p-Adic Physics: Physical Ideas

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Abstract

The most important p-adic concepts and ideas are p-adic fractality, spin glass analogy, p-adic length scale hypothesis, p-adic realization of the Slaving Principle, p-adic criticality, and the non-determinism of the p-adic differential equations justifying the interpretation of the p-adic space-time regions as cognitive representations. These ideas are discussed in this chapter in a more concrete level than in previous chapters in the hope that this might help the reader to assimilate the material more easily. Some of the considerations might be a little bit out of date since the chapter is written much earlier than the preceding chapters.

a) The criticality of quantum TGD and the need to generalize conformal invariance to the 4-dimensional context were the original motivations of the p-adic approach. It however turned out that quaternion conformal invariance, rather than p-adic conformal invariance for the space-time surface regarded as an algebraic extension of p-adics, is the correct manner to realize conformal invariance. In TGD as a generalized number theory approach p-adic space-time regions emerge completely naturally and have interpretation as cognitive representations of the real physics. If this occurs already at the level of elementary particles, one can understand p-adic physics as a model for a cognitive model about physics provided by Nature itself. The basic motivation for this assumption is the p-adic non-determinism of the p-adic field equations making them ideal for the simulation purposes. The p-adic–real phase transitions are the second basic concept allowing to understand how intention is transformed to action and vice versa: the occurrence of this process even at elementary particle level explains why p-adic length scale hypothesis works. This picture is consistent with the idea about evolution occurring already at the level of elementary particles and allowing the survival of the systems with largest cognitive resources.

b) Spin glass analogy, which was the original motivation for p-adicization before the discovery that p-adic regions of space-time emerge automatically from TGD as a generalized number theory approach, is discussed at configuration space level. The basic idea is that the maximum (several of them are possible) of the exponential of the Kähler function with respect to the fiber degrees of freedom as function of zero modes is p-adic fractal. This together with spin glass analogy suggest p-adic ultra-metricity of the reduced configuration space CH_{red}, the TGD counterpart of the energy landscape.

c) Slaving Principle states that there exists a hierarchy of dynamics with increasing characteristic length (time) scales and the dynamical variables of a given length scale obey dynamics, where the dynamical variables of the longer length (time) scale serve as "masters" that is effectively as external parameters or integration constants. The dynam-

ics of the "slave" corresponds to a rapid adaptation to the conditions posed by the "master". p-Adic length scale hypothesis allows a concrete quantification of this principle predicting a hierarchy of preferred length, time, energy and frequency scales.

d) Critical systems are fractals and the natural guess is that p-adic topology serves also as an effective topology of real space-time sheets in some length scale range and that real non-determinism of Kähler action mimics p-adic non-determinism for some value of prime p. This motivates some qualitative p-adic ideas about criticality.

e) The properties of the CP_2 type extremals providing TGD based model for elementary particles and topological sum contacts, are discussed in detail. CP_2 type extremals are for TGD what black holes are for General Relativity. Black hole elementary particle analogy is discussed in detail and the generalization of the Hawking-Bekenstein formula is shown to lead to a prediction for the radius of the elementary particle horizon and to a justification for the p-adic length scale hypothesis. A deeper justification for the p-adic length scale hypothesis comes from the assumption that systems with maximal cognitive resources are winners in the fight for survival even in elementary particle length scales.

f) Quantum criticality allows the dependence of the Kähler coupling strength on zero modes. It would be nice if α_K were RG invariant in strong sense but the expression for gravitational coupling constant implies that it increases rapidly as a function of p-adic length scale in this case. This led to the hypothesis that G is RG invariant. The hypothesis fixes the p-adic evolution of α_K completely and implies logarithmic dependence of α_K on p-adic length scale. It has however turned out that the RG invariance might after all be possible and is actually strongly favored by different physical arguments. The point is that M_{127} is the largest Mersenne prime for which p-adic length scale is non-super-astronomical. If gravitational interaction is mediated by space-time sheets labelled by Mersenne prime, gravitational constant is effective RG invariant even if α_K is RG invariant in strong sense. This option is also ideal concerning the p-adicization of the theory.

1 Introduction

The basic implication of 'TGD as a generalized number theory' philosophy is that p-adic regions of the space-time surface result dynamically. Space-time surface is defined by the vanishing condition of a rational function of two quaternion-valued variables q_1 and p_1. This condition gives p_1 as a function of q_1. It can however happen that some components of the quaternion p_1 fail to be real numbers and become complex. It might be however possible to perform the completion of the rational space-time surface to a p-adic space-time surface and for some values of the p-adic prime the series defining the power series representing $p_1 = f(q_1)$ can converge to a number in some algebraic extension of the ordinary p-adic numbers. It is also quite possible that p-adic and real power roots $p_1 = f(q_1)$ converge simultaneously. Even more general rational-adic topologies in which norm is a power of a rational number are possible: rational-adic numbers do not however form a ring. p-Adic numbers are thus very closely related with quaternion-conformal invariance and criticality.

p-Adic topologies form an infinite hierarchy and p-adic physics leads to a vision about many-sheeted space-time as a hierarchical structure consisting of p-adic and real space-time sheetse of increasing size and increasing value of prime p. These surfaces are glued together using topological sum or join along boundaries bonds. Contrary to the original expectations, p-adic space-time regions represent 'mind-stuff' rather than 'matter' which is also present and represented by real and infinite-p p-adic regions. Thus p-adic provide 'cognitive representations' for matter like regions and this is why their physics provides a manner to understand real physics. If p-adic-to-real phase transitions are possible, one can understand why it is possible to assign p-adic prime even to real regions. In fact, the hypothesis that p-adic regions provide a cognitive model for real physics, poses very strong constraints on real physics.

There is a "holy trinity" of non-determinisms in TGD in the sense that there is the non-determinism associated with the quantum jumps, the classical non-determinism of the Kähler action and p-adic non-determinism. The non-determinism of quantum jumps can involve also a selection between various multifurcations for various absolute minima of the Kähler action in which case it represents a genuine volitional act. p-Adic non-determinism in turn corresponds to the non-determinism of pure imagination with no material consequences. Also real space-time sheets with finite time duration are also possible and they might represent what might be called 'sensory space-time sheets' as opposed to cognitive space-time sheets. Cognitive space-time sheets can be transformed to real ones in quantum jumps inducing change of control parameters of the polynomial defining space-time surface: if the change is such that the p-adic root is replaced with a real root, one can say that thought is transformed into action. The reverse of this process is the transformation of sensory input into cognition.

"Holy trinity" implies that it should be possible to determine the p-adic prime characterizing a given space-time region (or space-time sheet) by observing a large number of quantum time developments of this system. The characteristic p-adic fractality, that is the presence of time scales $T(p,k) = p^k T_p$, should become manifest in the statistical properties of the cognitive time developments which in should turn reflect the properties of the real physics since cognitive representations are in question. For instance, quantum jumps with especially large amplitude would tend to occur at time scales $T(p,k) = p^k T_p$. $T(p,k)$ could also provide series of characteristic correlation times. Needless to say, this prediction means definite departure from the non-determinism of ordinary quantum mechanics and only at the limit of infinite p the predictions should be identical. An interesting possibility is that $1/f$ noise [22] is a direct manifestation of the classical non-determinism: if this is the case, it should be possible to associate a definite value of p to $1/f$ noise. Also transformations of the p-adic cognitive space-time sheets to real space-time sheets of a finite time duration and vice versa might be involved with the $1/f$ noise so that $1/f$ noise would be a direct signature of cognitive consciousness.

The 'physical' building blocks of p-adic TGD, as opposed to the philosophical mathematical ones briefly summarized above, and in more detail in previous chapters, are spin glass analogy leading to the general picture about how finite-p p-adicity emerges from quantum TGD, the identification of elementary particles as CP_2 type extremals, and elementary particle black hole analogy. These building blocks have been present as stable pieces of theory from beginning whereas philosophical ideas and interpretations have undergone rather wild fluctuations during an almost last decade of p-adic TGD.

2 p-Adic numbers and spin glass analogy

Spin glass phase decomposes into regions in which the direction of the magnetization varies randomly with respect to spatial coordinates but remains constant in time. What makes spin glass special is that the boundary regions between regions of different magnetization do not give rise to large surface energies. Spin glass structure emerges in two manners in TGD framework.

a) Spin glass behavior at the level of real physics is encountered in TGD framework because of the classical non-determinism of the Kähler action. The classical non-determinism of CP_2 type extremals represents the manifestation of the spin glass analogy at the level of elementary particle physics. In macroscopic length scales real physics spin glass analogy makes possible 'real world engineering'.

b) Spin glass behavior at the level of cognition is encountered because of the p-adic non-determinism and makes possible what might be called imagination or 'cognitive engineering'. The point is that any piecewise constant function has a vanishing p-adic derivative. Therefore any function of the spatial coordinates depending on a finite number of the pinary digits is a pseudo constant. The discontinuities of this kind in the field variables do not lead to infinite surface energies in the p-adic context as they would in the real context.

Spin glass energy landscape is characterized by an ultra-metric distance function. The reduced configuration space CH_{red} consisting of the maxima of the Kähler function with respect to quantum fluctuating degrees of freedom and zero modes defines the TGD counter part of the spin glass energy landscape. This notion makes sense only in real context since p-adic space-time regions do not contribute to the Kähler function and all p-adic configurations are equally probable. The original vision was that if the ultra-metric distance function in CH_{red} is induced from a p-adic norm, a connection between p-adic physics and real physics also at the level of space-time might emerge somehow. It seems however that the ultra-metricity of CH_{red} need not directly relate to the p-adicity at the space-time level which can be understood if p-adic space-time regions give rise to cognitive representations of the real regions. Of course, it *might* be that the p-adic prime characterizing cognitive representation of a real region characterizes also the reduced configuration space associated with the region in question (one must of course assume that the reduced configuration space approximately decomposes into a Cartesian product of the reduced configuration spaces associated with real regions).

2.1 General view about how p-adicity emerges

In TGD classical theory is exact part of the quantum theory and in a well defined sense appears already at the level of the configuration space geometry: the definition of the configuration space Kähler metric [B1] associates a unique space-time surface to a given 3-surface. The vacuum functional of the theory (exponent of the Kähler function) is analogous to the exponent $exp(H/T_c)$ appearing in the definition of the partition function of a critical system so that the Universe described by TGD is quantum critical system. Critical system is characterized by the presence of two phases, which can be present in arbitrary large volumes. The TGD:eish counter part of this seems to be the presence of two kinds of 3-surfaces for which either Kähler electric or Kähler magnetic field energy dominates. These 3-surfaces have outer boundaries for purely topological reasons and these boundaries can be of a macroscopic size. Therefore it seems that 3-space should be regarded as what could be called topological condensate with a hierarchical, fractal like structure: there are 3-surfaces (with boundaries) condensed on 3-surfaces condensed on...... .

This leads to a radically new manner to see the world around us. The outer surfaces of the macroscopic bodies correspond to the boundaries of 3-surfaces in the condensate so that one can see the 3-topology in all its complexity just by opening one's eyes! A rather compelling evidence for the basic ideas of TGD if one is willing to give up the nebulous concept of "material object in topologically trivial 3-space" and to allow nontrivial 3-topology in macroscopic length scales. A second rather radical departure from the conventional picture of the 3-space is that TGD:eish 3-space is not connected but contains arbitrary many disjoint components. In fact the actual Universe should consist of infinitely many 3-surfaces condensed on each other.

In two-dimensional critical systems conformal transformations act as symmetries and conformal invariance implies the Universality of critical systems. This suggests that one should try to find the generalization of the conformal invariance to higher dimensional, in particular, 4-dimensional case. If finally turned out that quaternion-conformal invariance realizes quantum criticality four 4-surfaces imbedded to 8-dimensional space. As a by product an explanation for space-time and imbedding space dimensions results.

In this approach the p-adic regions of the space-time surface result dynamically. Space-time surface is defined by the vanishing condition of a polynomial of two quaternion-valued variables q and p. This condition gives p as a function of q. It can however occur that some components of p become complex numbers. They must be however real so that the solution fails to exist in the real sense. It might be however possible to perform the completion of the rational space-time surface to a p-adic space-time surface and for some values of the p-adic prime the series defining the power series representing $p = f(q)$ might converge to a number in some algebraic extension of the ordinary p-adic numbers. Even more general rational-adic topologies in which norm is power of a rational number are possible. p-Adic numbers would thus be very closely related with quaternion-conformal invariance and criticality.

p-Adic topologies form an infinite hierarchy and p-adic physics leads to a vision about many-sheeted space-time as a hierarchical structure consisting of p-adic 4-surfaces of increasing size and increasing value of prime p. These surfaces are glued together using topological sum operation. Contrary to the original expectations, this hierachy is the hierarchy for the regions of space-time representing 'mind-stuff' rather than 'matter' which is also present and represented by real and infinite-p p-adic regions. p-Adic provide 'cognitive representations' for matterlike regions and this is why their physics provides a manner to understand real physics.

2.2 p-Adic numbers and the analogy of TGD with spin-glass

The vacuum degeneracy of the Kähler action leads to precise spin glass analogy at the level of the configuration space geometry and the generalization of the energy landscape concept to the TGD:eish context leads to the hypothesis about how p-adicity is realized at the level of the configuration space. Also the concept of p-adic space-time surface emerges rather naturally.

2.2.1 Spin glass briefly

The basic characteristic of the spin glass phase [23] is that the direction of the magnetization varies spatially, being constant inside a given spatial region, but does not depend on time. In the real context this usually leads to large surface energies on the surfaces at which the magnetization direction changes. Regions with different direction of magnetization clearly correspond non-vacuum regions separated by almost vacuum regions. Amusingly, if 3-space is effectively p-adic and if magnetization direction is p-adic pseudo constant, no surface energies are generated so that p-adics might be useful even in the context of the ordinary spin glasses.

Spin glass phase allows a great number of different ground states minimizing the free energy. For the ordinary spin glass, the partition function is the average over a probability distribution of the coupling constants for the partition function with Hamiltonian depending on the coupling constants. Free energy as a function of the coupling constants defines 'energy landscape' and the set of free energy minima can be endowed with an ultra-metric distance function using a standard construction [24].

2.2.2 Vacuum degeneracy of the Kähler action

The Kähler action defining configuration space geometry allows enormous vacuum degeneracy: any four-surface for which the induced Kähler form vanishes, is an extremal of the Kähler action. Induced Kähler form vanishes if the CP_2 projection of the space-time surface is Lagrange manifold of CP_2: these manifolds are at most two-dimensional and any canonical transformation of CP_2 creates a new Lagrange manifold. An explicit representation for

Lagrange manifolds is obtained using some canonical coordinates P_i, Q_i for CP_2: by assuming

$$P_i = \partial_i f(Q_1, Q_2) \ ,$$

where f arbitrary function of its arguments. One obtains a 2-dimensional sub-manifold of CP_2 for which the induced Kähler form proportional to $dP_i \wedge dQ^i$ vanishes. The roles of P_i and Q_i can obviously be interchanged. A familiar example of Lagrange manifolds are $p_i = constant$ surfaces of the ordinary (p_i, q_i) phase space.

Since vacuum degeneracy is removed only by classical gravitational interaction there are good reasons to expect large ground state degeneracy, when system corresponds to a small deformation of a vacuum extremal. This degeneracy is very much analogous to the ground state degeneracy of spin glass.

2.2.3 Vacuum degeneracy of the Kähler action and physical spin glass analogy

Quite generally, the dynamical reason for the physical spin glass degeneracy is the fact that Kähler action has a huge vacuum degeneracy. Any 4-surface with CP_2 projection, which is a Legendre sub-manifold (generically two-dimensional), is vacuum extremal. This implies that space-time decomposes into non-vacuum regions characterized by non-vanishing Kähler magnetic and electric fields such that the (presumably thin) regions between the the non-vacuum regions are vacuum extremals. Therefore no surface energies are generated. Also the fact that various charges and momentum and energy can flow to larger space-time sheets via wormholes is an important factor making possible strong field gradients without introducing large surfaces energies. From a given absolute minimum of Kähler action one obtains a new one by adding arbitrary space-time surfaces which is vacuum exremal. Uniqueness of the absolute minima in the sense that real regions of space-time $X^4(X^3)$ are unique could be acieved by requiring that vacuum regions are p-adic and represent thus cognitive regions whereas real regions carry non-vanishing induced Kähler field.

The canonical invariance of the Kähler action for vacuum extremals allows a further understanding of the vacuum degeneracy. The presence of the classical gravitational interaction spoils the canonical group $Can(CP_2)$ as gauge symmetries of the action and transforms it to the isometry group of CH. As a consequence, the $U(1)$ gauge degeneracy is transformed to a spin glass type degeneracy and several, perhaps even infinite number of maxima of Kähler function for given values of the zero modes, become possible. Thus locally, the space maxima of Kähler function should look like a union of copies of the space of zero modes. Given sheet has naturally as its boundary the 3-surfaces for which two maxima of the Kähler function coalesce or are created from single maximum by a cusp catastrophe. In catastrophe regions there are several sheets and the value of the maximum Kähler function determines which give a measure for the importance of various sheets. The quantum jumps selecting one of these sheets can be regarded as phase transitions.

In TGD framework classical non-determinism forces to generalize the notion of the 3-surface by replacing it with a sequence of space like 3-surfaces having time like separations such that the sequence characterizes uniquely one branch of multifurcation. This characterization works when non-determinism has discrete nature. For CP_2 type extremals which are bosonic vacua, basic objects are essentially four-dimensional since M^4_+ projection of CP_2 type extremal is random light like curve. This effective four-dimensionality of the basic objects makes it possible to topologize Feynman diagrammatics of quantum field theories by replacing the lines of Feynman diagrams with CP_2 type extremals.

In TGD framework spin glass analogy holds true also in the time direction, which reflects the fact that the vacuum extremals are non-deterministic. For instance, by gluing vacuum extremals with a finite space-time extension (also in time direction!) to a non-vacuum extremal and deforming slightly, one obtains good candidates for the degenerate absolute minima. This non-determinism is expected to make the absolute minima of the Kähler action highly degenerate. The construction of S-matrix at the high energy limit suggests that since a localization selecting one degenerate maximum occurs, one must accept as a fact that each choice of the parameters corresponds to a particular S-matrix and one must average over these choices to get scattering rates. This averaging for scattering rates corresponds to the averaging over the thermodynamical partition functions for spin glass. A more general is that one allows final state wave functions to depend on the zero modes which affect S-matrix elements: in the limit that wave functions are completely localized, one ends up with the simpler scenario.

The real effective action is expected to be Einstein-Yang-Mills action for the induced gauge fields. This action does not possess any vacuum degeneracy. The space-time surfaces are certainly absolute minima of the Kähler action and EYM-action could take a dynamical role only in the sense that extremality with respect to classical part of EYM action selects one of the degenerate absolute minima of the Kähler action. On the other hand, the construction of S-matrix suggests that the choice of particular parameter values characterizing zero modes affects only the coupling constants and propagators of the effective Einstein-Yang-Mills theory, and that one must perform averaging over the predictions of these theories. Thus EYM action could at most fix a gauge.

2.2.4 p-Adic non-determinism and spin glass analogy

One must carefully distinguish between cognitive and physical spin-glass analogy. Cognitive spin-glass analogy is due to the p-adic non-determinism. p-Adic pseudo constants induce a non-determinism which essentially means that p-adic extrema depend on the p-adic pseudo constants which depend on a finite number of positive pinary digits of their arguments only. Thus p-adic extremals are glued from pieces for which the values of the integration constants are genuine constants. Obviously, an optimal cognitive representation is achieved if pseudo constants reduce to ordinary constants.

More precisely, any function

$$\begin{aligned} f(x) &= f(x_N) \ , \\ x_N &= \sum_{k\leq N} x_k p^k \ , \end{aligned} \tag{1}$$

which does not depend on the pinary digits x_n, $n > N$ has a vanishing p-adic derivative and is thus a pseudo constant. These functions are piecewise constant below some length scale, which in principle can be arbitrary small but finite. The result means that the constants appearing in the solutions the p-adic field equations are constants functions only below some length scale. For instance, for linear differential equations integration constants are arbitrary pseudo constants. In particular, the p-adic counterparts of the absolute minima (defined by the correspondence with infinite primes) are highly degenerate because of the presence of the pseudo constants. This in turn means a characteristic randomness of the spin glass also in the time direction since the surfaces at which the pseudo constants change their values do not give rise to infinite surface energy densities as they would do in the real context.

The basic character of cognition would be spin glass like nature making possible 'engineering' at the level of thoughts (planning) whereas classical non-determinism of the Kähler action would make possible 'engineering' at the level of the real world.

2.2.5 Localization in zero modes

The Kähler function defining configuration space metric possesses infinite number of zero modes which represent non-quantum-fluctuating degrees of freedom. The requirement that physics is local at the level of zero modes implies that each quantum jump involves a localization in zero modes. This localization could be complete or in a region whose size is determined by the p-adic length scale hypothesis.

Localization would mean an enormous calculational simplification: functional integral reduces into ordinary functional integral over the quantum-fluctuating degrees of freedom and there is no need to integrate over the zero modes. The complete or partial localization in zero modes would explain why the world of conscious experience looks classical. Perhaps the complete localization is however too much to wish for: it could however be that one must use wave functionals in the zero modes only in the case that one is interested in a comparison of the transition rates associated with different values of zero modes rather than in transition rates with the condition that a localization has occurred to definite values of zero modes.

The functional integral over the fiber degrees of freedom can be approximated by a Gaussian integrals around maxima. Classical non-determinism would suggest the possibility of several maxima in fiber degrees of freedom but the symmetric space property of the fiber suggests that there is only single maximum of Kähler function. The existence of single maximum gives good hopes that the configuration space integration reduces effectively to Gaussian integration of free field theory.

2.3 The notion of the reduced configuration space

Quantum jumps occur with highest probability to those values of zero modes which correspond to the maxima of the Kähler function and a simplified description of the situation is obtained by considering the reduced configuration space CH_{red} consisting of the maxima of Kähler function with respect to both zero modes and and quantum fluctuating degrees of freedom.

The hypothesis that the space CH_{red} is an enumerable set is a natural first guess. In macroscopic length scales, one might indeed hope that the generation of Kähler electric fields reducing the vacuum degeneracy could imply a discrete degeneracy for the maxima of the Kähler action.

In elementary particle length scales this hypothesis fails and it is good to analyze the situation in more detail since it gives some about how complex the situation can be. For the so called CP_2 type extremals the classical non-determinism gives rise to a functional continuum of degenerate maxima of the Kähler function. The degenerate maxima correspond to random zitterbewegung orbits for which the 'time parameter' u is an arbitrary function of CP_2 coordinates. In this case however zero modes characterizing light like random curve representing the zitterbewegung orbit behave exactly like conformal gauge degrees of freedom. The choice of the 'time parameter' u however affects S-matrix elements: dependence is very weak and only through the volumes of the propagator lines determined by the selection of u (Kähler action for CP_2 type extremal is proportional to its volume) occurring in quantum jump. Effectively the functional continuum is replaced with the real continuum of the volume of the propagator line varying from zero to the volume of CP_2.

A localization for the positions of the vertices of the Feynman diagrams defined by CP_2 type extremals cannot however be assumed. Neither can one assume that only single Feynman diagram is selected if one wants that a generalization of ordinary Feynman diagrammatics results. There are several alternative identifications.

a) The degrees represented by Feynman diagrams with varying positions of vertices represent fiber degrees of freedom so that there would be slight dependence of the Kähler function on the positions of the vertices. Certainly the Feynman diagrams with different topologies have different value of Kähler action and must correspond to fiber degrees of freedom. The reason is that vertex regions of the Feynman diagrams must involve deformations of CP_2 extremals since otherwise Feynman diagrams are singular as 4-manifolds. Note that the idea about localization in fiber degrees of freedom is not favored by this example.

b) The positions for the vertices of the Feynman diagram are excellent candidates for zero modes and localization is not possible now. The fact that these degrees of freedom correspond to center of mass degrees of freedom related to the isometries of the theory might distinguish between them and other zero modes. One can consider also a refinement for localization in the zero modes hypothesis: localization occurs only in length scale resolution defined by the p-adic length scale. In fact, the assumption that CP_2 type extremals have suffered topological condensation on space-time sheets with size of order p-adic length scale characterizing the elementary particle implies this.

Whether the notion of CH_{red} makes sense for the p-adic space-time regions is not at all obvious. For the proposed construction of the configuration space metric p-adic regions do not contribute to the Kähler function which is real-valued. Only in case that the p-adic contribution is rational number, it could be interpreted as a real valued contribution to the Kähler function. In case of CP_2 type extremals this is not the case although the exponent of the Kähler function for a full CP_2 type extremal is a rational number if the proposed model for the p-adic evolution of Kähler coupling strength is correct. If it does not make sense to distinguish between the maxima of the Kähler function in the p-adic context, one cannot define CH_{red} on basis of this criterion. From the point of view of cognition this means maximal freedom of imagination.

An interesting question is whether one must count the cognitive degeneracy as a degeneracy of physical states. If localization occurs in each quantum jump with respect to both real and p-adic zero mode degeneracy, and if all cognitive options are equally probable, then the only conclusion seems to be that space-time surfaces for which the cognitive degeneracy is highest, represent the most probable final states. This would mean that the systems with the highest cognitive resources would be winners in the struggle for survival. An alternative manner to see the same thing is that systems with a high cognitive degeneracy are able to undergo a rich repertoire of p-adic-to-real phase transitions and thus to adapt with the environment.

2.3.1 Explicit definition of the ultra-metric distance function for energy landscape

The points of CH_{red} are completely analogous to the minima of the free energy and the precise analogy with spin glass suggests that CH_{red} must possess naturally an ultra-metric topology. One can quite generally construct an explicit ultra-metric distance function for the set of energy minima in a given energy landscape describing energy as a function of the coordinates of some configuration space using existing recipes [25]. The concept is useful when the energy landscape has fractal like structure. An attractive metaphor is to regard energy as a height function for a landscape with mountains.

The distance function between two energy minima should describe the difficulty of getting from a given minimum to another one. A concrete measure for this difficulty is obtained by considering all possible paths from x to y. The height for the highest point on this path, absolute maximum $h_{max}(\gamma)$ of the height function on this path gives the measure for the difficulty for reaching y along the path γ. There exists some easiest path from x to y. The difficulty to reach y from x can be defined as the height of the highest point associated with the easiest path and hence the minimum of $h_{max}(\gamma)$ in the set of all possible paths from x to y:

$$d(x,y) = Min(h_{max}(\gamma(x,y)) \ .$$

It is easy check that this distance function is ultra-metric:

$$d(x,z) \leq Max\{d(x,y), d(y,z)\} \ .$$

All what is needed is to notice that for any path $x \rightarrow z$ going through y highest point of the path is either the highest point associated with the path from $x \rightarrow y$ or $y \rightarrow z$: from this the inequality follows trivially since one can in principle find also easier paths.

2.3.2 Identification of the height function in the case of the reduced configuration space?

Obviously the negative for the maximum of Kähler function as function of zero modes is the counterpart of free energy. This function could well be many valued but this is an unessential complication. It is not clear whether K is negative definite (there are strong reasons to believe that this is the case). One can however consider any positive definite function of K as a height function defining an ultra-metric norm in the manner suggested. The requirement that p-adic norm results should fix the definition uniquely.

The exponential $exp(-K_{max})$ of the maximum of Kähler function as function of the zero modes, which is the inverse for the vacuum functional of the theory, is the first guess for the height function defining the ultra-metric norm (the wandering from 3-surface X^3 to Y^3 corresponds to quantum tunnelling physically.). The justification for this identification is that the integration over the fiber degrees of freedom gives Gaussian determinant cancelling the metric determinant and leaves on the exponent of Kähler function to the functional integral over zero modes. The intuitive expectation is that ultra-metric norm is p-adic for some p and that the space of zero modes decomposes into regions D_p In order to get a power of p as required by p-adicity, one can expand h as powers of p and identify p-adic norm as p^n for the highest pinary digit n with non-vanishing coefficient.

The height function can have a normalization factor and this factor could be chosen so that the ultra-metric norm is a power of p for CP_2 type extremals, which are certainly very important building blocks of absolute minimum space-time surfaces. The argument relating the gravitational coupling constant to the Kähler coupling strength and fixing the dependence of the Kähler coupling strength on the prime p, suggests that one must define the height function as

$$h_p = \frac{exp(-K(p))}{exp(-K(p=1))} \ ,$$

where the Kähler function at $p = 1$ is formally obtained by regarding the value of the Kähler coupling strength as a function in the set of all natural numbers.

2.3.3 Does the proposed height function h_p define p-adic topology?

The great question is whether one can obtain p-adic ultra-metricity in this manner. There is some evidence for this.

a) Criticality and spin glass analogy suggests that $exp(K)$ as a function of zero modes is fractal. If it is p-adic fractal then p-adic topology is expected to be a natural consequence: in this case the map of CH_{red} to its p-adic counterpart could make it possible to replaced CH_{red} with a smooth function.

b) CP_2 type extremals, the counterparts of black holes and a model of elementary particle in TGD, have finite negative Kähler action. One can glue CP_2 type extremals to any space-time surface to lower the Kähler action. 3-surfaces Z^3 on path from X^3 to Y^3 containing CP_2 extremals on $X^4(Z^3)$ are excellent candidates for 'mountains' in the landscape metaphor. The height of Z^3 is roughly described by the number of CP_2 type extremals glued on $X^4(Z^3)$.

c) The argument leading to a correct prediction of gravitational constant in terms of assuming that Kähler coupling strength α_K depends on zero modes only through the p-adic prime assumed to characterize a given region D_p of the configuration space for which the set of maxima of Kähler function as function of zero modes should obey has p-adic topology. The crucial input is the relationship

$$exp(K_p(CP_2))\frac{R^2}{G} = \frac{1}{p} \ ,$$

which is equivalent with $G = exp(K_p(CP_2)L_p^2$, where $L_p \simeq \sqrt{p} \times R$ is the p-adic length scale and $R \simeq 10^4\sqrt{G}$ is CP_2 size and the fundamental p-adic length scale. This formula is a dimensional estimate for gravitational coupling strength in terms of the p-adic length scale squared and the exponential of Kähler function for CP_2 type extremal describing graviton. The exponent gives the probability for the appearance of one virtual graviton in a given quantum state. The probability is very small since the exponent is negative for CP_2type extremal and gravitation is consequently a very weak interaction.

d) If one makes the identification

$$\frac{R^2}{G}(\sim 10^8) = exp(-K_{p=1}),$$

then the function

$$h_p = \frac{exp(-K_p)}{exp(-K_{p=1})}/$$

is the n:th power of p for a vacuum extremal to which n CP_2 type extremals are glued. This is just the p-adic norm p^n! If h_p were p^n-valued in the general case it would be a p-adic pseudo constant and rather tame as a fractal. Very probably, this is not true in the general case and the p-adic norm of the p-adic counterpart of h_p in the canonical identification

$$N_p \equiv |Id(h_p)|_p \ ,$$
$$Id(\sum x_n p^n) = \sum_n x_n p^{-n} \ .$$

depending on the most significant pinary digit of h_p only, is a good candidate for a p-adically ultra-metric height function having also a correct normalization. In any case, it seems that the number of virtual CP_2 type extremals (gravitons!) glued to an absolute minimum space-time surface $X^4(X^3)$ could define the height function. p-Adicity would emerge naturally and would have a direct physical meaning. Of course, this identification works for $n \geq 0$ only: the physical interpretation of the p-adic norm in $n < 0$ case is open.

A possible interpretation in terms of virtual graviton emission suggests the interpretation of the factor $\frac{R^2}{G} = exp(-K_{p=1})$ as a Gaussian determinant $\sqrt{det_G}$ associated with the integration over the zero modes around the maximum. The definition of Gaussian determinant in the real context is problematic and p-adicization plus adelic decomposition of the functional integral might provide a precise definition of $\sqrt{det_G}$. The divergence of the Gaussian determinant in the real context would lead to the vanishing of the gravitational constant. This picture is in accordance with the assumption that gravitational constant does not appear in quantum TGD as a fundamental constant and that the curvature scalar term in the low energy effective action essentially results from radiative corrections and hence derives from the logarithm of det_G.

3 p-Adic numbers and quantum criticality

TGD Universe is quantum critical in the sense that the value of Kähler coupling constant is completely analogous to critical temperature. Therefore the obvious question is how p-adicity might relate to quantum criticality.

3.1 Connection with quantum criticality

p-Adicization of the reduced configuration space relates in an interesting manner to quantum criticality. At quantum criticality the number of the absolute minima of Kähler action for a surface Y^3 belonging to light cone boundary measures the cognitive resources of this surface and of its diffeomorphs. N_d is assumed to behave as $N_d \sim exp(-K_{cr})$, where Kähler function is evaluated for the critical value α_{cr} of the Kähler coupling strength. α_{cr} is like Hagedorn temperature appearing in the thermodynamics of strings. Above α_{cr} the theory might not be mathematically well defined since (at least real) the sum over the configuration space integrals associated with the maxima of Kähler function would diverge exponentially at the limit when the value of Kähler function increases. In string thermodynamics this corresponds to the growth of number $g(E)$ of the states of given energy more rapidly than the inverse of the Boltzmann factor $exp(-E/T_H)$. Below α_{cr} the theory is certainly well defined but in TGD framework the cognitive resources of the Universe would not be maximal since vacuum functional would differ significantly from zero for very few space-time surfaces only. At quantum criticality the situation is optimal but it is not clear whether the real theory makes sense at quantum criticality: at least in string thermodynamics the partition function diverges also at Hagedorn temperature.

The cognitive resources of p-adic space-time sheet are measured by the entropy type quantity $log(N_d)/log(2)$ having lower bound $log(p)/log(2)$ bits for the 3-surfaces allowed by the vacuum functional. For instance, the maximal cognitive resources of electronic space-time sheet ($M_{127} = 2^{127} - 1$) would be 127 bits. In TGD one must allow even infinite primes and for these cognitive resources can be literally infinite.

3.2 Geometric description of the critical phenomena?

The idea that critical systems might have a geometric description is not new. There is a lot of evidence that simple, purely geometric lattice models based on the bond concept reproduce same critical exponents as the thermal models [20]. The probability for a bond to exist corresponds to temperature in these models. For example, in a bond percolation model it is possible to relate the critical exponents to various fractal dimensions. This provides a nice manner to reduce the problem of predicting critical temperature to that of predicting the critical probability for the bond. This problem is local and once the temperature dependence of the bond probability and critical bond probability are known one can calculate the critical temperature.

What is new that in TGD approach the concept of bond ceases to be a phenomenological concept related to the simple modelling of the critical systems. TGD predicts that the boundaries of 3-surfaces can have arbitrarily large sizes. Furthermore, the formation of the join along boundaries bonds connecting the boundaries of two disjoint 3-surfaces seems to provide the basic mechanism for the formation of macroscopic quantum systems with long range correlations. This means that phase transitions should basically correspond to changes in the connectedness of the boundary of the 3-space. The description of the super fluidity, super conductivity and Quantum Hall effect based on the join along boundaries bond concept is suggested in [D7, E9] and also other phase transitions might be describable in the same manner. In hadronic length scale join along boundaries bonds correspond to color flux tubes connecting valence quarks. In nuclear length scale the short range part of the nuclear force corresponds to the formation of join along boundaries bonds between nucleons.

p-Adic approach suggests a concrete description for the phase transition changing the connectedness of the 3-surface. Disjoint 3-surfaces are labelled by p-adic numbers, whose p-adic expansion does not contain powers p^n with $n > N$, where N is some finite integer: the larger the value of N the larger the degree of disjointness. This means that phase transitions (say evaporation or condensation) changing the connectedness of the 3-surface should correspond to transitions changing the value of N. In evaporation process N increases and in condensation process N decreases. Also catastrophic processes like the breaking of a solid object to pieces might correspond to increase in N. Typical self organization processes such as biological growth and healing might correspond to a gradual decrease of N.

Fractal like configurations with a discrete scale invariance are known to play important role in the description of the critical phenomena: they are the most probable configurations at the critical point. The idea that fractal corresponds to a fixed point of a discrete scaling transformation, is in accordance with the definition of the fractals as fixed points for a set of affine transformations acting on subsets of some metric space [21]. A natural candidate for the discrete scaling transformation is the transformation of the 4-surface induced by the multiplication of the p-adic argument

Z of H-coordinate $h(Z)$ by a power of p: $Z \to p^n Z$. A tempting idea is that most probable 3-spaces indeed are invariant under these scalings. This even suggests that something, which might be called "Mandelbrot cosmology", might provide a description of the Universe in all length scales as a 4-dimensional analog of Mandelbrot set. The breaking of the discrete scaling invariance is bound to occur, when one considers finite subsystem instead of the whole Universe. p-Adic cutoff might provide an elegant description for the breaking of the exact scaling invariance: 3-surface in question depends on finite number of the pinary digits of Z only.

3.3 Initial value sensitivity and p-adic differentiability

Initial value sensitivity is one of the basic properties of the critical systems and implies unpredictability in practice. p-Adic differentiability seems to be related to this property in a very general manner. Consider a configuration of an initial value sensitive system, which can possess very high dimension. For definiteness, assume that the dynamics is described by some differential equations, which can be reduced to equations of first order for the configuration space coordinates X (we do not bother to write indices):

$$\frac{dX}{dt} = J(X) \ . \tag{2}$$

Space-time coordinate is a p-adic number one can assume that time coordinate is a p-adic number, too.

The purely p-adic feature of this differential equation follows from the fact that any function depending on a finite number of pinary digits of a p-adic number possesses a vanishing p-adic derivative! This implies that the integration constants are not just ordinary constants but functions of the p-adic number t depending on finite number of pinary digits of t! Obviously this implies classical non-determinism in long time scales! One can construct solutions of the differential equation in the form $X(t) = X_0(t) + X_1(t)$, where $X_0(t)$ depends on a finite number of pinary digits of the p-adic time t and equations reduce to

$$\frac{dX_1}{dt} = J(X_0 + X_1) \ . \tag{3}$$

Of course, one must be careful in defining what "finite number of pinary digits" means, when p-adic cutoff is actually present. The simplest integration constants depend on the p-adic norm of t (or on the lowest pinary digit of t) only.

The result is in accordance with the so called Slaving Principle [18]. One can think that the dynamics in long time scales (low pinary digits of p-adic number t) is given by the integration constants having arbitrary dependence on these pinary digits and the dynamics in short length scales is determined by the differential equations in the "background" given by these time dependent integration constants.

Initial value sensitivity implies effectively non-deterministic behavior and p-adic numbers perhaps provide a possibility to describe it properly. The properties of the Kähler function suggests that the classical non-determinism might be in fact actual. The point is that the classical space time surface associated with a given 3-surface need not be unique. This surface is determined as an absolute minimum of the so called Kähler action and Kähler action possesses enormous vacuum degeneracy [D1]: the most general vacuum extremal has 2- dimensional CP_2 projection, which is so called Lagrange manifold possessing a vanishing induced Kähler form. Canonical transformations and $Diff(M^4)$ act as exact dynamical symmetries of the vacuum extremals and $Diff(M^4)$ contains p-adically analytic transformations of M^4 as subgroup. It might well happen that those absolute minima, which are obtainable as small deformations of the vacuum extremals inherit the characteristic degeneracy of the vacuum extremals.

The classical macroscopic non-determinism might be essential to the possibility of the quantum measurements. In TGD the state function reduction is described as 'jump between histories' that is two deterministic time developments [H1]. In quantum measurement microscopic and macroscopic system are strongly correlated and microscopic transition induces a phase transition like phenomenon in a macroscopic critical system. The general belief is that quantum effects become unimportant in macroscopic systems. The situation need not be this if macroscopic system is critical, or even non-deterministic.

In the TGD inspired theory of 'thinking systems', conscious thoughts correspond to quantum jumps selecting one of the possible time developments in the quantum superposition of several quantum average effective space-time times allowed by the non-determinism. p-Adic pseudo constants could provide a mathematical description for this non-determinism. These 'cognitive' quantum jumps are certainly involved with a realistic description of a quantum measurement modelling also the presence of the observer quantum mechanically.

In turns out that quantum non-determinism, classical non-determinism of Kähler action and p-adic non-determinism are very closely related in quantum TGD: one could even speak of a holy trinity of non-determinisms.

Quantum non-determinism corresponds closely to the classical non-determinism of Kähler action: quantum jumps select between various branches of the branches of multifurcations of classical space-time surface. The p-adic counterparts of these branches are in turn obtained by varying pseudo constants in the solution of the p-adic Euler-Lagrange equations for the Kähler action: this requirement in fact makes it possible to assign unique p-adic prime to a given, sufficiently small space-time region.

3.4 There are very many p-adic critical orbits

An interesting connection between the p-adicity and initial value sensitive systems is related to the possibility to replace also the configuration space (possibly infinite dimensional) with an algebraic extension of the p-adic numbers. The underlying motivation is the need to get a proper mathematical description of the finite accuracy for the observables and p-adic cutoff provides this description.

This in turn suggests Universality in some aspects of the dynamical behavior. The dynamical equations $dX/dt = J(X)$ define a flow that is a diffeomorphism $X \rightarrow F(X,t)$ of configuration space. This flow contains as integration constants arbitrary functions of the p-adic time coordinate t depending on a finite number of pinary digits of t so that classical non-determinism is present. By p-adic conformal invariance this diffeomorphism ought to be p-adically analytic map that is representable as a power series of the algebraically extended p-adic numbers x and t.

The p-adic analyticity of the dynamic diffeomorphism gives strong constraints on the properties of the dynamic map. A particularly interesting map is in this respect Poincare map. One can ask several interesting questions. How does the Universal behavior of one- dimensional and 2-dimensional analytic iterated maps generalize to the p-adic case? What do attractors look like? What are the counterparts of Julia set and Mandelbrot set? What about routes to chaos? Could p-adic hypothesis provide deeper explanation for the fact that period doubling seems to be a rather general mechanism for the transition to turbulence. It might be possible to answer these questions since p-adic analyticity is very strong constraint on the behavior of the maps.

Already the study of the simplest p-adic complex maps reveal some surprises. The simplest map to study is the map $Z \rightarrow Z^n$ for any extension of p-adic numbers (dimension is arbitrary!). The repeller consists of the points p-adic norm equal to one. Due to the roughness of the p-adic topology, the real counterpart of the repeller is of same dimension as the configuration space itself so that the critical orbits form a set with a non-vanishing measure! For example, in the 2-dimensional case and for the 2-adic extension, the set of the critical orbits corresponds in the real plane to a square $(1/2, 1] \times (1/2, 1]$.

How do the small deformations of $Z \rightarrow Z^n$ of form $Z \rightarrow Z^n + \epsilon Z^m$ affect the set of the critical orbits? If the norm of the parameter ϵ is sufficiently small, the previous repeller belongs to the repeller also now. Also new points can appear in repeller. These considerations suggest that the repellers/attractors of the p-adically analytic maps have rather simple structure as compared to their real and complex counter parts. An interesting possibility is that in general case these sets are fractal like objects resembling the fractals associated with p-adic order parameters.

The fact that set of critical orbits is n-dimensional rather than $(n-1)$ or lower-dimensional in the p-adic case suggests an interesting physical interpretation in accordance with the general idea that p-adic topology corresponds to criticality. In ordinary situation these orbits are not very interesting because a small deformation spoils their criticality. In p-adic case the situation is different since the critical orbits are meta-stable and their are very many of them. In TGD one can even identify good candidates for the set of of these meta-stable critical orbits as small deformations of the vacuum extremals of the Kähler action. Needless to emphasize, this vacuum degeneracy is a phenomenon not encountered in the standard field theories.

4 p-Adic Slaving Principle and elementary particle mass scales

The understanding of the elementary particle mass scales is a fundamental problem in the unified field theories. The attempts to understand the generation of the mass scales dynamically have not been successful. The basic problem is the fine tuning difficulty: the predicted mass scale hierarchy is not stable under the small changes of the model parameters. A possible explanation for the failure is that the fundamental mass scales are really fundamental and therefore cannot depend on the details of the dynamical model.

Criticality is known to imply Universality and criticality indeed is the fundamental property of Kähler action. Therefore the derivation of the elementary particle length scale(s) should be based on a proper formulation of the criticality concept. p-Adic numbers indeed provide a promising tool in this respect and the following arguments show that it is possible not only to understand some general elementary particle length scale but leptonic, hadronic and intermediate gauge boson length scales plus a small number of shorter length scales in terms of primes near prime powers of two. The most important length scales correspond to Mersenne primes: there are only sixteen Mersenne primes below electron length scale and the remaining Mersenne primes correspond to super astronomical length scales.

What is nice that the p-adic hypothesis makes possible to express these length scales as square roots of Mersenne primes and possibly Fermat primes, that is prime numbers of type $p = 2^m \pm 1$. What is amusing is that Mersenne primes are closely related to the so called Perfect Numbers $n = 2^{m-1}(2^m - 1)$ representable not only as a product of their prime factors but also as a sum of their proper divisors. The ancient number mystics believed that this property makes these numbers very exceptional in the World Order!

4.1 p-Adic length scale hypothesis

p-Adic length scale hypothesis has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales $L_p = \sqrt{p}l$, $l = 1.376 \cdot 10^4 \sqrt{G}$ are fundamental length scale at p-adic condensate level p. The original interpretation of the hypothesis was following:

a) Above the length scale L_p p-adicity sets on and effective course grained space-time topology is p-adic rather than ordinary real topology.

b) The length scale L_p serves as a p-adic length scale cutoff for the field theory description of particles. This means that space-time begins to look like Minkowski space so that quantum field theory $M^4 \rightarrow CP_2$ becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces dominate.

c) It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime p there corresponds a cutoff length scale L_p above which p-adic quantum field theory $M^4 \rightarrow CP_2$ makes sense and one has a hierarchy of p-adic quantum field theories. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering $< p_1 < p_2 < ...$ means that only the surface $p_1 < p_2$ can condense on the surface p_2. The condensed surface can in practice be regarded as a point like particle at level p_2 described by the p-adic conformal field theory below length scale L_{p_2}.

The work with p-adic QFT has however demonstrated that the hypothesis a) and b) are probably wrong and the following interpretation is closer to the truth.

a) The length scale $L_p = \sqrt{p}l$ defines an *infrared* cutoff rather than ultraviolet cutoff for a p-adic quantum field theory formulated in terms of quarks and leptons and gauge bosons. For instance, for hadrons this length scale is of order hadron size and L_p defines UV cutoff for possibly existing field theory describing hadrons as basic objects. Above L_p real topology effectively replaces the p-adic one (real continuity implies p-adic continuity) and if length scale resolution L_p is used real physics is excellent approximation.

b) p-Adic QFT is free of UV divergences with any UV cutoff and there is no need to assume that p-adicity fails below some length scale. Rather, p-adicity is completely general property of the effective quantum average space-time defined by the Quantum TGD, which is based on the real number field. The concept of the effective space-time, or topological condensate, is in turn necessary for the formulation of field theory limit of TGD. The analogy of Quantum TGD with spin glass phase gives strong support for the p-adic topological condensate consisting of p-adic regions with different p glued together along their boundaries.

p-Adic topologies form a hierarchy of increasingly coarser topologies. The p-adic norm $N(x_p)$ defines a function of a real argument via the canonical identification of the nonnegative real numbers and p-adic numbers. The p-adic norm is same as ordinary real norm for $x = p^k$ and is constant at each interval $[p^k, p^{k+1})$. This means that

i) p-adic topologies are coarser than real topologies so that the functions, which are continuous in the p-adic topology need not be continuous in the real topology.

ii) p-adic topologies are ordered: the larger the value of p, the coarser the topology in the long lenght scales. In short length scales the situation is just the opposite.

4.2 Slaving Principle and p-adic length scale hypothesis

Slaving Principle states that there exists a hierarchy of dynamics with increasing characteristic length (time) scales and the dynamical variables of a given length scale obey dynamics, where the dynamical variables of the longer length (time) scale serve as "masters" that is effectively as external parameters or integration constants. The dynamics of the "slave" corresponds to a rapid adaptation to the conditions posed by the "master".

p-Adic length scale hierarchy suggests a quantitive realization of this philosophy.

a) By the previous considerations there is an infinite hierarchy of length scales L_p such that the space-time surfaces below the length scale L_p look like Minkowski space and p-adic quantum field theory $M^4 \rightarrow CP_2$ makes sense below the length scale L_p. These length scales are associated with the different condensation levels present in the topological condensate and define the typical size of the p-adic surface in absence of the collective quantum effects, which should correspond to the formation of the join along boundaries bonds between objects with size of order L_p. The reason why the typical size is just this is that the imbedding of the p-adic coordinate space into space H has strongest discontinuities in the real topology, when coordinate values correspond to powers of p so that a typical imbedding decomposes into separate pieces with size of order L_p. Of course, this kind of discontinuity is possible for all powers of p but is not observable in shorter length scales for the physically most interesting values of p due to the extreme smallness of the corresponding length scales.

b) The lowest level of the hierarchy corresponds to 2-adic dynamics and this field theory makes sense below the cutoff length scale $L_2 = \sqrt{2}l$ defining the typical size for a 2-adic surface. Solutions of the 2-adic field equations are non-deterministic due to the possibility of the integration constants depending on finite number of binary digits. The dependence on a finite number of positive bits of the real coordinates only means that they are genuine constants below some length scale $L_2(lower) < L_2$, which in principle depends on the state of the system.

c) 2-adic pseudo-constants are analogous to external parameters and should be determined by the dynamics associated with the longer length and time scales. The properties of the p-adic numbers suggest that these constants in turn are p-adically differentiable functions of their argument with some value of $p_1 > 2$ determined by the p_1-adic dynamics describing the interaction between $p = 2$ surface condensed on $p = p_1$ level and $p = p_1$ background surface. The p_1-adic integration constants associated with these functions are actual constants above the length scale $L_{p_1}(lower) \geq L_2(lower)$ but also these in principle depend on a finite number of pinary digits and their values are determined by the interaction of p_1 level with the next level in the condensation hierarchy.

d) At the next level p_1 one encounters p_1-adic dynamics and new p-adic integration constants. The net effect is that one obtains a hierarchy of p-adic numbers $2 < p_1 < p_2 < ...$ in correspondence with the length and time scales $L_2 < L_{p_1} < L_{p_2} < ...$: the higher the boss the larger the p. In TGD it is very tempting to interpret the various levels of the slaving hierarchy as the levels of the topological condensate so that the surfaces at level p are condensed on the surfaces of level $p_1 > p$ (see Fig. 4.2). Not all values of p need be present in the hierarchy and it might well happen that certain values of p are in an exceptional position physically.

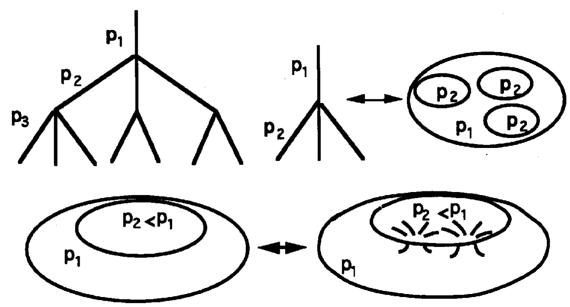

Figure 1: Two-dimensional visualization of topological condensate concept

4.3 Primes near powers of two and Slaving Hierarchy: Mersenne primes

All values of p are in principle present in the Slaving Hierarchy but the assumption that all values of p are equally important physically is not realistic. The point is that the number $N(n)$ of primes smaller than n behaves as $N(n) \sim n/ln(n)$ and there are just too many prime numbers. For example, for $n = 10^{38}$ there are about one prime number per 87 natural numbers!

A natural looking assumption is that a new physically important length scale emerges, when a fixed number of powers of 2 combine to form a new length scale. The reason is that a given interval $[2^k, 2^{k+1})$ forms an independent fractal unit (for the simplest fractals these intervals are related by a similarity, see figures in [E4] and it is therefore unnatural to cut this unit into pieces as would happen if p were far from a power of two. This breaking would indeed happen since p-adically differentiable functions have sharp gradients at points p^k. This non-breaking or "synergy" is reached provided the allowed primes are as close as possible to powers of 2: $p \simeq 2^m$. It should be noticed that this condition also guarantees that the frequency peaks associated with various powers of p in good approximation correspond to period doubling frequencies characteristic to fractal and chaotic systems.

The best approximation achievable corresponds to Fermat and Mersenne primes

$$p = 2^m \pm 1 \ . \tag{4}$$

It can be shown that for Fermat primes (+) the condition $m = 2^k$ must be satisfied and for Mersenne primes (-) m must be itself prime.

How abundant are the prime numbers of type $p = 2^m \pm 1$? The great surprise was that there are very few numbers of this kind!

a) The primes of type $2^m + 1$, Fermat primes, are very rare: only 5 numbers in the range $1 < n < 2^{2^{21}} \simeq 10^{10^6}$(!) [17] and there are good arguments suggesting that the number of the Fermat primes is finite! The known Fermat primes correspond to $m = 2^k$, with $k = 0, 1, 2, 3, 4$. The corresponding primes are $p = 3, 5, 17, 257, 65537$. Note that the lowest Fermat prime 3 is also a Mersenne prime. It will be later found that p-adic conformal invariance is in TGD possible for primes p satisfying the condition $p \ mod \ 4 = 3$ and this condition is not satisfied by Fermat primes $F > 3$.

b) The primes of form $2^m - 1$, Mersenne primes, are also there as follows from the requirement that m is prime. The list of allowed exponents of m consists of the following numbers:

$$2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, ...$$

.

One can make two observations about these numbers:

a) $m = 127$ corresponds to the number 10^{38} fundamental to Physics. The square root of this number gives the ratio of the proton length scale to Planck length scale. This suggests the possibility that fundamental physical length scales are given by square roots of Mersenne and possibly Fermat primes using some length scale of order Planck scale as a unit.

b) $m = 61$ corresponds to the number of order 10^{19}: this in turn allows the possibility that fundamental physical length scales are linearly related to Fermat and Mersenne primes. This alternative however turns out to be not the correct one.

These observations lead to following scenario for the fundamental length scales:

a) The p-adic length scale L_p, below which p-adic quantum field theory approximation makes sense, is proportional to the square root of p and these length scales are p-adically the most interesting length scales:

$$
\begin{aligned}
L_p &= \sqrt{p}l \ , \\
l &\sim k \cdot 10^4 \sqrt{G} \ , \\
k &\simeq 1.376 \ .
\end{aligned} \tag{5}
$$

Only quite recently the physical interpretation of the length scale l was found. Contrary to the original expectations, CP_2 is not of order Planck length but of order l. At this length scale Euclidian regions of space-time, in particular CP_2 type extremals representing elementary particles, become important. Above this length scale a field theory in Minkowski space is expected to be a good approximation to quantum physics.

b) Physically the most interesting length scales correspond to the p-adic cutoff length scales L_p associated with the Mersenne primes M_n.

c) The fact that l is of the same order of magnitude as the length scale at which the coupling constants of the standard model become approximately equal, is not probably an accident. Below l it is not anymore sensible to speak about the topological condensation of CP_2 type extremals since CP_2 type extremals themselves have size of order l. Hence the symmetry breaking effects caused by the topological condensation cannot be present in the string model type desciption applying below l.

The predictions are as follows:

i) $m = 127$ corresponds to electron Compton length.

ii) $m = 107$ corresponds to proton Compton length L_P.

iii) $m = 89$ corresponds to length scale of order $1/256$ times proton Compton length and is identifiable approximately as $L_W/2\sqrt{2}$, where L_W is intermediate boson length scale of about $L_P/100$.

iv) $m = 61$ corresponds to length scale of the order of $10^{-6}L_P$ is not reachable by the present day accelerators.

v) $m = 521$ corresponds to a completely super-astronomical length scale of order 10^{27} light years!

It seems that the proposed scenario might have catched something essential in the problem of the elementary particle mass scales: it predicts correctly 3 fundamental length scales associated with leptons, hadrons and intermediate gauge bosons from number theory; there is extremely large gap in the length scale hierarchy after electron Compton length and new shorter length scales exist but unfortunately they are outside the reach of the present day experiments. The calculations of the third part of the book show that not only the mass scales can be understood but also particle masses can be predicted with errors below one per cent using the length scale hypothesis combined with the p-adic Super Virasoro invariance and p-adic thermodynamics.

4.4 Length scales defined by prime powers of two and Finite Fields

Above M_{127} there is an extremely large gap for Mersenne primes and this suggests that there must be also other physically important primes. Certainly all primes near powers of two define physically interesting length scales by 2-adic fractality but there are two many of them. The first thing, which comes into mind is to consider the set of primes near prime powers of two containing as special case Mersenne primes. The following argument is one of the many arguments in favor of these length scales developed during last years.

TGD Universe is critical at quantum level and criticality is related closely to the scaling invariance. This suggests that unitary irreducible representations of p-adic scalings $x \rightarrow p^m x$, $m \in Z$ should play central role in quantum theory. Unitarity requires that scalings are represented by a multiplication with phase factor and the reduction to a representation of a finite cyclic group Z_m requires that scalings $x \rightarrow p^m x$, m some integer, act trivially. In ordinary complex case the representations in question correspond to the phase factors $\Psi_k(x) = |x|^{(\frac{ik2\pi}{ln(p)})} = exp(iln(|x|)\frac{k2\pi}{ln(p)})$, $k \in Z$ and the reduction to a representation of Z_m is also possible but there is no good reason for restricting the consideration to discrete scalings.

a) The Schrödinger amplitudes in question are p-adic counterparts of the ordinary complex functions $\Psi_k(x) = exp(iln(|x|)k\frac{ik2\pi}{ln(p)})$, $k \in Z$. They have a unit p-adic norm, they are analogous to plane waves, they depend on p-adic norm only and satisfy the scaling invariance condition

$$
\begin{aligned}
\Psi_k(p^m x|p \rightarrow p_1) &= \Psi_k(x|p \rightarrow p_1) \ , \\
\Psi_k(x|p \rightarrow p_1) &= \Psi_k(|x|_p|p \rightarrow p_1) \ , \\
|\Psi_k(x|p \rightarrow p_1)|_p &= 1 \ ,
\end{aligned} \tag{6}
$$

which guarantees that these functions are effectively functions on the set of the p-adic numbers with cutoff performed in m:th power.

b) The solution to the conditions is suggested by the analogy with the real case:

$$
\Psi_k(x|p \rightarrow p_1) = exp(i\frac{kn(x)2\pi}{m}) \ ,
$$

$$n(x) = ln_p(N(x)) \in N \ , \tag{7}$$

where $n(x)$ is integer (the exponent of the lowest power of the p-adic number) and $k = 0, 1, ..., m-1$ is integer. The existence of the functions is however not obvious. It will be shortly found that the functions in question exist in $p > 2$-adic for all m relatively prime with respect to p but exist for all odd m and $m = 2$ in the 2-adic case.

c) If m is prime (!) the functions $K = \Psi_k$ form a finite field $G(m,1) = Z_m$ with respect to the p-adic sum defined as the p-adic product of the Schrödinger amplitudes

$$K + L = \Psi_{k+l} = \Psi_k \Psi_l \ , \tag{8}$$

and multiplication defined as

$$KL = \Psi_{kl} \ . \tag{9}$$

Hence, if the proposed Schödinger amplitudes possessing definite scaling invariance properties are physically important, then the length scales defined by the prime powers of two must be physically special since Schrödinger amplitudes or equivalently, the p-adic scaling momenta k labeling them, have a natural finite field structure. By the Slaving Hierarchy Hypothesis, also the p-adic length scales near prime powers of two (and perhaps of prime $p > 2$, too) are therefore physically interesting. p-Adic scalings correspond to p-adic translations if p-adic coordinates correspond to exponentials of the ordinary linear coordinates so that translations are represented by scalings.

The generalized plane waves exist p-adically if nontrivial $N = p$:th root of the quantity $exp(i2\pi) = 1$ exists.

a) $N = 2$:th roots of 1 exist trivially for all values of p.

b) In 2-adic case the roots exist always for odd values of N and especially so for prime values of N: the trick is to write $1^{1/N} = -(-1)^{1/N} = -(1-2)^{1/N}$ and use the Taylor series

$$\begin{aligned} (1+x)^{1/N} &= \sum_n \frac{A_n}{n!} x^n \ , \\ A_n &= \prod_{k=0}^{n-1} (\frac{1}{N} - k)(-1)^n \ , \\ x &= -2 \ . \end{aligned} \tag{10}$$

to show the existence of one root different from the trivial root. In 2-adic case the powers of $x = 2$ converge to zero rapidly and compensate the powers of 2 coming from $n!$ in the denominator. The coefficients A_n possess 2-adic norm not larger than 1.

c) For $p > 2$ nontrivial $N = p$:th roots do not allow representation as plane waves for the simple reason that only the trivial p:th root of 1 exists p-adically. Roots of unity must have p-adic norm equal to one and by writing the condition modulo p one obtains a condition $a^N\ mod\ p = 1$ in $G(p, 1)$. The roots of unity in $G(p, 1)$ satisfy always $a^{p-1} = 1$ and the possible orders N are factors of $p - 1$. In particular, prime roots with $p_1 > p - 1$ are not possible. The number of prime factors is typically quite small. For instance, for primes of order $p = 2^{127}$ the number of prime roots is of order 6.

The conclusion is that for $p > 2$ only those finite fields $G(p_1, 1)$ for which p_1 is factor of $p - 1$ are realizable as representation of phasefactors whereas for $p = 2$ all fields $G(p_1, 1)$ allow this kind of representation. Therefore $p = 2$-adic numbers are clearly exceptional. In the p-adic case the functions $\Psi_p(x, |p \rightarrow p_1)$ give irreducible representations for the group of p-adic scalings $x \rightarrow p^m x$, $m \in Z$ and the integers k can be regarded as scaling momenta. This suggests that these functions should play the role of the ordinary momentum eigenstates in the quantum theory of fractal structures. The result motivates the hypothesis that prime powers of two and also of p define physically especially interesting p-adic length scales: this hypothesis will be of utmost importance in future applications of TGD.

The ordinary (number theoretic) p-adic plane waves associated with the translations can be constructed as functions $f_k(x) = a^{kx}$, $k = 0, ..., n$, $a^n = 1$. For $p > 2$ these plane waves are periodic with period n, which is factor of $p - 1$ so that wavelengths correspond to factors of $p - 1$ and generate a finite number of physically favored length scales. The p-adic plane waves with the momenta $k = 0, ..., p-2$ form finite field $G(p, 1)$, when p-adic arithmetics is replaced with the modulo p arithmetics, that is to accuracy $O(p)$ (note that the definition of the arithmetic operations is *not* the same as in the previous case). The square roots of the p-adic plane waves are also well defined

The important property of the p-adic plane waves is that they are pseudo constants: this property played profound role in the earlier formulations of the p-adic QFT limit. It took a considerable time to discover that the counterparts of the ordinary real plane waves providing representations for translation group exists and satisfy the appropriate orthogonality relations. Therefore number theoretic plane waves do not play so essential role in p-adic QFT as was originally believed.

5 CP_2 type extremals

CP_2 type extremals are perhaps the most important vacuum extremals of the Kähler action. The reason is that they are vacuum extremals with a negative and finite Kähler action and hence favored by the absolute minimization of the Kähler action. On the other hand, maximization of Kähler function does not favor CP_2 type extremals because the virtual CP_2 type extremals are exponentially suppressed. CP_2 type extremals seem to play the same role as black holes possess in General Relativity. p-Adic thermodynamics, leading to excellent predictions for the masses of the elementary particles, predicts that elementary particles should possess p-adic entropy and Hawking-Bekenstein law for the entropy generalizes.

In GRT based cosmology black holes populate the most probable Universe, which is of course a problem: in TGD black holes are replaced by elementary particles. The second law of thermodynamics requires that the very early Universe should have a low entropy and hence that black holes should populate the recent day Universe: in TGD the very early cosmology is dominated by cosmic strings, which is a low entropy state. By the absolute minimization of the Kähler action, most cosmic strings however decay to elementary particles and produce p-adic entropy. To get a grasp of the orders of magnitude, it is good to notice that electron, which corresponds to $p = M_{127} = 2^{127} - 1$, has entropy equal to 127 bits.

The basic observation is that the M^4_+ projection of the CP_2 type extremal corresponds to a light like random curve and the quantization of this motion leads to Virasoro algebra and Kac Moody algebra characterizing quantized transversal motion superposed with the cm motion. CP_2 type extremals allow covariantly constant right handed neutrino spinors as solutions of the Dirac equation for the induced spinors in the interior and this leads to $N = 1$ super symmetry and a generalization of the Virasoro invariance to Super Virasoro invariance.

The previous p-adic mass calculations were based on this picture but it turned out that the Super Virasoro invariance and related Kac Moody symmetries generalize to the level of the configuration space geometry and in an extended form provide the basic symmetries of the quantum TGD. Although the quantization of the zitterbewegung motion of the CP_2 type extremals is a phenomenological procedure only, and is not needed in the fundamental theory, it deserves to be described because of its key role in the development of quantum TGD. There were however some strange features involved: for instance, $N = 1$ super-symmetry generated by righthanded neutrino was exact only for minimal surfaces.

The realization that super-symmetry requires modified Dirac action led to the final breakthrough. CP_2 type extremals allow quaternion-conformal symmetries and the super-generators associated with quark and lepton numbers are non-vanishing despite the fact that vacuum extremals are in question. Even Super-Kac-Moody generators are non-vanishing. Even more, CP_2 type extremals cease to be vacua for Dirac action. Especially beautiful feature of CP_2 type extremals is that they can describe also massive states and zitterbewegung is the geometric correlate of massivation.

5.1 Zitterbewegung motion classically

The M^4_+ projection of a CP_2 type extremal is a random light like curve. Also Dirac equation, which gives also classically rise to a motion with light velocity and this motivates the term 'zitterbewebung'. Zitterbewegung occurs at the light of velocity and any given 3-velocity gives rise to the solution of light likeness condition if one fixes the time component of velocity to be

$$\frac{dm^0}{d\tau} = \sqrt{m_{ij}\frac{dm^i}{d\tau}\frac{dm^j}{d\tau}} \ . \tag{11}$$

The vanishing of CP_2 part of the second fundamental form requires that velocity and acceleration are orthogonal:

$$m_{kl}\frac{dm^k}{d\tau}\frac{d^2m^l}{d\tau^2} = 0 \ . \tag{12}$$

This condition is identically satisfied.

A very general solution to the conditions is provided by the equations

$$\frac{d^2m^k}{d\tau^2} = F^{kl}\frac{dm^l}{d\tau} \ , \tag{13}$$

describing the motion the of massless charged particle in external Maxwell field.

5.2 Basic properties of CP_2 type extremals

CP_2 type extremal has the following explicit representation

$$m^k = f^k(u(s^k)) \ , \quad m_{kl}\frac{df^k}{du}\frac{df^l}{du} = 0 \ . \tag{14}$$

The function $u(s^k)$ is an arbitrary function of CP_2 coordinates and serves effectively as a time parameter in CP_2 defining a slicing of CP_2 to time=constant sections. The functions f^k are arbitrary apart from the restriction coming from the light likeness. When one expands the functions f^k to Fourier series with respect to the parameter u, light likeness conditions reduce to classical Virasoro conditions $L_n = 0$.

It is possible to write the expression for m^k in a physically more transparent form by separating the center of mass motion and by introducing p-adic length scale L_p as a normalization factor.

$$\frac{m^k}{L_p} = m_0^k + p_0^k u + \sum_n \frac{1}{\sqrt{n}} a_n^k exp(i2\pi nu) + c.c. \ . \tag{15}$$

The first term corresponds to the center of mass term responsible for rectilinear motion along geodesic line and second term corresponds to the zitterbewegung motion. p^k serves as an effective classical momentum which can be normalized as $p_k p^k = \epsilon$, $\epsilon = \pm 1$ or $\epsilon = 0$. What has significance is whether p^k is time like, light like, or space like. Conformal invariance corresponds to the freedom to replace u with a new 'time parameter' $f(u)$.

The physically most natural representation of u is as a function $f(U)$ of the fractional volume U for a 4-dimensional sub-manifold of CP_2 spanned by the 3-surfaces $X^3(U=0)$ and $X^3(U)$:

$$u = f(U) \ , \quad U = \frac{V(s^k)}{V(CP_2)} = \frac{S_K(u)}{S_K(CP_2)} \ . \tag{16}$$

The range of the values for U is bounded from above: $U \leq V_{max}/V(CP_2)$ and the value $U = 1$ is possible only if CP_2 type extremal begins and ends as a point. U represents also Kähler action using the value of the Kähler action for CP_2 as a unit.

The requirement that CP_2 type extremal extends over an infinite time and spatial scale implies the requirement

$$f(U_{max}) = \infty \ . \tag{17}$$

For $f(U_{max}) < \infty$ CP_2 type extremal can exist only in a finite temporal and spatial interval for finite values of 'momentum ' components p^k. This suggest a precise geometric distinction between real and virtual particles: virtual particles correspond to the functions $f(U_{max}) < \infty$ in contrast to the incoming and outgoing particles for which one has $f(U_{max}) = \infty$. This hypothesis, although it looks like an ad hoc assumption, is at least worth of studying.

The mere requirement that virtual CP_2 type extremal extends over a temporal or spatial distance of order $L > L_p$ implies that for $L < L_p$ the value of U is smaller than one. Kähler action, which is given by

$$S_K(X^4) = U \times S_K(CP_2) \ , \tag{18}$$

remains small for distances much smaller than L. For $f(U_{max}) = \infty$ this is even more true. This has an important implication: below a certain length scale the exponential of the Kähler action associated with the internal line of a Feynman diagram does not give rise to a suppression factor whereas above some characteristic length L and time scale there is an exponential suppression of the propagator by the factor $exp(-S_K(CP_2))$ practically hindering the propagation over distances larger than this length scale.

The presence of the exponential obviously introduces an effective infrared cutoff: this cutoff is prediction of the fundamental theory rather than ad hoc input as in quantum field theories. Of course, infrared cutoff results also from the condition $f(U_{max}) < \infty$. Physically the infrared cutoff results from the topological condensation of the CP_2 type extremals to larger space-time sheets. These could correspond to massless extremals (MEs). p-Adic length scale L_p is an excellent candidate for the cutoff length scale in the directions transversal to ME.

The suppression factor coming from the exponent of the Kähler action implies a distance dependent renormalization of the propagators. In the long length scale limit the suppression factor approaches to a constant value

$$exp\left[-\frac{V_{max}}{V(CP_2)} S_K(CP_2)\right] \ ,$$

and can be absorbed to the coupling constant so that the dependence on the maximal length of the internal lines can be interpreted as an effective coupling constant evolution. For instance, the smallness of the gravitational constant could be understood as follows. Since gravitons propagate over macroscopic distances, the virtual CP_2 type extremals develops a full Kähler action and there is huge suppression factor reducing the value of the gravitational coupling to its observed value: at short length scales the values of the gravitational coupling approaches to $G_{short} = L_p^2$ which means strong gravitation for momentum transfers $Q^2 > 1/L_p^2$. The values of V_{max} and thus those of the suppression factor can vary: only at the limit when

CP_2 extremal has point like contact with the lines it joins together, one has $V_{max} = V(CP_2)$. If the boundary component characterizing elementary particle family belongs to CP_2 type extremal (it could be associated with a larger space-time sheet), CP_2 type extremal contains a hole: also this reduces the maximal volume of the CP_2 extremal.

5.3 Quantized zitterbewegung and Super Virasoro algebra

Calculating various Fourier components of right left hand side of the light likeness condition $m_{kl}p^kp^l = 0$ for $p^k = dm^k/du$ explicitly using the general expansion for m^k separating center of mass motion from zitterbewegung, one obtains classical Virasoro conditions

$$\begin{aligned} p_0^2 &= L_0 \ , \\ L_n|phys\rangle &= 0 \ , \ . \end{aligned} \tag{19}$$

where L_n are defined by by their classical expressions as bi-linears of the Fourier coefficients. Therefore interior degrees of freedom give Virasoro algebra and zitterbewegung is more or less equivalent with the classical string dynamics.

It is not however not obvious whether a quantization of this dynamics is needed. If quantization is needed (perhaps to formulate the unitarity conditions in zero modes properly), it corresponds to the construction of the bosonic wave functionals in zero modes defined by the zitterbewegung degrees of freedom. Quantization could be carried out in the same manner as in string models.

a) The simplest assumption motivated by the Euclidian metric of CP_2 type extremal is that the commutator of p^k and m^k is proportional to a delta function as in ordinary quantization. One can Fourier expand m^k and p_k in the form

$$\begin{aligned} m^k &= m_0^k + p_0^k s + \frac{1}{K}\sum \frac{1}{n} a_n^{k,\dagger} exp(inKs) + \sum \frac{1}{n} a_n^k exp(-inKs) \ , \\ p^k &= p_0^k + i\sum a_n^{k,\dagger} exp(inKs) - i\sum a_n^k exp(-inKs) \ . \end{aligned} \tag{20}$$

Here cm motion has been extracted and the formula is identical with the formula expressing the motion for a fixed point of string. The parameter K is Kac Moody central charge. Note that the exponents $exp(iKns)$ exist provided that Ks is p-adically of order $O(p)$ or, if algebraic extension by introducing $\sqrt{p}$ is allowed, of order $O(\sqrt{p})$.

The commutator of p_i and m^j is of the standard form if the oscillator operators obey Kac-Moody algebra

$$\begin{aligned}
\left[p_{i,0}, m_0^j\right] &= m_i^{\ j} \ , \\
Comm(a_{i,m}^{\dagger}, a_n^j) &= Km\delta(m,n)m_i^{\ j} \ .
\end{aligned} \tag{21}$$

Here K appears Kac-Moody central charge, which must be integer in the real context at least.

Expressing the light likeness condition as quantum condition, one obtains an infinite series of conditions, which give the quantum counterparts of the Virasoro conditions

$$\begin{aligned}
p_0^2 &= kL_0 \ , \\
L_n|phys\rangle &= 0 \ , \ n < 0 \ .
\end{aligned} \tag{22}$$

k is some proportionality constant. One can solve these conditions by going to the transverse gauge in which physical states are created by oscillator operators orthogonal to an arbitrarily chosen light like vector. What quantization means physically is that zitterbewegung amplitudes are constrained by a Gaussian vacuum functional. A good guess motivated by the p-adic considerations is that the width of the ground state Gaussian is given by a p-adic length scale L_p: this is achieved if m^k is replaced with m^k/L_p in the general expression for $m^k(u)$. The experience with string models would suggests that vacuum functionals might be crucial for the understanding of graviton emission.

5.4 Zitterbewegung at the level of the modified Dirac action

At the level of the modified Dirac action zitterbewegung motion implies that the conserved momentum associated with CP_2 type extremal, besides being conserved and non-vanishing, is also time like. This means that zitterbewegung creates massive particles besides massless particles as well as off-mass-shell versions of both and Super Virasoro conditions imply the quantization of the mass squared spectrum.

This means that in quantum TGD Feynman diagrammatics is topologized in the sense that the lines of Feynman diagram correspond to CP_2 type extremals which in general performing zitterbewegung. The non-determinism of the CP_2 type extremals means that one obtains a sum over over all possible diagrams with vertices at arbitrary space-time locations just as in quantum field theory approach. What is so nice that the time-development operator associated with an individual line of the diagram is the exponent of the Hamiltonian operator identified as the Poincare energy associated with the modified Dirac action. This operator is that associated with a free theory and contains no nonlinear terms. Interactions result from absolute minimization of Kähler action. In particular, one gets rid of the divergences of the interacting quantum field theories by the topologization of the Feynman diagrammatics.

6 Black-hole-elementary particle analogy

String models have provided considerable insights into black hole thermodynamics by reducing it to ordinary thermodynamics for stringy black holes [19] although one still does not understand, which is the mechanism of the thermalization. In TGD context elementary particles are regarded as thermodynamical systems in p-adic sense. This is something new since the standard theories of particle physics describe elementary particles as pure quantum states. The resulting thermal description of the the particle massivation is extremely successful. The fact that one can associate a well defined entropy to an elementary particle, suggests an analogy between black holes and elementary particles and this analogy indeed exists in a quite precise form as will be found. It also leads to a partial explanation for the p-adic length scale hypothesis serving as the corner stone of the p-adic mass calculations. The identification of the CP_2 type extremal as a cognitive representation of elementary particle suggests that p-adic entropy characterizes information associated with a cognitive representation provided by CP_2 type extremal.

6.1 Generalization of the Hawking-Bekenstein law briefly

In TGD elementary particles are modelled as so called CP_2 type extremals, which are surfaces with a size of order Planck length having metric with Euclidian signature. These vacuum surfaces are isometric with CP_2 itself and have a one-dimensional, random light like curve as the M^4_+ projection. A natural candidate for the TGD:eish counterpart of the black hole horizon is the surface at which the Euclidian signature of the metric associated with the CP_2 type extremal is changed to the Minkowskian signature of the background space-time. The radius r of this surface is the crucial length scale for the topological condensation and the simplest guess is that it is of the order of the size of the CP_2 radius and hence of the fundamental p-adic length scale. The hope is that the generalization of the black hole thermodynamics, with r replacing the radius of the black hole horizon, could give this information.

p-Adic mass calculations indeed give the p-adic counterpart of the Hawking-Bekenstein formula $S \propto GM^2$ as an identity at p-adic level:

$$S_p = -\frac{1}{T_p}(M_p^2/m_0^2) \ ,$$

where $1/T_p = n$ is the the integer valued inverse of the p-adic temperature and the mass scale $m_0^2/3$ corresponds to unit p-adic number in the unit used. The peculiar looking sign of S_p does not have in the p-adic context the same significance as in real context since the real counterpart of S_p is positive. Although p-adic entropy and mass squared are linearly related, the real counterparts are not in such a simple relation. In case of massive particles the real counterpart of the entropy is in excellent approximation equal to $S = log(p)$ whereas the mass is of order $1/p$ (p is of order 10^{38} for electron!). For massless (or nearly massless) particles one has $S \leq log(p)/p$. The large difference between fermionic and photonic entropies does not favor pair annihilation and this suggests that matter antimatter asymmetry is generated thermodynamically. For instance, via the topological condensation of fermions and anti-fermions on different space-time sheets during the early cosmology.

The generalization of the Hawking-Bekenstein formula in the form of the area law $S = A/4G$ reads as

$$S = \frac{xA}{4l^2} \ ,$$

where the fundamental p-adic length scale $l \simeq 1.376 \cdot 10^4 \sqrt{G}$ replaces Planck length $\sqrt{G}$ and x is a numerical constant near unity. The radius of the elementary particle horizon is in an excellent approximation given by $r(p) = \sqrt{\frac{log(p)}{\pi x}} l$. Particles are thus surrounded by an Euclidian region of the space-time with radius r. Thus the fundamental p-adic length scale l of order CP_2 size has a direct geometric meaning. For instance, in the energy scales below $1/l$ the induced metric of the space-time becomes Euclidian and it might be possible to describe particle physics using Euclidian field theory: essentially QFT in a small deformation of CP_2 would be in question. It is encouraging, that l is also the length scale at which the standard model couplings become identical and super symmetry is expected to become manifest.

The p-adic length scale hypothesis stating that the primes p near prime powers of two are the physically most interesting p-adic primes, is the cornerstone of p-adic mass calculations but there is no really convincing argument for why should it be so. The proportionality of r to $\sqrt{log(p)}$ suggests an explanation for the p-adic length scale hypothesis. The point is that for $p \simeq 2^k$, k prime, one has $r \propto L(k)$ and if the numerical constant x is chosen to be $x = \frac{log(2)}{\pi}$, the radius of elementary particle horizon is in excellent approximation $r(p \simeq 2^k) = L(k)$. Note also that the area of the elementary particle horizon becomes quantized in multiples of prime. This suggests that the precise value of $p \simeq 2^k$ is such that this condition is satisfied optimally and that physics is k-adic below r and $p \simeq 2^k$-adic above r.

$M^4_+ \times CP_2$ allows the imbedding of Schwartshild metric in the region below Schwartchild radius but the imbedding fails for too small values of the radial variable [D3]. An interesting possibility is that black hole entropy is just the sum of the elementary particle entropies topologically condensed below the horizon. This would give $S_{TGD} \propto \sum m_i^2 < S_{GRT} \propto (\sum m_i)^2$. An interesting problem is related to the detailed definition of p-adic entropy: are the entropies of particles with same value of p additive as p-adic numbers or does the additivity hold true for the real counterparts of the p-adic entropies. A related question is whether it might be that also in case of black holes additivity holds true, not for the mass as it is usually assumed, but for the p-adic mass squared for a given p (in TGD inspired model of hadron this is true for quark masses). This could be understood as a result of strong gravitational interactions. The additivity with respect to mass squared would give an upper bound of order $10^{-4}/\sqrt{G}$ for the contribution of a given p-adic prime to the total mass. For instance, the total contribution of electrons to the mass would be always below this mass irrespective of the number of electrons!

6.2 In what sense CP_2 type extremals behave like black holes?

CP_2 type extremals are in some respects classically black hole like objects since their metric is Euclidian. When this kind of surface is glued to Minkowskian background there must exist a two-dimensional surface, where the signature of the induced metric changes from the Minkowskian $(1,-1,-1,-1)$ to the Euclidian $(-1,-1,-1,-1)$. On this surface, which could be called elementary particle horizon, the metric is degenerate and has the signature $(0,-1,-1,-1)$. Physically elementary particle horizon can be visualized as the throat of the wormhole feeding the elementary particle gauge fluxes to the background space-time. Of course, one cannot exclude the presence of several wormholes for a given space-time sheet.

This surface indeed behaves in certain respects like horizon. Time like geodesic lines cannot go through this surface. The reason is that the square of the four velocity associated with the geodesic is conserved:

$$v_\mu v^\mu = 1 \ , \ 0 \ \text{or} \ -1 \ ,$$

depending on whether the geodesic is time like, light like or space like. Clearly, a time like geodesic cannot enter from the external world to the interior of the CP_2 type extremal. If a space like geodesic starts from the interior of the CP_2 type extremal it can in principle continue as a space like geodesic into the exterior. These analogies should not be taken too seriously: it does not make sense to identify particles orbits as geodesics in these length scales shorter than the actual sizes of particle.

These analogies suggest that Hawking-Bekenstein formula $S = A/4G$ relating black hole entropy to the area of the black hole horizon, might have a generalization to the elementary particle context with the radius of the elementary particle horizon replacing the black hole horizon. The unit of the area need not be determined by Planck length $\sqrt{G}$, it could be replaced by the fundamental p-adic length scale $l \sim 10^4\sqrt{G}$: this length scale indeed replaces Planck length as a fundamental length scale in TGD.

6.3 Elementary particles as p-adically thermal objects?

In the p-adic mass calculations elementary particles were assumed to be thermal objects in the p-adic sense. What is new that energy is replaced with mass squared and the thermalization is believed to result from the interactions of a topologically condensed CP_2 type extremal with the background space-time surface of a much larger size. The thermalization mixes massless states with Planck mass states and gives rise to particle massivation. Super Virasoro invariance $-$ abstracted from the Virasoro invariance of the CP_2 type extremals $-$ together with the general symmetry considerations based on the symmetries of $M^4_+ \times CP_2$, leads to the realization of the mass squared operator essentially as the Virasoro generator L_0 in certain representations of the Super Virasoro algebra constructed using the representations of various Kac Moody algebras associated with Lorentz group, electro-weak group and color group.

$-L_0$ takes thus the role of a Hamiltonian in the partition function:

$$exp(-H/T) \rightarrow p^{L_0/T_p} \ ,$$

where T_p is the p-adic temperature, which by number theoretic reasons is quantized to $1/T_p = n$, n a positive integer. Mass squared is essentially the thermal expectation of L_0. The real mass squared is the real counterpart of the p-adic mass squared in the canonical identification $x = \sum x_n p^n \rightarrow \sum x_n p^{-n} \equiv x_R$ mapping p-adics to reals. Assuming that elementary particles correspond to p-adic primes near prime powers of two, one obtains excellent predictions, not only for the mass scales of elementary particles but also for the particle mass ratios. For instance, electron corresponds to the Mersenne prime $M_{127} = 2^{127} - 1$.

It should be noticed that the real counterpart of the p-adic inverse temperature $1/T_p$ is naturally defined as

$$(\frac{1}{T_p})_r = (\frac{1}{T_p})_R log(p) \ ,$$

where $log(p)$ factor results from the definition of Boltzmann weights as powers of p rather than power of e. The real counterpart T_r of T_p can be identified as

$$T_r = \frac{1}{nlog(p)} \ . \qquad (23)$$

One might wonder about whether the sign of T_p should be taken as negative since positive exponent of L^0 appears in the Boltzman weights. The sign is correct; for the opposite sign T_r would be in good approximation equal to $\frac{1}{(p-n)log(p)}$, which is not consistent with the fact that physically temperature decreases when n increases.

As already explained, the new vision about p-adics and cognition forces to modify this early vision by interpreting CP_2 type extremals as cognitive representations of elementary particles rather than genuine elementary particles.

6.3.1 p-Adic mass squared

The thermal expectation of the p-adic mass squared operator is proportional to the thermal expectation of the Virasoro generator L_0:

$$\begin{aligned} M_p^2 &= k\langle L_0 \rangle \ , \\ k &= 1 \ . \end{aligned} \qquad (24)$$

The correct choice for the value of the rational number k is $k = 1$ as became clear in the recent reconstruction of the quantum TGD [F2].

The real mass squared M^2 is identified as

$$\begin{aligned} M^2 &= \frac{M_R^2 \pi^2}{l^2} \ , \\ l &\simeq 1.376 \cdot 10^4 \sqrt{G} \ , \end{aligned} \tag{25}$$

where l is the fundamental p-adic length scale and M_R^2 is the real counterpart of M_p^2 in the canonical identification. $\sqrt{G}$ is Planck length scale.

6.3.2 p-Adic entropy is proportional to p-adic mass squared

The definition of the p-adic entropy involves some number theory. The general definition

$$S = -p_n log(p_n) \ ,$$

in terms of the probabilities p_n of various states does not work as such since the e-based logarithm $log(p_n)$ does not exist p-adically. Since p-adic Boltzmann weights are integer powers of p it is natural to modify somehow the p-based logarithm $log_p(x)$ so that the resulting logarithm $Log_p(x)$ exists for any p-adic number and has the basic property

$$Log_p(xy) = Log_p(x) + Log_p(y) \ ,$$

guaranteing the additivity of the p-adic entropy for non-interacting systems. The definition satisfying these constraints is

$$Log_p(x = \sum_{n \geq n_0} x_n p^n) \equiv n_0 \ . \tag{26}$$

The lowest power in the expansion of x in powers of p fixes the value of the logarithm in the same way as it determines also the norm of the p-adic number. This leads to the definition of p-adic entropy as

$$S_p = -\sum_p p_n Log_p(p_n) \ . \tag{27}$$

In p-adic thermodynamics the p-adic probabilities have the general form

$$p_n = \frac{p^{L_0(n)/T_p}}{Z} \ .$$

Here $L_0(n)$ denotes the eigenvalue of the Virasoro generator L_0, which is integer. The partition function $Z = trace(p^{L_0/T_p})$ has unit p-adic norm if the ground state is massless, so that its p-adic logarithm vanishes in this case: $Log_p(Z) = 0$. This implies $Log_p(p_n) = Log_p(p^{L_0(n)/T_p}) = L_0(n)/T_p$ so that the p-adic entropy reduces to

$$S_p = \frac{1}{T_p}\langle L_0\rangle \ , \qquad (28)$$

ane hence that the p-adic mass squared and p-adic entropy are proportional to each other

$$S_p = -\frac{1}{kT_p}M_p^2 \ . \qquad (29)$$

By noticing that the entropy for Schwartschild black hole is given by

$$S = 4\pi GM^2 \ , \qquad (30)$$

one finds that in the p-adic context the analog of the Hawking-Bekenstein formula indeed holds as an identity.

The proposed identification of the entropy is in accordance with the formula $dE = TdS$. In the p-adic context E should clearly be replaced by $\langle -L_0\rangle$ and T by T_p. The differentials do not however make sense since the thermodynamical quantities are now discrete. Since only $\langle -L_0\rangle$ and T_p appear as variables one could define

$$\langle -L_0\rangle = T_pS_p \ .$$

This definition gives $S_p = -\frac{1}{kT_p}M_p^2$ and is in accordance with the standard definition of the Shannon entropy. The definition for the real counterpart of the p-adic entropy is

$$S = log(p)S_R \ .$$

The inclusion of $log(p)$-factor maximizes the resemblance with the usual Shannon entropy defined in terms of the e-based logarithm and makes it possible to compare the real counterpart of entropy with other kind of entropies.

6.3.3 The real counterparts of entropy and mass squared are not linearly related

Due to the delicacies related to the canonical identification, the real counterparts of entropy and mass squared differ drastically from each other and there is no simple relationship between the two quantities. The reason is that the vacuum expectation of $-L_0$ is of order $-np$ for particles having $T_p = 1$ and, essentially due to the presence of minus sign, one has $S_R(p) = 1$ in an excellent approximation, whereas the real counterpart of M_p^2 is of order n/p. For photon and other (nearly) massless bosons the entropy vanishes or is very small.

The fundamental difference in the thermal properties of fermions and massless bosons should have observable consequences. For instance, the annihilation of fermion-anti-fermion pair to massless particles means a considerable reduction of the p-adic entropy and would not be a favorable process thermodynamically. Thus the second law of thermodynamics would favor the presence of net fermion and anti-fermion number densities. For instance, fermions and anti-fermions could suffer a topological condensation on different space-time sheets to avoid annihilation during early cosmology or anti-fermions could even suffer topological evaporation as suggested in [D2, F6]. This in turn would lead to the generation of matter-antimatter asymmetry. It should be noticed that lare entropies are in accordance with the second law of thermodynamics.

6.3.4 Hawking-Bekenstein area formula in elementary particle context

Hawking-Bekenstein formula in the p-adic form $S_p \propto M_p^2$ holds true on basis of the previous considerations although ther are no hopes of deriving the area law from the first principles at this stage. Hawking-Bekenstein formula can be also written in the form

$$S = \frac{A}{4G} \ ,$$

relating black hole entropy to the area of the black hole horizon. One might hope that in the real context a generalization of the area law to the form

$$S = x\frac{A}{4L^2} \ ,$$

where L is some fundamental length scale analogous to the gravitational constant G and x is some numerical constant near unity, would hold true.

Since the size of CP_2 defines the fundamental p-adic length scale and replaces $\sqrt{G}$ as a fundamental length scale in TGD, it is conceivable that L is of the order of the CP_2 size $l \sim 10^4\sqrt{G}$. The area in question would be most naturally the area of the elementary particle horizon, where the signature of the induced metric for the topologically condensed CP_2 type extremal changes from Euclidian to Minkowskian. It is well known that l is also the length scale at which the couplings of the standard model become identical and super-symmetry is expected to become manifest. This is what is expected since above cm energy $1/l$ one would have an Euclidian quantum field theory in CP_2.

The radius r of the elementary particle horizon is of order

$$r \simeq \sqrt{log(p)}L \ . \tag{31}$$

This means that the # contacts connecting the CP_2 type extremal to the background space-time are surrounded by an Euclidian region with a size of order L.

It is interesting to look for the detailed form of the Hawking-Bekenstein law for elementary particles. One obtains the following general relationship

$$\begin{aligned} S &\equiv log(p)S_R = log(p)(\langle\frac{-L_0}{T_p}\rangle)_R == Xlog(p)M_R^2 = X \times log(p)\frac{l^2}{\pi^2}M^2 \ , \\ X &\equiv \frac{M_R^2}{S_R} \ . \end{aligned} \tag{32}$$

For massive particles $X \sim p$ holds true. Hence the entropy is related by a factor $p \cdot 10^8$ to the corresponding black hole entropy:

$$\begin{aligned} S &= a^2 S_{BH} \ , \\ S_{BH} &= 4\pi G M^2 \\ a &= \sqrt{\frac{log(p)X}{4\pi^3}}\frac{l}{\sqrt{G}} \sim 10^4 \ , \\ l &\simeq 1.376 \cdot 10^4\sqrt{G} \ . \end{aligned} \tag{33}$$

6.4 p-Adic length scale hypothesis and p-adic thermodynamics

The basic assumption of p-adic mass calculations is that physically interesting p-adic primes correspond to prime powers of two:

$$p \simeq 2^k \ , \ k \text{ prime } .$$

There are several arguments in favor of this hypothesis but no really convincing argument. The area law however leads to a very attractive, if not even convincing, explanation of the p-adic length scale hypothesis.

The proportionality of the elementary particle horizon radius to $\sqrt{log(p)}$ suggests quite attractive partial explanation for the p-adic length scale hypothesis. The point is that for $p \simeq 2^k$, k prime one has $r \propto L(k)$. Thus, if the numerical constant x is chosen suitably, it is possible to obtain very precisely

$$r(p \simeq 2^k) = L(k) \ .$$

The reason is that the p-adic entropy is in thermal equilibrium very near to its maximum value. The required value of the coefficient x is

$$x = \frac{log(2)}{\pi} \ . \tag{34}$$

The requirement that r_F (r_B) is as near as possible to the appropriate p-adic length scale $L(k)$ ($L(k)\sqrt{p}$) fixes also the precise value of the p-adic prime $p \simeq 2^k$.

This hypothesis means that the area of the elementary particle horizon is quantized in the multiples of prime k:

$$A = kA_1 \ . \tag{35}$$

The quantization law for the area has been proposed also in the context of the non-perturbative quantum gravity. A suggestive possibility is that physics is k-adic below the elementary particle horizon and $p \simeq 2^k$-adic above it. The appearance of an additional k-adic length scale suggests that for $p \simeq 2^k$ the degeneracy of the effective space-time surfaces is especially large due to the additional k-adic degeneracy and that the p-adic scattering amplitudes are be especially large for this reason. Hence the favored p-adic primes would emerge purely dynamically.

It must be noticed that k-adic fractality allows also more general primes of type $p \simeq 2^{k^n}$, where k is prime and n is integer. For these primes the radius of the elementary particle horizon is $\sqrt{k^{n-1}}L(k)$ and hence also a natural k-adic length scale. There are very few physically interesting length scales of this type. As the p-adic mass calculations show, the best fit to the neutrino mass squared differences is obtained for $p_\nu \simeq 2^{13^2=169}$ rather than $p \simeq 2^{167}$. The length scale $L(p_\nu)$ is also the natural length scale associated with the double cell layers appearing very frequently in bio-systems ($k = 167$ corresponds to the typical size of a cell)!

6.5 Black hole entropy as elementary particle entropy?

In TGD Schwartshild metric does not allow a global imbedding as a surface in $M^4_+ \times CP_2$. One can however find imbeddings, which extend also below the Schwartshild radius. This suggests that particles in the interior of the black hole are topologically condensed below the radius r_s. The problem is whether the single particle entropies are additive as real numbers or as p-adic numbers.

6.5.1 Additivity of real entropies?

Consider first the additivity as real numbers. With this assumption the sum for the real counterparts of the p-adic entropies of various particles gives a lower bound for the black hole entropy:

$$S = \sum_i S(i) = \sum_i k m_i^2 \ .$$

This entropy is by a factor is $10^8 \cdot p$ larger than the corresponding black hole entropy so that black hole-elementary particle analogy does not work at quantitative level. For sufficiently large particle numbers elementary particle entropy becomes smaller than the black hole entropy, which behaves as $(\sum m_i)^2$. In case of protons $p = M_{107} = 2^{107} - 1$ the critical value of N would be roughly $N \sim 10^{32}$, which would mean black hole with a mass of order 100 kilograms.

6.5.2 Additivity of the p-adic entropies?

One can consider also a different definition of the black hole entropy. In p-adic thermodynamics the natural additive quantity for many particle systems is the Virasoro generator L_0 (mass squared essentially) rather than energy. The additivity works quite nicely for the TGD based model of a hadron as a bound state of quarks. Therefore one could consider the possibility that also for black holes the mass squared of elementary particles with same value of p-adic prime p is p-adically additive

$$(m_p^2)_R = (\sum_i m_p^2(i))_R \text{ rather than } m = \sum m_i \ .$$

Therefore for a black hole containing only particles with single value of the p-adic prime p, the Hawking-Bekenstein formula in the form

$$S_p \propto M_p^2$$

would hold true. For the real counterparts this proportionality does not hold.

When the particle number N exceeds p/n, the mass squared of the system reduces from its upper bound $10^{-4}/\sqrt{G}$ by a factor of order $1/\sqrt{p}$. Thus the mass of, say, the electrons inside black hole, is always below this upper bound irrespective of the number of the electrons!

If particles with several p-adic primes are present inside the black hole then the formula for the black hole entropy reads as

$$S = \sum_p S(p) = \sum_p k(p)M^2(p) \ ,$$

so that the proportionality to the total mass squared does not hold true except approximately (in the case that the mass is in good approximation given by the total mass of a particular particle species).

6.6 Why primes near prime powers of two?

The great challenge of TGD is to predict the p-adic prime associated with a given elementary particle. The problem decomposes into the following subproblems.

a) One must understand why there is a definite value of the p-adic prime associated with a given real region of space-time surface (in particular, the space-time time surface describing elementary particle) and how this prime is determined. The new view about p-adicity allows to understand the possibility to label elementary particles by p-adic primes if p-adic–real phase transitions occur already at elementary particle level or if real elementary particle regions are accompanied by p-adic space-time sheets possible providing some kind of a cognitive model of particle. The great question mark is the correlation of the p-adic prime characterizing the particle with the quantum numbers of the particle: is this correlation due to the intrinsic properties of the particle or perhaps a result of some kind of adaptation at elementary particle length scales. In the latter case sub-cosmologies with quite different elementary particle mass spectra are possible. On the other and, quantum self-organization does not allow too many final state patterns, so that elementary particle mass spectrum could be more or less a constant of Nature.

b) One must understand why quantum evolution by quantum jumps has led to a situation in which elementary particle like surfaces correspond to some preferred primes. It indeed seems that an evolution at elementary particle level is in question (how p-adic evolution follows from simple number theoretic consistency conditions is discussed in the [E6]. It seems that the degeneracy due to the p-adic space-time regions associated with the system must be counted as giving rise to different final states in a quantum jump between quantum histories. If the number $N_d(X^3)$ of the physically equivalent cognitive variants of the space-time surface is especially high, this particular physical state dominates over the other final states of the quantum jump. Highly cognitive systems are winners in the fight for survival. Thus in TGD framework evolution is also, and perhaps basically, evolution of cognition.

d) One should also understand why the primes $p \simeq 2^k$ near prime powers of two are favored physically and to predict the value of k for an elementary particle with given quantum numbers. The analogy between elementary particles and black holes suggests only a partial explanation for the prime powers of 2 and the real explanation should probably involve enhanced cognitive resources for these primes.

In order to formulate the argument supporting p-adic length scale hypothesis one must first describe the general conceptual background.

a) Configuration space of the 3-surfaces decomposes into regions D_P labelled by infinite p-adic primes. In each quantum jump localization of CH spinor field to single sector D_P must occur if localization in zero modes occurs. Quantum time development corresponds to a sequence of quantum jumps between quantum histories and the value of the infinite-p p-adic prime P characterizing the 3-surface associated with the entire universe increases in a statistical sense. This has natural interpretation as evolution. In a well defined sense the infinite prime characterizing infinitely large universe is a composite of finite p-adic primes characterizing various real regions (space-time sheets) of the space-time. The effective infinite-p p-adic topology associated with this infinite prime is very much like real topology since canonical identification mapping infinite number to its real counterpart just drops the infinitesimals of infinite-p p-adic number. Therefore real physics is an excellent approximation at this level. If the S-matrix is complex rational, the approximation is in fact exact. Note that real topology is quite possible also at the level of configuration space and configuration space might consist of both real and infinite-P p-adic regions.

b) The requirement that quantum jumps correspond to quantum measurements in the sense of QFT, implies that also localization in zero modes occurs in each quantum jump: localization could occur also in the length scale resolution defined by the p-adic length scale L_p. The strongest hypothesis suggested by the properties of thermodynamical spin glasses is that quantum jump occurs to a state localized around single maximum of the Kähler function.

c) This picture suggests that evolution has occurred already at the elementary particle level and selected preferred p-adic primes characterizing the space-time regions associated with the elementary particles. A crucial question is whether this evolution could have occurred for isolated elementary particles or whether the interaction of the elementary like space-time regions with the surrounding space-time has served as a selective pressure. It might well be that the latter option is the correct one. If this is the case, one can say that the winners in the fight for survival correspond to infinite primes, which are composites of preferred finite primes, perhaps the finite primes given by the p-adic length scale hypothesis.

d) In TGD framework evolution is also evolution of cognition and the most plausible guess is that p-adic non-determinism is what makes cognition possible. Of course, also the classical non-determinism of Kähler action is also present and also important. Perhaps one should call the space-time sheets of finite time duration made possible by this non-determinism as 'sensory space-time sheets' as opposed to p-adic space-time sheets. Certainly this non-determinism should be responsible for volition. In any case, the degenerate space-time sheets are not physically equivalent in this case as they are in case of the p-adic non-determinism. The number $N_d(X^3)$ of the p-adically degenerate and physically equivalent absolute minima $X^4(X^3)$ of Kähler action is the measure for the cognitive resources of the 3-surface. The basic idea is simple: if $N_d(X^3)$ is very large then quantum jumps lead with high probability to some degenerate physically equivalent maximum of the Kähler function associated with given value of p. One can see this also from the point of view of an elementary particle: the high cognitive degeneracy plus the possibility of p-adic–real phase transitions mean that the particle can adapt to the environment: the surviving elementary particles would be the most intelligent ones! What one should be able to show is that cognitive degeneracy is especially large for some preferred primes so that evolution selects these primes as the most intelligent ones.

In this conceptual framework one can develop more precise variants for arguments supporting the p-adic length scales hypothesis.

a) The simplest possibility is that single maximum of Kähler function is selected in the quantum jump. In this case the relative rate for quantum jumps to a given physical final state with fixed physical configuration is proportional to the p-adic cognitive degeneracy $N_d(N)$, where N denotes the infinite primes characterizing the interacting space-time surface associated with the final state. N decomposes into a product of infinite primes p and $N_d(N)$ decomposes decomposes into a product $N = \prod_P N_d(P)$ $N_d(N)$ is maximized if $N_d(P)$ is maximzes. The elementary systems for which $N_d(P)$ is especially large are winners.

b) The situation reduces to the level of finite p-adic primes if takes seriously the argument allowing to estimate the value of the gravitational constant. The argument was based on the assumption that P decomposes in a well defined sense into passive primes p_i and active prime p characterizing elementary particle: thus there would be the correspondence $P \leftrightarrow p$. This suggests that it is possible to understand the finite p-adic prime p associated with the elementary particle by restricting the consideration to the 3-surfaces describing topologically condensed elementary particles: that is, CP_2 type extremals glued to a space-time sheet with size of order Compton length. p-Adic cognitive degeneracy $N_d(p)$ should be especially high for p-adic primes predicted by the p-adic length scale hypothesis.

b) The interpretation of p-adic regions as cognitive regions suggests a more concrete explanation for the p-adic length scale hypothesis. The degeneracy due to p-adic non-determinism for the p-adic CP_2 type extremals presumably depends on the value of the p-adic prime characterizing the cognitive version of elementary particle. If p-adic–real phase transitions representing transformation of thought-to-action and viceversa are possible for CP_2 type extremals, one could understand the origin of the p-adic length scale hypothesis. p-Adic primes near prime powers of two are winners because the the degeneracy due to p-adic non-determinism is especially larger for them. The observed elementary particles would thus dominate in the Universe simply because the thoughts about them are winners in the fight for survival.

c) The black hole-elementary particle analogy suggests that the primes $p \simeq 2^k$, k prime, are especially interesting since the radius of the elementary particle horizon is the p-adic length scale $L(k)$. This could be understood since k-adicity provides an additional cognitive degeneracy for the absolute minima of Kähler function coming from the region of size $L(k)$ surrounding a topologically condensed elementary particle and any # contact. This enhances the value of $N_d(p)$ further by a multiplicative factor $N_d(k)$ so that $N_d(P)$ becomes especially large.

d) These arguments do not yet tell how to deduce the prime k associated with a given elementary particle. Cognitive resources are measured by a negative on an negentropy type quantity proportional to $N_c = log(N_d(p))$. A natural guess is that N_c is dominated by a term proportional to $log(p)$: $N_c = A(p) + log(p)$. For $p \simeq 2^k$ one has an additional source of cognitive degeneracy which gives $N_c = log(k) + log(p)$ instead of $N_c = log(p)$ and these primes thus correspond to the local maxima of cognitive resources as a function of p. Quite generally, the larger the p, the more probable is its appearance as elementary particle prime (neglecting the constraints coming from, say, the cosmic temperature). Hence it seems that the p-adic evolution of a given elementary particle is frozen to some local maximum of $N_d(p(k))$, with $p(k)$ given by the p-adic length scale hypothesis.

e) Freezing can be understood if the transition probabilities $P(k \rightarrow k_1)$ are so small that further evolution by quantum jumps is impossible. A possible interpretation of the transition $k_i \rightarrow k_j$ is a p-adic phase transition changing the elementary particle horizon from radius L_{k_i} to L_{k_j} so that $P(k_i \rightarrow k_j)$ would describe the probability of this phase transition. For neutrinos the transition probabilities $P(k_i \rightarrow k_j)$ between different sectors allowed by the p-adic length scale hypothesis seem to be largest whereas for higher quark generations they seem to be smallest. Furthermore, k is smaller for higher generations. In particular, $P(k_i \rightarrow k_j)$ seems to be largest for spherical boundary topology. This suggests that the (phase) transition probabilities $P(k_i \rightarrow k_j)$ decrease as a function of the strength of the dominating particle interaction and of the genus of the particle (reflecting itself via the modular contribution to the particle mass increasing as a function of genus).

7 The evolution of Kähler coupling as a function of the p-adic prime

The simplest hypothesis is that Kähler coupling strength as the analog of critical temperature is renormalization group invariant. For long time it however seemed that this option cannot be true since it would predict rapid growth of the gravitational coupling G as a function of the p-adic length scale. In the following various arguments related to the problem are discussed and the cautious conclusion based on the understanding of dark matter [A9] and general mathematical structure and physical content of TGD [C6] is that α_K is RG invariant after all.

There are several manners to end up with the dependence of the Kähler coupling constant on the p-adic length scale L_p.

a) Require that the exponent of the Kähler action for CP_2 type extremal, which is analogous to Boltzmann weight, corresponds to a number theoretically allowed value of the Boltzmann weight in p-adic thermodynamics, that is a power of p.

b) Perform an order of magnitude estimate for the gravitational constant assuming that virtual graviton corresponds to a CP_2 type extremal and that the value of the gravitational constant does not depend on the prime p (or depends only weakly on p).

In the following a general definition of α_K is considered and the arguments allowing evaluate $\alpha_K(p)$ are represented in the order they have evolved. These arguments represent different views which are not mutually completely consistent and certainly are in conflict with the most recent understanding which favors the independence of α_K on p-adic length scale.

7.1 Kähler coupling strength as a functional of an infinite p-adic prime characterizing configuration space sector

If Kähler action is taken as the fundamental action instead of the modified Dirac action, Kähler coupling strength becomes fundamental coupling and one can only allow it to have very weak dependence on 3-surface. The natural guess of elementary particle physicist is that this dependence is on finite p-adic prime p characterizing elementary particle. This hypothesis indeed leads to arguments allowing to deduce the general dependence of the Kähler coupling on p. The assumption that gravitational coupling is renormalization group invariant parameter leads to an explicit formula for α_K.

There is however an objection against Kähler coupling identified as a functional of a three-surface depending on infinite prime P characterizing 3-surface X^3. The space-time surface $X^4(X^3)$ decomposes into regions representing various particles and each region of this kind should correspond to its own Kähler coupling strength. A way out of the paradox is that Kähler action results from the normal ordering of the modified Dirac action, so that also Kähler coupling can be different for various space-time sheets (for, say legs of the TGD counterpart of a Feynman diagram). At the level of infinite integers the space-time regions corresponding to different Kähler coupling strengths correspond to the prime factors of an infinite integers representing the interacting many-particle state.

The Kähler coupling is by definition critical since the super-conformal symmetry of the modified Dirac action gives can be seen as a mathematical characterization of the criticality in a complete analogy with the situation in two-dimensional conformal field theories describing 2-dimensional critical systems. Thus Kähler coupling strength is completely analogous to a critical temperature so that for various values of P a fixed point theory for those particular values of isometry invariants is obtained in accordance with the original coupling constant evolution hypothesis stating that Kähler coupling strength is in some sense renormalization group invariant.

The Kähler coupling strength, being determined by the normal ordering of the modified Dirac action, could be a complicated functional of a region of 3-surface. In particular, α_K could depend on the isometry invariants associated with the configuration space geometry. These invariants are cosmological in principle and only in the case that the conformal invariance makes sense as an approximate symmetry, when the dip of the light cone is at laboratory, this dependence has some practical meaning. There is a great temptation to assume that super canonical invariance makes sense also at laboratory since zero modes characterize the size and shape of the 3-surface and classical Kähler fields associated with it: these variables are indeed absolutely essential for characterizing physical system classically.

The simplest assumption is that the dependence on isometry invariants is only through the infinite prime P associated with the space-time sheet characterizing given elementary particle in the interacting manyparticle state. Since P decomposes in a well defined sense to finite primes, one expects that in the simplest situation P involves same passive primes p_i common to all elementary particles and single active prime p characterizing the particle and that effectively the dependence of α_K is only on p: one would thus effectively have a correspondence $P \leftrightarrow p$ and coupling constant evolution effectively reduces to the level of the finite p-adic primes. Since Kähler action is $U(1)$ action and p corresponds to the p-adic length scale $L(p) \propto \sqrt{p}$, one expects that α_K increases with the decreasing value of p and that the dependence on p is logarithmic. The task is to guess the precise dependence of α_K on p.

7.2 Analogy of Kähler action exponential with Boltzmann weight

CP_2 type extremals are certainly fundamental solutions of field equations and the exponent of Kähler action for these defines a pure number, which should have basic role in TGD. The construction of S-matrix suggests strongly that the CP_2 type extremals representing virtual particles are small pieces of CP_2 for all elementary particles expect graviton and this indeed explains the extreme weakness of the gravitational interaction. External lines of Feynman diagrams can however have quite large Kähler action. The intuitive explanation for the special role of the graviton is that since gravitons are spin-2 particles, they can be emitted only as almost point like 3-surfaces.

The analogy of the exponential of the Kähler function $exp(2S/\alpha_K)$ with the Boltzmann weight $exp(-E/T)$ plus the fact that in p-adic thermodynamics $exp(-E/T)$ corresponds to p^{L_0/T_p} , $1/T_p = n = 1, 2, ...$,that is power of p, suggests that one has

$$K^{-2}exp(2S_K(CP_2)/\alpha_K(p)) = p^n \ \ , \ \ n \ integer \ \ .$$

in the case of gravitonic line and perhaps The presence of the coefficient K turns out to be necessary in the formula. An interpretation for the presence of the coefficient K is that Boltzmann weight corresponds to the exponential of the subtracted Kähler action

$$\frac{2S_K(CP_2)}{\alpha_K(p)} - \frac{2S_K(CP_2)}{\alpha_K(p=1)} \ \ .$$

If α_K satisfies the Boltzmann condition., one obtains immediately an expression for α_K as a function of p:

$$\frac{1}{\alpha_K(p)} = \frac{1}{2S_K(CP_2)}(nlog(p) + 2log(K)) \ \ . \qquad (36)$$

$n = 1$ turns out to be the correct value of n. This represents a typical logarithmic coupling constant evolution as a function of the p-adic length scale. At high energies (small p) the Kähler coupling increases and approaches to

$$\frac{1}{\alpha_K(p=2)} = \frac{1}{2S_K(CP_2)}(nlog(2) + 2log(K)) \ \ .$$

The following nice analogies between 'thermodynamics' defined by the Kähler function and p-adic thermodynamics support the proposed evolution of α_K. $1/\alpha_K$ is analogous to temperature and the dependence of $1/\alpha_K - 1/\alpha_K(p = 1)$ on p is logarithmic as is also the dependence of the real counterpart of the inverse of the quantized p-adic temperature in p-adic thermodynamics. Kähler function is analogous to F/T, F free energy. $S_K(CP_2, p) - S(CP_2, p = 1)$ is analogous to entropy and proportional to

$log(p)$ as is also the p-adic entropy associated with elementary particles. Generalizing Hawking-Bekenstein formula for the radius of black hole in terms of black hole entropy one obtains expression for the radius of surface at which the Euclidian signature of the metric of CP_2 extremal changes to Minkowskian. By requiring that this length scale itself is near to a p-adic length scale one ends up with the requirement that physically interesting p:s are near to the prime powers of 2 and p-adic entropy is quantized in prime number of bits.

7.3 Does Kähler coupling strength approach fine structure constant at electron length scale?

The task is to fix the parameters appearing in the formula for the evolution of α_K. One achieves this by using the peculiar coincidence observed in the context of p-adic mass calculations (and interpreted in a different manner in that context). Assuming the relationship

$$L(127) = exp(S_K(CP_2))\sqrt{G} \ , \tag{37}$$

between gravitational constant, the p-adic length scale $L(127)$ associated with $p = M_{127} = 2^{127} - 1$ (electron Compton length roughly) and the Kähler action $S_K(CP_2)$ associated with the CP_2 type extremals defined by the formulas

$$\begin{aligned} L(127) &= \sqrt{M_{127}}L_0 \ , \\ S_K(CP(2)) &= \frac{\pi}{8\alpha_K} \ , \\ L_0 &= k10^4\sqrt{G} \ , \\ k &\simeq 1.376 \ , \end{aligned} \tag{38}$$

one obtains a prediction for the value of Kähler coupling strength $\alpha_K(M_{127})$. The predicted low energy value of α_K is

$$1/\alpha_K(M_{127}) = 136.3496143 \ ,$$

which is surprisingly near to the fine structure constant

$$\frac{1}{\alpha_{em}(m_e)} = 137.0360211$$

at m_e: the deviation is only .5 per cent.

This looks quite a reasonable result physically. At electron length scale classical Z^0 and W fields are absent. The requirement that Z^0 field vanishes implies that photon field reduces to Kähler field. This suggests that Kähler coupling strength should be very nearly equal to the fine structure constant. The two couplings might be identical at $L(127)$ for some deep reason: this might reflect the special role of M_{127} as the largest Mersenne prime which does not correspond to a super-astronomical length scale. Of course, in other length scales they evolve at quite different rates and this should reflect the nontrivial role of classical Z^0 fields in these length scales (also in macroscopic lengths scales). The couplings of the fermions to Higgs fields give some contribution to the fermion masses increasing them so that the fundamental mass scale is reduced and the value of the length scale $L(127)$ increases. From the equation 37 it is clear that α_K decreases as a consequence and this perhaps makes possible an exact identity of the fine structure- and Kähler coupling constants at electron lengths scale.

Summarizing, the argument suggests that the correct form for the dependence of α_K on p is

$$\frac{G}{L^2}exp(2S_K(CP_2)/\alpha_K(p)) = p \ , \tag{39}$$

where L is p-adic length scale. Note that using $S_K(CP_2) = \pi/8$, one obtains

$$\frac{1}{\alpha_K(p=2)} \simeq 25.15$$

for the high energy limit ($p = 2$) of α_K.

What is beautiful that this hypothesis also correctly predicts the value of the gravitational constant G in terms of the the fundamental p-adic length scale L (and thus CP_2 radius) in terms of the Kähler action of CP_2 type extremal. Note that gravitational constant is predicted to be invariant under the p-adic coupling constant evolution unless one assumes that gravitational interaction is mediated along space-time sheets characterized by M_{127} (this hypothesis found strong support years after writing the above lines!). In fact, the assumption of the typical logarithmic dependence of G on p implies only $log(log(p))$ corrections to the formula for $\alpha_K(p)$.

Consider now objections against this line of argument.

a) p-Adic mass calculations do not allow to fix the second order contribution to the electron mass so that the value of K can vary. Group theoretically a more natural identification of Kähler coupling strength would be as electro-weak U(1) coupling strength $\alpha_{U(1)} \simeq 1/104$ at electron length scale. This option is indeed allowed.

b) The Kähler action for topologically condensed CP_2 type extremal could be modified by the interaction with the larger space-time sheet and this would affect the prediction for α_K.

c) The strong RG invariance of α_K looks definitely more natural than that of G. If gravitational interaction is mediated along space-time sheets characterized by Mersenne prime then the fact that M_{127} defines the largest Mersenne prime for which p-adic length scale is not super-astronomical would mean that G is effectively RG invariant even in the case that α_K has this property.

7.4 Is G or α_K invariant in the p-adic length scale evolution?

The first guess would obviously be that α_K as the analog of critical temperature is invariant under p-adic coupling constant evolution. The dimensional estimate for gravitational coupling constant as $G = L_p^2 exp(-2S(P_2/\alpha_K)$ would however imply that gravitational constant increases rapidly as a function of p-adic length scale for this option. This led to an alternative proposal according to which G is RG invariant. This argument sounds rather convincing but the increased understanding of the mathematical structure of TGD led to the realization that α_K could after all be RG invariant.

7.4.1 Why G should be RG invariant?

Since CP_2 type vacuum extremals possess negative action, they are indeed stable. Virtual graviton could correspond to an entire CP_2 type extremal whereas other virtual particles would correspond to pieces of CP_2 type extremals. Since the exchange of the graviton is suppressed by the negative exponential of the Kähler action, the extreme weakness of the gravitational interaction follows as a consequence as also follows the proposed evolution of α_K by requiring exact or approximate renormalization group invariance of G. In fact, the assumption of the typical logarithmic dependence of G on p implies only $log(log(p))$ corrections to the formula for $\alpha_K(p)$. The model also explains in principle the values of other coupling constant strengths as determined by the exponents of Kähler actions for virtual lines. There are certainly fluctuations of coupling constants present since the volume of the CP_2 type extremal associated with the virtual line varies.

A more precise argument goes as follows:

a) Each gravitonic internal line of the Feynman diagram, which corresponds to a CP_2 type extremal, must be suppressed by a factor $exp(-S_K(CP_2)/\alpha_K)$ in Feynman diagram since it corresponds to an addition of one CP_2 type extremal to the dominating Feynman diagram representing free propagation. For other than gravitonic internal lines the suppression factor is not so small since these lines are not full CP_2 type extremals and the value of the Kähler action is relatively small.

b) The external lines of the Feynman diagram do not contain exponentials of the Kähler action since the continuity of the configuration space spinor fields in the set of 3-surfaces corresponding to the topologies intermediate between the topologies containing N and $N+1$ CP_2 type extremals forces a compensating factor cancelling the exponential. Thus one can associate to each internal line representing CP_2 type extremal an action exponential but no exponential to the external lines.

c) An order of magnitude estimate for the gravitational constant in terms of the p-adic length scale $L_p = \sqrt{p}l$ and of the suppression factor given by the action exponential, is obtained by studying the diagram representing an exchange of graviton and is given by

$$\begin{aligned} G &\simeq exp(-2S(CP_2)/\alpha_K)L_p^2 \ , \\ L_p &= \sqrt{p}l \ . \end{aligned} \tag{40}$$

As found, $\alpha_K(M_{127})$ very nearly the fine structure constant at m_e gives G correctly. In the absence of the suppression factor one would have strong gravitation. The requirement that G does not depend on the p-adic length scale leads to the evolution of α_K as a function of the p-adic length scale.

7.4.2 Could α_K be RG invariant?

The problem with the RG invariance of G is that it predicts so fast evolution of Kähler coupling strength that it becomes difficult to understand how this evolution could be consistent with the much slower electro-weak coupling constant evolution. That this should be the case is suggested by the fact that α_K is of same order of magnitude as fine structure constant at electron length scale.

The understanding came from the TGD based model of dark matter as quantum coherent phase characterized by large values of Planck constant [C6]. The original model was proposed by Nottale [26] and TGD predicts the basic dimensionless parameter v_0 appearing in the formula $\hbar_{gr} = GMm/v_0$

of gravitational Planck constant [D6, C6]. The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electroweak couplings: the importance of this result is difficult to over-estimate.

On basis of these findings there is temptation to regard the problem as settled. Of course, one must keep mind open for more complex options such as the assignment of a separate p-adic evolution of electro-weak and color coupling constants to each Mersenne prime characterizing gravitonic space-time sheet. If so one would have genuine evolution of α_K as a function of Mersenne prime.

References

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview.* http://www.helsinki.fi/~matpitka/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry.* http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time.* http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD.* http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory.* http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy.* http://www.helsinki.fi/~matpitka/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics.* http://www.helsinki.fi/~matpitka/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems.* http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter.* http://www.helsinki.fi/~matpitka/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness.* http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory.* http://www.helsinki.fi/~matpitka/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://www.helsinki.fi/~matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://www.helsinki.fi/~matpitka/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://www.helsinki.fi/~matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://www.helsinki.fi/~matpitka/magnconsc/mathconsc.html.

References to the chapters of books

[A9] The chapter *Does TGD Predict the Spectrum of Planck Constants?* of [1].
http://www.helsinki.fi/~matpitka/tgdview/tgdview.html#Planck.

[B1] The chapter *Identification of the Configuration Space Kähler Function* of [2].
http://www.helsinki.fi/~matpitka/tgdgeom/tgdgeom.html#kahler.

[C6] The chapter *Was von Neumann Right After All* of [4].
http://www.helsinki.fi/~matpitka/tgdquant/tgdquant.html#vNeumann.

[D1] The chapter *Basic Extremals of Kähler Action* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#class.

[D2] The chapter *General Ideas about Topological Condensation and Evaporation* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#topcond.

[D3] The chapter *The Relationship Between TGD and GRT* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#tgdgrt.

[D6] The chapter *TGD and Astrophysics* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#astro.

[D7] The chapter *Macroscopic Quantum Phenomena and CP_2 Geometry* of [3].
http://www.helsinki.fi/~matpitka/tgdclass/tgdclass.html#super.

[E4] The chapter *p-Adic Numbers and Generalization of Number Concept* of [5].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#padmat.

[E6] The chapter *Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory* of [5].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#mblocks.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [5].
http://www.helsinki.fi/~matpitka/tgdnumber/tgdnumber.html#tqc.

[F2] The chapter *Massless States and Particle Massivation* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#mless.

[F6] The chapter *Topological Condensation and Evaporation* of [6].
http://www.helsinki.fi/~matpitka/paddark/paddark.html#padaelem.

[H1] The chapter *Matter, Mind, Quantum* of [10].
http://www.helsinki.fi/~matpitka/tgdconsc/tgdconsc.html#conscic.

Mathematics related references

[16] V. S. Vladimirov and I. V. Volovich (1989), Comm. Math. Phys. , Vol 123, No 4, p. 659.

[17] U. Dudley (1969), *Elementary Number Theory*, W. H. Freeman and Company.

Theoretical physics

[18] H. Haken (1978), *Synergetics*, Springer Verlag.

[19] G. Horowitz, J. Maldacena and A. Strominger (1996), hep-th/9603019.

[20] S. Lundqvist, N. H. March, M. P. Tosi (1988), *Order and Chaos in Nonlinear Physical Systems*, p. 295, Plenum Press, New York.

[21] M. Barnsley (1988),*Fractals Everywhere*, Academic Press.

Physics related references

[22] *A Bibliography of* $1/f$ *noise.* http://linkage.rockefeller.edu/wli/1fnoise.

[23] D. Chowdbury (1986), *Spin Glasses and other Frustrated Systems*, World Scientific.

[24] Solla S., Sorkin G. and White S. (1986), "Configuration space analysis for optimization problems", in *Disordered Systems and Biological Organization*, E. Bienenstock et al. (Eds.), NATO ASI Series, ¡b¿ F20¡/b¿, Berlin: Springer Verlag, pp. 283-293.

[25] R. Lister (1997), *Simulated Annealing: Quasi-fractals and Quasi-failures.* http://www.csu.edu.au/ci/vol2/lister/node1.html.

References related to anomalies

[26] D. Da Roacha and L. Nottale (2003), *Gravitational Structure Formation in Scale Relativity*, astro-ph/0310036.