Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?

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Abstract

It is not yet possible to deduce the length scale evolution of gauge coupling constants from Quantum TGD proper although one can understand precisely the origin of the p-adic coupling constant evolution in the partonic formulation of quantum TGD as a almost topological superconformal field theory. Quantum classical correspondence however encourages the hope that it might be possible to achieve some understanding of the coupling constant evolution by using the classical theory.

This turns out to be the case and the earlier speculative picture about gauge coupling constants associated with a given space-time sheet as RG invariants finds support. It remains an open question whether gravitational coupling constant is RG invariant inside give space-time sheet. The discrete p-adic coupling constant evolution replacing in TGD framework the ordinary RG evolution allows also formulation at space-time level.

The quantum phases $q = exp(i\pi/n)$ characterizing Jones inclusions define p-adic phase resolution in terms of the algebraic extension of p-adic numbers needed to represent q. The evolution of $\hbar$ is naturally associated with the p-adic phase resolution and allows a beautiful topological formulation at space-time level. This however requires a further generalization of the notion of imbedding space. This generalization is natural if one accepts that space-time and imbedding space emerge from an infinite-dimensional Clifford algebra extended to local algebra in a manner which is unique and from which TGD emerges.

The understanding of the quantization of Planck constants in terms of Jones inclusions has non-trivial implications concerning the understanding of the coupling constant evolution at quantitative level. One non-trivial implication is that one can in fact assume Kähler coupling strength to be RG invariant in p-adic coupling constant, an attractive idea that I had been forced to give up. The observations about general properties of induced classical color gauge fields combined with quantum classical correspondence inspire a hypothesis allowing to reduce the evolution of color coupling strength α_s to that of electro-weak U(1) coupling strength $\alpha_{U(1)}$, a result which is of considerable practical value.

The chapter consists of two parts. In the first sections quantitative predictions, which I dare to take rather seriously, are discussed. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered.

1 Introduction

It is not yet possible to deduce the length scale evolution gauge coupling constants from Quantum TGD proper. Quantum classical correspondence however encourages the hope that it might be possible to achieve some understanding of the coupling constant evolution by using the classical theory.

This turns out to be the case and the earlier speculative picture about gauge coupling constants associated with a given space-time sheet as RG invariants finds support. It remains an open question whether gravitational coupling constant is RG invariant inside give space-time sheet. The discrete p-adic coupling constant evolution replacing in TGD framework the ordinary RG evolution allows also formulation at space-time level.

The quantum phases $q = exp(i\pi/n)$ characterizing Jones inclusions define p-adic phase resolution in terms of the algebraic extension of p-adic numbers needed to represent q. The evolution of $\hbar$ is naturally associated with the p-adic phase resolution and allows a beautiful topological formulation at space-time level [A9]. This however requires a further generalization of the notion of imbedding space. This generalization is natural if one accepts that space-time and imbedding space emerge from an infinite-dimensional Clifford algebra extended to local algebra in a manner which is unique and from which TGD emerges [C6].

The understanding of the quantization of Planck constants in terms of Jones inclusions has non-trivial implications concerning the understanding of the coupling constant evolution at quantitative level. One non-trivial implication is that one can in fact assume Kähler coupling strength to be RG invariant in p-adic coupling constant, an attractive idea that I had been forced to give up. The observations about general properties of induced classical color gauge fields combined with quantum classical correspondence inspire a hypothesis allowing to reduce the evolution of color coupling strength α_s to that of electroweak U(1) coupling strength $\alpha_{U(1)}$, a result which is of considerable practical value.

The chapter decomposes into two parts. In the first sections quantitative predictions, which I dare to take rather seriously, are discussed. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered.

1.1 p-Adic evolution in phase resolution and the spectrum of values for Planck constants

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The basic idea is that the the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1.2 The reduction of the evolution of α_s to that for $\alpha_{U(1)}$

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

1.3 The evolution of gauge couplings at single space-time sheet

The renormalization group equations of gauge coupling constants g_i follow from the following idea. The basic observation is that gauge currents have vanishing covariant divergences whereas ordinary divergence does not vanish except in the Abelian case. The classical gauge currents are however proportional to $1/g_i^2$ and if g_i^2 is allowed to depend on the space-time point, the divergences of currents can be made vanishing and the resulting flow equations are essentially renormalization group equations. The physical motivation for the hypothesis is that gauge charges are assumed to be conserved in perturbative QFT. The space-time dependence of coupling constants takes care of the conservation of charges.

A surprisingly detailed view about RG evolution emerges.

a) The UV fixed points of RG evolution correspond to CP_2 type extremals (elementary particles).

b) The Abelianity of the induced Kähler field means that Kähler coupling strength is RG invariant which has indeed been the basic postulate of quantum TGD. The only possible interpretation is that the coupling constant evolution in sense of QFT:s corresponds to the discrete p-adic coupling constant evolution.

c) IR fixed points correspond to space-time sheets with a 2-dimensional CP_2 projection for which the induced gauge fields are Abelian so that covariant divergence reduces to ordinary divergence. Examples are cosmic strings (, which could be also seen as UV fixed points), vacuum extremals, solutions of a sub-theory defined by $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere, and "massless extremals".

d) At the light-like boundaries of the space-time sheet gauge couplings are predicted to be constant by conformal invariance and by effective two-dimensionality implying Abelianity: note that the 4-dimensionality of the space-time surface is absolutely essential here.

e) In fact, all known extremals of Kähler action correspond to RG fixed points since gauge currents are light-like so that coupling constants are constant at a given space-time sheet. This is consistent with the earlier hypothesis that gauge couplings are renormalization group invariants and coupling constant evolution reduces to a discrete p-adic evolution. As a consequence also Weinberg angle, being determined by a ratio of $SU(2)$ and $U(1)$ couplings, is predicted to be RG invariant. A natural condition fixing its value would be the requirement that the net vacuum em charge of the space-time sheet vanishes. This would state that em charge is not screened like weak charges.

f) When the flow determined by the gauge current is not integrable in the sense that flow lines are identifiable as coordinate curves, the situation changes. If gauge currents are divergenceless for all solutions of field equations, one can assume that gauge couplings are constant at a given space-time sheet and thus continuous also in this case. Otherwise a natural guess is that the coupling constants obtained by integrating the renormalization group equations are continuous in the relevant p-adic topology below the p-adic length scale. Thus the effective p-adic topology would emerge directly from the hydrodynamics defined by gauge currents.

1.4 RG evolution of gravitational constant at single space-time sheet

Similar considerations apply in the case of gravitational and cosmological constants.

a) In this case the conservation of gravitational mass determines the RG equation (gravitational energy and momentum are not conserved in general).

b) The assumption that coupling cosmological Λ constant is proportional to $1/L_p^2$ (L_p denotes the relevant p-adic length scale) explains the mysterious smallness of the cosmological constant and leads to a RG equation which is of the same form as in the case of gauge couplings.

c) Asymptotic cosmologies for which gravitational four momentum is conserved correspond to the fixed points of coupling constant evolution now but there are much more general solutions satisfying the constraint that gravitational mass is conserved.

d) It seems that gravitational constant cannot be RG invariant in the general case and that effective p-adicity can be avoided only by a smoothing out procedure replacing the mass current with its average over a four-volume 4-volume of size of order p-adic length scale.

1.5 p-Adic length scale evolution of gauge couplings

Understanding the dependence of gauge couplings constants on p-adic prime is one of the basic challenges of quantum TGD. The problem has been poorly understood even at the conceptual level to say nothing about concrete calculations. The generalization of the motion of S-matrix to that of M-matrix changed however the situation [C2]. M-matrix is always defined with respect to measurement resolution characterized in terms of an inclusion of von Neumann algebra. Coupling constant evolution reduces to a discrete evolution involving only octaves of $T(k) = 2^k T_0$ of the fundamental time scale $T_0 = R$, where R CP_2 scale. p-Adic length scale $L(k)$ is related to $T(k)$ by $L^2(k) = T(k)T_0$. p-Adic length scale hypothesis $p \simeq 2^k$, k integer, is automatic prediction of the theory. There is also a close connection with the description of coupling constant evolution in terms of radiative corrections.

If RG invariance at given space-time sheet holds true, the question arises whether it is possible to understand p-adic coupling constant evolution at space-time level and why certain p-adic primes are favored.

a) Simple considerations lead to the idea that M^4 scalings of the intersections of 3-surfaces defined by the intersections of space-time surfaces with light-cone boundary induce transformations of space-time surface identifiable as RG transformations. If sufficiently small they leave gauge charges invariant: this seems to be the case for known extremals which form scaling invariant families. When the scaling corresponds to a ratio p_2/p_1, $p_2 > p_1$, bifurcation would become possible replacing p_1-adic effective topology with p_2-adic one.

b) Stability considerations determine whether p_2-adic topology is actually realized and could explain why primes near powers of 2 are favored. The renormalization of coupling constant would be dictated by the requirement that Q_i/g_i^2 remains invariant.

1.6 The most recent view about coupling constant evolution

The last section of the chapter represents the most recent view about coupling constant evolution. Zero energy ontology, the construction of M-matrix as time like entanglement coefficients defining Connes tensor product characterizing finite measurement resolution in terms of inclusion of hyper-finite factors of type II_1, the realization that symplectic invariance of N-point functions provides a detailed mechanism eliminating UV divergences, and the understanding of the relationship between super-canoniocal and super Kac-Moody symmetries: these are the pieces of the puzzle whose combination makes possible a rather concrete vision about coupling constant evolution in TGD Universe and one can even speak about rudimentary form of generalized Feynman rules. It must be added that these pieces were lacking in the earlier attempts so that reader willing to get grasp of the most recent situation could start reading from the last section.

2 General view about coupling constant evolution in TGD framework

This focus of attention in this section is in some general ideas about p-adic coupling constant evolution. The most recent picture is not discussed in this section.

2.1 A revised view about the interpretation and evolution of Kähler coupling strength

The original hypothesis was that Kähler coupling strength is invariant under p-adic coupling constant evolution. Later I gave up this hypothesis and replaced it with the invariance of gravitational coupling since otherwise the prediction would have been that gravitational coupling strength is proportional to p-adic length scale squared. The recent view means return to the roots: Kähler coupling strength is invariant under p-adic coupling constant evolution and has value spectrum dictated by the Chern-Simons coupling k defining the theory at the parton level. Gravitational coupling constant corresponds in this framework to the largest Mersenne prime M_{127} which does not correspond to a completely super-astronomical p-adic length scale.

2.1.1 Formula for Kähler coupling constant

To construct expression for gravitational constant one can use the following ingredients.

a) The exponent $exp(2S_K(CP_2))$ defining the value of Kähler function in terms of the Kähler action $S_K(CP_2)$ of CP_2 type extremal representing elementary particle expressible as

$$S_K(CP_2) \quad = \quad \frac{\pi}{8\alpha_K} \ . \tag{1}$$

Since CP_2 type extremals suffer topological condensation, one expects that the action is modified:

$$S_K(CP_2) \quad \rightarrow \quad a \times S_K(CP_2) \ . \tag{2}$$

Naively one would expect reduction of the action so that one would have $a \leq 1$. One must however keep mind open in this respect.

b) The p-adic length scale L_p assignable to the space-time sheet along which gravitational interactions are mediated. Since Mersenne primes seem to characterized elementary bosons and since the Mersenne prime $M_{127} = 2^{127} - 1$ defining electron length scale is the largest non-super-astronomical length scale it is natural to guess that M_{127} characterizes these space-time sheets.

The formula for gravitational constant would read as

$$\begin{aligned} G &= L_p^2 \times exp(-2aS_K(CP_2)) \ . \\ L_p &= \sqrt{p}R \ . \end{aligned} \tag{3}$$

Here R is CP_2 radius defined by the length $2\pi R$ of the geodesic circle. The relationship boils down to

$$\begin{aligned} \alpha_K &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{G} \ . \end{aligned} \tag{4}$$

The value of K is fixed by the requirement that electron mass scale comes out correctly in the p-adic mass calculations and minimal value of K is factor. The uncertainties related to second order contributions however leave the precise value open.

The earlier calculations contained two errors. First error was related to the value of the parameter $K = R^2/G$ believed to be in good approximation given by the product of primes smaller than 26. Second error was that $1/\alpha_K$ was by a factor 2 too large for $a = 1$. This led first to a conclusion that α_K is very near to fine structure constant and perhaps equal to it. The physically more plausible option turned out to corresponds to $1/\alpha_K = 104$, which corresponds in good approximation to the value of electro-weak U(1) coupling at electron length scale but gave $a > 1$ whereas $a < 1$ would be natural since the action for a wormhole contact formed by a piece of CP_2 type vacuum extremal is expected to be smaller than the full action of CP_2 type vacuum extremal.

The correct calculation gives $a < 1$ for $\alpha_K = 1/104$. From the table one finds that if the parameter a equals to $a = 1/2$ the value of α_K is about 133. It would require $a = .6432$ for $Y_e = 0$ favored by the value of top quark mass. This value of a conforms with the idea that a piece of CP_2 type extremal defining a wormhole contact is in question. Note that a proper choices of value of a can make $K = R^2/G$ rational. The table gives values of various quantities assuming

$$K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 2^{-3} * (15/17) \ . \tag{5}$$

giving simplest approximation as a rational for K producing K_R for $Y_e = 0$ with error of 2.7 per cent which is still marginally consistent with the mass of top quark. This approximation should not be taken too seriously.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$1/\alpha_K$	133.7850	133.9064	133.9696
a_{104}	0.6432	0.6438	0.6441
a_α	0.4881	0.4886	0.4888
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{G}) \times 10^{-4}$	3.1167	3.1167	3.1167
$1/\alpha_K$	133.9158	133.9158	133.9158
a_{104}	0.6438	0.6438	0.6438
a_α	.4886	0.4886	0.4886
K_R/K	1.0267	1.1293	1.1868

Table 1. Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 2^{-3} * (15/17)$ is rational number producing R^2/G approximately is given*[1]. The values of α_K deduced using the formula above are given for $a = 1/2$ and the values of $a = a_{104}$ giving $\alpha_K = 1/104$ are given. Also the values of $a = a_\alpha$ for which α_K equals to the fine structure constant $1/\alpha_{em} = 137.0360$ are given.

If one assumes that α_K is of order fine structure constant in electron length scale, the value of the parameter a is slightly below 1/2 cannot be far from unity. Symmetry principles do not favor the identification. Later it will be found that rather general arguments predict integer spectrum for $1/\alpha_K$ given by $1/\alpha_K = 4k$. For this option $\alpha_K = 1/137$ is not allowed whereas the $1/\alpha_K = 104 = 4 \times 26$ is.

2.1.2 Can one predict the value of gravitational constant?

A lot remains to be understood. The value of gravitational constant is one important example in this respect. For a given space-time sheet defined as a preferred extremal of Kähler action one can in principle calculate the value of G_{class}. Physical gravitational constant G is however expected to quantum average of G_{class} for a given quantum state.

For years ago I found a nice formula relating G to CP_2 length scale, the p-adic prime p characterizing gravitons and equal to M_{127} in the case of ordinary graviton, and Kähler coupling strength. Quantum formula is in question since the exponent for the Kähler action for CP_2 type vacuum extremals appears in it. The task would be to calculate explicitly the G_{class} and its quantum expectation value.

[1] The earlier calculations giving $K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23$ in a good approximation contained an error

What seems clear is that G is state dependent. For instance, for quantum states concentrated around almost vacuum extremals (such as hadronic strings) G should be large since they are almost Kähler vacua and the model for hadrons indeed leads to the identification of strong gravitons with G_{strong} characterized by corresponding p-adic length scale.

One can write the basic formula for gravitational constant as

$$\frac{exp(-2S_K(CP_2)}{G(p)} = \frac{1}{pR^2} \ .$$

$S_K(CP_2)$ is Kähler action for CP_2 type vacuum extremals with small renormalization reflecting the fact that entire free CP_2 type extremal is not in question topological condensation. The two sides of this equation suggest an interpretation in terms of two thermodynamics. Vacuum functional defined by Kähler function defines the thermodynamics of the left hand side and Planck mass $M_{Pl}(p) = 1/\sqrt{G(p)}$ defining the fundamental mass equal to Planck mass for $p = M_{127}$ but depending on p as $1/\sqrt{p}$. Right hand side would correspond to p-adic thermodynamics with CP_2 mass defining the fundamental mass.

2.1.3 Formula relating v_0 to α_K and R^2/G

If v_0 is identified as the rotation velocity of distant stars in galactic plane, one can use the Newtonian model for the motion of mass in the gravitational field of long straight string giving $v_0 = \sqrt{TG}$. String tension T can be expressed in terms of Kähler coupling strength as

$$T = \frac{b}{2\alpha_K R^2} \ ,$$

where R is the radius of geodesic circle. The factor $b \leq 1$ would explain reduction of string tension in topological condensation caused by the fact that not entire geodesic sphere contributes to the action.

This gives

$$\begin{aligned} v_0 &= \frac{b}{2\sqrt{\alpha_K K}} \ , \\ \alpha_K(p) &= \frac{a\pi}{4log(pK)} \ , \\ K &= \frac{R^2}{G} \ . \end{aligned} \quad (6)$$

The condition that α_K has the desired value for $p = M_{127} = 2^{127} - 1$ defining the p-adic length scale of electron fixes the value of b for given value of a. The value of b should be smaller than 1 corresponding to the reduction of string tension in topological condensation.

The condition 6 for $v_0 = 2^{-m}$, say $m = 11$, allows to deduce the value of a/b as

$$\frac{a}{b} = \frac{4 * log(pK)}{\pi} \frac{2^{2m-1}}{K} \quad . \tag{7}$$

The table gives the values of b calculated assuming $a = a_{104}$ producing $\alpha_K = 1/104$ for various values of Y_e.

Y_e	0	.5	.7798
$b_{9,104}$	0.9266	1.0193	1.0711
$b_{11,104}$	0.0579	0.0637	0.0669
$b_{9,\alpha}$	0.7032	0.7736	0.81291
$b_{11,\alpha}$	0.0440	0.0483	0.050

Table 2. Table gives the values of b for $Y_e \in \{0, .5, .7798\}$ assuming the values $a = a_{104}$ guaranteing $\alpha_K = 1/104$ and $\alpha_K = \alpha_{em}$. $b_{9,...}$ corresponds to $m = 9$ and $b_{11,...}$ corresponds to $m = 11$.

From the table one finds that for $\alpha_K = 1/104$ $m = 9$ corresponds to the almost full action for topological condensed cosmic string. $m = 11$ corresponds to much smaller action smaller by a factor of order 1/16. The interpretation would be that as m increases the action of the topologically condensed cosmic string decreases. This would correspond to a gradual transformation of the cosmic string to a magnetic flux tube.

2.1.4 Does α_K correspond α_{em} or weak coupling strength $\alpha_{U(1)}$ at electron length scale?

The preceding arguments allow the original identification $\alpha_K \simeq 1/137$. On the other hand, group theoretical arguments encourage the identification of α_K as weak $U(1)$ coupling strength $\alpha_{U(1)}$:

$$\begin{aligned} \alpha_K &= \alpha_{U(1)} = \frac{\alpha_{em}}{cos^2(\theta_W)} \simeq \frac{1}{104.1867} \quad , \\ sin^2(\theta_W)_{|10\ MeV} &\simeq 0.2397(13) \quad , \\ \alpha_{em}(M_{127}) &= 0.00729735253327 \quad . \end{aligned} \tag{8}$$

Here Weinberg angle corresponds to 10 MeV energy is reasonably near to the value at electron mass scale. The value $sin^2(\theta_W) = 0.2397(13)$ corresponding to 10 MeV mass scale [33] is used.

Later it will be found that general argument predicts that $1/\alpha_K$ is integer valued: $1/\alpha_K = 4k$. This option excludes identification as $\alpha_{em}(127)$ but encourages strongly the identification as $\alpha_{U(1)}(127)$.

2.1.5 Is gravitational constant an approximate RG invariant?

The original model for the p-adic evolution of α_K was based on the p-adic renormalization group invariance of gravitational constant. The starting point was the observation that on purely dimension analytic basis one can write $G = exp(-2S_K(CP_2))L_p^2$. If α_K is p-adic RG invariant, G scales like L_p^2 which looked completely non-sensible at that time so that the identification $\alpha_K = \alpha_{U(1)}$ was given up. Discrete p-adic evolution for α_K is consistent with RG invariance and quantum criticality at a given p-adic space-time sheet.

This view however leads to problems with the identification $\alpha_K = \alpha_{U(1)}$ since the evolution of α_K dictated by RG invariance of G is much faster than that of $\alpha_{U(1)}$. The condition

$$cos^2(\theta_W)(89) = \frac{log(M_{127}K)}{log(M_{89}K)} \times \frac{\alpha_{em}(M_{127})}{\alpha_{em}(M_{89})} \times cos^2(\theta_W)(127) \ . \quad (9)$$

together with the experimental value $1/\alpha_{em}(M_{89}) \simeq 128$ as predicted by standard model, gives $sin^2(\theta_W)(89) = .0474$ to be compared with the measured valued .23120(15) at intermediate boson mass scale [33]!

Furthermore, if α_K evolves with p then v_0 is predicted to evolve too but $v_0 = 2^{-11}$ is consistent with experimental facts (apart from possible presence of sub-harmonics which can be however understood in TGD framework).

2.1.6 Or is α_K RG invariant?

One is forced to ask whether one must give up the existing scenario for the p-adic evolution of α_K and identify it with the evolution of $\alpha_{U(1)}$ or perhaps even p-adic RG invariance of α_K. The predicted very fast evolution $G \propto L_p^2$ in the approximation that α_K is RG invariant makes sense only if L_p characterizes the space-time sheets carrying gravitational interaction or even to gravitons and if these space-time sheet corresponds to $p = M_{127}$ under normal conditions.

If bosons correspond to Mersenne primes, this would be naturally the case since the Mersenne prime next to M_{127} corresponds to a completely super-astrophysical length scale. In this case p-adic length scale hypothesis would predict $v_0^{-2}(L(k)) = 2^{-22}2^{-k+127}$ if α_K is RG invariant so that it would behave as a power of 2. $\hbar_{gr}$ would scale as 2^{-k+127} and approach rapidly to zero as $L(k)$ increases whereas gravitational force would become strong.

If same p_0 characterizes all ordinary gauge bosons with their dark variants included, one would have $p_0 = M_{89} = 2^{89} - 1$. In this case however the gravitational coupling strength would be weaker by a factor 2^{-38}. M_{127} also defines a dark length scale in TGD inspired quantum model of living matter [F9, J6].

A further nice feature of this identification is that one can also allow the scaling of CP_2 metric and thus R^2 by $\lambda^2 = (\hbar/\hbar_0)^2$ inducing $K \rightarrow \lambda^2 K$. $1/v_0 \rightarrow \lambda/v_0$ implies that $\hbar_{gr}$ scales in the same manner as $\hbar$. Hence it would seem that $\hbar$ corresponds to M^4- and $\hbar_{gr}$ to CP_2 degrees of freedom and the huge value of $\hbar_{gr}$ would mean that there is that cosmology has quantal lattice like structure in cosmological length scales with H_a/G, $G \subset SL(2,C)$, serving as a basic lattice cell (here H_a denotes $a = constant$ hyperboloid of M^4_+). The observed sub-harmonics of v_0 could thus be understood in terms of scalings of CP_2 gravitational constant. This structure is supported also by the quantization of cosmological red shifts [32].

The huge value of h_{gr} assignable to color algebra does not mean that colored states would have huge values of color charges since fractionization of color quantum numbers occurs. It however means that dark color charges are de-localized in huge length scales and cosmological color could be seen as responsible for a macroscopic quantum coherence in astrophysical length scales.

2.1.7 What about color coupling strength?

Classical theory should be also able to say something non-trivial about color coupling strength α_s too at the general level. The basic observations are following.

a) Both classical color YM action and electro-weak U(1) action reduce to Kähler action.

b) Classical color holonomy is Abelian which is consistent also with the fact that the only signature of color that induced spinor fields carry is anomalous color hyper charge identifiable as an electro-weak hyper charge.

Suppose that α_K is a strict RG invariant. The first idea would be that the sum of classical color action and electro-weak U(1) action is RG invariant and thus equals to its asymptotic value obtained for $\alpha_{U(1)} = \alpha_s = 2\alpha_K$. Asymptotically the couplings approach to a fixed point defined by $2\alpha_K$ rather than to zero as in asymptotically free gauge theories.

Thus one would have

$$\frac{1}{\alpha_{U(1)}} + \frac{1}{\alpha_s} = \frac{1}{\alpha_K} \quad . \tag{10}$$

The formula implies that the beta functions for color and $U(1)$ degrees of freedom are apart from sign identical and the increase of $U(1)$ coupling compensates the decrease of the color coupling. This gives the formula

$$\alpha_s = \frac{1}{\frac{1}{\alpha_K} - \frac{1}{\alpha_{U(1)}}} \quad . \tag{11}$$

At least formally $\alpha_s(QCD)$ could become negative below the confinement length scale so that $\alpha_K < \alpha_{U(1)}$ for M_{127} is consistent with this formula. For M_{89} $\alpha_{em} \simeq 1/127$ gives $1/\alpha_{U(1)}(89) = 1/97.6374$.

a) $\alpha_K = \alpha_{em}(127)$ option does not work. Confinement length scale corresponds to the point at which one has $\alpha_{U(1)} = \alpha_K$ and in principle can be predicted precisely using standard model. In the case that $\alpha_s(107)$ diverges, one has

$$\alpha_{em}(107) = cos^2(\theta_W)\alpha_{U(1)} = cos^2(\theta_W)\alpha_K = \frac{cos^2(\theta_W)}{136} \ .$$

The resulting value of α_{em} is too small and the situation worsens for $k > 107$ since $\alpha_{U(1)}$ decreases. Hence this option is excluded.

b) TGD predicts that also M_{127} copy of QCD should exist and that M_{127} quarks should play a key role in nuclear physics [F8]. Hence one could argue that color coupling strength diverges at M_{127} (the largest not completely super-astrophysical Mersenne prime) so that one would have $\alpha_K = \alpha_{U(1)}(M_{127})$. Therefore the precise knowledge of $\alpha_{U(1)}(M_{127})$ in principle fixes the value of parameter $K = R^2/G$ and thus also the second order contribution to the mass of electron. On ther other hand, quite a general argument to be discussed later predicts $\alpha_K = 1/104$ so that an exact prediction for $U(1)$ coupling follows.

The predicted value of $\alpha_s(M_{89})$ follows from $sin^2(\theta_W) = .23120$ and $\alpha_{em} \simeq 1/127$ at intermediate boson mass scale using $\alpha_{U(1)} = \alpha_{em}/cos^2(\theta_W)$ and $1/\alpha_s = 1/\alpha_K - 1/\alpha_{U(1)}$. The predicted value $\alpha_s(89) = 0.1572$ is quite reasonable although somewhat larger than QCD value. For $1/\alpha_K = 108 = 4 \times 27$ one would have $\alpha_s(89) = 0.0965$.

The new vision about the value spectrum of Kähler coupling strength and hadronic space-time sheet suggests $\alpha_K = \alpha_s = \alpha_s = 1/4$ at hadronic space-time sheet labelled by M_{107}. α_s here refers however to super-canonical gluons which do not consist of quark-antiquark pairs. If the two values of α_s are identical at $k = 107$ (ordinary gluons might be perhaps mix strongly with super-canonical ones at this length scale), one has $\alpha_{U(1)}(107) = 1/100$. Using $sint^2(\theta_W) = 2397$ at 10 MeV this predicts $\alpha_{em}(107) = 1/131.53$.

To sum up, the proposed formula would dictate the evolution of α_s from the evolution of the electro-weak parameters without any need for perturbative computations and number theoretical prediction for U(1) coupling at electron length scale would be exact. Although the formula of proposed kind is encouraged by the strong constraints between classical gauge fields in TGD framework, it should be deduced in a rigorous manner from the basic assumptions of TGD before it can be taken seriously.

2.2 Does the quantization of Kähler coupling strength reduce to the quantization of Chern-Simons coupling at partonic level?

Kähler coupling strength associated with Kähler action (Maxwell action for the induced Kähler form) is the only coupling constant parameter in quantum TGD, and its value (or values) is in principle fixed by the condition of quantum criticality since Kähler coupling strength is completely analogous to critical temperature. The quantum TGD at parton level reduces to almost topological QFT for light-like 3-surfaces. This almost TQFT involves Abelian Chern-Simons action for the induced Kähler form.

This raises the question whether the integer valued quantization of the Chern-Simons coupling k could predict the values of the Kähler coupling strength.

I considered this kind of possibility already for more than 15 years ago but only the reading of the introduction of the [30] about his new approach to 3-D quantum gravity led to the discovery of a childishly simple argument that the inverse of Kähler coupling strength could indeed be proportional to the integer valued Chern-Simons coupling k: $1/\alpha_K = 4k$. $k = 26$ is forced by the comparison with some physical input. Also p-adic temperature could be identified as $T_p = 1/k$.

2.2.1 Quantization of Chern-Simons coupling strength

For Chern-Simons action the quantization of the coupling constant guaranteing so called holomorphic factorization is implied by the integer valuedness of the Chern-Simons coupling strength k. As Witten explains, this follows from the quantization of the first Chern-Simons class for closed 4-manifolds plus the requirement that the phase defined by Chern-Simons action equals to 1 for a boundaryless 4-manifold obtained by gluing together two 4-manifolds along their boundaries. As explained by Witten in his paper, one can consider also "anyonic" situation in which k has spectrum Z/n^2 for n-fold covering of the gauge group and in dark matter sector one can consider this kind of quantization.

2.2.2 Formula for the Kähler coupling strength

The quantization argument for k seems to generalize to the case of TGD. What is clear that this quantization should closely relate to the quantization of the Kähler coupling strength appearing in the 4-D Kähler action defining Kähler function for the world of classical worlds and conjectured to result as a Dirac determinant. The conjecture has been that g_K^2 has only single value. With some physical input one can make educated guesses about this value. The connection with the quantization of Chern-Simons coupling would however suggest a spectrum of values. This spectrum is easy to guess.

1. *Wick rotation argument*

The U(1) counterpart of Chern-Simons action is obtained as the analog of the "instanton" density obtained from Maxwell action by replacing $J \wedge *J$ with $J \wedge J$. This looks natural since for self dual J associated with CP_2 type vacuum extremals Maxwell action reduces to instanton density and therefore to Chern-Simons term. Also the interpretation as Chern-Simons action associated with the classical SU(3) color gauge field defined by Killing vector fields of CP_2 and having Abelian holonomy is possible. Note however that *instanton density is multiplied by imaginary unit in the action exponential of path integral.* One should find justification for this "Wick rotation" not changing the value of coupling strength and later this kind of justification will be proposed.

Wick rotation argument suggests the correspondence $k/4\pi = 1/4g_K^2$ between Chern-Simons coupling strength and the Kähler coupling strength g_K appearing in 4-D Kähler action. This would give

$$g_K^2 = \frac{\pi}{k} \ , \frac{1}{\alpha_K} = 4k \ . \qquad (12)$$

The spectrum of $1/\alpha_K$would be integer valued. The result is very nice from the point of number theoretic vision since the powers of α_K appearing in perturbative expansions would be rational numbers (ironically, radiative corrections should vanish by number theoretic universality but this might happen only for these rational values ofα_K!).

2. Are more general values of k possible

Note however that if k is allowed to have values in Z/n^2, the strongest possible coupling strength is scaled to $n^2/4$ unless $\hbar$ is not scaled: already for $n = 2$ the resulting perturbative expansion might fail to converge. In the scalings of $\hbar$ associated with M^4 degrees of freedom $\hbar$ however scales as $1/n^2$ so that the spectrum of α_K would remain invariant.

3. Experimental constraints on α_K

It is interesting to compare the prediction with the experimental constraints on the value of $1/\alpha_K$. As already found, there are two options to consider.

a) $\alpha_K = \alpha_{em}$ option suggests $1/\alpha_K = 137$ inconsistent with $1/\alpha_K = 4k$ condition. $1/\alpha_K = 136 = 4 \times 34$ combined with the formula $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K$ leads to nonsensical predictions.

b) For $1/\alpha_s + 1/\alpha_{U(1)} = 1/\alpha_K = 104$ option option the basic empirical input is that electro-weak $U(1)$ coupling strength reduces to Kähler coupling at electron length scale. This gives $\alpha_K = \alpha_{U(1)}(M_{127}) \simeq 104.1867$, which corresponds to $k = 26.0467$. $k = 26$ would give $\alpha_K = 104$: the difference would be only .2 per cent and one would obtain exact prediction for $\alpha_{U(1)}(M_{127})$. Together with electro-weak coupling constant evolution this would also explain why the inverse of the fine structure constant is so near to 137 but not quite. Amusingly, $k = 26$ is the critical space-time dimension of the bosonic string model. Also the conjectured formula for the gravitational constant in terms of α_K and p-adic prime p involves all primes smaller than 26.

2.2.3 Justification for Wick rotation

It is not too difficult to believe to the formula $1/\alpha_K = qk$, q some rational. $q = 4$ however requires a justification for the Wick rotation bringing the imaginary unit to Chern-Simons action exponential lacking from Kähler function exponential.

In this kind of situation one might hope that an additional symmetry might come in rescue. The guess is that number theoretic vision could justify this symmetry.

a) To see what this symmetry might be consider the generalization of the [31] obtained by combining theta angle and gauge coupling to single complex number via the formula

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} \ . \quad (13)$$

What this means in the recent case that for CP_2 type vacuum extremals [D1] Kähler action and instanton term reduce by self duality to Kähler action obtained by the replacement g^2 with $-i\tau/4\pi$. The first duality $\tau \rightarrow \tau + 1$ corresponds to the periodicity of the theta angle. Second duality $\tau \rightarrow -1/\tau$ corresponds to the generalization of Montonen-Olive duality $\alpha \rightarrow 1/\alpha$. These dualities are definitely not symmetries of the theory in the recent case.

b) Despite the failure of dualities, it is interesting to write the formula for τ in the case of Chern-Simons theory assuming $g_K^2 = \pi/k$ with $k > 0$ holding true for Kac-Moody representations. What one obtains is

$$\tau = 4k(1-i) \ . \quad (14)$$

The allowed values of τ are integer spaced along a line whose direction angle corresponds to the phase $exp(i2\pi/n)$, $n = 4$. The transformations $\tau \rightarrow \tau + 4(1-i)$ generate a dynamical symmetry and as Lorentz transformations define a subgroup of the group E^2 leaving invariant light-like momentum (this brings in mind quantum criticality!). One should understand why this line is so special.

c) This formula conforms with the number theoretic vision suggesting that the allowed values of τ belong to an integer spaced lattice. Indeed, if one requires that the phase angles are proportional to vectors with rational components then only phase angles associated with orthogonal triangles with short sides having integer valued lengths m and n are possible. The additional condition that the phase angles correspond to *roots of unity*! This leaves only $m = n$ and $m = -n > 0$ into consideration so that one would have $\tau = n(1-i)$ from $k > 0$.

d) Notice that theta angle is a multiple of $8k\pi$ so that a trivial strong CP breaking results and no QCD axion is needed (this of one takes seriously the equivalence of Kähler action to the classical color YM action).

2.2.4 Is p-adicization needed and possible only in 3-D sense?

The action of CP_2 type extremal is given as $S = \pi/8\alpha_K = k\pi/2$. Therefore the exponent of Kähler action appearing in the vacuum functional would be $exp(k\pi)$ - Gelfond's constant - known to be a transcendental number [17]. Also its powers are transcendental. If one wants to p-adicize also in 4-D sense, this raises a problem.

Before considering this problem, consider first the 4-D p-adicization more generally.

a) The definition of Kähler action and Kähler function in p-adic case can be obtained only by algebraic continuation from the real case since no satisfactory definition of p-adic definite integral exists. These difficulties are even more serious at the level of configuration space unless algebraic continuation allows to reduce everything to real context. If TGD is integrable theory in the sense that functional integral over 3-surfaces reduces to calculable functional integrals around the maxima of Kähler function, one might dream of achieving the algebraic continuation of real formulas. Note however that for light-like 3-surface the restriction to a category of algebraic surfaces essential for the re-interpretation of real equations of 3-surface as p-adic equations. It is far from clear whether also preferred extremals of Kähler action have this property.

b) Is 4-D p-adicization the really needed? The extension of light-like partonic 3-surfaces to 4-D space-time surfaces brings in classical dynamical variables necessary for quantum measurement theory. p-Adic physics defines correlates for cognition and intentionality. One can argue that these are not quantum measured in the conventional sense so that 4-D p-adic space-time sheets would not be needed at all. The p-adic variant for the exponent of Chern-Simons action can make sense using a finite-D algebraic extension defined by $q = exp(i2\pi/n)$ and restricting the allowed light-like partonic 3-surfaces so that the exponent of Chern-Simons form belongs to this extension of p-adic numbers. This restriction is very natural from the point of view of dark matter hierarchy involving extensions of p-adics by quantum phase q.

If one remains optimistic and wants to p-adicize also in 4-D sense, the transcendental value of the vacuum functional for CP_2 type vacuum extremals poses a problem (not the only one since the p-adic norm of the exponent of Kähler action can become completely unpredictable).

a) One can also consider extending p-adic numbers by introducing $exp(\pi)$ and its powers and possibly also π. This would make the extension of p-adics infinite-dimensional which does not conform with the basic ideas about cognition. Note that e^p is not p-adic transcendental so that extension of p-adics by powers e is finite-dimensional and if p-adics are first extended by powers of π then further extension by $exp(\pi)$ is p-dimensional.

b) A more tricky manner to overcome the problem posed by the CP_2 extremals is to notice CP_2 type extremals are necessarily deformed and contain a hole corresponding to the light-like 3-surface or several of them. This would reduce the value of Kähler action and one could argue that the allowed p-adic deformations are such that the exponent of Kähler action is a p-adic number in a finite extension of p-adics. This option does not look promising.

2.2.5 Is the p-adic temperature proportional to the Kähler coupling strength?

Kähler coupling strength would have the same spectrum as p-adic temperature T_p apart from a multiplicative factor. The identification $T_p = 1/k$ is indeed very natural since also g_K^2 is a temperature like parameter. The simplest guess is

$$T_p = \frac{1}{k} \ . \tag{15}$$

Also gauge couplings strengths are expected to be proportional to g_K^2 and thus to $1/k$ apart from a factor characterizing p-adic coupling constant evolution. That all basic parameters of theory would have simple expressions in terms of k would be very nice from the point of view quantum classical correspondence.

If U(1) coupling constant strength at electron length scales equals $\alpha_K = 1/104$, this would give $1/T_p = 1/26$. This means that photon, graviton, and gluons would be massless in an excellent approximation for say $p = M_{89} = 2^{89} - 1$, which characterizes electro-weak gauge bosons receiving their masses from their coupling to Higgs boson. For fermions one has $T_p = 1$ so that fermionic light-like wormhole throats would correspond to the strongest possible coupling strength $\alpha_K = 1/4$ whereas gauge bosons identified as pairs of light-like wormhole throats associated with wormhole contacts would correspond to $\alpha_K = 1/104$. Perhaps $T_p = 1/26$ is the highest p-adic temperature at which gauge boson wormhole contacts are stable against splitting to fermion-antifermion pair. Fermions and possible exotic bosons created by bosonic generators of super-canonical algebra would correspond to single wormhole throat and could also naturally correspond to the maximal value of p-adic temperature since there is nothing to which they can decay.

A fascinating problem is whether $k = 26$ defines internally consistent conformal field theory and is there something very special in it. Also the thermal stability argument for gauge bosons should be checked.

What could go wrong with this picture? The different value for the fermionic and bosonic α_K makes sense only if the 4-D space-time sheets associated with fermions and bosons can be regarded as disjoint space-time regions. Gauge bosons correspond to wormhole contacts connecting (deformed pieces of CP_2 type extremal) positive and negative energy space-time sheets whereas fermions would correspond to deformed CP_2 type extremal glued to single space-time sheet having either positive or negative energy. These space-time sheets should make contact only in interaction vertices of the generalized Feynman diagrams, where partonic 3-surfaces are glued together along their ends. If this gluing together occurs only in these vertices, fermionic and bosonic space-time sheets are disjoint. For stringy diagrams this picture would fail.

To sum up, the resulting overall vision seems to be internally consistent and is consistent with generalized Feynman graphics, predicts exactly the spectrum of α_K, allows to identify the inverse of p-adic temperature with k, allows to understand the differences between fermionic and bosonic massivation, and reduces Wick rotation to a number theoretic symmetry. One might hope that the additional objections (to be found sooner or later!) could allow to develop a more detailed picture.

2.3 Why gravitation is so weak as compared to gauge interactions?

The weakness of gravitational interaction in contrast to other gauge interactions is definitely a fundamental test for the proposed picture. The heuristic argument allowing to understand the value of gravitational constant is based on the assumption that graviton exchange corresponds to the exchange of CP_2 type extremal for which vacuum functional implies huge reduction of the gravitational constant from the value $\sim L_p^2$ implied by dimensional considerations based on p-adic length scale hypothesis to a value $G = exp(-2S_K)L_p^2$ which for $p = M_{127}$ gives gravitational constant for $\alpha_K = \pi a/log(M_{127} \times K)$, where a is near unity and $K = 2 \times 3 \times 5.... \times 23$ is a choice motivated by number theoretical arguments. The value of K is fixed rather precisely from electron mass scale and the proposed scenario for coupling constant evolution fixes both α_K and K completely in terms of electron mass (using p-adic mass calculations) and electro-weak $U(1)$ coupling at electron length scale $L_{M_{127}}$ by the formula $\alpha_K = \alpha_{U(1)}$ [A9]. The interpretation would be that gravitational masses are measured using p-adic mass scale $M_p = \pi/L_p$ as a natural unit.

2.3.1 Why gravitational interaction is weak

The first problem is that CP_2 type extremal cannot represent the lowest order contribution to the interaction since otherwise the normalization of the configuration space vacuum functional would give $exp[-2S_K(CP_2)]$ factor cancelling the exponential in the propagator so that one would have $G = L_p^2$. The following observations allow to understand the solution of the problem.

a) As already found, the key feature of CP_2 type extremals distinguishing them from other 3-surfaces is their non-deterministic behavior allowing them to carry off mass shell four-momenta. Other 3-surfaces can give rise only to scattering involving exchange of on mass shell particles and for space-like momentum exchanges there is no contribution.

b) All possible light-like 3-surfaces must be allowed as propagator portions of surfaces X^3_V but in absence of non-determinism they can give rise to massless exchanges which are typically non-allowed.

c) The contributions of CP_2 type extremals are suppressed by $exp[-2NS_K(CP_2)]$ factor in presence of N CP_2 type extremals with maximal action. CP_2 type extremals are vacuum extremals and interact with surrounding world only via the topological condensation generating 3-D CP_2 projection near the throat of the wormhole contact. This motivates the assumption that the sector of the configuration space containing N CP_2 type extremals has the approximate structure $CH(N) = CH(0) \times CP^N$, where $CH(0)$ corresponds to the situation without CP_2 type extremals and CP to the degrees of freedom associated with single CP_2 type extremal. With this assumption the functional integral gives a result of form $X \times exp(-2NS_K(CP_2)$ for N CP_2 type extremals. This factorization allows to forget all the complexities of the world of classical worlds which on the first sight seem to destroy all hopes about calculating something and the normalization factor is in lowest order equal to $X(0)$ whereas single CP_2 type extremal gives $exp[-2S_K(CP_2)]$ factor. This argument generalizes also to the case when CP_2 type extremals are allowed to have varying value of action (the distance travelled by the virtual particle can vary).

Massless extremals (MEs) define a natural candidate for the lowest order contribution since for them Kähler action vanishes. MEs describes a dispersion free on-mass shell propagation of massless modes of both induced gauge fields and metric. Hence they can describe only on mass shell massless exchanges of bosons and gravitons which typically vanishes for kinematical reasons except for collinear scattering in the case of massless particles so that CP_2 type extremals would give the leading contribution to the S-matrix element.

There are however exceptional situations in which exchange of ordinary CP_2 type extremals makes kinematically possible the emission of MEs as brehmstrahlung in turn giving rise to exchange of light-like momentum. Since MEs carry also classical gravitational fields, one can wonder whether this kind of exchanges could make possible strong on mass shell gravitation made kinematically possible by ordinary gauge boson exchanges inside interacting systems.

2.3.2 What differentiates between gravitons and gauge bosons?

The simplest explanation for the difference between gauge bosons and gravitons is that for virtual gauge bosons the volume of CP_2 type extremals is reduced dramatically from its maximal value so that $exp(-2S_K)$ brings in only a small reduction factor. The reason would be that for virtual gauge bosons the length of a typical CP_2 type extremal is far from the value giving rise to the saturation of the Kähler action. For gravitational interactions in astrophysical length scales CP_2 type extremals must indeed be very long.

Gravitational interaction should become strong sufficiently below the saturation length scale with gravitational constant approaching its stringy value L_p^2. According to the argument discussed in [A9], this length scale corresponds to the Mersenne prime M_{127} characterizing gravitonic space-time sheets so that gravitation should become strong below electron's Compton length. This suggests a connection with stringy description of graviton. M_{127} quarks connected by the corresponding strings are indeed a basic element of TGD based model of nuclei [F8]. TGD suggests also the existence of lepto-hadrons as bound state of color excited leptons in length scale M_{127} [F7]. Also gravitons corresponding to smaller Mersenne primes are possible but corresponding forces are much weaker than ordinary gravitation. On the other hand, M_{127} is the largest Mersenne prime which does not give rise to super-astronomical p-adic length scale so that stronger gravitational forces are not be predicted in experimentally accessible length scales.

More generally, the saturation length scale should relate very closely to the p-adic length scale L_p characterizing the particle. The amount of zitterbewegung determines the amount dS_K/dl of Kähler action per unit length along the orbit of virtual particle. L_p would naturally define the length scale below which the particle moves in a good approximation along M^4 geodesic. The shorter this length scale is, the larger the value of dS_K/dl is.

If the Kähler action of CP_2 type extremal increases linearly with the distance (in a statistical sense at least), an exponential Yukawa screening results at distances much shorter than saturation length. Therefore CP_2 extremals would provide a fundamental description of particle massivation at space-time level. p-Adic thermodynamics would characterize what happens for a topologically condensed CP_2 type extremal carrying given quantum numbers at the resulting light-like CD. Besides p-adic length scale also the quantized value $T_p = 1/n$ of the p-adic temperature would be decisive. For weak bosons Mersenne prime M_{89} would define the saturation length scale. For photons the p-adic length scale defining the Yukawa screening should be rather long. An n-ary p-adic length scale $L_{M_{89}}(n) = p^{(n-1)/2} L_{M_{89}}$ would most naturally be in question so that the p-adic temperature associated with photon would be $T_p = 1/n$, $n > 1$ [F3]. In the case of gluons confinement length scale should be much shorter than the scale at which the Yukawa screening becomes visible. If also gluons correspond to $n > 1$ this is certainly the case.

All gauge interactions would give rise to ultra-weak long ranged interactions, which are extremely weak compared to the gravitational interaction: the ratio for the strengths of these interactions would be of order $\alpha Q_1 Q_2 m_e^2 / M_1 M_2$ and very small for particles whose masses are above electron mass. Note however that MEs give rise to arbitrarily unscreened long ranged weak and color interactions restricted to light-like momentum transfers and these interactions play a key role in the TGD based model of living matter [J6, M3]. This prediction is in principle testable.

3 Some number theoretical ideas related to p-adicization

This section is a digression from basic theme being about some number theoretical ideas related to p-adicization. The justification for its inclusion is that p-adicization poses strong constraints on coupling constants and actually led to the parton level formulation of the p-adic coupling constant evolution.

3.1 Fusion of p-adic and real physics to single coherent whole by algebraic continuation

The development of the TGD inspired theory of consciousness theory and the vision about physics as a generalized number theory led to a general philosophy which provides powerful conceptual tools in attempts to answer the questions stated above.

3.1.1 Physics as a generalized number theory

The basic ideas behind physics as a generalized number theory approach deserve a brief summary.

1. The interpretation of p-adicity

Various p-adic versions of quantum TGD are interpreted as kind of cognitive representations of the real theory. p-Adic space-time sheets appear also at space-time level. Also real space-time sheets decompose into regions obeying effective p-adic topologies with the value of p characterizing the non-determinism of

Kähler action in a particular region. This explains why p-adic thermodynamics predicts particle masses. Also algebraic extensions of the number fields R_p are important.

2. The construction of real and p-adic physics by algebraic continuation of rational number based physics

Real number based quantum TGD can be algebraically continued to various number fields [E1]. It seems that the generalization of the notion of number obtained by gluing reals and various p-adic number fields along common rationals might be crucial in this respect. In the spirit of manifold theory also more general gluing maps than gluing along common rationals can be imagined and canonical identification and its variants seem to be natural as far as probabilities are considered.

The algebraic continuation applies to all mathematical structures involved. Two continuations share some set of points which can be regarded as common to the number field involved. Examples of the structures involved are real and p-adic variants of imbedding space, of configuration space of 3-surfaces, and of Hilbert space of quantum states.

The continuation from rationals to various number fields abstracts the basic facts about the relationship between physical world and theories about it. Rational points correspond to physical data or numerical predictions of mathematical theories. Physical world represents algebraic continuation to reals and various p-adic continuations a hierarchy of increasingly refined theories.

3. Number theoretical existence

Number theoretical existence requirement becomes a leading guide line in the construction of the theory. p-Adic mass calculations and the construction of topological mixing matrices U, D and CKM matrix $V = U^{\dagger}D$ provide an example of a successful application of the number theoretical existence requirements [F4]. Coupling constant evolution represents second obvious application yet to be developed. Here the challenges relate to the realization of non-algebraic functions like logarithm appearing typically in the formulas.

Two rather general number theoretical conjectures are inspired by the physics as a generalized number theory vision [E8, E1, E2, E3].
i) The ratios of logarithms of rationals are rationals. In particular, $log_2(q) = log(q)/log(2)$ is always rational so that pits are rational multiples of bits. Among other things this makes possible to construct rational valued iterated 2-based logarithms $log_2(...(log_2(q))...)$ expected to appear in running coupling constants.
ii) The numbers p^{iy} exist in finite dimensional extensions of rational numbers for each value of prime p and each zero $z = 1/2 + iy$ of Riemann Zeta. The obvious implication is that the exponents $q^{i\sum n_k y_k}$ satisfy the same condition. The construction of p-adic variant of Teichmueller parameters and moduli space provides an application of the conjecture [F2]. The implication is that two-dimensional shapes obey linear superposition and that the maxima of Kähler function correspond to these quantized 2-dimensional shapes.

As far as coupling strength evolution is considered the question whether one should require g^2 or $\alpha = g^2/4\pi$ or some other combination to be rational or in an finite-dimensional algebraic extension of R_p is of fundamental importance. The existence of the exponent of Kähler action for CP_2 support the view that g_K^2, and presumably all coupling constants should be proportional to π^2. Unless an infinite-dimensional extension of p-adic numbers defined by powers of π (possibly making sense) is allowed, a combination of $\alpha/2\pi$ or something equivalent with it should appear in Feynman diagrams.

3.1.2 Is it possible to introduce infinite-dimensional extension of p-adic numbers containing π?

The assumption that only finite-dimensional extensions of p-adic numbers are possible, is only a convenient working hypothesis. π appears in the basic formulas of geometry, in particular in the geometry of CP_2. π appears also in the Feynmann rules of quantum field theories and expressions for reaction rates, and the idea about the algebraic continuation of real integrals as a manner to define various momentum space integrals p-adically is very attractive. These observations strongly encourage to consider the possibility of an infinite-dimensional extension of p-adic numbers containing both positive and negative powers of π with additional constraints coming from conditions like $exp(i\pi/2) = i$.

The definition of p-adic norm should obey the usual conditions, in particular the requirement that the norm of product is product of norms. One can imagine two alternative definitions of the p-adic norm.

a) The first definition is as $N_p(x) = |det(x)|_p$, where $det(x)$ is the determinant of the linear map of the infinite-dimensional linear space spanned by powers of π defined by $x = \sum_{k=m_x}^{n_x} x_k \pi^k$. This definition is straightforward generalization of the usual definition and guarantees that norm is indeed algebraic homomorphism respecting product.

b) The second definition is as the limit of the $N_p(x) = lim_{N\rightarrow\infty}|(det(x)_N^{1/N})|_p$, truncated to the subspace defined by basis $\{\pi^{-N}, ...\pi^N\}$. The motivation for this definition is that first definition tends to give vanishing of infinite norm. This norm does not however define an algebraic homomorphism.

The linear map is represented by a matrix for which non-vanishing elements form a band parallel to the diagonal having width $n_x - m_x + 1$ with each vertical entry in band equal to the column vector $(x_{m_x}, ..., x_{n_x})^T$ defined by the components of x. Diagonal entries are equal to x_0. The determinant is a sum over all downward direct paths along the this band with the product of components x_i along the path assigned with a given path. The paths are not allowed to visit the same horizontal point twice so that an analog of a functional integral associated with a self-avoiding random walk constrained inside the diagonal band in a discrete lattice along x-axis is in question. Quantum fluctuations are restricted to the interval $[m_x, n_x]$ surrounding the classical path.

The condition that $x_n\pi^n$ approaches zero p-adically is natural and requires that the p-adic norm of x_n approaches zero. The stronger condition

$$... < |x_{n+1}|_p < |x_n|_p < ... \ , \tag{16}$$

simplifies dramatically the calculation of the determinant since $x_{m_x}\pi^{m_x}$ can be factored out and $|det(x)|_p$ becomes a norm of the determinant of a lower triangular matrix with units at diagonal multiplied by an infinite power of $|x_{m_x}|_p$.

In case a) the norm is simply $|x_{m_x}|_p^{N\rightarrow\infty}$ quite generally and diverges for $|x_{m_x}| > 1$, vanishes for $|x_{m_x}| < 1$ and equals to one for $|x_{m_x}| = 1$. In case b) the norm is $|x_{m_x}|$.

Trigonometric functions $cos(\pi q)$ and $sin(\pi q)$ allow to test the sensibility of the proposed alternative definitions. For instance, it is possible to check whether the norm of $sin(n\pi)$ vanishes for allowed values of n.

Option a): $x = cos(\pi q)$ would correspond to a lower triangular matrix with units along the diagonal so that the norm would be equal to 1 irrespective of the value of q. This is not consistent with $cos(\pi/2) = 0$. This is not a catastrophe, since $q = 1/2$ has p-adic norm $(1/2)_p \geq 1$ so that the series is not p-adically converging and does not satisfy the condition posed above. The minimum requirement is $q = rp$, r rational with unit p-adic norm. By the product decomposition $sin(\pi q)$ for $|q| < 1$ has a vanishing norm. Thus the condition $|x_n|_p \leq p^{-n}$ guarantees the consistency with the basic trigonometric formulas.

Option b: The p-adic norm of $cos(\pi q)$ is equal to 1 whereas the p-adic norm of $sin(\pi q)$ is equal to $|q|_p$ from product decomposition. In particular, the norm is non-vanishing for $x = np\pi$ so that an inconsistency results.

If the conjecture that $log(p) = x_p/\pi$, where x_p belongs to some finite-dimensional extension holds true, then also the powers of logarithms of rationals would belong to the extension.

3.1.3 What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

a) The naivest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

b) A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [E3].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

The physics of quarks and hadrons provides an immediate test for this interpretation. The surprising and poorly understood conclusion from the p-adic mass calculations was that the p-adic primes characterizing light quarks u,d,s satisfy $k_q < 107$, where $k = 107$ characterizes hadronic space-time sheet [F4].

a) The interpretation of $k = 107$ space-time sheet as a hadronic space-time sheet implies that quarks topologically condense at this space-time sheet so that $k = 107$ cannot belong to the collection of primes characterizing quark.

b) Quark space-time sheets must satisfy $k_q < 107$ unless $\hbar$ is large for the hadronic space-time sheet so that one has $k_{eff} = 107 + 22 = 129$. This predicts two kinds of hadrons. Low energy hadrons consists of u, d, and s quarks with $k_q < 107$ so that hadronic space-time sheet must correspond to $k_{eff} = 129$ and large value of $\hbar$. One can speak of confined phase. This allows also $k = 127$ light variants of quarks appearing in the model of atomic nucleus [F8]. The hadrons consisting of c,t,b and the p-adically scaled up variants of u,d,s having $k_q > 107$, $\hbar$ has its ordinary value in accordance with the idea about asymptotic freedom and the view that the states in question correspond to short-lived resonances.

3.1.4 Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [E3], hierarchy of Jones inclusions [C6], hierarchy of dark matters with increasing values of $\hbar$ [F9, J6], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [E3]. Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

a) The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

b) Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [E3] coding information too subtle to be catched by ordinary physical measurements [E10].

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

3.2 The number theoretical universality of Riemann Zeta

The conjecture about number theoretical universality of Riemann Zeta involves several conjectures.

The first conjecture emerged from the idea that at least the building blocks $1/(1-p^{-s})$ of Riemann Zeta for zeros of Riemann Zeta should make sense in all number fields provided that algebraic extensions of p-adic numbers are allowed [E8]. This kind of universality would guarantee that the radial logarithmic waves $r^{\zeta^{-1}(z)}$ assigned to the Hamiltonians of $\delta M^4_\pm \times CP_2$ exist for the rational values of the argument r as algebraic numbers for points $z = \sum_k n_k s_k$, where s_k is zero of ζ and z is the projection of partonic 2-surface to a geodesic sphere of CP_2 or to a geodesic sphere S^2 assignable to the light cone boundary. This condition emerges naturally in the p-adicization of quantum TGD using the notion of number theoretical braid [C1] defined as a subset of the intersection of real and p-adic variants of partonic 2-surface obeying same algebraic equations. The intersection consists of points in the algebraic extension of p-adic numbers used.

This form of number theoretical universality can be extended by requiring that also $\zeta(s)$ at points $s = \sum_k n_k s_k$, where n_k are non-negative numbers is an algebraic number. This version of conjecture has several forms of varying strength.

An strongest form of the conjecture requires that also the zeros s_k themselves are algebraic numbers and is motivated by the requirement that vacuum functional, which is conjectured to reduce to an exponent of Kähler function and identified as Dirac determinant, is an algebraic number for the maxima of Kähler function. The question what Dirac determinant means is far from trivial and the definition based on the notion of number theoretic braid is discussed in [C1].

To sum up, one can say that these conjectures would fit very nicely with the general spirit of TGD but it would be wrong to say that TGD is lost if these conjectures are wrong.

3.3 Some wrong number theoretical conjectures

Further speculative number theoretical conjectures inspired by TGD state that the ratio $log(q_1)/log(q_2)$ is rational for any pair or rational numbers and that $\pi/log(q)$ is rational for any rational q. It will be found that these conjectures are wrong. Also the conjecture stating that there is single rational q such that $\pi/log(q)$ is rational turns out to be wrong. This destroys a rather fascinating possibility to fix the basic parameters of quantum TGD.

3.3.1 About the action of group of rationals in the group of reals

These conjectures can be looked in a wider context by studying the orbits for the group of rationals in the group of reals. Two numbers whose ratio is rational belong to the same orbit. Rationals form trivially a normal subgroup so that the orbits form also a group. This means that the additive (or multiplicative) invariants $D(u)$ associated with orbits form an additive group so that $D(uv) = D(u) + D(v)$ and $D(u/v) = D(u) - D(v)$ hold true.

This brings in mind irreducible representations of the translation group: D(u) could be seen as a kind of number theoretic momentum. The invariant could be constructed by identifying the generators of the orbit group and picking up one representative from each generator orbit, take its logarithm, and use the additive group property to deduce the rest. These number theoretic momenta would like basic units of a momentum lattice: now however the dimension of lattice would be uncountably infinite. This construction would be very much like a gauge choice with rationals acting as a gauge group. This picture generalizes to the algebraic extensions of rationals too. The rational powers of any generator are generators and rational powers map orbits into orbits and therefore respect number theoretical "gauge invariance". Since 2^x covers all non-negative values of reals, x defines a convenient invariant labelling the orbits. At least x and $x + log(q)/log(2)$ correspond to the same orbit.

3.3.2 The failure of the conjectures

The rationality of $log(q_1)/log(q_2)$ would mean that the additive "gauge group" formed by logarithms of rationals responsible for the non-uniqueness of additive invariants D(u) would form single orbit itself! Logarithm would respect the orbit defining the unit element. This gives some aesthetic support for $log(q_1)/log(q_2)$ conjecture.

a) One could ask whether the ratio $log(x_1)/log(x_2)$ for numbers x_1 and x_2 at *any* orbit is rational. Besides rational powers logarithm would map orbits to orbits and respect the number theoretical "gauge invariance". The answer to this question is negative. Consider the orbit of e. The ratio of logarithms of e and $q\times e$ is $1/(1+log(q))$ and definitely not rational.

b) Conjecture fails also in the less general case. Assume that one has $log(q_2)/log(2) = q_1 = m_1/n_1$ such that q_2 is not rational power of 2. This implies $q_2 = 2^{log(q_2)/log(2)} = 2^{q_1}$. By taking n_1:th power one obtains $q_2^{n_1} = 2^{m_1}$. Since q_2 is not a power of 2, the decomposition of q_2 to produce of powers of primes would not be unique so that the conjecture cannot hold true.

c) Also the conjecture that $log(q)/\pi$ is rational for all rationals q fails. Assume that the conjecture holds true for two rationals q_1 and q_2. Taking the ratio one obtains that $log(q_1)/log(q_2)$ is always rational which cannot hold true.

d) Even the rationality of $\pi/log(q)$ for single q leads to a contradiction since it implies that $exp(\pi)$ is an algebraic number. This would in fact look extremely nice since the algebraic character of $exp(\pi)$ would conform with the algebraic character of the phases $exp(i\pi/n)$. Unfortunately this is not the case [16]. The argument showing this is based on the representation $i^{2i} = exp(\pi)$ and to the theorem that non-rational exponent of an algebraic number is transcendental. Hence one loses the attractive possibility to fix the basic parameters of theory completely from number theory unless one is somehow able to say something highly non-trivial about the parameter a.

4 p-Adic coupling constant evolution

p-Adic coupling constant evolution is one of the genuinely new elements of quantum TGD. In the following some aspects of the evolution will be discussed. The discussion is a little bit obsolete as far as the role of canonical identification is considered. The most recent view about p-adic coupling constant evolution is discussed at the end of the section.

4.1 p-Adic coupling constant evolution associated with length scale resolution at space-time level

If gauge couplings are indeed RG invariants inside a given space-time sheet, gauge couplings must be regarded as being characterized by the p-adic prime associated with the space-time sheet. The question is whether it is possible to understand also the p-adic coupling constant evolution at space-time level.

A natural view about p-adic length scale evolution is as an existence of a dynamical symmetry mapping the preferred extremal space-time sheet of Kähler action characterized by a p-adic prime p_1 to a space-time sheet characterized by p-adic prime $p_2 > p_1$ sufficiently near to p_1. The simplest guess is that the symmetry transformation corresponds to a scaling of M^4 coordinates in the intersection X^3 of the space-time surface with light-cone boundary $\delta M^4_+ \times CP_2$ by a scaling factor p_2/p_1, which in turn induces a transformation of $X^4(X^3)$, which in general does not reduce to M^4 scaling outside X^3 since scalings are not symmetries of the Kähler action.

This transformation induces a change of the vacuum gauge charges: $Q_i \rightarrow Q_i + \Delta Q_i$, and the renormalization group evolution boils down to the condition

$$\frac{Q_i + \Delta Q_i}{g_i^2 + \Delta g_i^2} = \frac{Q_i}{g_i^2} \ . \tag{17}$$

The problem is that this transformation has a continuous variant so that p-adic length scale evolution could reduce to continuous one.

A possible resolution of the problem is based on the observation that the values of the gauge charges depend on the initial values of the time derivatives of the imbedding space coordinates. RG invariance at space-time level suggests that small scalings leave the gauge charge and thus also coupling constant invariant. As a matter fact, this seems to be the case for all known extremals since they form scaling invariant families. The scalings by p_2/p_1 for some $p_2 > p_1$ would correspond to critical points in which bi-furcations occur in the sense that two space-time surfaces $X^4(X^3)$ satisfying the minimization conditions for Kähler action and with different gauge charges appear.

The new space-time surface emerging in the bifurcation would obey effective p_2-adic topology in some length scale range instead of p_1-adic topology. Stability considerations would dictate whether $p_1 \rightarrow p_2$ transition occurs and could also explain why primes $p \simeq 2^k$, k integer, are favored. This kind of bifurcations or even multi-furcations are certainly possible by the breaking of the classical determinism.

4.2 p-Adic evolution in angular resolution and dynamical Planck constant

Quantum phases $q = exp(i\pi/n)$ characterized Jones inclusions which have turned out to play key role in the understanding of macroscopic quantum phases in TGD framework. The basic idea is that the different values of Planck constant correspond to evolution in angular resolution in p-adic context characterized by quantum phase $q = exp(i\pi/n)$ characterizing Jones inclusion is. The higher the value of n, the better the angular resolution since the number of different complex phases in extension of p-adic numbers increases with n.

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

4.2.1 Jones inclusions and quantization of Planck constants

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

a) The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

b) In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of the scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups: $\hbar(M^4) = n_a$ and $\hbar(CP_2) = n_b$ whereas scaling factors of M^4 and CP_2 metrics are n_b^2 and n_a^2 respectively.

At the level of Schrödinger equation this means that Planck constant $\hbar$ correspodns to the effective Planck consant $\hbar_{eff} = (\hbar(M^4)/\hbar(CP_2))\hbar_0 = (n_a/n_b)\hbar_0$, which thus can have all possible positive rational values. For some time I believed on the scaling of metrics of M^4 *resp.* CP_2 as n_b^2 *resp.* n_a^2: this would imply invariance of Schrödinger equation under the scalings but would not be consistent with the explanation of the quantization of radii of planetary orbits requiring huge Planck constant [D7]. Poincare invariance is however achieved in the sense that mass spectrum is invariant under the scalings of Planck constants. That the ratio n_a/n_b defines effective Planck constant conforms with the fact that the value of Kähler action involves only this ratio (quantum-classical correspondence). Also the value of gravitational constant is invariant under the scalings of Planck constant since one has $G \propto g_K^2 R^2$, R radius of CP_2 for $n_a = 1$.

c) This predicts automatically arbitrarily large values of effective Planck constant n_a/n_b and they correspond to coverings of CP_2 points by large number of M^4 points which can have large distance and have precisely correlated behavior due to the G_a symmetry. One can assign preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter seem to correspond to these special values of Planck constants. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

d) These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_{2n}, E_6, E_8 are possible so that TGD seems to be able to mimic these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

4.2.2 The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant.

Gravitational Planck constant $\hbar_{gr}$ can be interpreted as effective Planck constant $\hbar_{eff} = (n_a/n_b)\hbar_0$ so that the Planck constant associated with M^4 degrees of freedom (rather than CP_2 degrees of freedom as in the original wrong picture) must be very large in this kind of situation.

The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 3 per cent in solar system to be compared with the accepted cosmological value of 4 per cent [D7].

If so, its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom are involved meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

There is however strong objection based on the observation that the radius of CP_2 would become gigantic. Surprisingly, this need not have any dramatic implications as will be found. It is also quite possible that the biomolecules subgroups of rotation group as symmetries could correspond to $n_a > 1$. For instance, the tedrahedral and icosahedral molecular structures appearing in water would correspond to E_6 with $n_a = 3$ and E_8 with $n_a = 5$. Note that $n_a = 5$ is minimal value of n_a allowing universal topological quantum computation.

4.3 Large values of Planck constant and electro-weak and strong coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

4.4 Super-canonical gluons and non-perturbative aspects of hadron physics

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [A7, E9] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase

[A9, D7]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [F4].

According to the model of hadron masses [F4], in the case of light pseudoscalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudoscalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of configuration space degrees of freedom ("world of classical worlds") in which super-canonical algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-canonical gluons. It turns out the model assuming same topological mixing of super-canonical bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-canonical particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \ .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-canonical colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [A9]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

4.5 Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

4.5.1 First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection againt this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-canonical bosons or the interaction between super-canonical bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-canonical color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labelling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have join along boundaries bonds directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for join along boundaries bonds of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

4.5.2 Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labelled by primes near power of two couple strongly with Mersenne primes.

4.6 The formula for the hadronic string tension

It is far from clear whether the strong gravitational coupling constant has same relation to the parameter $M_0^2 = 16m_0^2 = 1/\alpha' = 2\pi T$ as it would have in string model.

a) One could estimate the strong gravitational constant from the fundamental formula for the gravitational constant expressed in terms of exponent of Kähler action in the case that one has $\alpha_K = 1/4$. The formula reads as

$$\frac{L_p^2}{G_p} = exp(2aS_K(CP_2)) = exp(\pi/4\alpha_K) = e^{\pi} \quad . \tag{18}$$

a is a parameter telling which fraction the action of wormhole contact is about the full action for CP_2 type vacuum extremal and $a \sim 1/2$ holds true. The presence of a can take care that the exponent is rational number. For $a = 1$ The number at the right hand side is Gelfond constant and one obtains

$$G_p = exp(-\pi) \times L_p^2 \quad . \tag{19}$$

b) One could relate the value of the strong gravitational constant to the parameter $M_0^2(k) = 16m(k)^2$, $p \simeq 2^k$ also assuming that string model formula generalizes as such. The basic formulas can be written in terms of gravitational constant G, string tension T, and $M_0^2(k)$ as

$$\frac{1}{8\pi G(k)} = \frac{1}{\alpha'} = 2\pi T(k) = \frac{1}{M_0^2(k)} = \frac{1}{16m(k)^2} \quad . \tag{20}$$

This allows to express G in terms of the hadronic length scale $L(k) = 2\pi/m(k)$ as

$$G(k) = \frac{1}{16^2\pi^2}L(k)^2 \simeq 3.9 \times 10^{-4}L(k)^2 \quad . \tag{21}$$

The value of gravitational coupling would be by two orders of magnitude smaller than for the first option.

4.7 How p-adic and real coupling constant evolutions are related to each other?

It must be emphasized that part of this section was written before the realization that the generalized eigenvalue equation for the modified Dirac operator provides a fundamental definition of the p-adic coupling constant evolution and some of the considerations are therefore only heuristic. For instance, the relationship between p-adic and real coupling constant evolutions more or less trivializes since S-matrix elements in the approach based on number theoretical braids are algebraic numbers and thus make sense in any number field. The real and p-adic coupling constants are thus identical algebraic numbers.

4.7.1 Questions

One can pose many questions about p-adic coupling constant evolution. How do p-adic and corresponding real coupling constant evolution relate to each other? Why Mersenne primes and primes near prime (integer) powers of two seem to be in a special position physically? Could one say something about phase transition between perturbative and non-perturbative phases of QCD?

4.7.2 How p-adic amplitudes are mapped to real ones?

Before the realization that p-adic and real amplitudes could be algebraic numbers the question of the title was very relevant. If the recent picture is correct, the following considerations are to some degree obsolete.

The real and p-adic coupling constant evolutions should be consistent with each other. This means that the coupling constants $g(p_1, p_2, p_3)$ as functions of p-adic primes characterizing particles of the vertex should have the same qualitative behavior as real and p-adic functions. Hence the p-adic norms of complex rational valued (or those in algebraic extension) amplitudes must give a good estimate for the behavior of the real vertex. Hence a restriction of a continuous correspondence between p-adics and reals to rationals is highly suggestive. The restriction of the canonical identification to rationals would define this kind of correspondence but this correspondence respects neither symmetries nor unitarity in its basic form. Some kind of compromize between correspondence via common rationals and canonical identification should be found.

The compromise might be achieved by using a modification of canonical identification $I_{R_p \rightarrow R}$. Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R \rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n / \sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n} / \sum s_n p^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} \qquad (22)$$

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q = r/s$ have no common factors. For integers $n < p$ it reduces to direct correspondence. R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R \rightarrow R_{p_2}} I_{R_{p_1} \rightarrow R}$.

Instead of a re-interpretation of the p-adic number $g(p_1, p_2, p_3)$ as a real number or vice versa would be continued by using this variant of canonical identification. The nice feature of the map would be that continuity would be respected to high degree and something which is small in real sense would be small also in p-adic sense.

4.7.3 How to achieve consistency with the unitarity of topological mixing matrices and of CKM matrix?

It is easy to invent an objection against the proposed relationship between p-adic and real coupling constants. Topological mixing matrices U, D and CKM matrix $V = U^\dagger D$ define an important part of the electro-weak coupling constant structure and appear also in coupling constants. The problem is that canonical identification does not respect unitarity and does not commute with the matrix multiplication in the general case unlike gluing along common rationals. Even if matrices U and D which contain only ratios of integers smaller than p are constructed, the construction of V might be problematic since the products of two rationals can give a rational $q = r/s$ for which r or s or both are larger than p.

One might hope that the objection could be circumvented if the ratios of the integers of the algebraic extension defining the matrix elements of CKM matrix are such that the integer components of algebraic integers are smaller than p in U and D and even the products of integers in $U^\dagger D$ satisfy this condition so that modulo p arithmetics is avoided.

In the standard parametrization all matrix elements of the unitarity matrix can be expressed in terms of real and imaginary parts of complex phases ($p \ mod \ 4 = 3$ guarantees that $\sqrt{-1}$ is not an ordinary p-adic number involving infinite expansion in powers of p). These phases are expressible as products of Pythagorean phases and phases in some algebraic extension of rationals.

i) Pythagorean phases defined as complex rationals $[r^2 - s^2 + i2rs]/(r^2 + s^2)$ are an obvious source of potential trouble. However, if the products of complex integers appearing in the numerators and denominators of the phases have real and imaginary parts smaller than p it seems to be possible to avoid difficulties in the definition of $V = U^{\dagger}D$.

ii) Pythagorean phases are not periodic phases. Algebraic extensions allow to introduce periodic phases of type $exp(i\pi m/n)$ expressible in terms of p-adic numbers in a finite-dimensional algebraic extension involving various roots of rationals. Also in this case the product $U^{\dagger}D$ poses conditions on the size of integers appearing in the numerators and denominators of the rationals involved.

If the expectation that topological mixing matrices and CKM matrix characterize the dynamics at the level $p \simeq 2^k$, $k = 107$, is correct, number theoretical constraints are not expected to bring much new to what is already predicted. Situation changes if these matrices appear already at the level k. For $k = 89$ hadron physics the restrictions would be even stronger and might force much simpler U, D and CKM matrices.

k-adicity constraint would have even stronger implications for S-matrix and could give very powerful constraints to the S-matrix of color interactions. Quite generally, the constraints would imply a p-adic hierarchy of increasingly complex S-matrices: kind of a physical realization for number theoretic emergence. The work with CKM matrix has shown how powerful the number theoretical constraints are, and there are no reasons to doubt that this could not be the case also more generally since in the lowest order the construction would be carried out in finite (Galois) fields $G(p, k)$.

4.7.4 How generally the hybrid of canonical identification and identification via common rationals can apply?

The proposed gluing procedure, if applied universally, has non-trivial implications which need not be consistent with all previous ideas.

a) The basic objection against the new kind of identification is that it does not commute with symmetries. Therefore its application at imbedding space and space-time level is questionable.

b) The mapping of p-adic probabilities by canonical identification to their real counterparts requires a separate normalization of the resulting probabilities. Also the new variant of canonical identification requires this since it does not commute with the sum.

c) The direct correspondence of reals and p-adics by common rationals at space-time level implies that the intersections of cognitive space-time sheets with real space-time sheet have literally infinite size (p-adically infinitesimal corresponds to infinite in real sense for rational) and consist of discrete points in general. If the new gluing procedure is adopted also at space-time level, it would considerably de-dramatize the radical idea that the size for the space-time correlates of cognition is literally infinite and cognition is a literally cosmic phenomenon.

Of course, the new kind of correspondence could be also seen as a manner to construct cognitive representations by mapping rational points to rational points in the real sense and thus as a formation of cognitive representations at space-time level mapping points close to each other in real sense to points close to each other p-adically but arbitrarily far away in real sense. The image would be a completely chaotic looking set of points in the wrong topology and would realize the idea of Bohm about hidden order in a very concrete manner. This kind of mapping might be used to code visual information using the value of p as a part of the code key.

b) In p-adic thermodynamics p-adic particle mass squared is mapped to its real counterpart by canonical identification. The objection against the use of the new variant of canonical identification is that the predictions of p-adic thermodynamics for mass squared are not rational numbers but infinite power series. p-Adic thermodynamics itself however defines a unique representation of probabilities as ratios of generalized Boltzmann weights and partition function and thus the variant of canonical identification indeed generalizes and at the same time raises worries about the fate of the earlier predictions of the p-adic thermodynamics.

Quite generally, the thermodynamical contribution to the particle mass squared is in the lowest p-adic order of form rp/s, where r is the number of excitations with conformal weight 1 and s the number of massless excitations with vanishing conformal weight. The real counterpart of mass squared for the ordinary canonical identification is of order CP_2 mass by $r/s = R + r_1p + ...$ with $R < p$ near to p. Hence the states for which massless state is degenerate become ultra heavy if r is not divisible by s. For the new variant of canonical identification these states would be light. It is not actually clear how many states of this kind the generalized construction unifying super-canonical and super Kac-Moody algebras predicts.

A less dramatic implication would be that the second order contribution to the mass squared from p-adic thermodynamics is always very small unless the integer characterizing it is a considerable fraction of p. When ordinary canonical identification is used, the second order term of form rp^2/s can give term of form Rp^2, $R < p$ of order p. This occurs only in the case of left handed neutrinos.

The assumption that the second order term to the mass squared coming from other than thermodynamical sources gives a significant contribution is made in the most recent calculations of leptonic masses [F3]. It poses constraints on CP_2 mass which in turn are used as a guideline in the construction of a model for hadrons [F4]. This kind of contribution is possible also now and corresponds to a contribution Rp^2, $R < p$ near p.

The new variant of the canonical correspondence resolves the long standing problems related to the calculation of Z and W masses. The mass squared for intermediate gauge bosons is smaller than one unit when m_0^2 is used as a fundamental mass squared unit. The standard form of the canonical identification requires $M^2 = (m/n)p^2$ whereas in the new approach $M^2 = (m/n)p$ is allowed. Second difficult problem has been the p-adic description of the group theoretical model for m_W^2/m_Z^2 ratio. In the new framework this is not a problem anymore [F3] since canonical identification respects the ratios of small integers.

On the other hand, the basic assumption of the successful model for topological mixing of quarks [F4] is that the modular contribution to the masses is of form np. This assumption loses its original justification for this option and some other justification is needed. The first guess is that the conditions on mass squared plus probability conservation might not be consistent with unitarity unless the modular contribution to the mass squared remains integer valued in the mixing (note that all integer values are not possible [F4]). Direct numerical experimentation however shows that that this is not the case.

4.8 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge from quantum TGD proper?

What p-adic coupling constant evolution really means has remained for a long time more or less open. The progress made in the understanding of the S-matrix of theory has however changed the situation dramatically.

4.8.1 M-matrix and coupling constant evolution

The final breakthrough in the understanding of p-adic coupling constant evolution came through the understanding of S-matrix, or actually M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy states in zero energy ontology [C2]. M-matrix has interpretation as a "complex square root" of density matrix and thus provides a unification of thermodynamics and quantum theory. S-matrix is analogous to the phase of Schrödinger amplitude multiplying positive and real square root of density matrix analogous to modulus of Schrödinger amplitude.

The notion of finite measurement resolution realized in terms of inclusions of von Neumann algebras allows to demonstrate that the irreducible components of M-matrix are unique and possesses huge symmetries in the sense that the hermitian elements of included factor $\mathcal{N} \subset \mathcal{M}$ defining the measurement resolution act as symmetries of M-matrix, which suggests a connection with integrable quantum field theories.

It is also possible to understand coupling constant evolution as a discretized evolution associated with time scales T_n, which come as octaves of a fundamental time scale: $T_n = 2^n T_0$. Number theoretic universality requires that renormalized coupling constants are rational or at most algebraic numbers and this is achieved by this discretization since the logarithms of discretized mass scale appearing in the expressions of renormalized coupling constants reduce to the form $log(2^n) = nlog(2)$ and with a proper choice of the coefficient of logarithm log(2) dependence disappears so that rational number results.

4.8.2 p-Adic coupling constant evolution

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

a) The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

b) p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1.\times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

c) In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

d) The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \quad ,$$

it maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with $T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

4.9 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

5 The evolution of gauge and gravitational couplings at space-time level

The question is whether the RG evolution of all coupling constant parameters could have interpretation as flows at space-time level. This seems to be the case.

5.1 Renormalization group flow as a conservation of gauge current in the interior of space-time sheet

The induced gauge potentials relate to the gauge potentials A_i of perturbative gauge theory by the scaling $g_i \rightarrow g_i A_i$. Hence the gauge currents correspond to the scaled currents

$$\begin{aligned} J_i^\mu &= \frac{1}{g_i^2} \times J_{i,0}^\mu \ , \\ J_{i,0}^\mu &= (D_\nu F^{\mu\nu})_i \sqrt{g} \ . \end{aligned} \tag{23}$$

The simplest guess for the coupling constant evolution associated with g_i^2 is that the covariant gauge current J_i^a is conserved in ordinary sense (its is identically conserved in covariant sense). This gives meaning to the perturbative approach in which gauge charges are indeed conserved. Thus one would have:

$$\partial_\mu J_i^\mu = 0 \ . \tag{24}$$

or

$$J_{i,0}^\mu \partial_\mu log(g_i^2) = \partial_\mu J_{i,0}^\mu \ . \tag{25}$$

Note that the non-constancy of the Weinberg angle gives an additional term to the em current given by

$$\frac{1}{2} Z_0^{\mu\nu} \partial_\nu p \ . \tag{26}$$

This equation can be solved along the flow lines of the gauge current. When the flow is integrable:

$$J^{\mu}_{i,0} = \phi \partial^{\mu} t \ ,$$

one obtains

$$\frac{dlog(g_i^2)}{dt} = \frac{\partial_{\mu} J^{\mu}_{i,0}}{\phi} = \nabla(log(\phi)) \cdot \nabla t + \nabla^2 t \ . \quad (27)$$

When this flow is not integrable coupling constants become discontinuous functions with respect to the real topology but can be continuous or even smooth with respect to some p-adic topology and the previous discussion applies as such.

The ordinary divergence of the gauge current takes the role of beta function. RG evolution is trivial in the Abelian case since in this case ordinary divergence vanishes identically. This implies that Kähler coupling strength is indeed renormalization group invariant which has been the basic hypothesis of quantum TGD.

The natural boundary conditions to the coupling constant evolution state the vanishing of the normal components of the gauge currents at boundaries

$$J_i^n = \frac{D_{\beta} F_i^{n\beta} \sqrt{g_4}}{g_i^2} = 0 \ . \quad (28)$$

and guarantee that the flow approaches asymptotically the boundaries. These conditions can become trivial if the four-metric at the boundary component becomes singular (effectively 2-dimensional) so that $D_{\beta} F_i^{n\beta}$ can approach to finite or even infinite value. This might happen in case of color gauge coupling strength if it approaches infinity near the boundary. Otherwise the conditions says nothing about coupling constants at the boundary.

5.2 Is the renormalization group evolution at the light-like boundaries trivial?

One can ask whether it is possible to define coupling constant evolution also for the gauge fields induced at light-like boundary components. The technical problems are caused by the vanishing of the determinant of the induced metric and the non-existence of contravariant metric but it is quite conceivable that the restriction to the 2-dimensional sections makes sense if one defines a contravariant metric as the inverse of the induced metric in the 2-D section.

Since CP_2 projection is 2-dimensional, RG equations suggest that coupling constants are constants on the 2-dimensional sections and that conformal invariance in the light-like direction implies constancy over the entire boundary component. Since boundary components are identifiable as parton like objects, the result would look highly satisfactory.

If the right hand side of Eq. 27 vanishes at the boundary of space-time surface g_i^2 approaches to a finite value. When the left hand side is finite and

t becomes infinite as boundary is approached g_i^2 increases without limit. This happens for a finite value of t when the right hand side diverges. Classical color gauge fields are proportional to $H_A J$, where H_A are the Hamiltonians of the color isometries and J denotes the induced Kähler form. The non-triviality of renormalization group evolution is solely due to the presence of Hamiltonians. QCD would suggest that α_s diverges at the outer boundary or that at least approaches to a very large value at the outer boundaries of the hadronic 4-surface.

5.3 Fixed points of coupling constant evolution

Consider now the fixed points of the coupling constant evolution.

a) The first class of fixed points corresponds to CP_2 type extremals. In this case however also gauge currents vanish so that the RG equation says nothing.

b) The second class of fixed points of the coupling constant evolution corresponds to space-time regions in which gauge fields become Abelian. This is the case for all space-time surfaces with 2-dimensional CP_2 projection: this includes vacuum extremals, massless extremals, solutions for which CP_2 projection corresponds to a homologically non-trivial geodesic sphere, and cosmic strings. This supports the view that these extremals correspond to asymptotic self-organization patters.

5.4 Are all gauge couplings RG invariants within a given space-time sheet

No extremals for which the gauge currents would have non-vanishing ordinary divergence are known at this moment (gauge currents are light-like always). Therefore one cannot exclude the possibility that all gauge coupling constants rather than only Kähler coupling strength are renormalization group invariants in TGD framework, so that the hypothesis that RG evolution reduces to a discrete p-adic coupling constant evolution would be correct.

This implies that also Weinberg angle, being determined by the ratio of $SU(2)$ and $U(1)$ couplings, is constant inside a given space-time sheet. Its value in this case is determined most naturally by the requirement that the net vacuum em charge of the space-time sheet vanishes.

The fixed point property as an implication of Abelianity is obviously in conflict with the standard picture about gauge coupling evolution and supports the view that this evolution corresponds to a discrete p-adic gauge coupling evolution.

5.5 RG equation for gravitational coupling constant

In the case of gravitational coupling constant the renormalization group equation must be formulated the current representing the contribution of Einstein tensor to the gravitational mass being defined by Einstein tensor as

$$G^\alpha = \frac{1}{16\pi G} \times G^{\alpha\beta}\partial_\beta a\sqrt{g} \ , \tag{29}$$

where a refers the proper time of future light cone (or possibly to some other preferred time coordinate determined by dynamics). In the case of cosmological constant the corresponding contribution is

$$g^\alpha = \frac{\Lambda}{16\pi G} \times g^{\alpha\beta}\partial_\beta a\sqrt{g} \ . \tag{30}$$

A natural hypothesis is that the variation of G guarantees the conservation of gravitational mass. This does not mean that gravitational energy or four-momentum would be conserved or that conservation of gravitational mass would hold true except at a given space-time sheet. One can also assume that the two contributions to the gravitational mass are not independent. This means that there is a constraint between cosmological and gravitational constants. There are two options.

a) One has

$$\Lambda = \frac{x}{G} \ . \tag{31}$$

where x is renormalization group invariant of no other length scales are involved. The RG equation would in this case read as

$$\left(G^\alpha - \frac{2x}{G}g^\alpha\right) D_\alpha log(G) = D_\alpha \left(G^\alpha + \frac{x}{G}g^\alpha\right) \ . \tag{32}$$

b) On the other hand, if p-adic length scale hypothesis is accepted, one has

$$\Lambda = \frac{x}{L_p^2} \ , \tag{33}$$

where L_p is a p-adic length scale of order of cosmic time a: $L_p \sim a$ [D5]. This would mean that Λ is RG invariant. This option resolves the mysterious smallness of the cosmological constant so that it is the most plausible option in TGD framework.

The RG equations in this case is given by

$$\left(G^\alpha + \frac{x}{L_p^2}g^\alpha\right) D_\alpha log(G) = D_\alpha \left(G^\alpha + \frac{x}{L_p^2}g^\alpha\right) \ . \tag{34}$$

and of the same general form as in the case of gauge couplings, which also supports option b).

Vacuum extremals which correspond to asymptotic cosmologies with cosmological constant satisfying

$$D_\alpha \left(G^\alpha + \frac{x}{L_p^2} g^\alpha \right) = 0 \qquad (35)$$

represent examples of the fixed points of the coupling constant evolution with conserved gravitational four-momentum. Obviously much weaker conditions guarantee fixed point property.

For Schwartschild metric having imbedding as a vacuum extremal Einstein tensor vanishes so that the RG equations would say nothing about G for option a). For Reissner-Nordstöm metric also having embedding as a vacuum extremal Einstein tensor corresponds to the energy momentum tensor of Abelian gauge field and the length scale evolution of G would be non-trivial in both cases.

6 About electro-weak coupling constant evolution

The classical space-time correlates for electro-weak coupling constant evolution deserve a separate discussion.

6.1 How to determine the value of Weinberg angle for a given space-time sheet?

The general picture about the massivation of electro-weak bosons and electro-weak gauge bosons based on the notion of induced gauge field allows to determined Weinberg angle from the condition that electromagnetic vacuum charge for a given space-time sheet vanishes.

The basic idea is that electro-weak vacuum charge densities are generated and screen weak charges transforming $1/r$ Coulomb potentials to exponentially screened ones. The massivation of fermions occurs by a different mechanism in TGD [F2, F3, F4, F5] and they can be massive even in the case that electro-weak bosons are massless.

In gauge theories the screening of weak charges occurs in differential manner. In TGD framework RG invariance inside a given space-time sheet and p-adic coupling constant evolution support the view that this screening occurs in discrete manner in the sense that the weak fields would behave like massless fields inside a given space-time sheet but the net weak charges of the space-time sheets cause the screening of the weak charges and massivation in average sense. The masslessness of photons means that the vacuum em charge for a given space-time sheet vanishes. This condition allows to determine the value of Weinberg angle for a given space-time sheet.

6.2 Smoothed out position dependent Weinberg angle from the vanishing of vacuum density of em charge

A practical variant about the condition determining Weinberg angle for a given space-time sheet is obtained by a smoothing out procedure in which the distribution of discrete values of Weinberg angle is replaced with a continuous distribution interpreted as a constant below the typical size scale of space-time sheets involved.

The condition that the em charge density defined by the covariant divergence of electro-weak current vanishes, gives a differential equation allowing to solve for Weinberg angle. Using M^4_+ proper time a as a preferred time coordinate (identifiable as cosmic time and playing key role in the construction of configuration space geometry and quantum TGD [B2, B3])) this condition can be made general coordinate invariant. One can hope that with a proper choice of boundary conditions (fixed actually the the minimization of Kähler action) Weinberg angle can always have a physical value. Since gauge current is defined as the covariant divergence of gauge field the condition involves for $D > 2$ besides the ordinary divergence also a term proportional to $W_{+,\nu}W^{\mu\nu}_- - W_{-,\nu}W^{\mu\nu}_+$.

6.2.1 Simple special cases

For vacuum extremals ordinary em current vanishes for $p = sin^2(\theta_W) = 0$. In this case the 2-dimensionality of CP_2 projection guarantees that ordinary divergence equals to the covariant one. Hence $p = 0$ guarantees trivially the vanishing of em charge density also now but there are also other solutions.

For solutions with CP_2 projection belong to a homologically non-trivial geodesic sphere of CP_2 the condition determining the Weinberg angle reduces to the vanishing of the divergence of pJ^{0i} whereas the vanishing of γ would imply a non-physical value of p.

6.2.2 General solution of the conditions

The explicit expressions for classical em and Z^0 are given by

$$\begin{aligned} \gamma &= 3J - pR_{03} \ , \quad p \equiv sin^2(\theta_W) \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{36}$$

CP_2 Kähler form J and spinor curvature component R_{03} are given in terms of vierbein by

$$\begin{aligned} J &= 2\left[e_1 \wedge e_2 + e_0 \wedge e_3\right] \ , \\ R_{03} &= 2e_1 \wedge e_2 + 4e_0 \wedge e_3 \ . \end{aligned} \tag{37}$$

The general form of the condition determining Weinberg angle is given by

$$E_Z \cdot \nabla p + (\nabla \cdot E_Z) p = F \ ,$$
$$F = -6\nabla \cdot E_K - 2F_1 \ . \tag{38}$$

Here E_Z corresponds R_{03} term in em field and E_K to Kähler electric field and F_1 corresponds to the $W_{+,\nu}W_-^{\mu\nu} - W_{-,\nu}W_+^{\mu\nu}$ term. It is assumed that $1/e^2$ factor multiplying em current is constant. If this is not the case, the replacement $F \rightarrow F + 2E_{em}\nabla 2log(e^2)$ must be made on the right hand side.

These differential equations are of the same form as renormalization group equations and continuous solutions exist if one can introduce a coordinate system in which the flow lines of Kähler electric field correspond to one coordinate. This is possible if Z^0 electric field is of the form

$$E_Z = \phi dt \ . \tag{39}$$

This implies the integrability condition $dE_Z = d\phi \wedge dt$ implying

$$dE_Z \wedge E_Z = 0 \ . \tag{40}$$

By introducing space-time coordinates (x, t) (t does not refer to time now) the equation can be written in the form

$$\frac{dp}{dt} + \frac{\nabla \cdot E_Z}{\phi} p = \frac{F}{\phi} \ . \tag{41}$$

solutions can be written as

$$\begin{aligned} p &= p_0 + p_1 \ , \\ \frac{dp_0}{dt} + \frac{\nabla \cdot E_Z}{\phi} p_0 &= 0 \ , \\ \frac{dp_1}{dt} + \frac{\nabla \cdot E_Z}{\phi} p_1 &= \frac{F}{\phi} \ . \end{aligned} \tag{42}$$

p_0 and p_1 are given by

$$\begin{aligned} p_0(x,t) &= p_{00}(x) + exp\left(-\int_0^t du \frac{\nabla \cdot E_Z(x,u)}{\phi}\right) , \\ p_1(x,t) &= p_0(x,t) \int_0^t du \frac{F}{p_0 \phi}(x,u) \ . \end{aligned} \tag{43}$$

Whether $p_{00}(x) = constant$ is consistent with field equations is an open question.

3. What happens when the integrability condition fails?

The failure of the integrability condition has interpretation as failure of the smoothing out procedure. A natural guess is that in this case the coupling constant is continuous or perhaps even smooth with respect to p-adic topology below the p-adic length scale for some prime p. Non-integrability would provide a rather satisfactory differential-topological understanding of how effective p-adic topology emerges.

6.3 The role of # contacts in electro-weak massivation

For all known extremals of Kähler action except CP_2 type extremals classical weak gauge fields behave like massless fields inside a given space-time sheet. This raises the question whether topological condensation could characterize the essence of gauge boson massivation.

contacts (or wormhole contacts) and $\#_B$ contacts (join along boundaries bonds) discussed in detail in [F6] mediate the transfer of gauge fluxes to larger space-time sheets. # contacts correspond to CP_2 type extremals, which have suffered simultaneous topological condensation at two space-time sheets with Minkowskian signature. The first sheet is associated with the boson and second one with the background space-time. # contact is accompanied by two light-like causal horizons having interpretation as partons and thus the interpretation as an exotic 2-parton state is possible. If net gauge quantum numbers vanish, the two-parton system has strict interpretation as mediator of gauge fluxes expressible in terms of parton quantum numbers. Note that the space-time sheets in the vicinity of # contact have CP_2 projection with dimension $D \geq 3$.

The CP_2 type extremal defining # contact defines an Euclidian time evolution. This evolution conserves Abelian Kähler charge and also color charges defined either as gauge fluxes since color gauge fields are proportional to Kähler field or as isometry charges. Electro-weak gauge fluxes are however not conserved by their genuinely non-Abelian character. This suggests that the loss of correlations in length scales longer than the Compton length of weak boson implying massivation is due to the fact that the weak gauge fluxes change in a random manner while flowing through # contact. Em field can be written as $\gamma = 3J - sin^2(\theta_W)Z_0/2$ so that also this gauge flux contains a part which is not conserved in # contact. The Kähler form contribution however guarantees that long range correlations are not lost.

6.4 The identification of Higgs as a weakly charged wormhole contact

Quantum classical correspondence suggests that electro-weak massivation should have simple space-time description allowing also to identify Higgs boson if it exists. This description indeed exists and allows also to understand the precise relationship between gravitational and inertial masses and how Equivalence Principle is weakened in TGD framework.

The basic observation is that gauge and gravitational fluxes flow to larger space-time sheets through # (wormhole) contacts. If gravitational energy can be regarded in the Newtonian limit as a gauge charge, the contacts feed the gravitational energy regarded as a gauge flux to the lower condensate levels. The non-conservation of gravitational gauge flux means that # contacts can carry gravitational four-momentum. Since CP_2 type vacuum extremals are the natural candidates for # contacts, the natural hypothesis is that the non-vanishing light-like gravitational four-momentum of # contacts is responsible for the non-conservation of gravitational four-momentum flux. The non-conservation of the light-like gravitational four-momentum of CP_2 type extremals is in turn responsible for the non-conservation of the net gravitational four-momentum.

contacts can be also carriers of inertial four-momentum which must be conserved in absence of four-momentum exchange between environment and wormhole contact. Therefore Equivalence Principle cannot hold true in strict sense. Equivalence Principle is satisfied in a weak sense if the inertial four-momentum is equal to the average four-momentum associated with the zitterbewegung motion and corresponds to the center of mass motion for the # contact.

The non-conservation of weak gauge currents for CP_2 type extremals implies a non-conservation of weak charges and the finite range of weak forces. If wormhole contacts correspond to pieces of CP_2 type vacuum extremal, electro-weak gauge currents are not conserved classically unlike color and Kähler current. The non-conservation of weak isospin corresponds to the presence of pairs of right/left handed fermion and left/right handed antifermion at wormhole contacts. These wormhole contacts are excellent candidates for the TGD counterpart of Higgs boson providing the most natural mechanism for the massivation of weak bosons. The finding that that CP_2 parts of the induced gamma matrices connect different M^4 chiralities of induced spinor fields provided the original motivation for the belief that Higgs mechanism is realized in some manner in TGD Universe. This coupling must be crucial for the formation of weakly charged wormhole contacts.

There are two contributions to the mass of elementary particle corresponding to the primary and secondary topological condensation.

a) The dominant contribution to the fermion masses would be due to p-adic thermodynamics describing primary topological condensation. If weak form of Equivalence Principle holds true, inertial mass would result simply as the average of non-conserved light-like gravitational four-momentum. This contribution to the inertial mass is generated in the topological condensation of CP_2 type extremal representing elementary particle involving only single light like elementary particle horizon, say fermion, and by randomness of the zitterbewegung corresponds naturally to the contribution given by p-adic thermodynamics.

b) For gauge bosons the contribution from primary condensation should be very small or vanishing if the radius of zitterbewegung orbit is larger than the size of the space-time sheet containing the topologically condensed boson so that the motion is along a light-like geodesic in a good approximation. The space-time sheet representing massless state suffered secondary topologically condensation at a larger space-time sheet and viewed as a particle can develop mass via Higgs mechanism since wormhole contacts cannot be regarded as moving along light like geodesics in the length and time scale involved. # contacts carrying net left handed weak isospin have interpretation as TGD counterparts of neutral Higgs bosons and the formation of a coherent state involving superposition of states with varying number of wormhole contacts corresponds to the generation of a vacuum expectation value of Higgs field.

6.5 Questions related to the physical interpretation

This picture raises several interesting questions related to the physical interpretation.

a) What is the TGD counterpart of Higgs=0 phase? The dimension of CP_2 projection is is analogous to temperature and one can argue that massivation is analogous to a loss of correlations due to the increase of D bringing in additional degrees of freedom. Massless extremals having $D = 2$ all induced gauge fields are massless so that they are excellent candidates for Higgs=0 phase. Indeed, the construction of S-matrix leads to the interpretation that MEs allow massless particle exchanges with arbitrary long range but the very fact that the scattering is limited to massless momentum exchanges it is difficult to detect. Note that this scattering is not possible in two-particle system. Does the result mean that already $D = 3$ space-time sheets correspond to a massive phase?

b) Why electro-weak length scale corresponding to Mersenne prime M_{89} is preferred [F3]? Are there also other length scales in which electro-weak massivation occurs and thus scaled copies of electro-weak bosons? These questions reduce to the questions about the stability of the proposed bifurcations.

c) The basic problem of TGD based model of condensed matter is to explain why classical long range gauge fields do not give rise to large parity breaking effects in atomic length scale but do so in cell length length scale at least in the case of living matter (bio-catalysis). The proposal has been that particles feed electro-weak and em gauge fluxes to different space-time sheets. Could it be that blocks of bio-matter with size larger than cell the space-time sheets at which em and weak charges are feeded can be in Higgs=0 phase whereas for smaller blocks screening occurs already at quark and lepton level.

This would be consistent with the fact that the dimension D of CP_2 projection tends to decrease with the size of the space-time sheet: the larger the space-time sheet, the nearer it is to a vacuum extremal. Robertson-Walker cosmologies are exact vacuum extremals carrying however non-vanishing gravitational 4-momentum densities. By previous argument W and Z masses are identical in this kind of phase if the vanishing of vacuum em field is used to fix p. The weakening of correlations caused by classical non-determinism might imply massivation.

d) Do long ranged non-screened vacuum Z^0 and W gauge fields have some quantum counterparts as quantum-classical correspondence would suggest? Does dark matter identified as a phase with large value of $\hbar$ [J6] correspond to a phase in which electro-weak symmetry breaking is absent in the bosonic sector?

This phase would differ from the ordinary one in that the weak charges of dark counterparts of leptons and quarks are not screened in electro-weak length scale but that their masses are very nearly the same as in Higgs=0 phase since the dominant contribution to the masses of elementary fermions is not given by a coupling to Higgs type particle but determined by p-adic thermodynamics [F2, F3]. According to the TGD based model of condensed matter developed in [F9], em charges would be feeded to space-time sheets of order atomic size in this phase.

Does bio-matter involve this kind of phase at larger space-time sheets as chirality selection suggests [F9]? Does this phase of condensed matter emerge only above length scale defined by the cell size or cell membrane thickness?

The possibility to assign separate spectrum of values of M^4 and CP_2 Planck constants means also spectrum of scale factors of metric for both M^4 and CP_2 with scaling of covariant metric given by the square of integer n characterizing the quantum phase. If gravitational Planck constant can be identified as CP_2 Planck constant, gigantic values of CP_2 radius are possible in the sectors of the imbedding space corresponding to the dark matter.

Even if one does not accept this identification, the conclusion would seem to be that CP_2 radius can be very large in these phases. Obviously the ranges of weak and color interactions in this kind of phases would be macroscopic and even astrophysical. Second implication would be the presence of precise quantal lattice like structure involving strict quantum correlations in macroscopic length scales. The unavoidable question is whether the extremely tiny size of CP_2 could be scaled up to a macroscopic length scale even at the level of living matter and whether even the science fictive notion of hyper-space travel (which I have never liked!) might make sense after all.

e) An interesting question relates to the predicted presence of long ranged classical color gauge fields in all length scales suggesting a hierarchy of QCD type physics if quantum classical correspondence is taken seriously. The possibility to define the color Hamiltonians apart from an additive constant in principle makes possible to have vanishing classical color isospin and hyper charges at a given space-time sheet without affecting the color transformation properties of Hamiltonians. It is however far from clear whether this trick is enough. A more natural approach is to take seriously the prediction of infinite p-adic hierarchy of QCD type physics and look what the implications are.

7 General vision about real and p-adic coupling constant evolution

The unification of super-canonical and Super Kac-Moody symmetries allows new view about p-adic aspects of the theory forcing a considerable modification and refinement of the almost decade old first picture about color coupling constant evolution.

Perhaps the most important questions about coupling constant evolution relate to the basic hypothesis about preferred role of primes $p \simeq 2^k$, k an integer. Why integer values of k are favored, why prime values are even more preferred, and why Mersenne primes $M_n = 2^n - 1$ and Gaussian Mersennes seem to be at the top of the hierarchy?

Second bundle of questions relates to the color coupling constant evolution. Do Mersenne primes really define a hierarchy of fixed points of color coupling constant evolution for a hierarchy of asymptotically non-free QCD type theories both in quark and lepton sector of the theory? How the transitions $M_n \rightarrow M_{n(next)}$ occur? What are the space-time correlates for the coupling constant evolution and for for these transitions and how space-time description relates to the usual description in terms of parton loops? How the condition that p-adic coupling constant evolution reflects the real coupling constant evolution can be satisfied and how strong conditions it poses on the coupling constant evolution?

7.1 A general view about coupling constant evolution

7.1.1 Zero energy ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state. Equivalence Principle is expected to hold true for elementary particles and their composites but not for the quantum states defined around non-vacuum extremals.

7.1.2 Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [C2] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [19] known as hyperfinite factor of type II_1

(HFF) [A9, C6, C2]. HFF [20, 26] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [27]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [28], anyons [23], quantum groups and conformal field theories[21, 22], and knots and topological quantum field theories [29, 25].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [C2]. The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. M-matrix is identifiable in terms of Connes tensor product [26] and therefore exists and is almost unique. Connes tensor product implies that the Hermitian elements of the included algebra commute with M-matrix and hence act like infinitesimal symmetries. A connection with integrable quantum field theories is suggestive. The remaining challenge is the calculation of M-matrix and the needed machinery might already exist.

The tension is present also now. The connection with visions should come from the discretization in terms of number theoretic braids providing space-time correlate for the finite measurement resolution and making p-adicization in terms of number theoretic braids possible. Number theoretic braids give a connection with the construction of configuration space geometry in terms of Dirac determinant and with TGD as almost TQFT and with conformal field theory approach. The mathematics for the inclusions of hyper-finite factors of type II_1 is also closely related to that for conformal field theories including quantum groups relating closely to Connes tensor product and non-commutativity.

7.1.3 How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

Zero energy ontology in which zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of future and past directed light-cones are power of 2 multiples of fundamental time scale implies in a natural manner coupling constant evolution.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond

to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

7.2 Both symplectic and conformal field theories are needed in TGD framework

Before one can say anything quantitative about coupling constant evolution, one must have a formulation for its TGD counterpart and thus also a more detailed formulation for how to calculate M-matrix elements. There is also the question about infinities. By very general arguments infinities of quantum field theories are predicted to cancel in TGD Universe - basically by the non-locality of Kähler function as a functional of 3-surface and by the general properties of the vacuum functional identified as the exponent of Kähler function. The precise mechanism leading to the cancellation of infinities of local quantum field theories has remained unspecified. Only the realization that the symplectic invariance of quantum TGD provides a mechanism regulating the short distance behavior of N-point functions changed the situation in this respect. This also leads to concrete view about the generalized Feynman diagrams giving M-matrix elements and rather close resemblance with ordinary Feynman diagrammatics.

7.2.1 Symplectic invariance

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. This section appears already in the previous chapter about symmetries of quantum TGD [C1] but because the results of the section provide the first concrete construction recipe of M-matrix in zero energy ontology, it is included also in this chapter.

7.2.2 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [D8]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1, s_2, s_3))\Phi_m(s)d\mu_s \quad . \tag{44}$$

Here the coefficients c_{kl}^m are constants and $A(s_1, s_2, s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1, s_2, s)) Id d\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1, s_2, s)) d\mu_s \quad . \tag{45}$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n = 1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1, s_2, s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

7.2.3 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \tag{46}$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle(\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle\Phi_r(s)\Phi_m(s_3)\rangle d\mu_s$$
$$= \quad (47)$$
$$c_{kl}^r c_{rm} \int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \quad . \quad (48)$$

   Associativity requires that this expression equals to $\langle\Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle\Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \quad (49)$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments coincide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

7.2.4 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars

obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [C2] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time [E10].

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried

over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge

interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. The coupling constant evolution is based on the same mechanism as in QFT and symplectic invariance replaces ad hoc UV cutoff with a genuine dynamical regulation mechanism. Causal diamond itself defines the physical IR cutoff. p-Adic and real coupling constant evolutions reflect the underlying evolution in powers of two for the temporal distance between the tips of the light-cones of the causal diamond and the association of macroscopic time scale as secondary p-adic time scale to elementary particles (.1 seconds for electron) serves as a first test for the picture. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules could be treated independently.

7.2.5 More detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints [C1].

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight determining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^{\dagger}/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^dagger$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number

is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k\gamma_k^\dagger$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

In this framework coupling constant evolution would have interpretation in terms of addition of intermediate zero energy states corresponding to the generalized Feynman diagrams obtained by the insertion of causal diamonds with a new shorter time scale $T = T_{prev}/2$ to the previous Feynman diagram. p-Adic length scale hypothesis follows naturally. A very close correspondence with ordinary Feynman diagrammatics arises and and ordinary vision about coupling constant evolutions arises. The absence of infinities follows from the symplectic invariance which is genuinely new element. p-Adic and real coupling constant evolutions can be seen as completions of coupling constant evolutions for physics based on rationals and their algebraic extensions.

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