave. This flow is in general not integrable in the sense that one could assign a global coordinate varying along the flow lines. By identifying the points of X^2_i and X^2_f having the same value of the complex coordinate z, the flow $X^2_i \rightarrow X^2_f$ defines a map $w : (x, y) \rightarrow (u(x, y), v(x, y))$ mixing the points of X^2_i. The inner products of the states created by the local operator $\Psi_n(x, y)$ creating one-parton state at X^2_i with the state created by the transformed operator $\Psi_n(u(x, y), v(x, y)$ define correlation functions, which vanish above some length scale determining the mass of the parton. Massivation occurs if this map fails to be a conformal transformation.

b) By quantum classical correspondence this braiding can be also regarded as a braiding for the points of X^2_i, which correspond to the super-canonical conformal weights just like the points of the celestial sphere correspond to momenta. If the number of operators creating the parton state is larger than two, the minimal number three of threads in the braid is present and the conformal weights become "off mass shell" conformal weights. The massivation however can in principle occur always. In [O3, C5] the proposal was made that the bound state conformal weights could correspond to the zeros of poly-zetas: obviously this is a very strong prediction. Riemann Zeta would correspond to Higgs zero phase with the minimum of 2 operators with conjugate super-canonical conformal weights creating the state.

c) The successful description of particle massivation in terms of p-adic thermodynamics for the Virasoro generator L_0 (with p^2 not included) of the partons encourages to think that the change of the parton conformal weight could be understood as a generation of a thermal conformal weight by the flow induced by an ergodic braiding flow. This interpretation would allow to circumvent the problem created by the fact that the thermalization for mass squared is not consistent with the Lorentz invariance.

10.3 Particle massivation and anyonic shydrodynamics

It took quite a long time to understand how the strange hydrodynamic character of the field equations for both Kähler action and modified Dirac action relates to the particle massivation and S-matrix. It was the interaction with M-theory, which led to the discovery of 7–3 duality and effective 2-dimensionality, which in turn made possible the breakthrough in the attempts to understand how p-adic thermodynamics follows from the fundamental theory.

10.3.1 What is the flow defining the braiding?

The basic condition on the braiding is that it contains information about the interior of X^4 and thus about interactions with other partons. Second constraint is that the braiding flow is trivial for massless particles such as photon for which the space-time correlate should correspond to X_l^3 carrying a light-like energy momentum current.

The components $X^{n\alpha}$ of some tensor field with α restricted to X_l^3 define the most natural candidate for the braiding flow. The existence of the preferred light-like normal coordinate x^n constant at X_l^3 (in the case of δM^4_+ the light like coordinates would be $x^\pm = t \pm r$) is essential to achieve general coordinate invariance.

The discontinuity of the normal component $T^{n\alpha}$ of the energy momentum tensor associated with the Kähler action is a good candidate. At light like CDs the discontinuity of $T^{n\alpha}$ could be non-vanishing if allows light like CD to carry a shock wave also in imbedding space degrees of freedom as suggested by the super-symmetry. The discontinuity of $T^{n\alpha}$ would have an interpretation as a shock wave like hydrodynamic flow at the boundary. For massless particles the energy momentum current would have only a longitudinal component, the braiding would be trivial and particle would remain massless. The appearence of the energy momentum tensor in the definition of the S-matrix conforms with the hydrodynamic character of field equations and with the fact that the theory must be also a quantum theory of gravitation.

This guess is supported by the modified Dirac equation. By multiplying the modified Dirac equation at X_l^3 for shock waves localized at X_l^3 with the o_t defined by the light like gamma matrix along X_l^3 and doing the anti-commutations with the modified Dirac operator D, one finds

$$T^{\alpha n} D_\alpha \Psi \;=\; 0 \; . \tag{25}$$

The equation states that Ψ is covariantly constant along the flow lines of the flow defined by $T^{\alpha n}$. The equation can be written as

$$\begin{aligned} D_t\Psi + v^i D_i \Psi &= 0 \; , \\ v_i &= \frac{T^{ni}}{T^{nt}} \; . \end{aligned} \tag{26}$$

This differs from a standard flow equation for a quantity Ψ moving along field lines only by the fact that ordinary derivatives ∂_α are replaced by covariant derivatives D_α. This means that Ψ suffers a braiding transformation in spin and electro-weak degrees of freedom. Obviously, this equation states super-conformal invariance in the sense that it is not possible to poses the values of Ψ arbitrarily in entire X_l^3 but only at X^2. One could regard TGD as anyonic shydrodynamics at the space-time level. The usual dispersion characterizing Schödinger equation emerges only at the level of imbedding space when one assigns wave equations to propagators defined by S-matrix elements.

If spinor connection is continuous at X_l^3, the discontinuity for this equation is the ordinary hydrodynamic equation

$$\begin{aligned} \partial_t\Psi + w^i \partial_i \Psi &= 0 \; , \\ w_i &= \Delta \left[\frac{T^{ni}}{kT^{nt}} \right] \; . \end{aligned} \tag{27}$$

10.3.2 The correspondence with Higgs mechanism

A priori one can imagine three sources of mass.

a) The mixing caused by the flow reduces correlations unless it induces a conformal transformation leaving the physical state invariant. This mechanism could obviously relate to the massivation described in terms of p-adic thermodynamics.

b) The lacking covariant constancy of the charge matrices appearing in the definition of electro-weak gauge bosons except photon can induce a loss of correlations, and is a good candidate for TGD counterpart of Higgs coupling. Also fermionic states involve this kind of dependences inducing a Higgs type contribution to the mass squared.

c) In contrast to the first guess, the holonomy transformation associated with braiding does not lead to a loss of correlations. The 2-dimensionality of X^2 implies that the holonomy group defining the braiding is Abelian. In spin degrees of freedom spinor connection vanishes by the flatness of M^4 so that the braiding cannot make neither graviton nor gluons massive. The vanishing of photon mass requires that the braiding in elecro-weak spin degrees of freedom is generated by an element of the holonomy algebra commuting with the electromagnetic charge. This however means that for charge eigen states the braiding brings in only a phase factor and would not lead to a loss of correlations.

These observations lead to a view about how TGD description relates to the standard description in terms of the phases defined by the vacuum expectation values of the Higgs field.

The model for the modular contribution to the mass squared to be discussed suggests that the p-adic prime p characterizes the size and effective p-adic topology of X^2. This raises the question why the states created by super-canonical operators O_q, $q \neq p$ do not appear in the spectrum. The intuitive guess is that the states corresponding to "wrong" p-adic topology are very massive. For instance, for $q \neq p$ the states could have inverse p-adic temperature $T_q = n > 1$ possible if roots of q are allowed in the extension of R_q used. Various phases would be characterized by the primes labelling various operator basis O_q.

Higgs zero phase would correspond to CP_2 type extremals for which the operators O_p for various primes reduce to same operator apart from the normalization constant since the projection of X^2 to $\delta M^4_\pm$ becomes zero-dimensional. This phase would be unstable but the assumption that elementary particles are near to this phase allows to identify a mechanism eliminating light exotics.

One can consider also an alternative interpretation based on the notion of conformal homotopy class implying that p-adic prime p characterizes X^3_l rather than X^2.

a) Homotopy theory could generalize in the sense that there are several conformal equivalence classes of space-time sheets connecting given partonic 2-surfaces X_i^2 at various 7-D CDs representing initial and final states. The derivatives of the imbedding space coordinates with respect to the light like coordinate t at X_i^2 are same for these surfaces. These conformal homotopy classes could define the TGD counterparts of the "phases" of gauge theories labelled by vacuum expectation values of Higgs fields.

b) In the trivial conformal equivalence class braiding flow reduces to a mere conformal transformation: photon, graviton, and gluons remain massless. The thermal massivation of photon might be describable in terms of a non-trivial conformal homotopy class.

c) The success of the p-adic mass calculations would suggest that p-adic primes label conformal homotopy classes and thus phases and characterize the mass scale as well as the effective p-adic topology for which p-adic non-determinism corresponds to the classical non-determinism of Kähler action. p-Adic prime would characterize X_l^3 rather than X^2.

There are however objections against this interpretation. First, the notion of conformal homotopy class is not consistent with the notion of generalized Feynman diagram requiring that even topologically different surfaces X_l^3 are equivalent. Secondly, the model for the modular contribution to the mass squared forces to assign p-adic prime p to X^2 rather than X_l^3.

11 Is it possible to understand coupling constant evolution at space-time level?

It is not yet possible to deduce the length scale evolution gauge coupling constants from Quantum TGD proper. Quantum classical correspondence however encourages the hope that it might be possible to achieve some understanding of the coupling constant evolution by using the classical theory.

This turns out to be the case and the earlier speculative picture about gauge coupling constants associated with a given space-time sheet as RG invariants finds support [C3]. It remains an open question whether gravitational coupling constant is RG invariant inside give space-time sheet. The discrete p-adic coupling constant evolution replacing in TGD framework the ordinary RG evolution allows also formulation at space-time level as also does the evolution of $\hbar$ associated with the phase resolution.

11.1 The evolution of gauge couplings at single space-time sheet

The renormalization group equations of gauge coupling constants g_i follow from the following idea. The basic observation is that gauge currents have vanishing covariant divergences whereas ordinary divergence does not vanish except in the Abelian case. The classical gauge currents are however proportional to $1/g_i^2$ and if g_i^2 is allowed to depend on the space-time point, the divergences of currents can be made vanishing and the resulting flow equations are essentially renormalization group equations. The physical motivation for the hypothesis is that gauge charges are assumed to be conserved in perturbative QFT. The space-time dependence of coupling constants takes care of the conservation of charges.

A surprisingly detailed view about RG evolution emerges.

a) The UV fixed points of RG evolution correspond to CP_2 type extremals (elementary particles).

b) The Abelianity of the induced Kähler field means that Kähler coupling strength is RG invariant which has indeed been the basic postulate of quantum TGD. The only possible interpretation is that the coupling constant evolution in sense of QFT:s corresponds to the discrete p-adic coupling constant evolution.

c) IR fixed points correspond to space-time sheets with a 2-dimensional CP_2 projection for which the induced gauge fields are Abelian so that covariant divergence reduces to ordinary divergence. Examples are cosmic strings (, which could be also seen as UV fixed points), vacuum extremals, solutions of a sub-theory defined by $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere, and "massless extremals".

d) At the light-like boundaries of the space-time sheet gauge couplings are predicted to be constant by conformal invariance and by effective two-dimensionality implying Abelianity: note that the 4-dimensionality of the space-time surface is absolutely essential here.

e) In fact, all known extremals of Kähler action correspond to RG fixed points since gauge currents are light-like so that coupling constants are constant at a given space-time sheet. This is consistent with the earlier hypothesis that gauge couplings are renormalization group invariants and coupling constant evolution reduces to a discrete p-adic evolution. As a consequence also Weinberg angle, being determined by a ratio of $SU(2)$ and $U(1)$ couplings, is predicted to be RG invariant. A natural condition fixing its value would be the requirement that the net vacuum em charge of the space-time sheet vanishes. This would state that em charge is not screened like weak charges.

f) When the flow determined by the gauge current is not integrable in the sense that flow lines are identifiable as coordinate curves, the situation changes. If gauge currents are divergenceless for all solutions of field equations, one can assume that gauge couplings are constant at a given space-time sheet and thus continuous also in this case. Otherwise a natural guess is that the coupling constants obtained by integrating the renormalization group equations are continuous in the relevant p-adic topology below the p-adic length scale. Thus the effective p-adic topology would emerge directly from the hydrodynamics defined by gauge currents.

11.2 RG evolution of gravitational constant at single space-time sheet

Similar considerations apply in the case of gravitational and cosmological constants.

a) In this case the conservation of gravitational mass determines the RG equation (gravitational energy and momentum are not conserved in general).

b) The assumption that coupling cosmological Λ constant is proportional to $1/L_p^2$ (L_p denotes the relevant p-adic length scale) explains the mysterious smallness of the cosmological constant and leads to a RG equation which is of the same form as in the case of gauge couplings.

c) Asymptotic cosmologies for which gravitational four momentum is conserved correspond to the fixed points of coupling constant evolution now but there are much more general solutions satisfying the constraint that gravitational mass is conserved.

d) It seems that gravitational constant cannot be RG invariant in the general case and that effective p-adicity can be avoided only by a smoothing out procedure replacing the mass current with its average over a four-volume 4-volume of size of order p-adic length scale.

11.3 p-Adic evolution of gauge couplings

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level.

a) Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

b) Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

11.4 p-Adic evolution in angular resolution and dynamical $\hbar$

For a given p-adic topology algebraic extensions of p-adic numbers define also a hierarchy ordered by the dimension of the extension and this hierarchy naturally corresponds to an increasing angular resolution so that RG flow would be associated also with it.

a) A characterization of angular scalings consistent with the identification of $\hbar$ as a characterizer of the topological condensation of 3-surface X^3 to a larger 3-surface Y^3 is that angular scalings correspond to the transformations $\Phi \rightarrow r\Phi$, $r = m/n$ in the case of X^3 and $\Phi \rightarrow \Phi$ in case of Y^3 so that X^3 becomes analogous to an m-fold covering of Y^3. Rational coverings could also correspond to m-fold scalings for X^3 and n-fold scalings for Y^3.

b) The formation of these stable multiple coverings could be seen as an analog for a transition in chaos via a process in which a closed Bohr orbit regarded as a particle itself becomes an orbit closing only after m turns. TGD predicts a hierarchy of higher level zero energy states representing S-matrix of lower level as entanglement coefficients. Particles identified as "tracks" of particles at orbits closing after m turns might serve as space-time correlates for this kind of states. There is a direct connection with the fractional quantum numbers, anyon physics and quantum groups.

c) The simplest generalization from the p-adic length scale evolution consistent with the proposed role of Beraha numbers $B_n = 4cos^2(\pi/n)$ is that bifurcations can occur for integer values of r=m and change the value of $\hbar$. The interpretation would be that single 2π rotation in δM^4_+ corresponds to the angular resolution with respect to the angular coordinate ϕ of space-time surface varying in the range $(0, 2\pi)$ and is given by $\Delta\phi = 2\pi/m$.

d) The evidence for a gigantic but finite value of "gravitational" Planck constant suggests that large values of $\hbar$ corresponding to $3 < n < 4$ and defining a "generalized" Beraha number B_r are possible. For $n = 3$ corresponding to the minimal resolution of $\Delta\phi = 2\pi/3$ $\hbar$ would be infinite. This would allow to keep the formula for $\hbar(n)$ in its original form by replacing n with a rational number. This would mean that also rational values of r correspond to bifurcations in the range $3 < r < 4$ at least.

12 About the construction of S- and U-matrices

The enormous symmetries of quantum are bound to lead to a highly unique S-matrix but the practical construction of S-matrix is a formidable challenge and necessitates deep grasp about the physics involved so that one can make the needed approximations. The evolution of the ideas related to S-matrix involves several side-tracks and strange twists characteristic for a mathematical problem solving when a direct contact with the experimental reality is lacking. The work with S-matrix has taught me that principles are more important than formulas and that the only manner to proceed is from top to bottom by gradually solving the philosophical problems, identifying all the relevant symmetries and understanding the horribly nonlinear dynamics defined by the absolute minimization of Kähler action.

The poor understanding of the philosophical issues has led to frustratingly many candidates for S-matrix. TGD inspired theory of consciousness has however gradually led to a clarification of various issues and it seems that it is safer to distinguish between two matrices: U-matrix and S-matrix. Moreover, it seems that one must talk in plural: there is entire hierarchy of U-matrices and S-matrices correspond to higher levels of the hierarchy.

a) U-matrix is much more fundamental object than S-matrix conventionally defined as time-translation operator and characterizes what happens in single quantum jump $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$. A good candidate for U-matrix is as Glebsch-Gordan coefficients relating free and interacting Super Virasoro representations.

b) U-matrix decomposes into a tensor product of a U-matrix characterizing dispersion in zero modes and a U-matrix characterizing the dynamics in fiber degrees of freedom. The U-matrix associated with the fiber degrees of freedom in turn decomposes into a tensor product of the local U-matrices associated with various space-time sheets. The tensor factor of U describing dynamics in conformal degrees of freedom for a given space-time sheet could correspond to the TGD counterpart of the stringy S-matrix.

c) The new view about sub-system forced by the many-sheeted space-time and the integration of quantum jump sequences to single quantum as far as conscious experience is considered, suggests that one must introduce entire hierarchy of U-matrices corresponding to p-adic length scale hierarchy and hierarchy of durations for sequences of quantum jumps. The identification as S-matrices is natural since the duration of quantum jump sequence allows identification as counterpart for the duration of time evolution associated with S-matrix in standard physics. Actually subjective time duration is in question and corresponds only in a statistical sense to a definite duration of geometric time. In particular, one expects that these higher level U-matrices do not provide only an approximate description of something more fundamental but express all that can be said and tested by conscious observer.

d) The hierarchy of p-adic number fields and their extensions of increasing dimension should correspond to the hierarchy of U-matrices. This means that the matric elements of S-matrix should be in finite-dimensional extensions of rationals possibly involving transcendentals. This would mean that S-matrix theory becomes number theory at the deepest and most challenging level one can imagine. A typical problem can be formulated as a question whether some finite set of transcendentals is a closed system under the process of taking logarithms.

The lack of explicit formulas for S-matrix elements have been the basic weakness of quantum TGD approach as compared to the concrete perturbative formulas provided by super-string approach. Fortunately, the new number theoretic vision leads to concrete Feynman rules for S-matrix in the approximation that elementary particles can be regarded as CP_2 type extremals. Of course, this is only small piece of quantum TGD but certainly the most important one as far as the empirical testing of the theory is considered.

12.1 Inner product from divergence cancellation

The construction of a divergence free and unitary inner product for the configuration space spinor fields has been already considered in the first part of the book. The inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('light cone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{28}$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [19]. The scalar product for two complex valued representation functions is defined as

$$(f,g) \quad = \quad \int \overline{f} g exp(nK)\sqrt{g} dV \ . \qquad (29)$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is nonpositive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Divergence cancellation in the bosonic integration has been already demonstrated in the first part of the book. One can develop the Kähler function as a Taylor series around maximimum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x,\alpha \rightarrow y,\beta) = \sum_{r,s} |S(r,\alpha \rightarrow s,\beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r,m \rightarrow s,n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

a) Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models. The identification of all isometry invariants and corresponding integration measure in the configuration space integral was considered in the first part of the book.

b) α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parametrizing the zero modes.

c) Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. There are theorems stating that semiclassical approximation is exact for certain systems (for example for integrable systems [20]). An interesting question is whether the Kähler function might possess the properties guaranteing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ would reduce to an integration over the zero modes: this kind of reduction actually occurs in string models.

12.2 The fundamental identification of U- and S-matrices

Single quantum jump corresponds to the sequence

$$\Psi_i \rightarrow U\Psi_i \rightarrow ....\Psi_f \ .$$

U does not certainly correspond to a genuine time-development in the sense of a unitary time translation operator. A good guess is that U has interpretation as Glebsch-Gordan coefficients between free an interacting representations of Super Virasoro algebra associated with surfaces $\cup_i X^4(Y_i^3)$ and $X^4(\cup_i Y_i^3)$. For a given unentangled subsystem (subsystem in self-organizing self-state) the eigen states of the density matrix of the subsystem becoming unentangled in quantum jump determines what are the final states of the quantum jump. Negentropy Maximization Principle states that the subsystem of unentangled subsystems whose measurement gives rise to maximal entanglement negentropy gain, is quantum measured. U is much more fundamental object than S-matrix. If subsystem is entangled then in a reasonable approximation nothing happens to it during quantum jump sequence and the sub-quantum history remains unchanged.

What could then be the interpretation of S-matrix in this framework?

a) The first glance to the problem is based on macro-temporal quantum coherence. If the dissipative effects caused by the state function reductions and state preparations are absent completely or are not visible in the time resolution defined by the duration of macro-temporal quantum coherence, one might expect that S-matrix is product of U-matrices occurred during the period of the macro-temporal quantum coherence. The system in self state would indeed effectively behave like its own Universe. One could say that time-evolution is discretized with CP_2 time defining the duration of the chronon.

b) Observer is represented by a cognitive space-time sheet drifting towards the geometric future quantum jump by quantum jump along material space-time sheet. Observer is basically interested in the unitary time development induced by the time-translation operator P_0 associated with the modified Dirac operator. This time development can have any duration T and defines time evolution operator $exp(iP_0T)$ if subsystem develops as essentially free system. The measurement of the scattering probabilities defined by S-matrix corresponds to a construction of an empirical arrangement guaranteing that quantum measurement observed by (the sufficiently intelligent!) cognitive space-time sheet at time $t = T$ reduces the quantum state to some of the state of the initial state basis at time $t = 0$. In the ideal situation the measured system would develop unitarily during this interval and stay thus entangled so that it cannot self-organize by quantum jumps.

This picture would suggest that the S-matrix could be defined as a generalization of the exponential $exp(iP_0T)$ of the second quantized Poincare energy operator P_0 associated with the modified Dirac action for the interacting space-time surface $X^4(\cup_i Y_i^3)$ and acts on the tensor product of the state spaces associated with $X^4(Y_i^3)$. This picture might make sense at the limit when wave mechanics is a good approximation but does not work in elementary particle length scales where CP_2 type extremals whose M_+^4 projection is a random light like curve, are expected to dominated.

Interacting space-time surfaces defined by a connected sum of CP_2 type extremals can be regarded as a Feynman graph with lines thickened to 4-manifolds. This suggests the assignment of the exponent with the internal lines of the generalized Feynman graph acting as translation operators whereas vertices where lines join give rise to vertex operators which can be regarded as Glebch-Gordan coefficients for super-Kac-Moody representations.

Since the Hamiltonian in question is quadratic in oscillator operators, the theory is free in the standard sense of the word, and it is only the absolute minimization of Kähler action which induces interactions and makes the theory nontrivial. Feynman diagram structure has purely topological origin. For instance, topological sums of CP_2 type extremals can be regarded as an example of Feynman diagrams with lines thickened to 4-manifolds. The absence of interaction terms guarantees that there are no sources of divergences. For the CP_2 type extremals one can develop rather detailed form of the Feynman rules.

TGD inspired theory of consciousness suggests that one should not take too dogmatic view about S-matrix as a summary for the predictions of quantum theory. Even the idealizations about experimental situation involved with the S-matrix formalism might be quite too strong since experimental observations are always about subsystems.

12.3 7–3 duality and construction of S-matrix

The notion of 7–3 duality emerged from the (one might say violent) interaction between TGD and M-theory [A2]. The attempts to construct quantum TGD have gradually led to the conclusion that the geometry of the configuration space ("world of classical worlds") involves both 7-D and 3-D light like surfaces as causal determinants. 7-D light like surfaces X^7 are unions of future and past light cone boundaries and play a role somewhat resembling that of branes. Each 7-D CD determines one particular sector of the configuration space. 3-D light like surfaces X^3_l can correspond to boundaries of space-time sheets, regions separating two maximally deterministic space-time regions, and elementary particle horizons at which the signature of the induced metric changes.

7–3 duality states that it is possible to formulate the theory using either the data at 3-D space-like 3-surfaces resulting as intersections of the space-time surface with 7-D CDs or the data at 3-D light like CDs [B4]. This results if the data needed is actually contained by 2-D intersections $X^2 = X^3_l \cap X^7$. This effective 2-dimensionality has far-reaching implications. The notion simplifies dramatically the basic formulas related to the configuration space geometry and spinor structure and allows to understand the relationship between various conformal symmetries of TGD.

Effective 2-dimensionality also leads to an explicit identification of the generalized Feynman diagrams at space-time level as light like 3-D CDs.

The basic philosophy is that quantum-classical correspondence stating that space-time sheets provide a description for the physics associated with the configuration space spin degrees of freedom (fermionic degrees of freedom). This means kind of self-referentiality making it possible for quantum states to represent the dynamics of quantum jumps.

The generalized Feynman diagrammatics is simple. The propagator associated with 3-D CD is simply the inverse of the modified Dirac operator D, and vertices the inner products at X^2 for the positive energy states and negative energy states entering to the vertex and in principle computable. The equivalence of generalized Feynman diagrams with tree diagrams is expected on basis of the effective 2-dimensionality and gives very strong additional constraints to the vertices. It implies also unitarity. Since no loop summations are involved, the theory does not differ from effective theories defined by effective action.

The counterparts of loop sums are absent in TGD framework and p-adic number fields and their extensions defining an infinite hierarchy of fixed point values of Kähler coupling strength and thus of gauge coupling constants. The question is whether this discrete coupling constant evolution can mimic a QFT type coupling constant evolution (or vice versa). Is it possible to have renormalization without renormalization? The construction of quantum state using generalization of coset construction for super-canonical and super Kac-Moody algebra allows to answer this question. The counterparts of bare states are non-orthogonal and have a natural multi-grading. Gram-Schmidt orthogonalization procedure makes the bare states dressed and brings in TGD counterpart of loop corrections to the S-matrix. The counterparts of renormalization group equations result by formally regarding p-adic prime p as a continuous variable. Quantum field theory approximation results when the inner products defining simplest particle decays are described as coupling constants.

Certainly this generalized Feynman diagrammatics can be seen as the fulfillment of a long held dream and forms the basic for various approximate theories. This does not however exclude alternative approaches to S-matrix assuming that they are consistent with the fundamental equivalence of the generalized Feynman diagrams with tree diagrams.

12.4 Equivalence of loop diagrams with tree diagrams and cancellation of infinities in Quantum TGD

In [C5] a vision about how dual diagrams generalize in TGD context is developed. The vision is based on generalization of mathematical structures discovered in the construction of topological quantum field theories (TQFT) and conformal field theories (CQFT). In particular, the notions of Hopf algebras and quantum groups, and categories are central. The following gives a very concise summary of the basic ideas.

12.4.1 Feynman diagrams as generalized braid diagrams

The first key idea is that generalized Feynman diagrams are analogous to knot and link diagrams in the sense that they allow also "moves" allowing to identify classes of diagrams and that the diagrams containing loops are equivalent with tree diagrams, so that there would be no summation over diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of absolute minimum of Kähler action as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. At quantum level S-matrix element can be seen as a representation of a path defining continuation of configuration space (CH) spinor field between different sectors of CH corresponding to different 3-topologies. All continuations and corresponding Feynman diagrams are equivalent. The idea about Universe as a computer and algebraic hologram allows a concrete realization based on the notion of infinite primes, and space-time points become infinitely structured monads [O4]. The generalized Feynman diagrams differing only by loops are equivalent since they characterize equivalent computations.

12.4.2 Coupling constant evolution from infinite number of critical values of Kähler coupling strength

The basic objection against the new view about Feynman diagrams is that it is not consistent with the notion of coupling constant evolution involving loops in an essential manner. The objection can be circumvented. Quantum criticality requires that Kähler coupling constant α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that α_K has an infinite number of possible values labelled by p-adic length scales and also also by the dimensions of effective tensor factors defined hierarchy of II_1 factors (so called Beraha numbers) as found in [O4]. The dependence on p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to a finite angular resolution and to a hierarchy of finite-dimensional extensions of p-adic number fields R_p. The finiteness of the resolution is forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically. The higher the algebraic dimension of the extension and the higher the value of p-adic prime the better the angular (or phase) resolution and nearer the p-adic topology to that for real numbers.

12.4.3 R-matrices, complex numbers, quaternions, and octonions

A crucial observation is that physically equivalent R-matrices of 6-vertex models are labelled by points of CP_2 whereas maximal space of commuting R-matrices are labelled by points of 2-sphere S^2. CP_2 also labels maximal associative and thus quaternionic subspaces of 8-dimensional octonion space whereas S^2 labels the maximally commutative subspaces of quaternion space, which suggests that R-matrices and these structures correspond to each other.

Number theoretic vision leads to a number theoretic variant of spontaneous compactification meaning that space-time surfaces could be regarded either as hyper-quaternionic, and thus maximal associative, 4-surfaces in M^8 regarded as the space of hyper-octonions or as surfaces in $M^4 \times CP_2$ (the imaginary units of hyper-octonions are multiplied with $\sqrt{-1}$ so that the number theoretical norm has Minkowski signature). Associativity constraint is an essential element of also Yang-Baxter equations [27, 28].

These observations lead to a concrete proposal how quantum classical correspondence is realized classically at the space-time level. Each point of CP_2 corresponds to R-matrix and 3-surfaces can be identified as preferred sections of the space-time surface by requiring that unitary R-matrices are commuting in the section (micro-causality) whereas commutativity fails for R-matrices corresponding to different values of time coordinate defined by this foliation.

12.4.4 Ordinary conformal symmetries act on the space of super-canonical conformal weights

TGD predicts two kinds of super-conformal symmetries when space-time surfaces are regarded as sub-manifolds of $H = M^4 \times CP_2$ (in $OH = M^8$ picture hyper-quaternionic and hyper-octonionic conformal symmetries are relevant).

a) The ordinary super-conformal symmetries realized at the space-time level are associated with super Kac-Moody representations realized at light-like 3-surfaces appearing as boundaries of space-time sheets and boundaries between space-time regions with Euclidian and Minkowskian signature of metric. Conformal weights are half-integer valued in this case.

b) Super-canonical conformal invariance acts at the level of imbedding space and corresponds to light-like 7-surfaces of form $X_l^3 \times CP_2 \subset M^4 \times CP_2$ believed to appear as causal determinants too. In this case conformal weights are complex numbers of form $\Delta = n/2 + iy$ for the generators if super algebra, and I have proposed that the conformal weights of the physical states correspond are of form $1/2 + iy$, where y is zero of Riemann Zeta or possibly even superposition of them. In the latter case the imaginary parts of zeros would define a basis of an infinite-dimensional Abelian group spanning the weights.

The basic question concerns the interaction of these conformal symmetries.

a) Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of geodesic sphere of CP_2 (and thus also of space-time surface) labelling also mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. One can thus regard super-generators as conformal fields in space-time or complex plane having conformal weights as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories [28] induces by a pull-back a braid group action on the conformal labels of configuration space gamma matrices (super generators) and corresponding isometry generators.

b) The gamma matrices and isometry generators spanning the super-canonical algebra can be regarded as fields of a conformal field theory in the complex plane containing infinite number of punctures defined by the complex conformal weights. Quaternion conformal Super Virasoro algebra and Kac Moody algebra would act as symmetries of this theory and the S-matrix of TGD would involve the n-point functions of this conformal field theory.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors [29] having a close connection with the non-trivial representations of braid group and quantum groups. The sequence of non-trivial zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded an an infinite braid behind the von Neumann algebra [29]. Contrary to the expectations, also trivial zeros seem to be important. Purely algebraic considerations support the view that superposition for the imaginary parts of non-trivial zeros makes sense so that one would have entire hierarchy of one-dimensional lattices depending on the number of zeros included. The finite braids defined by subsets of zeros could be seen as a hierarchy of completely integrable 1-dimensional spin chains leading to quantum groups and braid groups [27, 28] naturally. In conformal field theories it is possible to construct explicitly the generators of quantum group in terms of operators creating screening charges [28]: interestingly, the charges are located going along a line parallel to imaginary axis to infinity.

It seems that not only Riemann's zeta but also polyzetas [30, 31, 32, 33] could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically. The physical justification for the hypothesis would be quantum criticality. Indeed, it has been found that the loop corrections of quantum field theory are expressible in terms of polyzetas [34]. If the arguments of polyzetas correspond to conformal weights of particles of many-particle bound state, loop corrections vanish when the super-canonical conformal weights correspond to the zeros of polyzetas including zeta. This argument does not allow superpositions of imaginary parts of zeros.

12.4.5 Equivalence of loop diagrams with tree diagrams from the axioms of generalized ribbon category

The fourth idea is that Hopf algebra related structures and appropriately generalized ribbon categories [27, 28] could provide a concrete realization of this picture. Generalized Feynman diagrams which are identified as braid diagrams with strands running in both directions of time and containing besides braid operations also boxes representing algebra morphisms with more than one incoming and outgoing strands. 3-particle vertex should be enough, and the fusion of 2-particles and $1 \rightarrow 2$ particle decay would correspond to generalizations of the algebra product μ and co-product Δ to morphisms of the category defined by the super-canonical algebras associated with 3-surfaces with various topologies and conformal structures. The basic axioms for this structure generalizing ribbon algebra axioms [27] would state that diagrams with self energy loops, vertex corrections, and box diagrams are equivalent with tree diagrams.

Tensor categories might provide a deeper understanding of p-adic length scale hypothesis. Tensor primes can be identified as vector/Hilbert spaces, whose real or complex dimension is prime. They serve as "elementary particles" of tensor category since they do not allow a decomposition to a tensor product of lower-dimensional vector spaces. The unit I of the tensor category would have an interpretation as a one-dimensional Hilbert space or as the number field associated with the Hilbert space and would act like identity with respect to tensor product. Quantum jump cannot decompose tensor prime system to an unentangled product of sub-systems. This elementary particle like aspect of tensor primes might directly relate to the origin of p-adicity. Also infinite primes are possible and could distinguish between different infinite-dimensional state spaces.

For quantum dimensions $[n]_q \equiv (q^n - q^{-n})/(q - q^{-1})$ [27] no decomposition into a product of prime quantum dimensions exist and one can say that all non-vanishing quantum integers $[n]_q$ are primes. For q an n^{th} root of unity, quantum integers form a finite set containing only the elements $0, 1, ..., [n-1]$ so that quantum dimension is always finite. The numbers $[2]_q^2$, for $q = exp(i\pi/n)$ define a hierarchy of Beraha numbers having an interpretation as a renormalized dimension $[2]_q^2 \leq 4$ for the spinor space of 4-dimensional space and appearing as effective dimensions of type II_1 sub-factors of von Neumann algebras.

12.4.6 Quantum criticality and renormalization group invariance

Quantum criticality means that renormalization group acts like isometry group at a fixed point rather than acting like a gauge symmetry as in the standard quantum field theory context. Despite this difference it is possible to understand how Feynman graph expansion with vanishing loop corrections relates to generalized Feynman graphs and a nice connection with the Hopf- and Lie algebra structures assigned by Connes and Kreimer to Feynman graphs emerges. It is possible to deduce an explicit representation for the universal momentum and p-adic length scale dependence of propagators in this picture. The condition that loop diagrams are equivalent with tree diagrams gives explicit equations which might fix completely also the p-adic length scale evolution of vertices. Quantum criticality in principle fixes completely the values of the masses and coupling constants as a function of p-adic length scale.

12.4.7 Modified Dirac action for the induced spinor fields as the QFT description of quantum TGD?

Quantum-classical correspondence suggests that quantum TGD should allow a quantum field theory formulation of some kind allowing to relate the abstract physics at the level of configuration space to the space-time physics defined by Kähler action. QFT formulation indeed seems to exist and the constraints satisfied by the QFT formulation of quantum TGD are so strong that the formulation is essentially unique.

a) The QFT in question should be determined by the absolute minimum $X^4(X^3)$ of Kähler action corresponding to a given causal determinant (7-dimensional light like surface $X_l^3 \times CP_2$) rather than in M^4. The averaging over all Poincare and color translates of this space-time surface would give rise to an S-matrix respecting the basic symmetries. Note that the theory would be 3-D quantum field theory at X^3. The only information needed about $X^4(X^3)$ would be the time derivatives of the imbedding space coordinates at X^3. Also the value of Kähler action seems to be needed but even the exponent of Kähler function might disappear from the Greens functions in normalization just as the exponent e^G of the generating functional G of connected Green's functions disappears in the quantum field theory.

The effective 3-dimensionality has an interesting connection to unresolved difficulties encountered in the attempt to formulate bound state problems in quantum field theory context. Non-relativistic, essentially 3-dimensional, Schrödinger equation works and yield correct predictions whereas Bethe-Salpeter equation in Minkowski space fails. The TGD based explanation of this failure is that bound state formation means that 3-surfaces of particles involved form a join along boundaries condensate. This means that non-relativistic 3-dimensional formulation is necessary in order to catch the essential aspects of the physics involved. In quantum field theory context point-likeness of the particles allows only the modelling of those aspects of particle interactions which do not involve bound states.

b) To calculate correlation functions one should expand the super-canonical generators as functional Taylor series around the maximum of the Kähler function at the 3-surface X^3. If super-canonical generators can be regarded as functionals of the second quantized induced spinor field ψ, also the functional series with respect to ψ is needed. This would make it possible to evaluate the correlation functions perturbatively in terms of the tree diagrams defined by the effective action using bosonic and fermionic propagators defined by it. One would calculate M^4 Fourier transforms of the correlation functions and integration over Poincare translates would give

Poincare invariant correlation functions.

c) The complete localization at configuration space level means the vanishing of the bosonic loops and effective freezing of configuration space degrees of freedom. This is achieved if the action is such that the bosonic part vanishes when the induced spinor fields vanish. I have represented in [B4] arguments that the action for the induced spinor fields treated as Grassman variables is all that is needed to define quantum physics. Kähler action would be the effective action associated with the Dirac action and its absolute minimization would be thus natural. The approach would also predict the possible values of Kähler coupling strength as part of data characterizing the effective action. This approach also conforms with the fact that elementary bosons are predicted to be bound states of fermion-anti-fermion pairs.

This approach would also bring induced electro-weak gauge potentials into play so that quantum-classical correspondence would be realized. The absence of quark color as spin like quantum number of induced spinor fields would not be a problem. Also the topologization of the family replication phenomenon in terms of the genus of 2-surface would result without representation as an additional spin like degeneracy of fermion fields.

d) The action would be the modified Dirac action for the induced spinor fields at the maximum of the Kähler function. Modified Dirac action is defined by replacing the induced gamma matrices $\Gamma_\alpha = \partial_\alpha h^k \Gamma_k$ by the modified gamma matrices

$$\hat{\Gamma}^\alpha = \frac{\partial(L\sqrt{|g|})}{\partial(\partial_\alpha h^k)}\Gamma^k \quad . \tag{30}$$

Here L denotes the action density of Kähler action and Γ^k denotes gamma matrices of the imbedding space H. Modified Dirac action is supersymmetric and shares the vacuum degeneracy of Kähler action [B4].

The fermionic propagator would be defined by the inverse of the modified Dirac operator. Bosonic kinetic term would be Grassmann algebra valued and vanish for $\psi = 0$ and would contribute nothing to the perturbation series. Since the fermionic action is free action, a divergence free quantum field theory would be in question irrespective of whether the fermionic action is interpreted as an action or an effective action. One could also see the action as a fixed point of the map sending action to effective action in a complete accordance with the idea that loop corrections vanish.

e) The nice feature of this approach is that the information needed about the absolute minimum would be minimal since one can restrict the consideration to 3-surface X^3 which can be selected arbitrarily. In fact, the outcome is a 3-dimensional free field theory in the fermionic degrees of freedom and the integral over Poincare and color translates guarantees isometry symmetries. Grassmannian functional integral would give exponent of Kähler function as the analog of the generating functional G for connected Green functions. The functional derivatives of the configuration space spinor field with respect to the induced spinor field are very simple by the conservation of fermion number. This approach could be seen as an alternative approach to calculate n-point functions by treating fermionic fields as Grassmann fields whereas in the super-algebra approach fermionic fields would be second quantized.

12.5 Various approaches to the construction of S-matrix

The gigantic symmetries of quantum TGD are bound to lead to a highly unique U-matrix but the practical construction of U-matrix remains still a formidable challenge. Despite this one can write Feynman rules for the S-matrix in the approximation that the consideration is restricted to elementary particles modelled as CP_2 type extremals. This approximation might well be all that is needed for practical purposes and leads to precise predictions.

12.5.1 7–3 duality as a key to the construction of S-matrix

The notion of 7–3 duality emerged from the (one might say violent) interaction between TGD and M-theory [A2]. The attempts to construct quantum TGD have gradually led to the conclusion that the geometry of the configuration space ("world of classical worlds") involves both 7-D and 3-D light like surfaces as causal determinants. 7-D light like surfaces X^7 are unions of future and past light cone boundaries and play a role somewhat resembling that of branes. 3-D light like surfaces X_l^3 can correspond to boundaries of space-time sheets, regions separating two maximally deterministic space-time regions, and elementary particle horizons at which the signature of the induced metric changes.

7–3 duality states that it is possible to formulate the theory using either the data at 3-D space-like 3-surfaces resulting as intersections of the space-time surface with 7-D CDs or the data at 3-D light like CDs [B4]. This results if the data needed is actually contained by 2-D intersections $X^2 = X_l^3 \cap X^7$. This effective 2-dimensionality has far-reaching implications. It simplifies dramatically the basic formulas related to the configuration space geometry and spinor structure, it leads to the explicit identification of the generalized

Feynman diagrams at space-time level as light like 3-D CDs. The basic philosophy is that quantum-classical correspondence stating that space-time sheets provide a description for the physics associated with the configuration space spin degrees of freedom (fermionic degrees of freedom).

The generalized Feynman diagrammatics is simple. The fermions do not carry four-momenta but are on mass shell particles characterized by the eigenvalues of the modified Dirac operator D. There is no propagator associated with 3-D CDs: only a unitary transformation U_λ representing braiding in spin and electroweak spin degrees of freedom can be present. Vertices are the inner products at X^2 for the positive energy states and negative energy states entering to the vertex, finite, and in principle computable. The equivalence of generalized Feynman diagrams with tree diagrams is expected on basis of the effective 2-dimensionality, and indeed follows from on mass shell property directly. Unitarity follows trivially. No loop summations are thus involved.

12.5.2 Quantum criticality and Hopf algebra approach to S-matrix

Quantum criticality leads to a generalization of duality symmetry of string models stating that the generalized Feynman diagrams with loops are equivalent with diagrams having no loops. This means that each S-matrix element correspond to a unique tree diagram. The conditions for this equivalence can be formulated as algebraic conditions characterizing a Hopf algebra like structure, and, using the language of ordinary Feynman diagrams, correspond to the vanishing of the loop corrections in the configuration space integral crucial for the p-adicization. This symmetry is expected to be of crucial importance for practical evaluation of S-matrix elements as should be also the reduction of the matrix elements of generators of the enveloping algebra of super-canonical algebra to n-point functions of conformal field theory in the complex plane of super-canonical conformal weights.

12.5.3 von Neumann algebras and S-matrix

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix [O5].

1. Inclusions of hyper-finite II_1 factors as a basic framework to formulate quantum TGD

a) The effective 2-dimensionality of the construction of quantum states and configuration space geometry in quantum TGD framework makes hyper-finite factors of type II_1 very natural as operator algebras of the state space. Indeed, the elements of conformal algebras are labelled by discrete numbers and also the modes of induced spinor fields are labelled by discrete label, which guarantees that the tangent space of the configuration space is a separable Hilbert space and Clifford algebra is thus a hyper-finite type II_1 factor. The same holds true also at the level of configuration space degrees of freedom so that bosonic degrees of freedom correspond to a factor of type I_∞ unless super-symmetry reduces it to a factor of type II_1.

b) Four-momenta relate to the positions of tips of future and past directed light cones appearing naturally in the construction of S-matrix. In fact, configuration space of 3-surfaces can be regarded as union of big-bang/big crunch type configuration spaces obtained as a union of light-cones with parameterized by the positions of their tips. The algebras of observables associated with bounded regions of M^4 are hyper-finite and of type III_1. The algebras of observables in the space spanned by the tips of these light-cones are not needed in the construction of S-matrix so that there are good hopes of avoiding infinities coming from infinite traces.

c) Many-sheeted space-time concept forces to refine the notion of sub-system. Jones inclusions $\mathcal{N} \subset \mathcal{M}$ for factors of type II_1 define in a generic manner imbedding interacting sub-systems to a universal II_1 factor which now corresponds naturally to infinite Clifford algebra of the tangent space of configuration space of 3-surfaces and contains interaction as $\mathcal{M} : \mathcal{N}$-dimensional analog of tensor factor. Topological condensation of space-time sheet to a larger space-time sheet, formation of bound states by the generation of join along boundaries bonds, interaction vertices in which space-time surface branches like a line of Feynman diagram: all these situations could be described by Jones inclusion characterized by the Jones index $\mathcal{M} : \mathcal{N}$ assigning to the inclusion also a minimal conformal field theory and conformal theory with k=1 Kac Moody for $\mathcal{M} : \mathcal{N} = 4$. $\mathcal{M} : \mathcal{N}$=4 option need not be realized physically as quantum field theory but as string like theory whereas the limit $D = 4 - \epsilon \rightarrow 4$ could correspond to $\mathcal{M} : \mathcal{N} \rightarrow 4$ limit. An entire hierarchy of conformal field theories is thus predicted besides quantum field theory.

d) von Neumann's somewhat artificial idea about identical a priori probabilities for states could replaced with the finiteness requirement of quantum theory. Indeed, it is traces which produce the infinities of quantum field theories. That $\mathcal{M} : \mathcal{N} = 4$ option is not realized physically as quantum field theory (it would rather correspond to string model type theory characterized by a Kac-Moody algebra instead of quantum group), could correspond to the fact that dimensional regularization works only in $D = 4 - \epsilon$. Dimensional regularization with space-time dimension $D = 4 - \epsilon \rightarrow 4$ could be interpreted as the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. $\mathcal{M}$ as an $\mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module would provide a concrete model for a quantum space with non-integral dimension as well as its Clifford algebra. An entire sequence of regularized theories corresponding to the allowed values of $\mathcal{M} : \mathcal{N}$ would be predicted.

2. Generalized Feynman diagrams are realized at the level of $\mathcal{M}$ as quantum space-time surfaces

The key idea is that generalized Feynman diagrams realized in terms of space-time sheets have counterparts at the level of $\mathcal{M}$ identifiable as the Clifford algebra associated with the entire space-time surface X^4. 4-D Feynman diagram as part of space-time surface is mapped to its $\beta = \mathcal{M} : \mathcal{N} \leq 4$-dimensional quantum counterpart.

a) von Neumann algebras allow a universal unitary automorphism $A \rightarrow \Delta^{it} A \Delta^{-it}$ fixed apart from inner automorphisms, and the time evolution of partonic 2-surfaces defining 3-D light-like causal determinant corresponds to the automorphism $\mathcal{N}_i \rightarrow \Delta^{it} \mathcal{N}_i \Delta^{-it}$ performing a time dependent unitary rotation for $\mathcal{N}_i$ along the line. At configuration space level however the sum over allowed values of t appear and should gives rise to the TGD counterpart of propagator as the analog of the stringy propagator $\int_0^t exp(iL_0 t)dt$. Number theoretical constraints from p-adicization suggest a quantization of t as $t = \sum_i n_i y_i > 0$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta.

b) At space-time level the "ends" of orbits of partonic 2-surfaces coincide at vertices so that also their images $\mathcal{N}_i \subset \mathcal{M}$ also coincide. The condition $\mathcal{N}_i = \mathcal{N}_j = ... = \mathcal{N}$, where the sub-factors $\mathcal{N}$ at different vertices differ only by automorphism, poses stringent conditions on the values t_i and Bohr quantization at the level of $\mathcal{M}$ results. Vertices can be obtained as a vacuum expectations of of the operators creating the states associated with the incoming lines (crossing symmetry is automatic).

c) The equivalence of loop diagrams with tree diagrams would be due to the possibility to move the ends of the internal lines along the lines of the diagram so that only diagrams containing 3-vertices and self energy loops remain. Self energy loops are trivial if the product associated with fusion vertex and co-product associated with annihilation compensate each other. The possibility to assign quantum group or Kac Moody group to the diagram gives good hopes of realizing product and co-product. Octonionic triality would be an essential prerequisite for transforming N-vertices to 3-vertices.

The equivalence allows to develop an argument proving the unitarity of S-matrix.

d) A formulation using category theoretical language suggests itself. The category of space sheets has as the most important arrow topological condensation via the formation of wormhole contacts. This category is mapped to the category of II_1 sub-factors of configurations space Clifford algebra having inclusion as the basic arrow. Space-time sheets are mapped to the category of Feynman diagrams in $\mathcal{M}$ with lines defined by unitary rotations of $\mathcal{N}_i$ induced by Δ^{it}.

e) The hierarchy of imbeddings for type II_1 factors generalizes to the hierarchy of generalized Feynman diagrams in which the particles of given level correspond to Feynman diagrams of the previous level. These Feynman diagrams provide representations for the projections of S-matrix to subspaces of incoming and outgoing states providing a hierarchy of self representations about the system. By crossing symmetry the entanglement defined by S-matrix corresponds to the so called Connes tensor product [38]. At space-time level it corresponds to time-like entanglement made possible by the failure of strict determinism. That the failure is only partial explains why the entanglement is so special.

The vertices for the interactions of these Feynman diagrams reduce to the interactions at the lowest level apart from the automorphisms $\Delta_{\mathcal{M}_n}$ defining free propagation. Also transitions between between different levels are possible. The interpretation is as an infinite cognitive hierarchy realizing theory about the material world as cognitive quantum states. This hierarchy corresponds to states with vanishing conserved net quantum numbers but having non-vanishing "gravitational" charges identifiable as classical charges.

12.5.4 Crossing symmetry and configuration space spinor fields as representations of S-matrix elements

In case of CP_2 type extremals the picture seems to be more complicated than this since the classical space-time surface is not unique. There is summation over the positions of the vertices of a Feynman diagram represented as a topological sum over CP_2 type extremals as well as sum over the Feynman diagrams. One could perhaps see these summations as an integral over the fiber of freedom and possible also zero modes of the configuration space surfaces $X^4(\cup_i Y_i^3)$: the same infinite prime indeed defines space-time surface for each values of the fiber coordinates and this would give rise to Feynman diagrams labelled by the positions of the vertices.

There is also a further problem involved: classical space-time surface represents *both* the incoming and outgoing states. The correspondence between Fock-space and classical descriptions of particle reactions suggests that also the outgoing space-time $X^4(\cup_i Y_i^3(out))$ surfaces appear as asymptotic space-time regions of $X^4(\cup_i Y_i^3)$. This might be possible thanks to the non-determinism of Kähler action.

The simple Glebch-Gordan vision fails in the case of CP_2 type extremals, since it neglects the non-determinism of the classical time evolution, which in case of CP_2 type extremals plays a crucial role in the construction of the theory. The construction of the S-matrix for CP_2 type extremals relies on the correspondence between Feynman diagrams as space-time surfaces constructed from the topological sums of CP_2 type extremals. Stringy Feynman rules are however not consistent with the Glebch-Gordan philosophy for the simple reason that space-time surface represents both the incoming and outgoing states rather than only the initial states.

The following interpretation inspired by crossing symmetry and the idea that configuration space spinor fields represent, not only physical states, but also S-matrix geometrically, however allows to understand why the Feynman rules make sense.

a) Feynman graphs with lines thickened to CP_2 type extremals represent classically quantum jumps between initial and final states characterized by incoming and outgoing CP_2 type extremals. Space-time surface containing topologically condensed CP_2 type extremals does not represent quantum state but a pair of quantum states and a quantum transition between these states. Incoming and outgoing states are represented as positive and negative energy particles and the net quantum numbers of the space-time sheet containing particles vanishes.

b) The initial state for quantum transition in question is vacuum and the final state $\Psi_f(m \rightarrow n)$ is represented by a which is superposition of all possible Feynman graphs for the transition in question. Thus S-matrix is coded into a physical state itself and the amplitudes $\langle 0|\Psi_f(m \rightarrow n)\rangle$ are proportional to S-matrix elements $S(m \rightarrow n)$. The amplitudes are indeed equal to S-matrix elements if crossing symmetry holds true.

Thus one ends up with the view that classical non-determinism and crossing symmetry make it possible for the configuration space spinor fields to represent not only physical states but also the U-matrix between them. This is possible also for other particles than those represented by CP_2 type extremals since one can construct space-time sheets containing arbitrary pairs of positive and negative energy states such that the net quantum numbers of the state pair cancel. The beauty of this representation is that it allows also to derive S-matrix easily as overlap integrals of vacuum state and zero energy states. Obviously this means huge simplification and gives a connection with ordinary Feynman diagrammatics.

The representation of entire sequences of quantum transitions as zero energy states of form $|\Psi_f(m_1 \rightarrow m_2... \rightarrow m_n)\rangle$ becomes possible so that classical non-determinism makes possible self-referential Universe able to represent the laws of physics in the structure of its own states. Also this idea supports the view that cognition is present already in elementary particle length scales.

12.5.5 U-matrix as Glebsch-Gordan coefficients

U-matrix relates 'free' and 'interacting' representations of the super-canonical and super Kac-Moody algebras acting as symmetries of quantum TGD. The construction is based on the association of 3-surfaces Y_i^3 and corresponding absolute minima $X^4(Y_i^3)$ to incoming states as well as the interacting four-surface $X^4(\cup_i Y_i^3)$ describing the interactions classically. The generators for various super-algebras associated with $X^4(\cup_i Y_i^3)$ are modified by interactions so that the generator basis is not just a union of the generator basis associated with $X^4(Y_i^3)$. U-matrix relates the tensor product for the representations associated with the incoming 'free' space-time surfaces $X^3(Y_i^3)$ and the interaction representation associated with $X^4(\cup Y_i^3)$: generalized Glebch-Gordan coefficients are clearly in question and unitarity is obvious.

12.5.6 Number theoretic approach to U-matrix

The task of assigning to the surfaces Y_i^3 the free space-time surfaces $X^4(Y_i^3)$ and interacting space-time surface $X^4(\cup_i Y_i^3)$ is the basic stumbling block for the construction of U-matrix. The super-algebra generators creating the excitations of the incoming ground states are super-algebra generators associated with $\cup X^4(Y_i^3)$ whereas the outgoing states are created by the super-algebra generators associated with $X^4(\cup_i Y_i^3)$. The surfaces $X^4(Y_i^3)$ correspond to the space-time surfaces associated with infinite primes P_i representing ground states of super-conformal representations whereas $X^4(\cup_i Y_i^3)$ corresponds to the space-time surface associated with the infinite integer $N = \prod_i P_i^{k_i}$. This means that the worst part of the problem is solved. The remaining challenge is to relate the super-algebra basis to each other.

12.5.7 Perturbation theoretic approach to U-matrix

This formal approach starts from the identification of U-matrix elements as Glebsch-Gordan coefficients relating free and interacting states and tries to construct U-matrix perturbatively by reducing it to stringy perturbation theory. The starting point is that U-matrix must follow from Super Virasoro invariance alone and that the condition $L_0(tot)\Psi = 0$ (plus the corresponding conditions for other super-Virasoro generators) must determine U-matrix. Here $L_0(tot)$ corresponds to the Virasoro generators associated with the interacting space-time surface $X^4(\cup_i Y_i^3)$ whereas $L_0(free, i)$ correspond to the free generators associated with $X^3(Y_i^3)$. It is however not at all obvious whether the generators $L_0(tot)$ are perturbatively related to the the generators $L_0(free, i)$ and whether U-matrix allows perturbative expansion.

12.5.8 Construction of the S-matrix at high energy limit

It is possible to write Feynman rules for the S-matrix in the approximation that only CP_2 type extremals appear as virtual and real particles. All CP_2 type extremals are locally isometric with CP_2 itself and only the random lightlike curve is dynamical. The classical dynamics is actually isomorphic with stringy dynamics since classical Virasoro conditions are satisfied. Fermions belong to the representations of Super-Kac-Moody algebra of $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. The classical nondeterminism of the dynamics implies that Feynman graph expansion is topologized. This saves from the troubles caused by fermionic divergences since the exponent of the momentum generator effecting translation along the line of the Feynman graph corresponds to that associated with the modified Dirac action and thus to a free quantum theory for fermions.

Vertex operators $V(a, b, c)$ are generalizations of the vertex operators of string theory: instead of strings 3-surface inside CP_2 type extremal fuse together. Propagator factors are products of the exponent of the Kähler action for CP_2 type extremal proportional to the volume of the CP_2 type extremal; the 'stringy' $1/(L_0+i\epsilon)$ factor, which comes from the vertices; and a unitary translation operator (counterpart of S-matrix as time translation operator) along the geodesic representing average cm motion.

The theory has some features which are characteristic for quantum TGD.

a) One can assume that each quantum jump involves localization in zitterbewegung degrees of freedom. The resulting S-matrix is independent of the choice of the representative for the zitterbewegung orbit as long as the cm motion connects the lines of the vertices. The predictions depend however on an arbitrary function of U of CP_2 coordinates giving rise to a decomposition of CP_2 to 'time slices'. The dependence of the propagator is only through the volume of CP_2 type extremal determined by U whereas coupling constants have more complicated, but presumably very mild dependence on U. The dependence on the function U means that one must average the scattering rates over the allowed spectrum of functions U. This dependence of the fundamental coupling constants on U is in accordance with spin glass analogy and means that fundamental coupling constants are not strictly speaking constants.

b) The volume of the internal line, which is a fraction of CP_2 volume determines the value of the exponent of Kähler action and provides thus a suppression factor serving as an infrared cutoff. A constraint to the allowed functions U results from the topological condensation of CP_2 in particle like space-time sheet (for instance, massless extremal), which implies that CP_2 type extremals cannot extend outside the region with size of order p-adic length scale L_p. The only plausible interpretation seems to be that the information about the infrared cutoff length scale is coded into the structure of particle: particle in the box is quite not the same as free particle. This suggests new view about color confinement: quarks and gluons correspond to CP_2 type extremals which cannot exist too long time as free particles and therefore cannot leave hadron.

13 Does TGD predict the values of Planck constant?

The idea that $\hbar$ is dynamical and can have arbitrarily large values is about one and half year old as a write this. A lot of progress has occurred during the last year but I have not yet been able to seriously pose the question whether and how TGD could predict the values of the Planck constant. In the following a proposal for how TGD predicts the value spectrum of $\hbar$ as one aspect of quantum criticality is discussed and number theoretical arguments are used to make a guess about the spectrum of $\hbar$. In very concise form the argument goes as follows.

a) The freedom to choose the value of $\hbar$ corresponds to the freedom to choose the overall scaling $\lambda = \hbar/\hbar_0$ of M^4 metric associated with various copies of M^4 obtained identified as various algebraic extensions of rational M^4 glued together along common set of rationals consistent with the isometric identification.

b) The dependence of λ on the algebraic extension for a given p-adic prime p is fixed by the quantum criticality condition stating that the critical Kähler coupling strength is same for various algebraic extensions associated with given p-adic prime p.

c) Number theoretic ideas allow to make good guesses concerning the dependence of λ on the algebraic extension. Simplest guess is that λ for union of linearly independent extensions is product or $1/\lambda$ a sum of λ:s for composites. It turns out that sum is the most plausible guess.

13.1 The basic ideas

In order to build a more coherent theoretical framework for the quantization of $\hbar$ let us summarize the basic vision as it exists now.

13.1.1 Four-momentum is invariant in the transition changing $\hbar$

The basic constraint is that four-momentum and other quantum numbers of the state are conserved in the scaling $\hbar \rightarrow \lambda\hbar$ whereas various quantum lengths and times are scaled up by λ so that one obtains the basic predictions such as macroscopic and macro-temporal quantum coherence in arbitrarily long scales. It is however not at all obvious how to realize this condition.

13.1.2 The identification of the value of the parameter v_0 in terms of Kähler coupling strength

The parameter $v_0 \simeq 2^{-11}$, which has has actually dimension of velocity unless on puts $c = 1$, and its harmonics and sub-harmonics appear in the scaling of $\hbar$. v_0 corresponds to the velocity of distant stars in the model of galactic dark matter. TGD allows to identify this parameter as the parameter

$$\begin{aligned} v_0 &= 2\sqrt{TG} = \sqrt{\frac{1}{2\alpha_K}}\sqrt{\frac{G}{R^2}} \ , \\ T &= \frac{1}{8\alpha_K}\frac{\hbar_0}{R^2} \ . \end{aligned} \tag{31}$$

Here T is the string tension of cosmic strings, R denotes the "radius" of CP_2 ($2R$ is the radius of geodesic sphere of CP_2). $\hbar_0$ corresponds to Beraha number B_∞. α_K is Kähler coupling strength whose evolution is dictated by the condition that G is invariant in coupling constant evolution and by number theoretical arguments to

$$\alpha_K = k\frac{1}{log(p) + log(K)} \ ,$$

$$
\begin{aligned}
K &= \frac{R^2}{\hbar_0 G} = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23 = 223,092,870 \ , \\
k &\simeq \pi/4 \ . \qquad (32)
\end{aligned}
$$

Equivalence Principle requires that $\hbar$ in the formula for K corresponds to the value $\hbar_0 \equiv 1$ [D6] so that gravitational coupling constant is invariant also with respect to coupling constant evolution associated with Beraha numbers. One can define "dynamical" Planck length as

$$
L_d = \sqrt{G\hbar_{gr}} = \sqrt{M_2/M_1}\frac{1}{v_0}GM_1 \ .
$$

The order of magnitude is not too far from Schwartschild radius.

Number theoretic constraints are expected to pose strong constraints on the value of k. A discrete version of a typical logarithmic evolution of U(1) coupling constant strength as a function of length scale is in question and for $k = 127$ (M_{127}) the value of α_K is very near to fine structure constant $\alpha \simeq 1/137$. Note that the ratio R^2/G is predicted to be constant so that G would be renormalization group invariant even in the sense that it would not depend on the p-adic length scale.

The resulting prediction for v_0 reads as

$$
\begin{aligned}
v_0 &= \sqrt{\frac{1}{2\left[k\times log(2) + log(K)\right]K}} \quad \text{for } p \simeq 2^k \ , \\
v_0 &\simeq .54412\times 10^{-3} \ \text{for } p = M_{127} \ . \qquad (33)
\end{aligned}
$$

v_0 approaches to zero at logarithmic rate for large values of k. For $k = 127$ it is rather near to $v_0 = 2^{-11}$. In a good approximation the value 2^{-11} is reached for $k = 135$ which corresponds to the p-adic length scale $L(k_{eff} = 113 + 22)$ associated with dark variants of $k = 113$ weak bosons appearing in the TGD based model of atomic nucleus. The mean experimental value of $1/v_0$ is $1/v_0 = 2174$ with an accuracy of 1 per cent.

TGD allows also to develop arguments explaining the harmonics and sub-harmonics of v_0. Higher harmonics would be due to wrapped magnetic flux tubes for which CP_2 coordinates are n-valued functions of M^4 coordinates defining local coordinates of the flux tube space-time sheet. Sub-harmonics would result when M^4 coordinates become n-valued functions of CP_2 coordinates.

The challenge is to understand why v_0 appears in the basic formula expressing the change of $\hbar$ in the transition increasing the value of $\hbar$. It would seem that $\hbar$ characterizes the magnetic flux tubes (join along boundaries bonds) connecting the interacting systems and serving as space-time correlates for the interaction giving rise to bound state.

The value of v_0 deduced for cosmic strings does not make sense in astrophysical or condensed matter context, where cosmic strings are replaced with magnetic flux tubes. v_0 remains invariant in this scaling down if R^2 is replaced by the p-adic length scale L_p^2 apart from a multiplicative factor in the formulas for G and T so that the product TG remains invariant. $T_m \propto 1/L_p^2$ characterizes the magnetic energy density of the magnetic flux tube and $G_m \rightarrow L_p^2$ is identifiable as a "strong" gravitational coupling strength characterizing the interactions of magnetic flux tubes behaving like string like objects.

13.1.3 The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician.

A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$ and predicts the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule of thumb has now been applied with success the interpretation of hadronic mass calculations and to build models for systems like atomic nucleus and high T_c superconductor and seems to work. Of course, the criterion for transition is primitively formulated and the understanding what really happens in the transition to large $\hbar$ phase behaving like dark matter.

13.1.4 What are the allowed values of $\hbar$?

From the beginning I had strong gut feeling that the allowed values of $\hbar$ are expressible in terms of Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$ related to Jones inclusion hierarchies of hyperfinite factors of type II_1, which correspond to von Neumann algebra naturally associated with configuration space spinors.

Consider the inclusion $N \subset M$ of these factors as von Neumann algebras. A deep result is that one can express M as $N : M$-dimensional module over N with fractal dimension $N : M = B_n$. $\sqrt{B_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{B_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra.

This observation leads to the idea about how one could predict the value spectrum of $\hbar$. There are two arguments based on similar reasoning but leading to quite different predictions.

Option I: Consider the trace of $\sigma_z^2 = \hbar^2 \times 1$ for $d = 2$-component spinors. Assume that the trace of σ_z^2 does not depend on $\hbar$ so that one has

$$Tr(\sigma_z^2) = \sqrt{M : N}\hbar_n^2 = 2\hbar_\infty^2 \ .$$

By comparing the two sides one obtains

$$\frac{\hbar_n}{\hbar_\infty} = \sqrt{\frac{1}{cos(\pi/n)}} \ , \ n \geq 3 \ . \tag{34}$$

This would predict that the values of $\hbar_n/\hbar_\infty$ vary in the range $[1, \sqrt{2}]$. This is not enough to explain the large values of $\hbar$ but the arguments represented in the sequel allow to consider this option seriously.

Option II: In order to understand the large values of $\hbar$ I developed an argument the outcome of which seems to be correct although the proposed explanation of large values of $\hbar$ need not be correct. The idea is that the dimension of the spinor space is given by $d = 2^{D/2}$, where D is the dimension of the space-time. By applying this formula in the recent fractal context one obtains

$$D_n = 2log_2(\sqrt{B_n}) = 2log_2(2cos(\pi/n)) \ .$$

In recent case the invariance for the trace of the scaling operator $L_0 = \hbar m^k d/dm^k$ acting in the representation defined by M^D coordinates implies that the trace $Trace(L_0) = D\hbar$ equals to $2\hbar_\infty$.

$$D_n\hbar_n = 2\hbar_\infty \tag{35}$$

allowing to identify the renormalized value of $\hbar$ as

$$\frac{\hbar_n}{\hbar_\infty} = \frac{2}{D_n} = \frac{1}{log_2(2cos(\pi/n))} \ . \tag{36}$$

This would give $D_3 = 1$ and $D_\infty = 2$ and $\hbar_3 = \infty$. The idea was that the gigantic value of $\hbar$ in astrophysical length scales would result as a small perturbation of $x = 1/\hbar$ at $x = 0$. The infinite value of $\hbar$ for $n = 3$ would have a nice interpretation in terms of vacuum degeneracy. $n = 3$ would correspond to vacuum extremals for which perturbation theory does not make sense. The situation would be saved by the vanishing value of gauge coupling strengths meaning that all quantum corrections to the classical prediction vanish. It will turn out that option II is indeed more natural in the TGD framework.

13.1.5 Do Jones inclusions correspond to inclusions of rationals to their algebraic extensions?

Jones inclusions for type II_1 factors have several interpretations and applications in TGD framework. The most obvious interpretation Jones inclusion is as sub-system-system inclusion. Jones inclusion could be also accompany the inclusions of rationals to algebraic extensions of rationals at the level of configuration space spinor fields. One can also consider the inclusions of p-adic number fields to their complex extensions.

The inclusion $Q \subset Q[exp(i\pi/n)]$ would correspond to Jones inclusion characterized by n at the level of configuration space spinors. In the framework of rational physics the hierarchy of algebraic extensions of rationals would define a hierarchy of Jones inclusions and these extensions would give rise to a hierarchy of Planck constants.

The problem is that the large values of $\hbar$ do not seem to be natural in either case except possibly for option II by allowing perturbative corrections to $x = 1/\hbar_3 = 0$. This need not to be the case. $M : N$ characterizes configuration space Clifford algebra assignable with an algebraic extension of rationals as N-module where N corresponds to CH Clifford algebra assignable to rationals. What comes in mind that M could be much higher-dimensional as N-module than in the case of standard Jones inclusions defined using reals or complex numbers as a coefficient field of the Clifford algebra. The simplest guess is that M is tensor power of $M : N$-dimensional spinor base as an N module indeed allowed by option II as will be found.

13.2 Mathematical constraints

In order to gain insight to how one might understand the hierarchy of Planck constants in TGD framework it is good to start from good old Schrödinger equation.

13.2.1 What is behind minimal substitution?

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term does not depend on it. This makes the scaling of $\hbar$ a non-trivial transformation. In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. Minimal substitution does not make sense in CP_2 degrees of freedom

The first crucial observation is that the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined naturally in such a manner that spinor connection corresponds to the quantity $eQ/\hbar$ and there is no natural manner to separate $e/\hbar$ from it.

The only reasonable conclusion is that the Dirac operator in $M^4 \times CP_2$ depends on $\lambda = \hbar/\hbar_0$ only via its M^4 part and that Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$. The values of k^2 are proportional to $1/\lambda^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

2. The dynamical character of $\hbar$ can be only due to the freedom to select the scale of M^4 metric

The second crucial observation is that the freedom to vary λ can be only due to the freedom to vary the overall scaling of M^4 metric which is indeed possible. Whether the freedom to choose the scaling of M^4 metric freely corresponds to a genuine dynamical degree of freedom has been a continual source of worried thoughts now and then. If $\hbar$ has only single value this freedom does not seem to have meaning but if it has spectrum the situation changes.

The question is how to realize this freedom to choose the scale of the M^4 metric realized as replacements $m_{kl} \rightarrow m_{kl}/\lambda$, where allowed values of λ define the spectrum of Planck constant. The number theoretic vision suggests an answer to the question.

TGD leads to a generalization of the notion of number by gluing reals and various extensions of p-adic number fields to a larger structure. The gluing is carried out along common rationals. On the other hand, the rational physics approach suggests that the physics in various number fields satisfy the enormously powerful constraint that they are obtained by an algebraic continuation from rational physics. Hence rationals and the algebraic extensions of rationals should play key role in TGD.

The obvious idea encouraged also by the observation about the role of Beraha numbers is that different algebraic extensions of rationals (or p-adic numbers) correspond to different scalings of M^4 metric. This would mean that the identification along common rationals would be replaced with the identification $y = \lambda x$, where y is rational and x is arbitrary number in extension, and λ characterizing the scaling of M^4 metric belongs to extension. If lambda is not rational, the values of x are obtained by multiplying rationals by $1/\lambda$. Rationality would give a strong constraint on λ, which might be however too strong as the guesses for the form of λ suggest. This identification would generalize the original $y = x$ identification and guarantee that the distances of points y and x from the origin of M^4 would be same in respective M^4 metrics.

3. Other Dirac operators and dynamical scale of M^4 metric

The number theoretically realized spectrum of scales for M^4 metric fits nicely with the properties of the Dirac operator of imbedding space, with the properties of the modified Dirac operator defined for induced metric and spinor structure, and with the properties super-conformal Dirac operators of configuration space since both these operators involve natural separation of M^4 and CP_2 degrees of freedom. Induced metric and spinor structure depend non-linearly on λ. Super-conformal mass squared formula is replaced by a formula for wave vector squared involving λ as a scaling factor and the oscillator operator algebra used depends on the value of λ since it is expressed in terms fermionic oscillator operators associated with second quantized induced spinor fields.

4. Quantum classical correspondence and the values of λ

Quantum classical correspondence suggests that the values of λ should be represented also at space-time level. The variational principle making space-time sheets counterparts of Bohr orbits indeed implies the quantization of Kähler magnetic flux and the quantum need not be the standard flux quantum. The generalized quantization condition would be $\int BdS = n\lambda\hbar_0$ and in principle it is possible to deduce the values of λ from the classical theory. The flux integral does not involve the induced metric so that there is no explicit dependence on λ. There is however an implicit dependence via field equations which involve λ via the induced metric. This should be the case since the information about algebraic extension must be coded to the classical theory somehow.

13.2.2 The invariance of angular momentum under the scaling of $\hbar$

The assumption that four-momentum is invariant in the scaling of M^4 metric by λ combined with Poincare invariance implies that also angular momentum is invariant under the scaling by λ. Hence the analog of wave vector defined as $L_z/\hbar$ would scale down by a factor $1/\lambda$ giving $L_z/\hbar = m/\lambda$. The one-valuedness of the wave function however requires $L_z/\hbar = m$.

A possible resolution of the paradox is based on following argument.

a) Consider first a situation in which λ is integer. Effective 2-dimensionality means that 2-dimensional partonic surfaces are basic structures to be considered. By conformal invariance scalings and rotations are generated by L_0 and iL_0. Hence it would not be surprising if the radial scaling by λ^k would be accompanied by a similar angular scaling. If M^4 projection of the partonic 2-surface has dimension $D \geq 1$, this scaling would at space-time level mean that partonic 2-surface becomes analogous to λ^k-sheeted Riemann surface defining λ^k-fold covering of $E^2 \subset M^4$. Using M^4 coordinates for the space-time sheet as local coordinates, one finds that induced spinor fields have fractional angular momentum eigenvalues $L_z = m/\lambda^k$.

b) In this picture the approach to quantum criticality would correspond to the emergence of classical chaos at space-time level by a step-wise process in which the step $\hbar \rightarrow \lambda\hbar$ can be regarded as generalization of the period doubling bifurcation. As $\hbar$ increases by a factor λ^k, the space-time sheet representing an orbit of particle closing after one turn transforms to an orbit closing only after λ^k turns. Note that the volume of space-time sheet remains finite only if the orbit closes after finite number of turns. The step $k \rightarrow k+1$ would correspond to a local fractal operation making each sheet of the λ^k sheeted surface λ-sheeted so that λ^{k+1} sheeted surface would result. Instead of period doubling one would have period λ-folding with the value of λ depending on p-adic prime $p \simeq 2^k$.

c) The model of Nottale for planetary orbits requires besides the harmonics of $\lambda_0 \simeq 2^{11}$ also some of its sub-harmonics, at least third and fifth one. This conflicts with the assumption that λ is integer unless λ_0 is divisible by 3 and 5. This would be the case for $\lambda_0 = 2175 = 3 \times 5^2 \times 29$ consistent with the mean value $\lambda = 2174$ deduced with one per cent accuracy from the model for planetary orbits [39]. Only integer factors of $\lambda_0(p \simeq 2^k)$ would be allowed as sub-harmonics for given p for this option and integer valuedness of λ_0 would also pose strong conditions on the values of p if the proposed formula for λ_0 is accepted.

d) If one allows the quantization of angular momentum using integer multiple of $\hbar_0$ as a unit, λ need not be an integer. In this case one could have $\lambda = (r/s) \times \lambda_0$, where $\lambda_0 \simeq 2^{11}$ is integer. The orbit represented by the space-time sheet would close after $(r\lambda_0)^k$ turns and angular momentum would be quantized as $L_z = ms^k\hbar_0$. Even in this case the proposed formula for $\lambda(p)$ would give strong constraints on the values of p.

13.2.3 Kähler function codes for a perturbative expansion in powers of λ

Suppose that one accepts the number theoretical interpretation for the spectrum of $\hbar$ in terms of a hierarchy of overall scalings of M^4 metric. The first implication of this picture is that the modified Dirac operator determined by the induced metric and spinor structure depends on the λ in a highly nonlinear manner. This in turn implies that the fermionic oscillator algebra used to define configuration space spinor structure and metric depends on the value of λ. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite powers series in powers of $\hbar$ replaced now with λ.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over configuration space defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact. This certainly cannot be the case always: consider only the photon-photon scattering. This paradox can be also regarded as an objection against the proposal that generalized Feynman diagrams are equivalent with tree diagrams or more generally, that each diagram is equivalent with a minimal loopy diagram allowing homologically non-trivial imbedding with non-intersecting lines to a higher genus Riemann surface.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant configuration space metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of λ. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semiclassical" approximation, which might be actually exact, automatically would give a full expansion in powers of $\hbar$. This is in a sharp contrast to the usual quantization approach.

13.2.4 How the spectrum of $\hbar$ could be predicted by quantum TGD?

Number theoretical vision combined quantum criticality could allow to determine the allowed values of λ to a high degree. Quantum criticality means that different values for $\hbar$ corresponding to algebraic extensions of rationals correspond to same form of Kähler function, that is same value of Kähler coupling constant. Number theoretical vision in turn has led to the proposal that the exponent of Kähler function is expressible as a Dirac determinant for the modified Dirac operator. As a matter fact, the ratio of Dirac determinants for space-time regions separated by a light-like causal determinant acting as causal horizon would give the exponent for the difference of Kähler functions of the two regions [B4].

The condition that the value of g_K^2, defined as the analog of critical temperature, is same for all algebraic extensions would fix the value of λ as a function of algebraic extension apart from possible multi-valuedness.

This condition does not require the restriction of the modified Dirac operator to the set of rationals or their algebraic extensions. The only thing that is required is that the subset of allowed eigenvalues of the modified Dirac operator belongs to the the extension considered. In this manner one can calculate a Dirac determinant for each extension as a function of λ. Note that λ could belong to the algebraic extension considered. By requiring that g_K^2 corresponds to its value for rationals, one can fix the value of λ. Even better, it is quite possible that the value of Dirac determinant, rather than only the ratio of Dirac determinants, is finite since the number theoretic restriction might be satisfied only by a finite number of eigenvalues. Hence nature would also take care of regularization of Dirac determinants by using the number theoretic hierarchy.

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