Negentropy Maximization Principle

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Abstract

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The localization in zero modes guarantees that the world of conscious experience looks classical. Together with the assumption that the unitary operator U acts effectively as a flow in zero modes, this implies standard quantum measurement theory with zero modes playing the role of macroscopic effectively classical variables and quantum fluctuating degrees of freedom correspond to quantum degrees of freedom. Contrary to the original belief there is however no need to assume that this localization occurs in each quantum jump and might also be governed by Negentropy Maximization Principle, whose formulation is the basic topic of this chapter.

The localization in zero modes (state function reduction) is assumed be followed by a sequence of self measurements in quantum fluctuating degrees of freedom. Self measurement is repeated again and again and eventually leads to a product state: only bound state entanglement is stable against this process. Obviously the process is equivalent with state preparation. Negentropy Maximization Principle provides the dynamical law governing state preparation and, as is has turned out, also state function reduction.

a) Consider a given unentangled system S. The basic assumption is that the density matrix of the subsystem of S, or equivalently, of its complement, is the fundamental observable measured in self measurement. NMP applies separately inside each system of this kind and states that for given system the quantum measurement occurs for that subsystem-complement pair for which the reduction of the entanglement entropy in self measurement is largest.

b) The original belief was that self measurement leads to an unentangled state. It is however possible to assign a negative entanglement entropy to an entanglement characterized by entanglement probabilities in finite extension of rationals. Thus NMP allows also a reduction to this kind of state. The natural interpretation of this kind of state is as a bound state. The density matrix must be unit matrix for the outcome if one requires that a measurement of density matrix is in question.

There are important technicalities involved with the formulation of NMP.

a) The definition of sub-system concept remains a highly nontrivial challenge for TGD. The reason is the classical non-determinism of Kähler action. A 3-surface acting as a causal determinant of Kähler action is the most general definition of the sub-system at space-time level. Causal determinants can can be light like surfaces $X^3_l \subset H$ (elementary particle horizons) or space-like 3-surfaces inside light like 7-surfaces $X^3_l \times CP_2 \subset M^4_+ \times CP_2$ analogous to the boundary $\delta M^4_+ \times CP_2$ of H. The reason is that these surface act as quantum holograms and representations of super-canonical and quaternion conformal algebras.

b) The many-sheeted space-time concept forces to modify the naive definition of subsystem as a tensor factor: two un-entangled systems can have sub-systems, which are entangled. The length scale dependent notion of subsystem allows to see this kind of entanglement as an entanglement invisible in the length scale resolution of the un-entangled systems.

c) Concerning the precise definition of negentropy there are three cases to be discussed.
i) In the situation in which entanglement probabilities reduce to a finite extension of rationals (discrete number field) a purely number theoretic definition of the entanglement entropy is possible using a p-adic variant of logarithm with argument replaced by its p-adic norm. Entanglement entropy can be defined as the maximally negative entanglement entropy S_p resulting in this manner: this assigns a unique p-adic prime p to the entanglement. The resulting real-valued entanglement entropy is negative and the entanglement is stable against self measurements and NMP. This negentropic entanglement could be identified as a correlate for the experience of understanding.
ii) In the second case entanglement probabilities are genuinely real or p-adic numbers. For real entanglement Shannon entropy works. The modification of p-based logarithm preserving the additivity of negentropy allows to define in p-adic case a p-adic valued entanglement entropy, which can be mapped to a non-negative real number by canonical identification.

d) The highly non-trivial observation is that the entanglement between systems belonging to different number fields is possible provided the states are orthonormalized. Furthermore, entanglement coefficients can belong to any number field. This means that the character of entanglement does not depend at all on the character of the entangled systems and is thus a typical category theoretic notion (relationship or "arrow" in the slang of category theory).

These findings lead to the idea state function reduction and preparation are number theoretic necessities. Unitary process U creates a formal superposition of states with entanglements in various number fields. State function reduction and preparation realized as a sequence of self measurements reduce the entanglement to a finitely extended rational entanglement interpreted as an information carrying bound state entanglement. Quantum jump can therefore be regarded as an elementary act of cognition in which unitary process is followed by analysis yielding as an outcome bound state entanglement giving rise to an experience of understanding. State function reduction and preparation can also occur in quantum parallel manner in various scales. This view modifies dramatically the interpretation of what de-coherence means. De-coherence removes only the entropic non-bound entanglement and preserves and even generates bound state entanglement. This obviously forces totally new view about second law of thermodynamics.

There are good reasons to expect that finitely extended rational entanglement is a basic characteristic of living and intelligent systems and crucial for the understanding of the information theoretic aspects of life. Negentropic bound state entanglement due to the quantum spin glass degeneracy provides mechanisms of macro-temporal quantum coherence making possible quantum computation type processes. The possibility of quantum parallel dissipation also forces to generalize quantum computation paradigm so that quantum parallel classical computations become possible.

1 Introduction

Quantum TGD involves 'holy trinity' of time developments. There is the geometric time development dictated by the absolute minimization of Kähler action crucial for the realization of General Coordinate Invariance. There is the unitary "time development" U: $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$, associated with each quantum jump, which is the counterpart of the Schrödinger time evolution $U(-t, t \rightarrow \infty)$. There is however no actual Schrödinger equation involved: situation is in practice same also in quantum field theories. Quantum jump sequence itself defines what might be called subjective time development.

Some dynamical principle governing subjective time evolution should exist and explain state function reduction with the characteristic one-one correlation between macroscopic measurement variables and quantum degrees of freedom and state preparation process. Negentropy Maximization Principle is the candidate for this principle, which I have been developing during last almost decade.

1.1 Evolution of ideas related to NMP

The evolution of ideas related to NMP has been slow and tortuous process and has basically evolution of ideas related to the anatomy of quantum jump and quantum TGD.

1. The first form of NMP was rather naive. There was no idea about the anatomy of quantum jump and NMP only stated that the allow quantum jumps are such that the information gain of conscious experienced measured by the reduction of entanglement entropy resulting in the reduction of entanglement between the subsystem of system and its complement is maximal. Already at this stage it however became clear that the special features of p-adic entropy in p-adic context might have deep implications.

2. Later it became clear that quantum jump has complex anatomy: unitary process is followed by the TGD counterpart of state function reduction and state preparation. State function reduction would naturally correspond to localization in zero modes strongly suggested by a mere mathematical consistency. There is no well-defined integration measure in the infinite-dimensional space of zero modes which by definition do not contribute the line element of the configuration space. This does not lead to difficulties if one assumes that complete localization in zero modes occurs in each quantum jump.

3. One of the last outcomes of the slow process has been the realization that complete localization in zero modes in every quantum jump is unnecessarily strong condition. It is enough that discretization in zero modes occurs in each quantum jump meaning that a state consisting of superposition of completely localized states in zero modes with quantum fluctuating degrees of freedom could survive under some conditions. This in fact is even suggested by the requirement that quantum measurement lasts for a

finite time and thus corresponds to a sequence of quantum jumps. This picture would suggest that NMP could be completely general principle applying to both state function reduction step and to state preparation and these two processes do not differ so much from each other as believed originally.

4. At some state the importance of the trivial fact that bound state entanglement must be stable against NMP, become obvious. The problem is however the characterization of bound state entanglement. At the level of space-time correlates the identification of join along boundaries bonds between space-time sheets as correlate for bound state entanglement suggests itself. The question is how to characterize bound state entanglement universally. NMP would suggest that bound state entanglement negentropy should be positive but this is impossible if entanglement negentropy is defined using Shannon's formula.

5. An important step in the process was the realization that the generation of macro-temporal quantum coherence means effective gluing of quantum jumps in quantum jump sequence to single quantum jump. This means that in appropriate degrees of freedom state function reduction and state preparation cease to occur during macro-temporal quantum coherence. This makes sense if macro-temporal quantum coherence means generation of bound state entanglement.

6. Many-sheeted space-time and p-adic length scale hierarchy force to generalize the notion of sub-system. The space-time correlate for the bound state entanglement is the formation of join along boundaries bonds connecting two space-time sheets. The basic realization is that two disjoint space-time sheets can containing smaller space-time sheets topologically condensed on them and connected by join along boundaries bonds. Thus systems un-entanglement at given level of p-adic hierarchy can contain entanglement subsystems at lower level. In TGD inspired theory of consciousness this makes possible sharing and fusion of mental images by entanglement. The resolution dependence for the notions of sub-system and bound state entanglement means that the entanglement between subsystems is not "seen" in the length scale resolution of unentangled systems. This phenomenon does not however result as an idealization of theoretician but is a genuine physical phenomenon. Obviously this generalized view about sub-system poses further challenges to the detailed formulation of NMP.

7. p-Adic length scale hypothesis leads to the view that there is entire hierarchy of durations for effective quantum jumps and this forces to ask whether the quantum jumps sequence decomposes into a hierarchy of effective quantum jumps of increasingly long duration just like physical systems form a hierarchy starting from the level of elementary particles and continuing through hadronic, nuclear, atomic and molecular physics up to

level where astrophysical objects take the role of particles. The usually un-noticed fact that hadrons can be regarded as quantum objects in long length and time scales whereas quark description treats hadrons as dissipative systems forces to make the following questions. Could state function reductions and preparations form a hierarchy and that the dissipative processes in short scales could occur in quantum parallel manner in longer scales. Using quantum computer language this would mean the possibility of quantum superposition of classical dissipative quantum computations.

8. The attempts to formulate NMP in p-adic physics led to the realization that one can distinguish between three kinds of information measures. In real physics the negative of the entanglement entropy defined by the standard Shannon formula defines a natural information measure, which is always non-positive. In p-adic physics one can generalize this information measure to p-adic valued information measure by replacing the logarithms of p-adic valued probabilities with the p-based logarithms $log_p(||_p)$ which are integer valued and can be interpreted as p-adic numbers. This p-adic valued entanglement entropy can be mapped to a non-negative real number by the so called canonical identification $x = \sum_n x_n p^n \rightarrow \sum_n x_n p^{-n}$. In both cases a non-positive information measure results.

   There is however interesting special case. When the entanglement probabilities are rational numbers or at most finitely extended rational numbers one can still define logarithms of probabilities as p-based logarithms $log_p(||_p)$ and interpret the entropy as real number. In this case the entropy can be however negative and positive definite information measure is possible. Irrespective of number field one can in this case define entanglement entropy as a maximum of number theoretic entropies S_p over the set of primes. This suggests that the notion of bound state could be number theoretic: perhaps bound state entanglement can be defied as entanglement with finitely extended rational entanglement probabilities and cannot be reduced in quantum jump since this would be in conflict with NMP. Bound state entanglement can of course be generated in quantum jump.

9. The fusion of real and p-adic physics to single physics has been a long standing challenge for TGD. The previous findings might have served as strong motivation for how to achieve this fusion. In fact, the motivations came from TGD inspired theory of consciousness, in fact the attempt to model how intentions having p-adic space-time sheets as space-time correlates are transformed to action having real space-time sheets as correlates. The proposal is that state function reduction and preparation number theoretical necessities. Unitary process creates a superposition of states in various number fields and the first step in the reduction process is to select number field, real number field or extension of p-adic number. After the process continues so long that a state contains only finitely extended rational entanglement stable under NMP and can be regarded as a product

of states with entanglement probabilities in discrete number fields defined by finite extensions of rationals. Thus the entire process leads from continuum to discrete and is consistent with basic aspects of cognition and analytic thought.

1.2 NMP, self measurements, cognition, state preparation, qualia

That the reduction of the standard quantum measurement theory to the effective classicality of zero modes results as a basic prediction of quantum TGD, was not at all obvious from the beginning. In fact, the original proposal was that subjective time development should be governed by the so called Negentropy Maximization Principle (NMP), which should replace the standard quantum measurement theory.

The realization that standard quantum measurement theory results from the localization in zero modes does not however mean that NMP should be thrown away. Rather, NMP could be seen as the variational principle governing the dynamics of *self measurements* giving rise to state preparation. It will be assumed that this self measurement process continues until the system decomposes into unentangled subsystems consisting of subsystems which for which entanglement is bound state entanglement.

NMP applies to any unentangled subsystem resulting in this cascade of self measurements and tells that self measurement is performed for the subsystem (or equivalently, its complement) which gives rise to maximum entanglement negentropy gain in the self measurement.

NMP applies to the anatomy of a single quantum so that there is actually no need to mention the notion of self at all in the context of NMP. Despite this it is useful to introduce the basic concepts related to self. Self is a subsystem able to remain unentangled in sequential quantum jumps and preserving its identity in some sense: presumably the p-adic prime characterizing self (and also the real space-time sheet associated with self) is what characterizes the self identity. One can define irreducible self as a self which does not decompose to further sub-selves in state preparation process, that is irreducible self is bound state of more elementary subsystems. A second reason for introducing the notion of self is that for a self in a state of macro-temporal quantum coherence the sequence of quantum jumps effectively fuses to single quantum jump representing single long lasting moment of consciousness.

Some comments about NMP are in order.

1. Standard quantum measurement theory does not allow a spontaneous reduction of entanglement between quantum fluctuating degrees of freedom of two subsystems associated with 3-surface. This kind of entanglement can be reduced only by introducing entanglement with zero mode degrees of freedom, that is 'observer'. Thus NMP is the principle governing the dynamics of spontaneous self measurements by selecting which subsystem-complement division of given unentangled subsystem is self measured.

2. Self measurement involves the division of unentangled subsystem (possibly self, mental image) into two unentangled subsystems. Analytical thought creates separations and comparisons so that this division could be identified as the basic mechanism of cognition. Also sensory experience generates separations and distinctions so that NMP should be identified as the variational principle governing the dynamics of cognition and perception. State preparation process makes the world of conscious experience to look completely classical since only bound state entanglement is stable against self measurement. One can thus say that state function reduction and the cascade of self measurements lead leads from a maximally entangled multiverse state $U\Psi_i$ to a maximally analyzed state: from quantum holism to classical reductionism. At the level of standard quantum measurement theory this process is equivalent with state preparation process yielding totally unentangled product state as incoming state of particle physics experiment.

3. Self measurements are distinguishable from standard quantum measurements. Thus it is in principle possible to experimentally detect the presence of self measurements. Their presence distinguish selves from nonselves. The fact that self measurement reduces entanglement entropy allows the system to fight against thermalization and self measurement could be also seen as a self repair mechanism.

4. Irreducible self *effectively* obeys in quantum fluctuating degrees of freedom a unitary time development defined by n:th power of U for a sequence of n quantum jumps, at least in reasonable approximation. This means fractality of consciousness: one can approximate sequences of quantum jumps with single quantum jump such as one can approximate molecules consisting of elementary particles with a point like particle. This observation is of crucial importance for understanding how quantum computing is possible in TGD universe despite that single quantum jump to an increment of psychological time equal to CP_2 time. Also Penrose-Hameroff hypothesis generalizes to TGD framework and one can understand the purely phenomenological notion of quantum de-coherence at fundamental level and also how the quantum spin glass nature of TGD Universe allows to circumvent the objections against Penrose-Hameroff hypothesis.

5. The fact that state preparation is not a deterministic process, forces a statistical modelling of the state of self using the ensemble formed by the prepared states defined by the sequence of quantum jumps in turn defining the contribution to the contents of consciousness of self as a statistical average. The simplest description is in terms of thermodynamics. Thermodynamical density matrix gives the probabilities for various states of a subsystem in the sequence of quantum jumps occurred after the last 'wake-up'. What is of paramount importance is that the contents of consciousness of self can be modelled using statistical thermodynamics. Non-geometric sensory qualia indeed have a close relationship with con-

jugate pairs of thermodynamical variables such as temperature-entropy, pressure-volume, chemical potential-particle number,... The sequence of quantum jumps also defines a sequence of quantum jumps in zero modes. Statistical averaging is not so natural for the values of zero modes characterizing the outcomes of the quantum measurements, which suggests that they could be experienced as separate ones by self and would correspond to geometric qualia experienced as being sharp and dynamical.

1.3 Quantum jump as number theoretic necessity

The hypothesis that state function reduction and preparation are essentially number theoretic necessities is a new idea inspired by the formulation of quantum TGD using generalized notion of number based on fusion of real and p-adic number fields to a larger structure by gluing them along common rationals. Unitary process generates a superposition of states in different number fields. Even more, entanglement between two systems can be real, p-adic or finitely extended rational entanglement (and thus correspond to discrete number field) irrespective of what are the number fields associated with entangled systems provided the states in question are orthonormalized. The sole goal of the state function reduction and preparation process is to lead to a final state containing only finitely extended rational entanglement identified as bound state entanglement. The process leads thus from continuum to discrete as far as number field is considered. Why this kind of entanglement is so special is that the number theoretic entanglement entropy associated with it is negative and has interpretation as a measure for genuine conscious information accompanying the experience of understanding. Thus NMP can be assumed to govern both state function reduction and preparation and force generation of bound state entanglement. The cognitive interpretation of quantum jump and its fractally scaled up counterparts would be as an analysis leading to an experience of understanding.

This vision forces to revise the interpretation of second law of thermodynamics. Only the coherence associated with non-bound entanglement is destroyed in quantum jump whereas bound state entanglement is preserved and even generated. Thus information is actually generated and one can give up with a sigh of relief the gloomy visions about the heat death of the universe.

The implications for quantum computing are especially important. Macrotemporal quantum coherence made possible by quantum spin glass degeneracy increasing the life times of bound states allows to circumvent standard objections against quantum computing in biological length scales. A generalization of quantum computation paradigm seems to be necessary since p-adic length scale hierarchy makes possible quantum parallel dissipation so that quantum parallel classical computations become possible.

Before continuing, I want to represent my apologies to the readers: the development of the TGD inspired theory of consciousness has been so fast that I simply have not had time to update the chapters accordingly: also this chapter documents some unavoidable side tracks and should be taken as a lab note book rather than a final documentation.

1.4 Hyper-finite factors of type II_1 and quantization of Planck constant

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of the configuration space of 3-surfaces ("world of classical worlds"). As a matter fact, it seems that the infinite-dimensional character of this algebra implies the rest of TGD. 4-D space-time, imbedding space $M^4 \times CP_2$, and the entire quantum TGD could emerge from the extension of the hyper-finite factor of type II_1 to a local algebra. This extension is local with respect to an octonionic coordinate whose non-associativity guarantees that the algebra does not reduce back to a mere hyper-finite factor of type II_1. The dynamics of quantum TGD would follow from the associativity condition: in particular, space-time surface would be maximal associative or co-associate sub-manifolds of imbedding space.

The quantization of Planck constants assignable to M^4 and CP_2 degrees of freedom as integer multiples of the ordinary Planck constant is strongly suggestive in this framework and the phases with large Planck constant are interpreted as a dark matter quantum controlling ordinary matter in living matter. The average geometric durations of quantum jumps are naturally quantized as multiples of the integer characterizing M^4 Planck constant. This allows the reduction of the notion of self to that of quantum jump at higher level of hierarchy. A strong quantitative prediction for the preferred geometric durations of quantum jumps emerges.

The topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a manner that unentangled systems can possess entangled sub-systems. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a justification from the quantum measurement theory for hyper-finite factors of type II_1.

Also the notions of resolution and monitoring pop up naturally in this framework. p-Adic probabilities relate very naturally to hyper-finite factors of type II_1 and extend the expressive power of the ordinary probability theory. p-Adic thermodynamics with conformal cutoff is very natural for hyper-finite factors of type II_1 and explains p-adic length scale hypothesis $p \simeq 2^k$, k prime characterizing exponentially smaller p-adic length scale.

2 General conceptual background

NMP was originally formulated to generalize quantum measurement theory and dictate which subsystems in a given quantum state suffer quantum jump interpretable as a measurement of the density matrix of subsystem.

What NMP states is that self measurement occurs for the quantum subsystem (or equivalently, its complement) for which the negentropy gain ΔN is largest. In the p-adic context the real counterpart of the total negentropy gain is maximum for the allowed quantum jump. NMP applies separately to each unentangled subsystem of the state resulting in state function reduction from the state $U\Psi_i$. Note that the values of zero modes do not change during state preparation process. NMP should specify more or less uniquely which quantum subsystem of unentangled system performs the self measurement. It could happen that there are several quantum subsystem-complement pairs giving the same maximum negentropy gain: the simplest possibility is that the selection among these alternatives takes place randomly.

It is convenient to introduce the concept of irreducible self as a self for which the entanglement between subsystems is always bound state entanglement which by definition is stable against self measurement. The notion of bound state is taken as granted in the real case. In p-adic case the identification of the bound state entanglement as entanglement with vanishing entanglement entropy is attractive (in this case NMP predicts nothing). Clearly, one can assign with a given subsystem a unique number telling to how many irreducible subsystems it decomposes. In the following the conceptual background of NMP will be discussed. before dwelling on the technical problems related with the precise formulation of NMP.

2.1 What happens in the quantum jump?

The detailed analysis of quantum jump between quantum histories leads to surprisingly strong general predictions as the following considerations intend to show.

2.1.1 The general structure of quantum jump

It has gradually become clear that TGD involves 'holy trinity' of dynamics.

1. The dynamics defined by absolute minimization of Kähler action corresponds to the dynamics of material existence, with matter defined as 'res extensa', three-surfaces.

2. The dynamics defined by the action of the unitary "time development" operator U can be regarded as informational "time development" occurring at the level of objective existence. The construction of U is completely analogous to the construction of the time evolution operator $U(-t,t)$, $t \to \infty$ associated with the scattering solutions of Schrödinger equation.

It seems however un-necessary and very probably also impossible to assign Schrödinger equation with U.

3. The dynamics of quantum jumps governed by the hypothesis about localization in zero modes and by NMP corresponds to the dynamics of subjective existence.

In accordance with this, quantum jump decomposes into informational time development

$$\Psi_i \rightarrow U\Psi_i \quad ,$$

followed by quantum jump

$$U\Psi_i \rightarrow \Psi_{f_0}$$

involving a localization into some sector D_P of the configuration space and a sequence of self measurements

$$\Psi_{f_0} \rightarrow \Psi_{f_1} ..... \rightarrow \Psi_f$$

governed by NMP.

A good metaphor for the quantum jump is as Djinn leaving the bottle (informational time development), fulfilling the wish (quantum jump involving choice) and returning to, possibly new, bottle (localization to the final sector D_p of the configuration space and self measurement cascade). One could formally regard each quantum jump as analogous to a quantum computation lasting infinitely long time interval $(-\infty, \infty)$.

A second useful metaphor is as generation of infinite number of quantum parallel potentialities in which entire universe is in a totally entangled holistic state of oneness followed by state function reduction and self measurement cascade analyzing the state into maximally unentangled subsystems. NMP states that the analysis produces maximum amount of conscious information. For irreducible selves analysis process do not continue and the sequences of quantum jumps effectively take the role of single quantum jump. Therefore this structure characterizes also conscious experience in macro-temporal time scales. Clearly, quantum measurement theory has fascinating parallels with Krishnamurti's philosophy of consciousness which underlines the competing holistic and reductionistic aspects of consciousness.

A third useful metaphor comes from particle physics. Moment of consciousness can be seen as elementary particle of consciousness and selves as the atoms, molecules, ...galaxies,... of consciousness. Fractality hypothesis allows to get general vision about structure of consciousness even in the time scale of human life.

2.1.2 Localization in zero modes is necessary

The detailed inspection of what happens in quantum jumps leads to the surprising result that quantum jump involves always a complete localization in the zero modes. The argument leading to this conclusion goes as follows.

1. QFT picture strongly suggests that sub-system must be defined as a tensor factor of the space of configuration space spinors at given point Y^3 of the configuration space. This suggests that subsystem should be defined as a function of Y^3 and should be a local concept. An important consequence of this definition is that entanglement entropy gives information about space-time geometry.

2. Configuration space spinor field can be formally expressed as a superposition of quantum states localized into the reduced configuration space consisting of 3-surfaces belonging to light cone boundary. Hence configuration space spinor field can be formally written as

$$\sum_{Y^3} C(Y^3)(n,N)|n\rangle|N\rangle$$

   for any subsystem-complement decomposition defined in Y^3. Clearly, configuration space coordinates appear in the role of additional indices with respect to which entanglement coefficients are diagonal. The requirement that final state is pure would suggest that the quantum jump reducing the entanglement must involve a complete localization of the configuration space spinor field to some Y^3 plus further quantum jump reducing the entanglement in Y^3. Complete localization in configuration space is however not physically acceptable option since the action of various gauge symmetries on quantum states does not commute with a complete localization operation. In particular, the requirement that physical states belong to the representations of Super Virasoro and Super Canonical algebras, is not consistent with this requirement.

3. Under rather reasonable assumptions one can however replace complete localization with the localization in zero modes. Configuration space has fiber space structure. Configuration space metric is non-vanishing in fiber degrees of freedom and since propagator for small fluctuations equals to the contravariant metric of configuration space, fiber degrees of freedom correspond to genuine quantum fluctuations. Configuration space metric vanishes in zero modes, which can be identified as fundamental order parameters in the spirit of Haken's theory of self-organization. Quantum entanglement occurs in fiber degrees of freedom. The requirement that various local symmetries act as gauge symmetries. provide good reasons to expect that *entanglement coefficients are gauge invariants and hence depend on the zero modes only*. If this is really the case then the localization in zero modes leads to a state for which entanglement coefficients

in the fiber degrees of freedom are constant so that localized quantum state reduces to a tensor product of nonlocalized states in fiber degrees of freedom.

4. Since the decomposition of the configuration space to sectors D_P is induced by the corresponding decomposition in zero modes, this hypothesis only strengthens the hypothesis about localization to D_P. The time development by quantum jumps in the zero modes is effectively classical: Universe is apparently hopping around in the space of zero modes. This looks very attractive physically since zero modes characterize the size, shape and classical Kähler fields associated with 3-surface. Therefore each quantum jump gives very precise conscious geometric information about the space-time geometry and about configuration space in zero modes. This also means that Haken's classical theory of self-organization generalizes almost as such to TGD context. The probability for the localization to given point of zero mode space is given by the reduced probability density Q defined by the integral of the probability density R defined by the configuration space spinor field over fiber degrees of freedom. The local maxima of Q appear as attractors for the time development by quantum jumps. Dissipative time development could be regarded as a sequence of quantum jumps leading to this kind of maximum.

5. The degrees of freedom characterizing the non-determinism of the Kähler action can be regarded as fiber degrees of freedom and the experience with standard quantum field theory suggests that only 4-surfaces around single maximum of Kähler function is selected in quantum jump: this might follow from the requirement of internal consistency and would correspond essentially to what happens in spontaneous symmetry breaking. Localization in the zero modes is completely analogous to Higgs mechanism in which scalar field attains vacuum expectation value. Thus the general structure of the configuration space spinor field together with TGD based quantum jump concept automatically implies spontaneous symmetry breaking in its TGD based version (note however that particle massivation relies on p-adic thermodynamics in TGD framework). Universe according to TGD is a superposition of parallel classical universes (3-surfaces). Therefore quantum entangled state can can be regarded as a superposition of parallel entangled states, one for each 3-surface. Formally entanglement coefficients can be regarded as coefficients having the configuration space coordinates of 3-surfaces as an additional index.

6. Localization in zero modes provides simple explation for why the universe of conscious experience looks classical: moment of consciousness makes it classical. It also explains why the physics treating space-time as a fixed arena of dynamics has been so successful.

7. Mathematical consistency requires complete localization in p-adic configuration space degrees of freedom which are thus zero modes. This means that the world of cognitive experience is completely classical.

2.1.3 U correlates completely zero modes with quantum fluctuating degrees of freedom

The localization in zero modes is consistent with the unitarity only if U itself in suitable basis can be regarded as inducing a flow in the zero modes. This means that for each values z of zero modes there exist preferred basis of the configuration space spinor fields in quantum fluctuating nonzero modes such that the action of U reads as

$$\begin{aligned} &|n,z\rangle \rightarrow U|m,z\rangle = |\hat{n}, z_1(z,n)\rangle \ , \\ &|\hat{n}\rangle = \textstyle\sum_m S^{\dagger}_{nm}|m\rangle \ . \end{aligned} \tag{1}$$

Clearly, U acts effectively as a flow in zero modes for any state $|n\rangle$. This assumption translates the basic assumption of classical quantum measurement theory about one-one correlation between classical macroscopic states and preferred basis of quantum states to TGD framework. The standard objection against the purely formal notion of S-matrix is that one can always go a state basis, in which S-matrix is diagonal and reduces to a multiplication by a phase factor. In TGD framework this objection does not apply since S-matrix is defined only in a preferred state basis for which the outgoing states are localized in zero modes.

The state basis $|\hat{n}\rangle$ is the natural basis for the outgoing states and S-matrix can be identified as the matrix relating the initial state basis $|n\rangle = S_{nm}|\hat{m}\rangle$ localized in z and the outgoing state basis $|\hat{n}\rangle$. Note that one must assume that the correlation between zero modes and quantum states is controllable: it is clearly absent in the initial state. This corresponds to the possibility to control whether the measurement interaction is on or off. In TGD framework measurement interaction has description as a coupling of the measured subsystems with larger space-time sheets representing measurement apparatus and observer.

The consistency with unitarity thus implies that also the action of U-matrix in zero modes is effectively classical in preferred states basis: the reason is that no dispersion which is characteristic of Schrödinger time evolution occurs. A further implication is that density matrix for the subsystems defined by quantum fluctuating degrees of freedom on one hand and zero modes on the other hand is diagonal. Therefore the localization in zero modes can be interpreted as a quantum measurement of this density matrix and since density matrix is a Hermitian operator, there is a complete consistency with standard quantum measurement theory.

The localization in zero modes has an interesting relationship with the proposals that the breaking of unitarity and the concept of symmetry breaking are necessary for understanding of bio-systems [23]. The localization in zero modes implies the breaking of quantum ergodicity since quantum jumps to states which are superpositions of states in zero modes are not possible final states of quantum jump. As will be found, this also implies evolution as the increase of the infinite prime characterizing Universe. This is in accordance with the fact that also thermal spin glasses break ergodicity. The breaking of unitarity can also be interpreted as the counterpart of symmetry breaking which also involves localization in zero mode type variables (say the direction of magnetization).

2.1.4 The localization in zero modes implies evolution

According to the number theoretic vision about quantum TGD [E3], there are reasons to consider the possibility that the configuration space of 3-surfaces decomposes into union of sectors D_P labelled by infinite primes and each infinite prime P. The notion of infinite-p p-adic topology indeed makes sense.

Infinite primes can be constructed by a process analogous to a repeated second quantization of an arithmetic quantum field theory for which states are labelled by primes. For given infinite prime P the number of infinite primes larger than P is infinitely larger than the number of infinite primes smaller than P. Zero modes characterize the value of the infinite prime and all points in the fiber of the configuration space correspond to same P for given values of zero modes. This means that the localization in zero modes implies a localization into some sector D_P and P must obviously increase in the long run.

Infinite prime P measures is in a well defined sense a composite of finite primes and defines a decomposition of space-time surface into regions labelled by finite primes. Thus the increase of P is achieved either by increase of the finite primes or by the emergence of new space-time sheets labelled by finite primes. Finite-p p-adic topologies in turn correspond to a hierarchy of increasingly refined topologies. Also the maximum increment of the p-adic entanglement negentropy in quantum jump increases with p. These features suggest that p serves as a kind of intelligence quotient for cognitive system. In the same manner also infinite primes serve as effective intelligence quotient. This obviously means that localization in zero modes implies evolution as increase of infinite prime P characterizing the universe.

Quantum jumps involve conscious choice of zero modes and thus the choice of some sector D_P. Since the increase/decrease of P corresponds to evolution/decline, the obvious interpretation is that this kind of choices are moral choices. The fact that the number of alternatives is infinite suggests however that our moral choices might basically involve comparison of the initial and final sectors D_P rather than comparison of alternative selections. The fact that we are moral agents forces to consider the possibility that our conscience corresponds to infinite prime contribution to our conscious experience coming from entire initial and final quantum histories. This in turn suggests that the mathematics of infinite might provide the proper tool for saying what one can say about such abstract concepts as moral, society and spirituality. One must however notice that an increase of infinite prime is induced by the increase of finite primes decomposing it and the appearance of new finite primes into decomposition. Thus local evolution implies global evolution. Therefore also our choices are basically between finite p-adic primes and infinite primes need not contribute to our conscious experience.

2.2 The concept of self

The introduction of dark matter matter hierarchy forces to also reconsider the definition of self and in the following the original definition and modified definition are discussed.

2.2.1 Original definition of self

According to the original definition prior to the introduction of the dark matter hierarchy self is identified as a subsystem able to remain unentangled during the informational 'time evolutions' U associated with the sequential quantum jumps. Or putting it differently: self is a subsystem behaving like its own sub-Universe (with respect to NMP). Space-time surface decomposes into regions corresponding to real or various p-adic topologies. In p-adic case only quaternion conformal spin degrees of freedom can entangle. If one allows rational entanglement between different number fields and if entanglement entropy is regarded as a rational number in this case, NMP dictates uniquely the dynamics of also this process which necessarily precedes self measurements inside various number fields. The interpretation as cognitive self measurements is natural.

Since connected space-time sheets correspond to irreducible selves, the entanglement reduced in these self measurements is between disjoint space-time sheets belonging to the same number field. Thus it is possible to have in the real context a situation in which some real entanglement entropies are infinite.

Self identity can be defined by the prime characterizing p-adic and also real space-time sheet. If this space-time sheet disappears, self loses consciousness. The formation of a join along boundaries bond with another space-time sheet corresponds to the generation of bound state entanglement, and also now self loses consciousness. Here one must however consider also a weaker condition: real self loses consciousness if it bound-state-entangles with a real self characterized by a larger p-adic prime. One could interpret the process as a phase transition in which the p-adic prime characterizing the smaller space-time sheet increases and corresponding self disappears.

The hypothesis that the experiences of self associated with the quantum jumps occurred after the last 'wake-up' sum up to single experience, implies that self can have memories about earlier moments of consciousness. Therefore self becomes an extended object with respect to subjective time and has a well defined 'personal history'. If temporal binding of experiences involves kind of averaging, quantum statistical determinism makes the total experience defined by the heap of the experiences associated with individual quantum jumps reliable. Subjective memory has natural identification as a short term memory.

A given self S behaves essentially as a separate sub-Universe with respect to NMP. If one postulates that the conscious experiences of sub-selves S_i of an self S integrate with the self experience of S to single experience, one obtains a filtered hierarchy of conscious experiences with increasingly richer contents and at the top of the hierarchy is entire universe, God, enjoying eternal self-consciousness since it cannot get entangled with any larger system.

An attractive hypothesis is that the experience of self is abstraction in the sense that the experiences of sub-selves S_{ij} of S_i are abstracted to average experience $\langle S_{ij} \rangle$. This implies that the experiences of sub-sub-...selves of S are effectively unconcious to S. This hierarchy obviously has extremely far-reaching consequences. Temporal binding implies that experiences of individual selves are reliable and abstraction brings in the possibility of quantum statistical determinism at the level of ensembles.

The binding of *experiencers* is also possible. The binding of selves by quantum entanglement however destroys the component selves (note however the comment about situation in which the p-adic primes are different for real entangling selves). This process could correspond to the formation as wholes from their parts, say the formation of the mental image representing word from the mental images representing letters, which are all represented as sub-selves. Associative learning might correspond to the generation of entanglement between selves representing objects of the sensory experience and conscious association would correspond to the reduction of this entanglement generating associated sub-selves. The entanglement of sub-selves of two selves is possible if one accepts the length scale dependent notion of subsystem and means sharing and fusion of mental images, binding of experiences. Entanglement might make possible communication between selves belonging to different levels of the self hierarchy and to different number fields: this entanglement would be reduced always in state function reduction step.

2.2.2 Dark matter hierarchy

The identification of dark matter as phases having large value of Planck constant [D6, J6, A8] led to a vigorous evolution of ideas still continuing while I am writing this addendum to the original text. Entire dark matter hierarchy with levels labelled by increasing values of Planck constant is predicted, and in principle TGD predicts the values of Planck constant if physics as a generalized number theory vision is accepted [A8]. Also a good educated guess for the spectrum of Planck constants emerges. The implications are non-trivial already at the level of hadron physics and nuclear physics and imply that condensed matter physics and nuclear physics are not completely disjoint disciplines as reductionism teaches us. One condensed matter application is a model of high T_c superconductivity predicting that the basic length scales of cell membrane and cell as scales are inherent to high T_c superconductors.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Jones inclusions and quantization of Planck constant

The Clifford algebra spanned by gamma matrices of infinite-dimensional space defines standard example of a von Neumann algebra known as hyper-finite factor of type II_1. The characteristic property of this algebra is that unit matrix has unit trace. Jones inclusions of hyperfinite factors of type II_1 combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant [A9].

1. Quantum TGD emerges from the infinite-dimensional Clifford algebra extended to an analog of a local gauge algebra with respect to hyper-octonionic coordinate [A8]. In particular, the notions space-time as a hyper-quaternionic four-surface of imbedding space emerges.

2. One can understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics as integer multiples of standard value $\hbar_0$ of Planck constant: $\hbar(M^4) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$. This in terms of a mild generalization of standard Jones inclusions [A8]. The emergence of imbedding space means that the scaling factor of these metrics given by the scaling factor of Planck constant have spectrum: their is no landscape as in M-theory. Also the fusion of real and various p-adic variants of imbedding space along common rational (algebraic) points is involved.

3. In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes

the values of scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups. Mass spectrum is invariant under these scalings.

4. This predicts automatically arbitrarily large values of Planck constant and assigns the preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible using only iterated square root operation: these correspond to polygons obtainable by compass and ruler construction with integer n expressible as $n = 2^k \prod_i F_{s_i}$, where $F_{s_i} = 2^{2^{s_i}} + 1$ are distinct Fermat primes: the lowest Fermat primes are given by $3, 5, 17, 127, 2^{16}+1$. In particular, experimentally favored values of $\hbar$ in living matter should correspond to these special values of Planck constant. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

5. These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_2n, E_6, E_8 are possible so that TGD seems to be able to mimick these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

2.2.3 Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy as a hierarchy defined by quantized Planck constants leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3].

The hierarchy of dark matter levels is labelled by the values of Planck constant having quantized but arbitrarily large values. It seems that the most important hierarchy comes as $\hbar(k) = \lambda^k \hbar_0$, where $\lambda \simeq 2^k$ is integer. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Dark matter hierarchy suggests a modification of the notion of self, in fact a reduction of the notion of self to that of quantum jump alone. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. This indeed looks extremely natural and the hypothesis that self remains un-entangled for a longer duration than single quantum jump un-necessary. It is perhaps un-necessary to emphasize that the reduction of the notion of self to that of quantum jump means conceptual economy and somewhat ironically, would also a return to the original hypothesis but with a quantized Planck constant.

The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

If one accepts the hypothesis that CP_2 time defines the typical geometric duration of quantum jump then moments of consciousness with duration longer than CP_2 time would be associated with dark matter. This would require quite huge value of n for human consciousness and does not seem a plausible option since the time scale of .1 seconds corresponds to integer $n \simeq 2^{256} \simeq 10^{38}$. A more reasonable looking option is that n-ary p-adic time scales $T(n,p)$ for a given value $\hbar = m\hbar_0$ define the typical geometric duration so that for a given prime p one would have the hierarchy $T(m,n,p) = mT_p(n) = m\sqrt{p}^n T_{CP_2}$ of geometric durations of moment of consciousness, with favored values of m given by $m = 2^k \prod_i F_{s_i}$: as already explained, $F_{s_i} = 2^{2^{s_i}} + 1$ are distinct Fermat primes and the lowest Fermat primes are given by $3, 5, 17, 127, 2^{16} + 1$. $m = 2^{11}$ seems to be favored in living matter [M3]. T_{CP_2} corresponds to CP_2 time about 10^4 Planck times. The geometric durations give a natural first guess for the duration of long term memories. Second interpretation is as the increase of geometric time coordinate in single quantum jump in the drift towards geometric future which should accompanying quantum jump making possible to understand the experience about flow of time.

2.3 Length scale dependent definition of subsystem

The challenge of defining the notion of sub-system is highly non-trivial in TGD framework. The notions of many-sheeted space-time and p-adic length scale hierarchy inspire a generalization of the sub-system concept. This generalization is already inherent in the renormalization group philosophy of quantum field theories relying on the notion of resolution.

2.3.1 Elementary particle black hole analogy

A highly nontrivial point is related to the fact that the space-time sheet glued to a larger space-time sheet is separated from the latter by wormhole contacts which have Euclidian signature of metric unlike the space-time sheets which have Minkowskian signature. This means the presence of 'elementary particle horizons' surrounding the wormhole throats such that the induced metric is degenerate at these horizons. This implies metric 2-dimensionality and conformal invariance identifiable as so called quaternion conformal invariance playing the same role in TGD as conformal invariance in string models.

What is of utmost importance is that the smaller space-time sheet is much like a black hole and elementary particle horizon plays the role of the black hole horizon. Also now only mass, angular momentum, charge and some other quantum numbers of subsystem are visible at the larger space-time sheet (hadronic physics is an excellent example of this) and Hawking-Bekenstein law generalizes and "explains" p-adic length scale hypothesis.

2.3.2 p-Adic length scale hypothesis and more abstract definition of sub-system-system relationship

Previous arguments suggests strongly that the ordinary description of the subsystem as a tensor factor of the larger system is not enough, and that one must adopt the length scale dependent description applied also in quantum field theories. p-Adic length scale hypothesis indeed gives a precise quantitative content for the notion of length scale cutoff and predicts a hierarchy of p-adic cutoff length scales.

Instead of one huge state space for which various space-time sheets correspond to tensor factors, one has a hierarchy of state spaces such that subsystem-system relation is described more abstractly. This has an important implication: two unentangled systems can have subsystems which are entangled (the entanglement is not visible at the level of systems). Therefore two selves can have sub-selves entangled to single common sub-self, which means fusion and sharing of mental images making possible quantum communications and telepathy.

One can wonder what might be the deeper mathematical formulation for this idea. The ordinary quantum measurement theory is formulated for von Neumann algebras of type I and does not work for hyper-finite factors of type II_1 since it the probability for a projection to single ray vanishes. Rather, the projection is always into an infinite-dimensional subspace. Therefore the quantum measurement is never ideal and gives information about finite number of degrees of freedom only with infinite number of them remain remaining untouched. Length scale cutoffs in quantum field theory represent a typical example of a finite experimental resolution of this kind.

The proposed generalization of quantum measurement theory discussed in [C2] is based on Jones inclusions $\mathcal{N} \subset \mathcal{M}$, where $\mathcal{N}$ represents the degrees of freedom about which quantum measurement does not provide any information. The quantum space $\mathcal{M}/\mathcal{N}$ defines the space of observables. In this situation quantum measurement reduces entanglement only in $\mathcal{M}/\mathcal{N}$ degrees of freedom. There is a strong temptation to assign the entanglement between subselves and sharing of mental images with $\mathcal{N}$.

2.3.3 The notion of causal determinant

How one could define precisely the notion of subsystem at space-time level so that one could apply this definition also at configuration space level? The identification of the subsystem as a 3-surface acting as a causal determinant of Kähler action is suggestive here. Causal determinant can be defined as a minimal set of 3-surfaces determining absolute minimum of Kähler action containing these 3-surfaces as sub-manifolds. In principle, these surfaces are defined apart from the action of 4-dimensional general coordinate transformations but one can select natural preferred representatives for them.

1. The space like 3-surfaces at the boundary of the future light cone (moment of big bang) would be natural representatives for the causal determinants of the Kähler action if Kähler action were deterministic. One could say that the boundary of imbedding space acts as quantum gravitational hologram coding all information about the geometric time development.

2. Kähler action is however non-deterministic and allows several absolute minima going through a given space like 3-surface at the light cone boundary. This leads to the necessity of allowing causal determinants consisting of sequences of space like 3-surfaces. But even this is probably not enough.

3. The sign of energy depends on time orientation of the space-time sheet. This means that space-time counterpart of pair creation is possible in the sense that pairs of space-time sheets with opposite energies can be generated from vacuum. The very special properties of light like 7-surfaces $X^3_l \times CP_2$ of $H = M^4_+ \times CP_2$, where X^3_l is light like 3-surface of M^4_+, suggests that pair creation of space-time sheets having opposite energies occurs at these surfaces analogous to the boundary of the imbedding space (TGD counterpart for the moment of big bang). In fact, the most elegant and predictive variant of TGD inspired cosmology is based on the assumption that the net flows of energy and other quantum numbers vanish also at $\delta H = \delta M^4_+ \times CP_2$. The net quantum numbers of the Universe would vanish, and one would avoid the philosophically painful questions like "What are the values of conserved fermion numbers of the Universe?". Big Bang would reduce to a "silent whisper amplifying to big bang" as TGD inspired cosmology assumes. All matter would be generated from vacuum, perhaps as an intentional process in which p-adic space-time sheets are transformed to real space-time sheets of opposite energies. Negative and positive energy regions would separate later from each other and the generation of matter antimatter asymmetry might relate to this process.

4. Even this is probably not enough. Also elementary particle horizons which are light like 3-surfaces $X^3_l \subset H$, seem to act as causal determinants in

the sense that one must specify also a set of elementary particle horizons to fix a given sector of the configuration space completely. These causal determinants are actually all that is needed to understand the elementary particle physics in TGD Universe -whereas the 7-dimensional causal determinants $X^3_l \times CP_2$ would represent new physics relevant for the intentional action.

Few years after the discovery of the importance of 3-D light like causal determinants X^3_l it became clear that its possible to formulate quantum TGD as almost-topological quantum field theory at these causal determinants having interpretation as orbits of partons [B4, C2]. The action determining X^3_l is Chern-Simons action for the induced Kähler gauge potential, and any surface having at most 2-dimensional CP_2 projection is solution of field equations. Also the modified Dirac equation can be solved explicitly.

The light-likeness condition involves induced metric and breaks almost-topological QFT property. The generalized Dirac determinant determined as the product of eigenvalues of the modified Dirac operator therefore depends on the entire induced metric and one can hope that it gives the exponent of Kähler action for the preferred extremals of Kähler action identifiable as Bohr orbits [B4]. What is nice that the resulting theory has all the expected super-conformal symmetries at parton level and one can also understand the proposed dualities easily. This formulation allows also to understand how to achieve the p-adicization of the theory both at the level of space-time level and of S-matrix [E1]. Also very profound insights about the concrete realization of cognition and intentionality present at the elementary particle level emerge [H8].

2.4 Basic definitions related to density matrix and entanglement entropy

In this sequel the detailed definitions of density matrix and entropy are discussed. It has become clear that one must distinguish between three kinds of systems systems.

1. Genuinely real systems for which entanglement probabilities are not rational numbers or finitely extended rational numbers. In this case one can regard the probabilities as limiting values of frequencies for outcomes of measurement defined by a time series. This is also the case when the entanglement coefficients are rational or algebraic numbers but the number of entangled state pairs is infinite so that the entanglement probabilities need not be algebraic numbers anymore.

2. A genuinely p-adic system is a p-adic system in which entanglement probabilities are not positive rational numbers so that one cannot interpret the entanglement probabilities as a limit for frequencies defined by any ensemble.

3. Finitely extended rational entanglement probabilities allow an interpretation as ordinary probabilities. In this case one can regard the probabilities as belonging to an extension of rationals or to any p-adic number field. What is essential is that the number field is now discrete whereas it is continuous in above mentioned cases.

   One must use different definition for the real counterpart of the entanglement entropy in these two cases. In the first case standard Shannon's entropy works. In the second case p-adic counterpart of the Shannon entropy mapped to a real number by the canonical identification is the only possibility. In the third case the number theoretic entropies S_p based on p-adic norm can be regarded as extended rational numbers as such. In this case S_p can be negative, and one can fix the value of p used to define the entropy by requiring that entropy is maximally negative and thus identifiable as a genuine information measure.

2.4.1 Density matrix

The density matrix of subsystem, call it A, can be defined using the standard formulas of QM: essentially trace over the degrees of freedom associated with the complement of A, call it B, is performed. B could effectively reduce to a sub-system of the complement. Density matrix is Hermitian matrix and can be diagonalized in the real context. Eigenvalues are real and give the weights for various eigen states in the superposition. There is important *duality* present: in the basis of A in which the density matrix for A is diagonal also the density matrix of B is diagonal.

Density matrix actually determines one-one-correspondence between certain states of the system A and system B. The state in eigen state basis can be written as

$$|A,B\rangle \ = \ \sum_m c_m |m\rangle \times |M(m)\rangle \ , \tag{2}$$

where the map $m \rightarrow M(m)$ defines identification of certain states of A with certain states of B.

Quantum measurement of density matrix means that subsystem goes to an eigen state of density matrix. In the p-adic context the diagonalization of the density matrix requires special assumptions about the form of the state since the p-adic number fields are not closed with respect to algebraic operations. There is an algebraic extension obtained by requiring that each 'real' p-adic number has square root [E4]. The extension is 4-dimensional for $p \geq 3$ and 8-dimensional for $p = 2$. It can quite well happen that density matrix can be diagonalized only partially in this extension since the eigenvalues of the density matrix are in general algebraic numbers determined as a solution of polynomial eigenvalue equation.

One can however allow the extension of the p-adic number field to allow eigenvalues in an algebraic extension. Unless this is allowed the concepts of density matrix and entropy are not well defined for a generic subsystem. Physically this would mean that quantum state can have irreducible number theoretic entanglement besides the entanglement related to the quantum statistics. The vision about TGD as a generalized number theory encourages the allowance of the algebraic extension. This means that quantum subsystems can be classified using as criterion the dimension of the p-adic algebraic extension needed to define the eigen states and eigenvalues of the density matrix. In well defined sense physical systems generate increasingly complicated number fields as algebraic extensions of the p-adic numbers.

An interesting possibility is that Hermiticity in the p-adic context must be defined so that the eigenvalues of the density matrix are *ordinary p-adic numbers*: if this is the case then the algebraic extension is needed only for the diagonalization of the density matrix but the diagonalized density matrix itself is 'p-adically real'. This option seems however un-necessarily restrictive and will not be considered in the sequel.

If entanglement coefficients are algebraic numbers then also entanglement probabilities are algebraic numbers in the case that the number of entanglement state pairs is finite. Even finite-dimensional extensions of p-adic number numbers involving transcendentals such as $e, e^2, ..., e^{p-1}$) can be allowed. If the number of entangled state pairs is infinite, entanglement probabilities need not belong to a finite extension of rationals and it seems that entanglement cannot be regarded as bound state entanglement in this case.

2.4.2 p-Adic entanglement negentropy

In the real context negentropy is defined using the standard formula for Shannon entropy:

$$N = \sum_k p_k \cdot ln(p_k) \ . \tag{3}$$

In the real context one could equally well replace the e-based logarithm $ln(x)$ by a-based logarithm (a could be any positive real) since this introduces only multiplicative factor ($log_a(x) = \frac{ln(x)}{ln(a)}$).

p-Adic thermodynamics has turned out to be surprisingly successful for the calculation of elementary particle masses. p-Adic thermodynamics is however naturally based on p-based logarithm log_p rather than the ordinary e-based logarithm since Boltzmann weights are powers of p rather than exponents. This would suggest the following definition

$$N = \sum_k p_k \cdot log_p(p_k) \ . \tag{4}$$

There are however two problems:

1. p-based logarithm exists only for $p_k = p^r$, that is power of p. One should somehow modify the definition of the logarithm so that it is defined for all p-adic numbers.

2. Since the probabilities p_k correspond to eigenvalues of density matrix, they in general belong to some algebraic extension of p-adic numbers. Thus the modified logarithm should also exist for any algebraic extension of p-adic numbers.

The definition of the modified p-based logarithm $Log_p(x)$ should satisfy following constraints.

1. If argument is power of p then modified logarithm must be equal to p-based logarithm:

$$Log_p(p^n) = log_p(p^n) \quad .$$

2. Modified logarithm must be additive in order to make negentropy additive for systems having no interactions:

$$Log_p(xy) = Log_p(x) + Log_p(y) \quad .$$

These requirements fix the definition of logarithm uniquely. The modified logarithm can depend on the p-adic norm of the argument only. Or in terms of canonical identification

$$I : \sum x_n p^n \rightarrow \sum x_n p^{-n} \quad ,$$

mapping p-adics to reals and p-adic norm $N_p(x)$ one must have

$$\begin{aligned} Log_p(x) &= log_p([x]) \quad , \\ [x] &= I^{-1}(N_p(x)) \quad , \\ &= \left[\sum_{n \geq n_0} x_n p^n \right] = p^{n_0} \quad . \end{aligned} \tag{5}$$

This definition works also for the algebraic extensions, for which p-adic norm is defined as the p-adic norm for the determinant of the linear map induced by a multiplication with z in algebraic extension: it is easy to see that the determinant of this map is indeed a power of p always (note that this norm is multiplicative, which implies the additivity of modified logarithm and entropy).

For the algebraic extensions of p-adic numbers one must define how the units e_k of algebraic extension $z = x + \sum_k y^k e_k$ are mapped to the reals in the canonical identification map. e_k are typically roots of integers in the range $-1, ..., p$. The rule is following: if e_k is not a root of p then it is mapped to e_k interpreted as a real number: for instance, $2^{1/3}$ is mapped to $2^{1/3}$ for $p \neq 2$ in case that $2^{1/3}$ does not exist as p-adic number. If e_k is root of p it is mapped to its inverse: for instance, $\sqrt{p}$ is mapped to $\frac{1}{\sqrt{p}}$.

Note that p-adic entanglement entropy can be also expressed as a sum over the derivatives of the p-adic entanglement probabilities with respect to p:

$$S = \sum_i \frac{d}{dp} p_i \quad . \tag{6}$$

The real counterpart of the p-adic entanglement entropy is obtained by canonical identification $x = \sum x_n p^n \rightarrow \sum x_n p^{-n} = x_R$

$$S_r = S_R \times log(p) \quad . \tag{7}$$

$log(p)$ factor must be included in order to make possible the comparison of entropies associated with different values of p.

The value of the p-adic entanglement entropy is always non-negative. It vanishes if the p-adic entanglement entropies have unit p-adic norm. Thus $S = 0$ p-adic entanglement is possible. This entanglement need not be stable since a direct sum of eigen spaces of density matrix with finitely extended rational entanglement probabilities has negative entanglement entropy.

Unless some p-adic probabilities do not have p-adic norm larger than one, p-adic entanglement entropy is of order $O(p)$ for genuinely p-adic systems so that negentropy gain is below $log(p)$ irrespective of the size of the system. This situation is realized in p-adic thermodynamics. There is a nice connection with p-adic mass calculations: p-adic thermal mass squared expectation value is essentially the p-adic entropy. This connection was noticed already earlier [E5] and it was suggested that p-adic primes associated with elementary particles could correspond to entropy maxima as function of p. This connection suggests that the proper definition of p-adic entropy is based on the canonical identification.

Remark: Statistics does not give rise to entanglement entropy as one might erratically conclude by considering the symbolic representation of tensor product suggesting the identification of 'left' and 'right' members of the tensor product as subsystems A and B: the concrete representation of the states using oscillator operators associated with Y^3 and its complement shows that there is no statistical entanglement entropy between the subsystem and its complement: if this were the case the entire universe should behave like a single conscious being and this would be a catastrophe as far as NMP is considered.

2.4.3 Systems with finitely extended rational entanglement

In the case of an finitely extended rational entanglement one can map the p-adic entropy to its real counterpart using the identification by common rationals instead of the canonical identification. This gives the formula

$$
\begin{aligned}
S_R &= S_p log(p) \ , \\
S_p &= \sum_n p_k Log_p(p_k) log(p) \ , \\
Log_p(x) &= log_p(|x|_p) \ .
\end{aligned} \tag{8}
$$

where the p-adic entropy which can be regarded as a rational number is re-interpreted as a real number. Note that the probabilities p_k are positive numbers. What is remarkable is that in this case entanglement entropy can be a negative rational number or a number in a finite extension of rational numbers. These states are obviously stable against self measurements and ideal for cognitive quantum computing. If these states are interpreted as bound states there is no need to postulate that bound state entanglement is stable against self measurements.

The consistency with the standard quantum measurement theory requires that the process corresponds to a measurement of the density matrix so that a projection must occur to an eigen space or sub-space of eigen space of the density matrix if this maximizes negentropy gain. The density matrix of the system would become

$$
\rho \rightarrow \frac{1}{D_i} P_i \ . \tag{9}
$$

Here D_i and P_i denote the dimension of the eigen space associated with p_i and corresponding projection operator. Assuming that D_i has the decomposition

$$
D_i = \prod_{i \in I} q_i^{n_i}
$$

to a product of powers of primes, the negentropy of the final state can be written as

$$
N_R = Max\{n_i log(q_i) | i \in I\} \ . \tag{10}
$$

The maximization of the increment of entanglement entropy gives a criterion selecting the final eigen space or its sub-space. Quantum classical correspondence suggests that one can assign similar inherent negentropy to the space-time sheet consisting of D strictly deterministic regions.

Quantum computers typically operate with systems for which entanglement probabilities are identical and the process would thus produce quantum computer type state. The eigen spaces of the density matrix with dimensions $D = p^N$ are of special interest. The entanglement negentropy for $D = p^N n_0$, n_0 integer not divisible by p, is $N_R = Nlog(p)$. The reduction to a sub-space of the eigen space can yield higher negentropy gain than the reduction to the entire eigen space and powers of prime are favored as dimensions of these sub-spaces.

The entanglement negentropy per single dimension of eigen space is $N_R/D = Nlog(p)p^{-N}/n_0$. For $D = p^N$ the entanglement negentropy per dimension of eigen space is $N_R/D = Nlog(p)/p^N = log(D)/D$ and maximum as a function of n_0. N_R/D as a function of D has a maximum $N_R/D = .3662$ for $D = 3$ rather than $D = 2$ as one might expect. For $D = 2$ and $D = 4$ one has $N_R/D = .3466$ (note that there are 4 DNA nucleotides). For other values of D N_R/D is smaller.

For extended rational entanglement the measurement of the density matrix can occur only in special cases. For instance, when the probabilities p_k belong to a finite extension of rational numbers and are different, the measurement of the density matrix would reduce the negentropy to zero and NMP does not therefore allow the measurement of density matrix to occur. Degenerate eigen spaces do not correspond to the maximum entanglement negentropy per dimension. $p_k = n_k/p^N$, n_k not divisible by p, gives $N_R = Nlog(p)$ irrespective of dimension D, and $N_R/D = Nlog(p)/2$ for $D = 2$ ($p_1 = m/p^N$ and $p_2 = (p^N - m)/p^N$, m not divisible by p) is the best one can achieve. Since there is no upper bound for N nor p even in the case of a 2-state system, the negentropy gain can be arbitrarily high. One could criticize this result as counter intuitive.

2.4.4 The number field characterizes entanglement rather than entangled systems

On basis of the preceding considerations it is clear that it is possible to assign number field to the entanglement irrespective of the properties of the systems. If orthonormalized state basis are used for entangled systems, it does not matter at all which number field one assigns with them. Therefore states orthonormalized but decomposing into entangled states in different number fields R_{p_1} and R_{p_2} can entangle in R_{p_3}. Entanglement seems to be a notion conforming with the spirit of category theory and thus allows a considerable generalization in the sense that the structure of states which are entangled does not matter at a given level of entanglement hierarchy.

For extended rational entanglement this number field can be regarded as a finite extension of p-adic numbers for any R_p. The requirement that entanglement entropy is maximally negative fixes the choice of p in this case and the interpretation as bound state entanglement is natural.

These observations are in line with the number theoretical ideas about quantum jump and number theoretic origin of the state function reduction and preparation to be discussed in the next section.

3 Physics as fusion of real and p-adic physics and NMP

In this section the vision about state function reduction and preparation processes as number theoretic necessities is developed: also the chapter "Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory" contains related topics. The proposal raises NMP to fundamental principle applying also to the state function reduction step.

3.1 Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = -\sum_n p_n log(p_n) \ . \tag{11}$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

3.1.1 p-Adic entropies

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)/log(2)$ so that the number of bits is expressible as a number defining a finite-dimensional extension. The character of this is extension is already discussed. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of x, where log_p denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) \ , \\ k(p_n) &= -log_p(|p_n|) \ . \end{aligned} \tag{12}$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n)log(p) + p^{k_n} log(p_n/p^{k_n}\right] \ . \tag{13}$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times log(p)) \ , \\ (\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ . \end{aligned} \tag{14}$$

The real counterpart of the p-adic entropy is non-negative.

3.1.2 Number theoretic entropies and bound states

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|)log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv Max\{-S_p, \ \ p \text{ prime}\} \ . \tag{15}$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP, and has a natural interpretation as bound state entanglement. The prediction would be that the bound states of real systems form a number theoretical hierarchy according to the prime p and and dimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes projecting the physical states from either real or p-adic sectors of the state space to their intersection. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the U-matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

3.1.3 Number theoretic information measures at the space-time level

Quantum classical correspondence suggests that the notion of entropy should have also space-time counterpart. Entropy requires ensemble and both the p-adic non-determinism and the non-determinism of Kähler action allow to define the required ensemble as the ensemble of strictly deterministic regions of the space-time sheet. One can measure various observables at these space-time regions, and the frequencies for the outcomes are rational numbers of form $p_k = n(k)/N$, where N is the number of strictly deterministic regions of the space-time sheet. The number theoretic entropies are well defined and negative if p divides the integer N. Maximum is expected to result for the largest prime power factor of N. This would mean the possibility to assign a unique prime to a given real space-time sheet.

The classical non-determinism resembles p-adic non-determinism in the sense that the space-time sheet obeys effective p-adic topology in some length and time scale range is consistent with this idea since p-adic fractality suggests that N is power of p.

3.2 Generalized Quantum Mechanics

One can consider two generalizations of quantum mechanics to a fusion of p-adic and real quantum mechanics.

1. For the first generalization the guiding principle for the generalization of quantum mechanics is that quantum mechanics in a given number field is obtained as an algebraic continuation of the quantum mechanics in the field of rational numbers common to all number fields or in finite-dimensional extensions of rational numbers. This means that U-matrices U_F for transitions from H_Q to H_F, where F refers to various completions of rationals, are obtained as algebraic continuations of the unitary U-matrix U_Q for H_Q. The generalization means enormously strong algebraic constraints on the form of the U-matrix.

2. A more radical option is that transitions from rational Hilbert space H_Q to the Hilbert spaces H_F associated with different number fields occur. This requires that U-process is followed by a process analogous to a state function reduction and preparation takes care that the resulting states become states in H_Q: this is what makes this generalization of a special interest. In this case one can speak about total scattering probability from H_Q to H_F. The U-matrices U_F are not anymore mere analytic continuations of U_Q. A possible interpretation of the unitary process $H_Q \rightarrow H_F$ is as generation of intention whereas the reduction and preparation means the transformation of the intention to action.

The assumption that H_Q allows an algebraic continuation to the spaces H_F is probably too strong an idealization in p-adic and even in the real case. For instance, one cannot allow all rational valued momenta in p-adic case for the simple reason that the continuation to the p-adic case involves always some momentum cutoff if the extension of p-adics remains finite. Even in the real case the summation over all rational momenta in the unitarity conditions of U-matrix fails to make sense and cutoff is needed. A hierarchy of cutoffs suggests itself and has a natural interpretation as number theoretical hierarchy of extensions of p-adics.

In order to avoid un-necessary complications the following formal discussion however uses H_Q as a universal Hilbert space contained by the various state spaces H_F.

3.2.1 Quantum mechanics in H_F as a algebraic continuation of quantum mechanics in H_Q

The rational Hilbert space H_Q is representable as the set of sequences of real or complex rationals of which only finite number are non-vanishing. Real and p-adic Hilbert spaces are obtained as the numbers in the sequences to become real or p-adic numbers and no limitations are posed to the number of non-vanishing elements. All these Hilbert spaces have rational Hilbert space H_Q as a common sub-space. Also momenta and other continuous quantum numbers are replaced by a discrete value set. Superposition principle holds true only in a restricted sense, and state function reduction and preparation leads always to a final state which corresponds to a state in H_Q. This picture differs from the earlier one in which p-adic and real Hilbert spaces were assumed to form a direct sum.

The notion of unitarity generalizes. Contrary to the earlier beliefs, U-matrix does not possess matrix elements between different number fields but between rational Hilbert space and Hilbert spaces associated with various completions of rationals. This makes sense since the final state of the quantum jump (and thus the initial state of the unitary process, is always in H_Q.

The U-matrix is a collection of matrices U_F having matrix elements in the number field F. U_F maps H_Q to H_F. Each of these U-matrices is unitary. Also U_Q is unitary and U_F is obtained by algebraic continuation in the quantum numbers labelling the states of U_Q to U_F.

Hermitian conjugation makes sense since the defining condition

$$\langle \alpha_F | U n_Q \rangle = \langle U^\dagger \alpha_F | n_Q \rangle \ . \tag{16}$$

allows to interpret $|n_Q\rangle$ also as an element of H_F. If U would map different completed number fields to each other, hermiticity conditions would not make sense.

The hermitian conjugate of U-matrix maps H_F to H_Q so that $UU^\dagger$ *resp.* $U^\dagger U$ maps H_F *resp.* H_Q to itself. This means that there are two independent unitarity conditions

$$\begin{aligned} U_F U_F^\dagger &= Id_F \ , \\ U_F^\dagger U_F &= Id_Q \ . \end{aligned} \tag{17}$$

One can write $U = P_Q + T_F$ and $U^\dagger = P_Q + T_F^\dagger$, where P_Q refers to the projection operator to H_Q.
This gives

$$\begin{aligned} T_F + T_F^\dagger &= -T_F T_F^\dagger \ , \\ P_Q T_F + T_F^\dagger P_Q &= -T_F^\dagger T_F \ . \end{aligned} \tag{18}$$

It is convenient to introduce the notations $T_Q = P_Q T_F$ and $T_Q^\dagger = T_F^\dagger P_Q$ with analogous notations for U and $U^\dagger$. The first condition, when multiplied from both sides by P_Q, gives together with the second equation unitarity conditions for T_Q

$$\begin{aligned} T_Q + T_Q^\dagger &= -T_Q T_Q^\dagger \ , \\ T_Q + T_Q^\dagger &= -T_F^\dagger T_F \ . \end{aligned} \tag{19}$$

This means that the restriction of the U-matrix to H_Q is unitary.

The difference between the right hand sides of the equation should vanish. The understanding of how this happens requires more delicate considerations. For instance, in the case of $F = C$ continuous sum over indices appears at the right hand side coming from four-momenta labelling the states. The restrictions of quantum numbers to Q and its subsets could be a process analogous to the momentum cutoff of quantum field theories. The continuation from discrete integer valued labels of, say discrete momenta, to continuous values is performed routinely in various physical models routinely, and it would seem that this process has cognitive and physical counterparts. This picture conforms with the vision that the rational (or extended rational) U-matrix U_Q gives the U-matrices U_F by an algebraic continuation in the quantum numbers labelling the states (say 4-momenta).

3.2.2 Could U_F describe dispersion from H_Q to the spaces H_F?

One can also consider a more general situation in which the states in H_Q can be said to disperse to the sectors H_F. In this case one can write

$$T \quad = \quad \text{"}\sum_F\text{"} T_F \ . \tag{20}$$

Here the sum has only a symbolic meaning since different number fields are in question and an actual summation is not possible. The T-matrix T_Q is the sum of the restrictions of T_F to H_Q and is the sum of rational valued T-matrices: $T_Q = \sum_F P_Q T_F$.

The T-matrices T_F are not anymore obtainable by algebraic continuation from same T-matrix T_Q. The unitarity conditions

$$\sum_F (P_Q T_F + T_F^\dagger P_Q) \quad = \quad -\sum_F T_F^\dagger T_F \tag{21}$$

make sense only if they are satisfied separately for each T_F, exactly as in the previous case. T

The diagonal elements

$$T_F^{mm} + \overline{T}_F^{mm} = \sum_\alpha T_F^{m\alpha}\overline{T}_F^{m\alpha} = \sum_r T_F^{mr}\overline{T}_F^{mr}$$

give essentially total scattering probabilities from the state $|m\rangle$ of H_Q to the sector H_F, and must be rational (or extended rational) numbers. One can therefore say that each U-process leads with a definite probability to a particular sector of the state space.

The fact that states which are superpositions of states in different spaces H_F does not make sense mathematically, forces the occurrence of a process, which might be regarded as a number theoretical counterpart of state function reduction and preparation. First a sector H_F is selected with probability p_F. Then F-valued (in particular complex valued) entanglement in H_F is reduced by state reduction and preparation type processes to a rational or extended rational entanglement having interpretation as bound state entanglement. It would be natural to assume that Negentropy Maximization Principle governs this process. Obviously the possibility to reduce state function reduction to number theory forces to consider quite seriously the proposed option.

3.2.3 Do state function reduction and state-preparation have number theoretical origin?

The foregoing considerations support the view that state function reduction and state preparation are number theoretical necessities so that there would be a deep connection between number theory and free will. One could even say that free will is a number theoretic necessity. The resulting more unified view provides the reason why for state function reduction, and preparation and allows to generalize previous views developed gradually by physics and consciousness inspired educated guess work.

1. Negentropy Maximization Principle as variational principle of cognition

It is useful to discuss the original view about Negentropy Maximization Principle (NMP) before considering the possible generalization of NMP inspired by the number theoretic vision.

NMP was originally motivated by the need to construct a TGD based quantum measurement theory. Gradually it however became clear that standard quantum measurement theory more or less follows from the assumption that the world of conscious experience is classical: this meant that NMP became a principle governing only state preparation.

State function reduction is achieved if a localization in zero modes occurs in each quantum jump, and if U matrix in zero modes corresponds to a flow in some orthogonal basis for the configuration space spinor fields in the quantum fluctuating fiber degrees of freedom of the configuration space. The requirement that U-matrix induces effectively a flow in zero modes is consistent with the effective classicality of the zero modes requiring that quantum evolution causes no dispersion. The one-one correlation between preferred quantum state basis in quantum fluctuating degrees of freedom and zero modes implies nothing but a one-one correspondence between quantum states and classical variables crucial for the interpretation of quantum theory. It seems that number theoretical vision forces to generalize this view, and to raise NMP to a completely general principle applying also to the state function reduction as the original proposal indeed was.

In its original form NMP governs the dynamics of self measurements and thus applies to the quantum jumps reducing the entanglement between quantum fluctuating degrees of freedom for given values of zero modes. Self measurements reduce the entanglement only between subsystems in quantum fluctuating degrees of freedom since they occur after the localization in the zero modes. Self measurement is repeated again and again for the unentangled subsystems resulting in each self measurement. This cascade of self measurements leads to a state possessing only extended rational entanglement identifiable as bound state entanglement and having negative number theoretic entanglement entropy. This process should be equivalent with the state preparation process assumed to be performed by a conscious observer in standard quantum measurement theory.

NMP states that the self measurement can be regarded as a quantum measurement of the subsystem's density matrix reducing the counterpart of the entanglement entropy of some subsystem to a smaller value, and that this occurs for the subsystem for which the reduction of the entanglement entropy is largest among all subsystems of the p-adic self. Inside each self NMP fixes some subsystem which is quantum measured in the quantum jump. One could perhaps say that self measurements make possible quantum level self repair since they allow the system in self state to fight against thermalization which results from the generation of unbound entanglement between subsystem-complement pairs.

2. NMP and number theory

The requirement the universe of conscious experience is classical is one manner to justify quantum jump. This hypothesis could be replaced by a postulate that state function reduction and preparation project quantum states to a definite number field and that only extended rational entanglement identifiable as bound state entanglement is stable. This is consistent with NMP since it is possible to assign to an extended rational entanglement a non-negative number theoretic negentropy as the maximum over entropies defined by various p-adic entropies $S_p = -\sum p_k log(|p_k|_p)$.

The unitary process U would thus start from a product of bound states for which entanglement coefficient are extended rationals, and would lead to a formal superposition of states belonging to different number fields. Both state function reduction and state preparation would begin with a localization to a definite number field. This localization would be followed by a self measurement cascade reducing the entanglement to extended rational entanglement.

This vision forces to challenge the earlier views about state function reduction.

1. There is no good reason for why NMP could not be applied to both state function reduction and preparation.

2. If the entanglement between zero modes and quantum fluctuating degrees of freedom involves only discrete values of zero modes, the problems caused by the fact that no well-defined functional integral measure over zero modes exists, find an automatic resolution. Since extended rational entanglement possesses negative entanglement entropy, it is stable also against reduction if NMP applies completely generally. A discrete entanglement involving transcendentals not contained to any *finite* extension of any p-adic number field is unstable and reduced.

3. The quantum measurement lasts for a time determined by the life-time of the bound state entanglement between zero modes and quantum fluctuating degrees of freedom. Physical considerations of course support the view that it takes more than single quantum jump (10^{-39} seconds of psychological time) for the state function reduction to take place. The notion of zero mode-zero mode bound state entanglement seems however to be self-contradictory. If join along boundaries bonds are space-time correlates for the bound state entanglement, their formation should transform roughly half of the zero modes associated with the two space-time sheets to quantum fluctuating degrees of freedom.

4. If p-adic length scale hierarchy has as its counterpart a hierarchy of state function reduction and preparation cascades, one must accept the quantum parallel occurrence of state function reduction and preparation processes in the parallel quantum universes corresponding to different p-adic length scales. This picture provides a justification for the modelling of hadron as a quantum system in long length and time scales and as a dissipative system consisting of quarks and gluons in shorter length and time scales. The bound state entanglement between subsystems of entangled systems having as a space-time correlate join along boundaries bonds connecting subsystem space-time sheets, is a second important implication of the new sub-system concept, and plays a central role in TGD inspired theory of consciousness.

4 Some consequences of NMP

In the sequel the most obvious consequences of self measurement and NMP are discussed from the point of view of physics, biology, cognition, and quantum computing.

4.1 NMP and physics

Since NMP relates to self measurement, which is something new from the point of view of standard physics, NMP has nontrivial implications also in real context.

4.1.1 NMP and the second law of thermodynamics

The relationship of NMP to the second law has been a longstanding open issue. One is also forced to ask whether TGD really predicts second law in its standard form or should one introduce a fractal version of the second law taking into account the p-adic length scale hypothesis.

1. NMP, de-coherence, and macro-temporal quantum coherence

NMP means maximization of the information content of conscious experience if interpreted as negentropy gain. In case of un-bound entanglement elimination of entanglement means information gain and this means also de-coherence which in turn corresponds to dissipation and is interpreted as implying second law. NMP can however also generate bound state entanglement and makes possible macroscopic quantum coherence since the sequence of quantum jumps corresponding to the lifetime of the bound state effectively integrates to single quantum jump. A possible conscious experience accompanying negentropic bound state entanglement would be as experience of understanding.

If finitely extended rational entanglement is identified as bound state entanglement, one can understand dissipation as a process eliminating unbound entanglement and leaving bound state entanglement or even strengthening it and thus leading to a genuine generation of order. De-coherence is loss of non-bound coherence. One could also say that state function reduction and preparation are analogous to Darwinian selection destroying what is unstable. Thus it seem that the interpretation of the second law is not quite correct. Only if one forgets bound state entanglement second law leads to the illusionary conclusion that universe is becoming gradually more and more disordered. The belief on the eventual heat death of Universe is perhaps the dramatic misunderstandings following from the neglect of bound state entanglement.

The cascade of state function function reductions and self measurements occurring in different scales and perhaps even in quantum parallel manner is very much analogous to a conscious analysis. This process is not however a mere decay process since the outcome of the process is stable finitely extended rational entanglement giving rise to an experience of understanding as a result of this analysis. By macro-temporal quantum coherence this process occurs in various time scales. The basic aspect of conscious thought and of theoretical description is that it can model the reality using only discrete mathematical structures and the outcome of the quantum jumps indeed is quantum entanglement in discrete number fields.

2. Thermodynamics for qualia and thermodynamics for matter

Second law seems to hold also at the level of conscious experience of self: the non-determinism of the state function reduction and preparation processes implies that conscious experience involves statistical aspects in the sense that the experienced qualia correspond to the averages of quantum number and zero mode increments over the sequence of quantum jumps. When the number of quantum jumps in the ensemble defining self increases, qualia get more entropic and qualia fuzzy unless macro-temporal quantum coherence changes the situation. Thus the maximization of information at the level of single quantum jump implies loss of information at the level of self and vice versa. NMP destroys non-bound quantum coherence but tends to generate bound quantum coherence.

Important difference between the statistical physics for qualia and material world is that qualia correspond to averages for the increments of quantum numbers and zero modes in the ensemble of quantum jumps defining sub-self (mental image) whereas in ordinary statistical physics measured quantities would correspond to zero modes and quantum numbers basically. Of course, sequence of quantum jumps defines also this kind of ensemble but the averages over this ensemble are not experienced directly as suggested by various arguments relying on symmetries (it is not possible to experience symmetry related quantum numbers and zero mode values as different). Thus also the statistical ensembles of thermodynamics could be identified at the fundamental level as ensembles defined by quantum jump sequences. The basic function of sensory organs is to relate thermodynamical ensembles to corresponding qualia ensembles by mapping quantum numbers to quantum number increments so that our sensory perception is in reasonable approximation about world rather than changes of the world.

3. Quantum classical correspondence

Quantum-classical correspondence requires that second law must have a classical space-time description. The classical non-determinism of Kähler action indeed makes it possible to represent quantum jump sequences symbolically at space-time level and therefore the increase of entropy at the level of conscious experience should have counterpart at the level of the symbolic representations of conscious experience. This makes sense also for the p-adic cognitive representations of conscious experience.

Intuitively it seems clear that the classical non-determinism of Kähler action is what allows to understand second law since without it everything would effectively reduce to the light cone boundary and entropy would be the same for all cosmic time constant sections as a general coordinate invariant.

To make this intuition more transparent, notice that entanglement entropy is associated with causal determinants which correspond to light like 3-surfaces $X_l^3 \subset H$ (elementary particle horizons) or to light like 7-surfaces $X_l^3 \times CP_2 \subset H$ (analogs of $\delta H = \delta M_+^4 \times CP_2$, initial moment of "big bang"), both playing the role of the role of quantum hologram. If Kähler action were deterministic, general coordinate invariance would imply that the whole physics could be reduced to δH and time would be lost. Kähler action is however not deterministic, and entropy in general increases since the presence of the topologically condensed space-time sheets inside space-time sheets implies additional additive contributions to the entropy. Topological light rays (MEs) represents perhaps the most important contributions of this kind. Thus an observer represented by a mind like space-time sheet drifting to the direction of future would find that entropy tends to increase. There are of course also other sources to what might be called classical space-time entropy.

The formal definition of entropy for a given space-time sheet has been already considered. Space-time sheet decomposing to N strictly deterministic regions defines in a natural manner ensemble and one can calculate thermodynamical averages for various observables in this ensemble. The probabilities are always proportional to $1/N$ and thus number theoretic entropies S_p are negative for primes p appearing as factors of N. This raises the question whether the entropy due to the non-determinism could be actually regarded information. This view is supported by the fact, that classical non-determinism makes possible for the system to represent symbolically other systems in its state. Perhaps it depends to some degree on observer whether the non-determinism corresponds to information or entropy.

4. Entropy of conscious experience, thermodynamical entropy, classical space-time entropy: how do they relate?

There are three kinds of entropies and the basic question is how these entropies relate.

1. Does the entropy characterizing the experience of self relate to the thermodynamical entropy of some system? The fact that non-geometric sensory qualia have a statistical interpretation, suggests that the entropy associated with the qualia of the mental image corresponds to the thermodynamical entropy for a system giving rise to the qualia via the sensory mapping. The thermodynamics of quantities in the external world would thus be mapped to the thermodynamics of qualia, increments of quantities, in the inner world. Selves could also represent the fundamental thermodynamical ensembles since they define also statistical averages of quantum numbers and zero modes although these are not directly experienced.

2. Could one interpret the entropies of the space-time sheets as entropies associated with the symbolic representations of conscious experiences of selves? Could one see the entire classical reality as a symbolic representation? Does the entropy of conscious experience correspond to the thermodynamical entropy of the perceived system, which in turn would correspond to the classical space-time entropy of the system representing the perceived system symbolically? Does this conclusion generalize to the case of p-adic entropy? Quantum-classical correspondence would encourage to cautiously think that the common answer to these questions might be yes.

4. Questions about second law

One can question also the second law.

1. Could it be that second law is an illusion created by the fact that we do not realize that besides us there is infinite hierarchy of selves which also have conscious information about world? What looks like a loss of information for us, could be gain of information for lower levels of consciousness.

2. Is the notion of p-adic evolution consistent with the second law? Is second law perhaps true only in time scales longer than the p-adic time scale $T_p = L_p/c$ at the space-time sheet characterized by the p-adic prime p? This would mean a fractal hierarchy of breakings of the second law.

3. Spin glasses are non-ergodic systems: does the quantum spin glass property of the TGD universe imply the breaking of the second law? Gravitation has been seen as one possible candidate for the breaking second law because of its long range nature. It is indeed classical gravitational energy which distinguishes between almost degenerate spin glass states.

4. How the possibility of irreducible selves in a state of oneness and able to quantum compute relates to second law? In this situation most of the zero modes are transformed to quantum fluctuating degrees of freedom and very many quantum jumps bind to a single effective quantum jump. State function reduction and self measurement cascade, which ought to take care of quantum de-coherence guaranteing that second law is obeyed, do not occur. Doesn't this mean the breaking of second law in the time scale defined by the duration of the bound state?

   Note that by the previous argument it is precisely the quantum spin glass degeneracy caused by classical gravitational energy differences which makes the lifetimes of bound states much longer than they should be otherwise.

5. What do experiments say?

That the status of the second law is far from settled is demonstrated by an experiment performed by a research group in Australian National University [24]. The group studied a system consisting of 100 small beads in water. One bead was shot by a laser beam so that it became charged and was trapped. The container holding the beads was then moved from side to side 1000 times per second so that the trapped bead dragged first one way and then another. The system was monitored and for monitoring times not longer than .1 seconds second law did not hold always: entropy could also decrease.

What is remarkable that .1 seconds defines the duration τ of the memetic code word and corresponds to the secondary p-adic time scale $T_p(2) = \sqrt{p}L_p/c$ associated with Mersenne prime $p = M_{127}$ characterizing electron. This correspondence follows solely from the model of genetic code predicting hierarchy of codes associated with $p = 3, 7, 127$ (genetic code), $p = M_{127}$,... τ should be the fundamental time scale of consciousness. For instance, average alpha frequency 10 Hz corresponds to this time scale and 'features' inside cortex representing sensory percepts have average duration of .1 seconds.

This raises some intriguing questions:

1. Could super-computing non-decohering irreducible selves/mental images in a state of oneness lasting for .1 seconds be involved? Could selves representing memetic codewords be generated also in the water outside our brain? Does this mean that formation of the clusters of water molecules is the fundamental mechanism giving rise to these super-conducting selves?

2. The experiment involves also the millisecond time scale, roughly the duration of single bit of the memetic codeword and the duration of the nerve pulse. Could this have some significance?

4.1.2 NMP and self-organization

The notion of self allows a nice solution to the problem by providing the fundamental mechanism of dissipation. Self is s dissipative structure and perceiving self maps the dissipation at the level of quantities in the external world to dissipation at the level of qualia in the internal world. Dissipation in turn reduces basically to the non-determinism of the Kähler action. In particular, the emergence of sub-selves inside self looks like dissipation from outside but corresponds to self-organization from the point of view of self.

Dissipation leads to self-organization patterns and in the absence of external energy feed to thermal equilibrium. Thus thermodynamics emerges as a description for an ensemble of selves or for the time average behavior or single self when external energy feed to system is absent. One can also understand how the dissipative universe characterized by the presence of parameters like diffusion constants, conductivities, viscosities, etc.. in the otherwise reversible equations of motion, emerges. Dissipative dynamics is in a well defined sense the envelope for the sequence of reversible dynamical evolutions modelling the sequence of final state quantum histories defined by quantum jumps.

Quantum self-organization leads to fixed point self-organization patterns analogous to the patterns emerging in Benard flow. Since selves approach 'asymptotic selves', dissipation can be regarded as a Darwinian selector of both genes and memes. Thus not only surviving physical systems but also stable conscious experiences of selves, habits, skills, behaviors, etc... are a result of Darwinian selection.

The new element in the picture is that even quantum jump itself can be seen as a self-organization process analogous to Darwinian selection, which eliminates all unbound entanglement and yields a state containing only bound state state entanglement and representing analog of the self-organization patterns. By macro-temporal quantum coherence effectively gluing quantum jumps sequences to single quantum jump this pattern replicates itself fractally in various time scales. Thus self-organization patterns can be identified as bound states and the development of the self-organization pattern as a fractally scaled up version of single quantum jump. Second new element is that dissipation is not mere destruction of order but producer of jewels. A further new element is that dissipation can occur in quantum parallel manner in various scales.

The arrow of psychological time is closely related to second law of thermodynamics. Psychological time can be identified as kind center of mass time for cognitive space-time sheet of finite time duration representing self and is zero mode with precisely defined value in each final state of the quantum jump. The arrow of psychological time derives from the drift of the mind like space-time sheet in future light cone: for a given point of future light cone there is much more room in the future light cone than in its past. A more precise view about the emergence of psychological time is based on p-adic-to-real phase transitions identified as transformation of intentions to actions.

4.1.3 NMP and p-adic length scale hypothesis

The original form of the p-adic length scale hypothesis stated that physically most interesting p-adic primes satisfy $p \simeq 2^k$, k prime or power of prime. It has however turned out that all positive integers k are possible. Surprisingly few new length scales are predicted by this generalization in physically interesting length scales. p-Adic length scale hypothesis leads to excellent predictions for elementary particle masses (note that the mass prediction is exponentially sensitive to the value of k) and explains also some interesting length scales of biology: for instance, the thicknesses of the cell membrane and of single lipid layer of cell membrane correspond to $k = 151$ and $k = 149$ respectively.

The big problem of p-adic TGD is to derive this hypothesis from the basic structure of the theory. The most convincing argument is based on black hole-elementary particle analogy [E5] leading to the generalization of the Hawking-Bekenstein formula: the requirement leading to the p-adic length scale hypothesis is that the radius of the so called elementary particle horizon is itself a p-adic length scale. This argument involves p-adic entropy essentially and it seems that information processing is somehow involved.

An exciting possibility, suggested already earlier half seriously, is that there is *evolution already at elementary particle level*. The identification of p-adic physics as physics of cognition indeed forces this interpretation. In particular, one can understand p-adic length scale hypothesis as reflecting the survival of the cognitively fittest p-adic topologies. It will be found that a model for learning as a transformation of the reflective level of consciousness to proto level supports the view that evolution and learning occur already at elementary particle level as indeed suggested by NMP: the p-adic primes near power of prime powers of two are the fittest ones. The core of the argument is the characterization of learning as a map from 2^N many-fermion states to M association sequences. The number of association sequences should be as near as possible equal to 2^N. If M is power of prime: $M = p^K$, association sequences can be given formally the structure of a finite field $G(p, K)$ and p-adic length scale hypothesis follows as a consequence of $K = 1$. NMP provides the reason for why $M = p^K$ is favored: in this case one can construct realization of quantum computer with entanglement probabilities $p_k = 1/M = 1/p^K$ and the negentropy gain in quantum jump is $Klog(p)$ while for M not divisible by p the negentropy gain is zero.

A possible physical reason for the primes near prime powers of 2 is that survival necessitates the ability to co-operate, to act in resonance: this requirement might force com-measurability of the length scales for p-adic space-time sheet (p_1) glued to larger space-time sheet ($p_2 > p_1$). The hierarchy would state from 2-adic level having characteristic fractal length scales coming as powers of $\sqrt{2}$. When $p > 2$ space-time sheet is generated during cosmological evolution $L(p)$ for it must correspond to power of $\sqrt{2}$ so that one must have $p \simeq 2^n$. This argument does not however explain why *power of prime* powers of 2 are favored.

4.1.4 NMP and dark matter hierarchy

The hierarchy of Planck constants seems to reduce the notion of self to that of quantum jump at the higher level of self hierarchy so that lower level quantum jumps correspond to sub-selves giving rise to mental images and experienced flow of time. The finite geometric duration of quantum jump corresponds to the time that subsystem remains un-entangled. NMP holds still true as a variational principle for the dynamics of quantum jumps at each level of the hierarchy and the quantum jump would be actually replaced by a process proceeding from long time and length scales to short length scales.

4.2 NMP and biology

The notion of self is crucial for the understanding of bio-systems and consciousness. By leaving cognition (p-adic physics) outside biology, one can restrict the considerations to the real form of NMP.

4.2.1 Life as islands of finitely extended rational numbers in the seas of real and p-adic continua?

If quantum jump is indeed number theoretic necessity, the generation of macro-temporal quantum coherence means generation of states with extended rational entanglement. Macro-temporal quantum coherence is basic aspect of life and consciousness and one could therefore say that life corresponds to an island of extended rationality in the seas of real and p-adic continua. One could also see evolution of cognition as gradual emergence of extensions of p-adic number fields (in particular, p-adic space-time sheets) of increasing value of p and increasing algebraic dimension.

The view about the crucial role of rational and finitely extended numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that also algebraic numbers rather besides rationals are essential for life.

Also finite-dimensional extensions of p-adic numbers involving transcendentals are possible and in fact necessary (say the extension containing $e, e^2, .., e^{p-1}$ as units (e^p is ordinary p-adic number). Thus one could see the discovery of transcendentals as an emergence of transcendentals like e and π to the finite-dimensional algebraic extension associated with p-adic space-time sheets serving as correlates for mathematical cognition.

It however turns out that $log(p)$ and π cannot belong to a finite-dimensional extension of p-adic numbers (see the chapter "Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory"). One might however hope that $log(p)/log(2)$ belongs to an extension of rationals defining finite-dimensional extension of p-adics so that p-adic entropy would be always extended rational multiple of bit. This is achieved if one has

$$log(p) = e^{q_1(p)} q_2(p) \times t \ , \ q_2(p_1) \neq q_2(p_2) \ \text{for} \ p_1 \neq p_2 \qquad (22)$$

such that t is a transcendental number other than root of e so that one does not get contradiction by exponentiating both sides of the above equation. This ansatz does not lead to any obvious contradictions. For instance, power of π is a reasonable candidate and for physical reasons $t = 1/\pi$ is a favored value of t.

4.2.2 How second law and evolution can be mutually compatible?

Second law is associated with psychological time characterizing the mind like space-time sheet of observer and reflects (or is reflected by) the classical non-determinism of Kähler action. With respect to subjective time evolution takes place since the localization in zero modes implies that the infinite p-adic prime of the universe increases in the long run since the number of primes larger than given infinite prime P is infinitely larger than the number of primes than P. The infinite prime P characterizing the entire universe decomposes in a well defined manner to finite primes and p-adic evolution at the level of entire universe is implied by local p-adic evolution at the level of selves. Therefore maximum entanglement entropy gain for p-adic self increases at least as $log(p)$ with p in the long run. This kind of relationship might hold true for real selves of p-adic physics is physics of cognitive representations of real physics as suggested by the success of p-adic mass calculations. Thus it should be possible to assign definite prime to each real space-time sheet.

The arguments leading to p-adic evolution and the arrow of psychological are very similar and are in accordance with TGD inspired generalization of 'Ontogeny Recapitulates Phylogeny' principle which says that the evolution at the level of space-time reflects the evolution at the level of the configuration space. The occurrence of phase transitions leading to the emergence of larger and larger space-time sheets characterized by increasingly larger p-adic primes indeed leads to the increase of the entropy.

p-Adic evolution is consistent with NMP. NMP forces the generation of bound state entanglement with maximal entanglement negentropy in given quantum jump. Therefore it would not be surprising if the entanglement negentropy per space-time volume would gradually increase in the evolution by quantum jumps.

A further new perspective to the second law is implied by the dark matter hierarchy predicting existence of an infinite hierarchy of quantum jumps with increasing geometric time durations. The dark matter hierarchy is not taken into account in standard physics based view about information. In this framework it is quite possible that second law is actually an illusion reflecting only the character of conscious experience rather than that of the underlying reality.

More precisely, the highest dark matter level associated with self corresponds to its geometric duration which can be arbitrarily long: the typical duration of the memory span gives an idea about the level of dark matter hierarchy involved. For instance, as the model for a hierarchy of generalized EEGs predicts, in the case of humans the highest level would be $k = 7$ corresponding to the time scale defined by the human lifetime [M3]. Mental images of self correspond always to the lower levels of dark matter hierarchy and to ensembles formed by the corresponding quantum jump sequences and the longer the sequence, the more entropic it is bound to be.

Second law is true as long as it characterizes what happens to the mental images. Second law however neglects completely the existence of all levels except the lowest level of the dark matter hierarchy and must therefore be illusory as a physical law. Second law can be seen as an unavoidable implication of the materialistic ontology allowing only the lowest level of the dark matter hierarchy and justifying itself by limiting all scientific tests to those which are consistent with it. It is possible to get temporarily rid of this illusion during experiences often characterized as enlightment experiences accompanied by the experiences of oneness and complete emptiness. During this kind of experiences the entropic contribution of the mental images to the conscious experience is absent.

4.2.3 Binding and quantum metabolism as different sides of the same coin

Only bound state entanglement is stable against the state preparation whose dynamics NMP dictates. Hence the fusion of the mental images implicates the formation of a bound state. This process is expected to involve a liberation of the binding energy as a usable energy. This process could perhaps be coined as quantum metabolism and one could say that quantum metabolism and binding are different sides of the same coin. It is known that an intense neural activity, although it is accompanied by an enhanced blood flow to the region surrounding the neural activity, does not involve an enhanced oxidative metabolism [29]. A possible explanation is that quantum metabolism accompanying the binding is involved. Note that the bound state is sooner or later destroyed by the thermal noise so that this mechanism would in a rather clever manner utilize thermal energy by applying what might be called buy now–pay later principle.

4.3 NMP, consciousness, and cognition

As already found NMP dictates the subjective time development of self and is therefore the basic law of consciousness. If p-adic physics is the physics of cognition, the most exotic implications of NMP relate to cognition rather than standard physics.

4.3.1 Quantum jump and cognition

The interpretation of quantum jump as a creation of a totally entangled holistic state $U\Psi_i$ which is then analyzed to stable bound state entangled pieces allows to interpret self measurement cascade as a conscious analysis: this interpretation generalizes by temporal fractality of consciousness. The resulting stable negentropic pieces give rise to experience of understanding.

One can see the situation also differently. The conscious experience of self is average over moments of consciousness and the eventual thermalization induced by the quantum jump sequence destroys all conscious information. To achieve macro-temporal quantum coherence self must be irreducible self for which self measurements and analysis do not occur. The self must also have large number of zero modes transformed to quantum fluctuating degrees of freedom and this achieved if self corresponds to a join along boundaries condensate able to carry out quantum computer like information processing which is the diametrical opposite of analysis. In this state self does not analyze. Thus the reductionistic and holistic modes of consciousness can be seen as different modes of consciously knowing and understanding.

There is obviously a conflict of interests. Selves want to achieve the state of oneness: this means bound state entanglement for sub-selves. On the other hand, my sub-selves want to stay conscious and are fighting against becoming bound state entangled.

There are rather interesting connections with altered states of consciousness and states of macro-temporal quantum coherence.

1. Making mind empty of mental images could perhaps be interpreted as a mechanism of achieving irreducible self state. In this state the sub-selves representing mental images fuse to single mental image which represents highly stable bound state. Effectively single long-lasting quantum jump results. The formation of bound state leads to the liberation of the binding energy utilizable as a useful energy. This might relate to the reports of meditators about lowered metabolic needs.

2. When mind is empty of mental images, self measurements do not occur. This means that quantum computations are also possible in good approximation in quantum fluctuating degrees of freedom. Thus state of oneness empty of mental images often paradoxically claimed to be source of infinite wisdom is a state in which quantum computation and direct experience of macroscopic multiverse become possible.

3. The ordinary wake-up consciousness is identifiable as the analytical mode in which each quantum jump is followed by a complete state preparation yielding a short-lived bound state so that effective moments of consciousness remain short. The basic reason for this would be sensory input. Krishnamurti has talked a lot about states of consciousness in which no separations and discriminations occur and timelessness prevails. These states could correspond to long-lived bound states giving rise to very long

effective moments of consciousness. In this kind of situation NMP does not force cognitive self measurements to occur (cognitively enlightened state cannot become more cognitively enlightened) and analysis and separations can thus be avoided.

4. Sharing and fusion of mental images by entanglement of sub-selves of separate selves makes possible quantum realization of telepathy and could be a universal element of altered states of consciousness.

4.3.2 The concepts of resolution and monitoring

When the fundamental observable (density matrix or entropy operator) has degenerate eigenvalues, one can only speak about probability for quantum jump to a particular eigen space of the the observable since there is no preferred basis in this eigen space. This leads to the concept of resolution: one cannot distinguish between states belonging to a given eigen space of density matrix and one can make predictions for the probabilities for quantum jumps to given eigen space only.

p-Adic probability concept implies additional exotic effect. The total real probability for quantum jump to degenerate subspace is the real counterpart for sum of p-adic probabilities rather than sum of the real counterparts of the p-adic probabilities. This can lead to rather dramatic effects: for instance, the sum of p-adic probabilities can be very small even when the sum of the real probabilities is large.

The notion of resolution is closely related to the notion of monitoring: resolution can be defined as a decomposition of the p-adic state space to a direct sum of subspaces such that the p-adic density matrix is degenerate inside each subspace. If p-adic probabilities are defined modulo $O(p)$ pinary cutoff this kind of degeneracy is bound to occur if the dimension of the state space is larger than p.

An interesting possibility is that the notions of resolution and monitoring could be important in the physics of cognition. Perhaps the well-known fact that the behavior of cognitive systems is sensitive to monitoring, might have something to do with the density matrix characterizing the entanglement between the monitoring and monitored systems. The behavior of monitored system would depend on the resolution of the monitoring, that is on how interested monitorer is about behavior of monitored system. In the limit that monitorer is not interested at all on the behavior, entanglement probabilities would in general be identical and unless the number of states is power of p, $S = 0$ state would result.

The total probability for a set of independent events to occur depends on the resolution of monitoring: not only the behavior of individual quantum system in ensemble but also the *statistical* behavior of the ensemble of systems characterized by same p-adic prime depends on the resolution of the monitoring.

Standard probability theory, which also lies at the root of the standard quantum theory, predicts that the probability for a certain outcome of experiment does not depend on how the system is monitored. For instance, if system has N

outcomes $o_1, o_2, ...o_N$ with probabilities $p_1, ..., p_N$ then the probability that o_1 or o_2 occurs does not depend on whether common signature is used for o_1 and o_2 or whether observer also detects which of these outcomes occurs. The crucial signature of p-adic probability theory is that monitoring affects the behavior of the system. NMP provides precise definition for the concept of monitoring. There are two forms of monitoring depending on whether the fundamental observable, denote it by O, is density matrix or entropy operator.

Consider first the situation in which all entanglement probabilities have p-adic norm different from unity. Physically monitoring is represented by quantum entanglement and differentiates between two eigen states of O (density matrix or entropy operator) only provided the eigenvalues of O are different. If there are several degenerate eigenvalues, quantum jump occurs to any state in the eigen space and one can predict only the total probability for the quantum jump into this eigen space. Hence the p-adic probability for a quantum jump to a given eigen space of density matrix is p-adic sum of probabilities over the eigen states belonging to this eigen space:

$$P_i = \frac{(n(i)P(i))_R}{\sum_j (n(j)P(j))_R} \quad .$$

Here n_i are dimensions of various eigen spaces.

If the degeneracy of the eigenvalues is removed by an arbitrary small perturbation, the total probability for the transition to the same subspace of states becomes the sum for the real counterparts of probabilities and one has in good approximation:

$$P^R = \frac{n(i)P(i)_R}{[\sum_{j\neq i}\sum_j (n(j)P(j))_R + n(i)P(i)_R]} \quad .$$

Rather dramatic effects could occur. Suppose that that the entanglement probability $P(i)$ is of form $P(i) = np$, $n \in \{0, p-1\}$ and that n is large so that $(np)_R = n/p$ is a considerable fraction of unity. Suppose that this state becomes degenerate with a degeneracy m and $mn > p$ as integer. In this kind of situation modular arithmetics comes into play and $(mnp)_R$ appearing in the real probability $P(1 \; or \; 2)$ can become very small. The simplest example is $n = (p+1)/2$: if two states i and j have *very nearly equal but not identical* entanglement probabilities $P(i) = (p+1)p/2+\epsilon$, $P(j) = (p+1)p/2-\epsilon$, monitoring distinguishes between them for arbitrary small values of ϵ and the total probability for the quantum jump to this subspace is in a good approximation given by

$$\begin{aligned} P(1 \; or \; 2) &\simeq \frac{x}{\left[\sum_{k\neq i,j}(P_k)_R + x\right]} \quad , \\ x &= 2\left[(p+1)p/2\right]_R \quad . \end{aligned} \tag{23}$$

and is rather large. For instance, for Mersenne primes $x \simeq 1/2$ holds true. If the two states become degenerate then one has for the total probability

$$
\begin{aligned}
P(1 \ or \ 2) &\simeq \frac{x}{\left[\sum_{k\neq i,j}(P_k)_R + x\right]} \ , \\
x &= \frac{1}{p} \ .
\end{aligned} \qquad (24)
$$

The order of magnitude for $P(1 \ or \ 2)$ is reduced by a factor of order $1/p$!

A test for the notion of p-adic quantum cognition would be provided by the study of the dependence of the transition rates of quantum systems on the resolution of monitoring defined by the dimensions of the degenerate eigen spaces of the subsystem density matrix (or entropy operator). One could even consider the possibility of measuring the value of the p-adic prime in this manner. The behavior of living systems is known to be sensitive to monitoring and an exciting possibility is that this sensitivity, if it really can be shown to have statistical nature, could be regarded as a direct evidence for TGD inspired theory of consciousness. Note that the mapping of the physical quantities to entanglement probabilities could provide an ideal manner to compare physical quantities with huge accuracy! Perhaps bio-systems have invented this possibility before physicists and this could explain the miraculous accuracy of biochemistry in realizing genetic code.

If some entanglement probabilities have unit norm so that their contributions to the p-adic entanglement entropy vanish, quantum jump to an entangled final state can occur: this is genuinely p-adic effect and serves as a second test for p-adic cognition. If density matrix is the fundamental observable, quantum jump can occur to an entangled final state, which corresponds to any $S = 0$ subspace of $S = 0$ eigen space of the entropy operator with is eigen space of the density matrix. If entropy operator is the fundamental observable, quantum jump can occur to any $S = 0$ subspace of entropy operator. Again the total probability for the transition is determined by the p-adic sum of the probabilities and dramatic 'interference' effects at the level of probabilities are possible.

The notion of resolution emerges naturally for the hyper-finite factors of type II_1. The trace of the unit operator is unit for the infinite-dimensional space in question so that any projector with a finite trace must project to an infinite dimensional space so that there would always an infinite-dimensional degeneracy involved with the eigenvalues of the measured observables.

4.3.3 Resolution and monitoring and hyperfinite factors of type II_1

One could however consider the formulation of the theory in terms of p-adic probabilities and for this formulation resolution and monitoring emerge naturally. One could go even further. For instance, if one can specify the infinite number of degrees of freedom as a p-adic integer, say $N = -1 = (p-1)\sum_{k=0}^{\infty} p^k$, which in a well-defined sense represents the largest p-adic integer, one can say that the p-adic probability for a given state is $1/N$ and finite as a p-adic number. It is finite also as a real number and equal to $1/p$ if canonical identification is used to map N to a real number. For a given finite-dimensional density matrix with finite number of distinct eigenvalues it would be possible to have projections to one-dimensional subspace but there would always infinitely degenerate eigenvalue present in accordance with the notion of finite resolution.

A natural question concerns the implications of the assumption that the map of p-adic probabilities to real ones conserves probabilities without additional normalization.

4.4 NMP and quantum computer type systems

TGD Universe can be regarded as an infinite quantum computer. Unitarity informational "time development" U is analogous to a quantum computation lasting infinitely long time. The last step in the quantum jump $U\Psi_i \rightarrow ....\Psi_f$ corresponds to the halting of the computation. The average duration of the quantum computation with respect to the psychological time of self is of order CP_2 time about 10^4 Planck times, if the simplest estimate is correct. Thus 10^{39} infinitely long quantum computations per second by infinite quantum computer occur! Selves, being subsystems able to remain unentangled, are effectively quantum sub-Universes and quantum computers in this very general sense.

The problem is however that conscious experience about the result of the computation is an average over the results of very many quantum computations and thus fuzzy. The statistical description of the conscious experience implies that mathematically the situation is very much analogous with that encountered in the standard quantum computation. Macro-temporal quantum coherence which basically corresponds to the formation of bound states made possible by the spin glass degeneracy of TGD Universe however resolves this problem. Long sequence of quantum jumps fuse effectively to single quantum jump in bound state entangled degrees of freedom and unitary time evolution is at least good approximation for the time evolution.

4.4.1 Number theory and quantum computation in TGD Universe

The notion of number theoretical entanglement entropy has deep implications for understanding what quantum computation really means.

1. For finitely extended rational entanglement entropy can be identified as the number theoretical entanglement entropy which is always negative. Otherwise ordinary Shannon entropy or its p-adic variant must be used. One must distinguish between two kinds of quantum computational modes corresponding to living and dead quantum computers. For dead quantum computers quantum coherence is extremely fragile and lasts for single quantum jump only. For living quantum computers entanglement is bound state entanglement and NMP takes care that quantum coherence lasts for the duration of the bound state. Thus bio-systems would be especially attractive candidates for performers of quantum computation like processes. The binding of molecules by lock and key mechanism is a

basic process in living matter and the binding of information molecules to receptors is a special case of this process. All these processes might involve new physics not taken into account in the standard physics based biochemistry.

2. Macro-temporal quantum coherence is what makes quantum computation like processes possible since a sequence of quantum jumps effectively binds to a single quantum jump with a duration, which corresponds to the lifetime of the bound state. Quantum computation like process starts, when the quantum bound state is generated and halts when it decays. Spin glass degeneracy increases the duration of the quantum computation to time scales which are sensible for human consciousness. In case of cognitive quantum computation like processes the quantum coherence is stabilized by NMP. Spin glass degeneracy also provides the needed huge number of degrees of freedom making quantum computations very effective. These degrees of freedom are associated with the join along boundaries bonds and are essentially gravitational so that a connection with Penrose-Hameroff hypothesis emerges.

3. The mechanism of macro-temporal quantum coherence is roughly following. In the formation of bound state entanglement join along boundaries bonds are generated between 3-space sheets involved and only the over all cm zero modes characterizing the shape and size of the resulting 3-surface remain zero modes. In these new quantum fluctuating degrees state function reduction does not occur anymore and the quantum de-coherence otherwise occurring in each quantum jump is absent. The number of these degrees of freedom is huge so that they are excellent candidates for quantum computation purposes (consider as an example tubulin molecules of micro-tubule or clusters of water molecules) and one ends up with TGD based variant of biological supercomputers. In p-adic context all configuration space variables are zero modes so that in this case this mechanism does not work. However, quaternion conformal spin degrees of freedom do not have any zero modes as correlates and they allow p-adic counterpart of quantum computation which halts when $S \leq 0$ system ends up to $S > 0$ state.

4. One must generalize the standard quantum computer paradigm since ordinary quantum computers represent only the lowest, 2-adic level of the p-adic intelligence. Qubits must be replaced by qupits since for algebraic entanglement two-state systems are naturally replaced with p-state systems. For primes of order say $p \simeq 2^{167}$ (the size of small bacterium) this means about 167 bits, which means gigantic quantum computational resources. The secondary p-adic time scale $T_2(127) \simeq .1$ seconds basic bit-like unit corresponds to $M_{127} = 2^{127} - 1$ M_{127}-qupits making about 254 bits. The idea about neuron as a classical bit might be little bit wrong!

5. Bio-systems are especially attractive candidates for performers of conscious quantum computation like processes. The binding of molecules by

lock and key mechanism is a basic process in living matter and the binding of information molecules to receptors is a special case of this process. All these processes would involve new physics not taken into account in the standard physics based biochemistry.

6. The number theoretic formulation of state function reduction and preparation encourages to think that there is an entire hierarchy of these processes labelled by p-adic length and time scales and that these processes occur in quantum parallel manner in different p-adic length and time scales. This would allow to understand why it is possible to describe hadrons as genuine quantum systems in long scales whereas perturbative QCD describes hadrons as dissipative systems using kinetic equations in short scales. This forces to ask whether quantum parallel dissipative computations each of them very similar to ordinary classical computation could be possible. If so then the strengths of classical and quantum computation might be combined.

7. It might be more appropriate to talk about conscious problem solving instead of quantum computation. In this framework the periods of macro-temporal quantum coherence replace the unitary time evolutions at the gates of the quantum computer as the basic information processing units and entanglement bridges between selves act as basic quantum communication units with the sharing of mental images providing a communication mode not possible in standard quantum mechanics.

4.4.2 Quantum computation and stereo consciousness

The fusion of two or more mental images to single one (binding) and the decomposition of this mental image back to the component mental images would be the counterpart of quantum computation like processes at the level of brain and could allow to think several thoughts or experience several sensory images simultaneously. Stereovision could be seen as basic example about the fusion of mental images to single mental image containing something essentially new.

Neuronal synchrony might be basic example of quantum computation like process. During neuronal synchrony oxidative metabolism is indeed lowered which has interpretation as a generation of bound state entanglement liberating binding energy as a usable energy so that oxidative metabolism is not needed. Bio-systems are especially attractive candidates for performers of quantum computation like processes. The binding of molecules by lock and key mechanism is basic processes in living matter and the binding of information molecules to receptors is a special case of this process. All these processes might involve new physics not taken into account in the standard physics based biochemistry.

4.4.3 Quantum computation, MEs, and time mirror mechanism

One can also consider more concrete ideas about quantum computation. Time mirror mechanism as basic mechanism of consciousness and bio-control is especially attractive in this respect.

1. Unitary process U from conformal invariance

For a quantum jump sequence lasting n steps under previously stated conditions the time development operator is in a good approximation n:th power of the operator U. Quantum-classical correspondence suggests that U has space-time representation. The natural guess inspired by string models is that U decomposes into stringy propagators and vertices. For the metric conformal degrees of freedom single particle time development operators at the light like boundaries of MEs along light like direction would give rise to stringy propagators whereas vertices would be identifiable as 3-surfaces connected by MEs. The reason why these propagators are non-trivial is that single particle conformal invariance ceases to be an exact symmetry just as single particle zero modes cease to be zero modes.

For quaternion conformal fermionic degrees of freedom, which do not have any obvious zero mode counterparts, single particle time evolution operator along the null direction of the elementary particle horizons would give rise to stringy propagator. The fusion of elementary particle horizons would in turn define the basic vertex possibly giving rise to annihilation and creation of wormhole contacts. Quaternion conformal time development operator would be realized also in p-adic fermionic degrees of freedom.

2. Long length scales are not a problem

A heavy objection against light like computation is that the length scales are so large for reasonable computation times. One can however consider the possibility that MEs are reflected again like in mirror and again in a finite volume and that each reflection preserves quantum information and is represented by a unitary transformation. In TGD framework also pairs of MEs with opposite time orientations are possible so that the result of computation in future and at distance of light years could be communicated backwards in time along the second space-time sheet. Also the reverse process might be possible. For instance, long term memory and anticipation could rely on this mechanism. In the chapter "Quantum antenna hypothesis" the idea about DNA as a sequence of MEs realizing laser mirror idea of [30] is discussed.

The reflection of ME could occur from curved almost vacuum space-time surface and have interpretation as topological realization of self energy diagram. The mirror mechanism of long term memories would also have interpretation in terms of the topological Feynman diagrams associated with the expression of the operator U. Thus the mechanism of long term memory would reduce to the fundamental quantum level.

In the p-adic context, p-adic non-determinism allows the reflection of ME to occur in both spatial and temporal directions since conserved quantities are only pseudo constants. Thus cognitive processes could use light like quantum computation routinely if one can assign quaternion conformal degrees of freedom to the light like boundaries of MEs (this is not obvious). The temporal and spatial reflection give also rise to p-adic teleportation and replication of p-adic

MEs and could be crucial element of cognition (see the chapter "Quantum model for cognition"). Temporal reflection is in fact the counterpart for the formation of phase conjugate of a hologram, and in TGD framework it corresponds to time reversed cognition.

3. Cognitive codes and quantum computation

p-Adic length scale hypothesis leads to the idea that each $p \simeq 2^k$, k integer, defines a hierarchy of cognitive codes with code word having duration given by the n-ary p-adic time scale $T(n,k)$ and number of bits given by any factor of k. Especially interesting codes are those for which the number of bits is prime factor or power of prime factor of k. This is a strong quantitative prediction since the duration of both the code word and bit correspond to definite frequencies serving as signatures for the occurrence of commutations utilizing these codes.

If k is prime, the amount of information carried by the codon is maximal but there is no obvious manner to detect errors. If k is not prime there are several codes with various numbers of bits: information content is not maximal but it is possible to detect errors. For instance, $k = 252$ gives rise to code words for which the number of bits is $k_1 = 252, 126, 63, 84, 42, 21_2, 9, 7, 6_2, 4, 3_2, 2$: the subscript $_2$ tells that there are two non-equivalent manners to get this number of bits. For instance, $126 = 42 \times 3$-bit codon can have 42 -bit parity codon: the bits of this codon would be products of three subsequent bits of 126-bit codon. This allows error detection by comparing the error codon for communicated codon and communicated error codon.

4.4.4 Dark matter hierarchy and quantum computation

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [A8]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [A8]. The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

From the point of view of quantum computations these levels would be of crucial importance since the durations of macroscopic quantum coherence increase like $\hbar$. For instance, EEG time scales corresponds to $k = 4$ and a time scale of .1 seconds [J6] and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Bio-systems might perform various kinds of quantum computations. Quantum parallel nerve pulse patterns could make possible quantum computation like activities, for instance during sleep [M2]. The braids formed by magnetic flux tubes are ideal for the realization of topological quantum computations. I have discussed some manners how DNA could act as a topological quantum computer in [E9].

4.4.5 Abstraction hierarchy and genetic code

Mersenne primes $M_n = 2^n - 1$, which seem to play fundamental role in elementary particle physics and it has been already found that their emergence is natural consequence of NMP. This would put primes 3, 7, 31, 127, etc. in a special position. Primes appear frequently in various bio-structures and this might reflect the underlying p-adicity for the association sequences providing 'plan' for the development of bio-system. For instance, we have actually 7 (!) fingers: two of them have degenerated during evolution but can be seen in the developing embryo. There are 31 subunits in our spinal chord, etc...

In the model of genetic code based on a simple model of abstraction process [L1] the so called Combinatorial Hierarchy $2, 3, 7, 127, 2^{127} - 1, ...$ of Mersenne primes emerges naturally. The construction for a model of abstraction process proceeds as follows.

1. At lowest level there are two digits. The statements Yes and No.

2. At the next level one considers all Boolean statements about these two statements which can be regarded as maps from 2-element set to 2-element set. There are 4 of them. Throw one away and you get 3 statements.

3. At the next level one considers all Boolean statements about these 3 statements and the total number of them is 2^3. Throw one away and you get 7 statements. And so on.

The mystery is why one statement must be thrown away at each level of the construction. The answer might relate to a concrete model of quantum computation.

1. A possible neurolevel realization of a quantum computation is following. Entangle in the proposed manner two memetic codewords represented as temporal sequences of 127 cognitive Z^0 magnetized antineutrino ensembles with bit represented as the magnetization direction. The phase transitions changing the direction of magnetization are assumed to involve classical non-determinism.

2. Nerve pulse (or pulse like membrane oscillation) results from each flip of the direction of the Z^0 magnetization. The temporal sequence for which all Z^0 magnetization are in the the direction of the external Z^0 magnetic field is excluded because this state does not give rise to a nerve pulse pattern (or membrane oscillation pattern). In this manner a quantum computer with $N = 1$ and $p = 2^{127} - 1$ results. Incoming nerve pulse patterns could

be taken to be identical memetic codewords and out would go a a pair of memetic codewords representing the initial memetic codeword and the result of the quantum computation. The duration of the computation is .1 seconds and involves $2^{127}-1$ quantum jumps effectively glued to single quantum jump by macro-temporal quantum coherence.

5 Generalization of NMP to the case of hyper-finite type II_1 factors

The intuitive notions about entanglement do not generalize trivially to the context of relativistic quantum field theories as the rigorous algebraic approach of [25] based on von Neumann algebras demonstrates. von Neumann algebras can be written as direct integrals of basic building blocks referred to as factors [16]. Factors can be classified to three basic types labelled as type I, II, and III. Factors of type I appear in non-relativistic quantum theory whereas factors of type III_1 in relativistic QFT [25]. Factors of type II_1 [17], believed by von Neumann to be fundamental, appear naturally in TGD framework [A8].

5.1 Factors of type I

The von Neuman factors of type I correspond to the algebras of bounded operators in finite or infinite-dimensional separable Hilbert spaces. In the finite-dimensional case the algebra reduces to the ordinary matrix algebra in the finite-dimensional case and to the algebra of bounded operators of a separable Hilbert space in the infinite-dimensional case. Trace is the ordinary matrix trace. The algebra of projection operators has one-dimensional projectors as basic building blocks (atoms), the notion of pure state is well-defined, and the decomposition of entangled state to a superposition of products of pure states is unique. This case corresponds to the ordinary non-relativistic quantum theory. Ordinary quantum measurement theory and also the theory of quantum computation has been formulated in terms of type I factors. Also the discussion of NMP has been formulated solely in terms of factors of type I.

5.2 Factors of type II_1

The so called hyper-finite type II_1 factors, which are especially natural in TGD framework, can be identified in terms of the Clifford algebra of an infinite-dimensional separable Hilbert space such that the unit operator has unit trace. Essentially the fermionic oscillator operator algebra associated with a separable state basis is in question. The theory of hyper-finite type II_1 factors is rich and has direct connections with conformal field theories [19], quantum groups [18], knot and 3-manifold invariants [20, 22, 21], and topological quantum computation [26, E9].

5.2.1 The origin of hyper-finite factors of type II_1 in TGD

Infinite-dimensional Clifford algebra corresponds in TGD framework to the super-algebra generated by complexified configuration space gamma matrices creating configuration space spinors from vacuum spinor which is the counterpart of Fock vacuum [A8]. By super-conformal symmetry also configuration space degrees of freedom correspond to a similar factor. For type hyper-finite II_1 factors the trace is by definition finite and normalized such that the unit operator has unit trace. As a consequence, the traces of projection operators have interpretation as probabilities.

Finite-dimensional projectors have vanishing traces so that the notion of pure state must be generalized. The natural generalization is obvious. Generalized pure states correspond to states for which density matrix reduces to a projector with a finite norm. The physical interpretation is that physical measurements are never able to resolve completely the infinite state degeneracy identifiable in TGD framework as spin glass degeneracy basically caused by the vacuum degeneracy implying non-determinism of Kähler action. An equivalent interpretation is in terms of state space resolution, which can never be complete.

In TGD framework the relevant algebra can also involve finite-dimensional type I factors as tensor factors. For instance, the entanglement between different space-time sheets could be of this kind and thus completely reducible whereas the entanglement in configuration space spin and "vibrational" degrees of freedom (essentially fermionic Fock space) would be of type II_1. The finite state-space resolution seems to effectively replace hyper-finite type II_1 factors with finite-dimensional factors of type I.

5.2.2 The new view about quantum measurement theory

This mathematical framework leads to a new kind of quantum measurement theory. The basic assumption is that only a finite number of degrees of freedom can be quantum measured in a given measurement and the rest remain untouched. What is known as Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras allow to realize mathematically this idea [A8]. $\mathcal{N}$ characterizes measurement resolution and quantum measurement reduces the entanglement in the non-commutative quantum space $\mathcal{M}/\mathcal{N}$. The outcome of the quantum measurement would still represented by a unitary S-matrix but in the space characterized by $\mathcal{N}$. It is not possible to end up with a pure state with a finite sequence of quantum measurements.

The measurement of components of quantum spinors does not make sense since it due to the non-commutativity it is not possible to talk about quantum spinor with single non-vanishing component. Therefore the measurements must be thought of as occurring in the state space associated with quantum spinors. The possible consequences of non-commutativity are considered from the point of view of cognition in [A8] by starting from the observation that the moduli squared of quantum spinor components are commuting Hermitian operators possessing a universal rational valued spectrum which suggests interpretation in terms of quantum version of fuzzy belief.

The obvious objection is that the replacement of a universal S-matrix coding entire physics with a state dependent unitary entanglement matrix is too heavy a price to be paid for the resolution of the above mentioned paradoxes. Situation could be saved if the S-matrices have fractal structure. The quantum criticality of TGD Universe indeed implies fractality. The possibility of an infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras expresses this fractal character algebraically. Thus one can hope that the S-matrix appearing as entanglement coefficients is more or less universal in the same manner as Mandelbrot fractal looks more or less the same in all length scales and for all resolutions. Whether this kind of universality must be posed as an additional condition on entanglement coefficients or is an automatic consequence of unitarity in type II_1 sense is an open question.

5.2.3 What happens in repeated measurements?

The assumption of the standard quantum measurement theory is that the outcome of state function reduction does not change in further measurements if the combined system consisting of measured system and performer of measurement is isolated. This hypothesis generalizes to the case of hyper-finite type II_1 factors. Suppose that the outcome of a quantum jump represented by a projection operator P. If the combined system is not isolated, P can replaced by an arbitrary projection operator in the next unitary process. If the combined system is isolated, the next unitary process leads to a state in which P is replaced by a state expressible in terms of projection operators P_i projecting to the sub-space defined by P, and one of them is selected in the next state function reduction or state preparation. A never-ending series of quantum jumps forcing the state to a smaller and smaller but always infinite-dimensional corner of the state-space would result in absence of the unitary process regenerating the entanglement. This process could be seen as a counterpart for the process in which state function reduction and state preparation processes propagate from long to short length scales.

The notion of rational entanglement has a natural type II_1 counterpart and corresponds to rational valued traces for the projection operators involved and rational valued coefficients for these projection operators in the expression of the density matrix. The idea about rational entanglement (or algebraic entanglement in algebraic extension of p-adics in question) as bound state entanglement carrying negative entanglement entropy generalizes.

Rational density matrices are in a special role since they can be thought of as being common to the real and p-adic variants of the state space. The information measures based on p-adic norm and allowing negative entanglement entropy make sense also now. The question whether there might be some deeper justification for the stability of the generalized rational (algebraic) entanglement against state function reduction/preparation reducing entanglement negentropy in the context of hyper-finite type II_1 factors, remains to be answered.

Consider a rationally entangled state characterized by projection operators

P_i such that the probabilities p_i are rational and remain stable in the unitary process. For factor of type I, a situation in which P_i are replaced by 1-dimensional projectors $Q_i < P_i$ is achieved sooner or later. In the infinite-dimensional case this situation can be approached but never reached.

5.2.4 p-Adic thermodynamics with conformal cutoff and hyper-finite factors of type II_1

For hyper-finite factors of type II_1 the unit matrix has unit trace. Hence real probabilities assignable to finite-dimensional projectors vanish so that the eigenvalues of the density matrix are always infinitely degenerate in the real context. p-Adic probabilities however make sense as finite p-adic numbers even if they vanish as real numbers. This raises the idea that p-adic probabilities are more natural for hyper-finite factors of type II_1 than real ones. Indeed, in p-adic context one could have finite probabilities for even one-dimensional sub-spaces, which would definitely mean an enhanced expressive power of the formalism. Thus hyper-finite factors of II_1 would give the reason why for p-adic thermodynamics [6].

The interpretation of p-adic probabilities is of overall importance from the point of view of physics. When probabilities are rational, the number field does not matter. If not, it seems necessary to map the p-adic probabilities to real ones. One can ask whether this mapping should respect probability conservation without normalization by hand. The variants of canonical identification with some additional conditions on probabilities satisfied for instance in p-adic thermodynamics provide a possible manner to perform this map (see [6]). In [E4, E1] it is found that so called canonical identification seems to provide a tool to achieve this.

Canonical identification in its basic form is defined as $I : \sum_{k=0}^{\infty} \alpha_k p^k \mapsto \sum_{k=0}^{\infty} \alpha_k p^{-k}$.

Canonical identification for rational numbers is defined using the unique representation $q = r/s$ as

$$I(\frac{r}{s}) = \frac{I(r)}{I(s)} \quad . \tag{25}$$

Canonical identification allows a further generalization to the case of p-adic thermodynamics where Boltzmann weights b_n are fundamental and their sum defines partition function as $Z = \sum_{n=0}^{\infty} g_n b_n$, where g_n is the degeneracy of the state with a given "energy" (or any conserved quantity whose thermal average is fixed). In real thermodynamics Boltzmann weights are given by

$$b(E_n) = g(E_n) exp(-E_n/T) \quad , \tag{26}$$

where E_n is "energy" and $g(E_n)$ the integer valued degeneracy of states with energy E_n. In p-Adic thermodynamics the partition function would not converges for this form of Boltzmann weights, which are therefore replaced by $b(E_n) = g(E_n)p^{E_n/T}$ and E_n/T is integer valued to guarantee the p-adic existence of the conformal weight. The quantization of E_n/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance in applications of topological geometrodynamics (see [E2, 6]), which is indeed a basic symmetry of the theory [C1]. Thus the mere number theoretical existence fixes the physics to a high degree and indeed leads to the understanding of elementary particle mass scales. For applications to the calculations of elementary particle masses see [6].

In p-adic thermodynamics the probabilities would be given by $p_n = b_n/Z$ and N_{max} would be replaced by Z. When b_n are integers it is natural to define the canonical identification as

$$I(p_n) = I(\frac{b_n}{Z}) \equiv \frac{I(b_n)}{I(Z)} \quad . \tag{27}$$

A physically very powerful additional constraint is that the additivity of probabilities for independent events holds true also for the *real* counterparts of the p-adic probabilities obtained by canonical identification so that one would obtain also a real probability theory without ad hoc normalization of the real images of p-adic probabilities. This condition is satisfied only if the Boltzman weights b_{n_1} and b_{n_2} for any pair (n_1, n_2) are p-adic integers having no common pinary digits so that no "interference" in the sum of the p-adic probabilities occurs.

The selection of a basis for independent events would correspond to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory such that quantum measurement can give only one of these states as an outcome. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product $\sum_n \alpha_n \beta_n p^n$ of integers $x = \alpha_n p^n$ and $y = \beta_n p^n$ defining analogs of wave functions in the space of pinary digits. Or putting it somewhat differently: Boltzman weights b_n for orthogonal quantum states represent them as orthogonal states in the space of binary digits with orthogonality realized as vanishing of the overlap for non-negative "wave functions". This map puts strong constraints on the probabilities of elementary independent events and is therefore highly interesting from the point of view of physics.

p-Adic thermodynamics satisfies the constraint that p-adic probabilities have no common pinary digits provided the degeneracies satisfy the condition $g(E_n) < p$ (later a somewhat more general conditions is deduced). For p-adic mass calculations (see [F3]) the degeneracies $g(n)$ of states with conformal weight $L_0 = n$

(taking the role of "energy") however increase exponentially so that the condition is not satisfied for very large values of n. Since $g(n)$ increases exponentially (say as 2^{nx}, where x is some parameter), probability conservation requires a cutoff of order $n_{max} \sim log_2(p)$ to the number of terms in the sum defining the partition function. In practice this cutoff has no implications since already the two lowest terms give excellent approximation to the elementary particle masses.

For instance, the value of p is $M_{127} = 2^{127} - 1 \sim 10^{38}$ in the case of electron so that higher terms in partition function Z are extremely small. The physical interpretation for the cutoff n_{max} would be in terms of p-adic length scale hypothesis (see [E4, E1] stating that the length scales $L_p \propto \sqrt{p}$ with primes $p \simeq 2^k$, k prime, are physically favored and the exponentially smaller p-adic length scale $L_k \propto \sqrt{k}$ defines the size scale of the elementary particle [F3].

For the ordinary thermodynamics of strings the exponential increase gives rise to Hagedorn temperature T_H as the maximal temperature possible for strings (see [27]). The interpretation is that the heat capacity of system grows without bound since the number of excited degrees of freedom increases without bound as T_H is approached. Clearly Hagedorn temperature is somewhat analogous to the pinary cutoff in p-adic thermodynamics.

The interpretation of the conformal cutoff in terms of factors of type II_1 factor would be that all conformal weights $n > n_{cr}$ correspond to the same p-adic probability so that it is not possible to distinguish experimentally between these states. This interpretation fits nicely with the notions of resolution and monitoring.

5.3 Factors of type III

For algebras of type III associated with non-separable Hilbert spaces all projectors have infinite trace so that the very notion of trace becomes obsolete. The factors of type III_1 are associated with quantum field theories in Minkowski space.

The highly counter-intuitive features of entanglement for type III factors are discussed in [25].

1. The von Neumann algebra defined by the observables restricted to an arbitrary small region of Minkowski space in principle generates the whole algebra. Expressed in a more technical jargon, any field state with a bound energy is cyclic for each local algebra of observables so that the field could be obtained in entire space-time from measurements in an arbitrary small region of space-time. This kind of quantum holography looks too strong an idealization.

   In TGD framework the replacement of Minkowski space-time with space-time sheet seems to restrict the quantum holography to the boundaries of the space-time sheet. Furthermore, in TGD framework the situation is nearer to the non-relativistic one since Poincare transformations are not symmetries of space-time and because 3-surface is the fundamental unit of dynamics. Also in TGD framework M^4 cm degrees of 3-surfaces are present but it would seem that they appear as labels of type II_1 factors in direct integral decomposition rather than as arguments of field operators.

2. The notion of pure state does not make sense in this case since the algebra lacks atoms and projector traces do not define probabilities. The generalization of the notion of pure state as in II_1 case does not make sense since projectors have infinite trace.

3. Entanglement makes sense but has very counter-intuitive properties. First of all, there is no decomposition of density matrix in terms of projectors to pure states nor any obvious generalization of pure states. There exists no measure for the degree of entanglement, which is easy to understand since one cannot assign probabilities to the projectors as their traces.

4. For any pair of space-like separated systems, a dense set of states violates Bell inequalities so that correlations cannot be regarded as classical. This is in a sharp contrast with elementary quantum mechanics, where "de-coherence effects" are believed to drive the states into a classically correlated states.

5. No local measurement can remove the entanglement between a local system and its environment. In TGD framework local operations would correspond to operations associated with a given space-time sheet. Irreducible type II_1 entanglement between different space-time sheets, if indeed present, might have an interpretation in terms of a finite resolution at state space level due to spin glass degeneracy.

On basis of these findings, one might well claim that the axiomatics of relativistic quantum field theories is not consistent with the basic physical intuitions.

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