Does TGD Predict a Spectrum of Planck Constants?

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Contents

Abstract

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1. Jones inclusions and quantization of Planck constant

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

a) The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

b) In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups. Mass spectrum is invariant under these scalings.

c) This predicts automatically arbitrarily large values of Planck constant and assigns the preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ using only iterated square root operation: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter correspond to these special values of Planck constant. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

d) These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_2n, E_6, E_8 are possible so that TGD seems to be able to mimick these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

2. The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant. The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of iterated square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 6 per cent in solar system to be compared with the accepted cosmological value of 4 per cent.

3. Identification of gravitational Planck constant as CP_2 Planck constant

$\hbar_{gr}$ can be interpreted as Planck constant associated with CP_2 degrees of freedom and its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

4. Large values of Planck constant and coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

1 Introduction

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with $M^4_\pm$ and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1.1 Jones inclusions and quantization of Planck constant

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appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

2. In ordinary phase Planck constants of $M^4_\pm$ and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of $M^4_\pm$ and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of the scaling factors of $M^4_\pm$ and CP_2 Planck constants to orders of the maximal cyclic sub-groups: $\hbar(M^4_\pm) = n_a$ and $\hbar(CP_2) = n_b$. The scaling factors of $M^4_\pm$ metric is naturally n_b^2. If one accepts symmetry argument, the scaling factor of CP_2 metric would be n_a^2. Later it will be found that more natural option is that there is no scaling of CP_2 metric.

   At the level of Schrödinger equation this means that Planck constant $\hbar$ corresponds to the effective Planck constant $\hbar_{eff} = (\hbar(M^4_\pm)/\hbar(CP_2))\hbar_0 = (n_a/n_b)\hbar_0$, which thus can have all possible positive rational values. For some time I believed on the scaling of metrics of $M^4_\pm$ *resp.* CP_2 as n_b^2 *resp.* n_a^2: this would imply invariance of Schrödinger equation under the scalings but would not be consistent with the explanation of the quantization of radii of planetary orbits requiring huge Planck constant [D6]. Poincare invariance is however achieved in the sense that mass spectrum is invariant under the scalings of Planck constants. That the ratio n_a/n_b defines effective Planck constant conforms with the fact that the value of Kähler action involves only this ratio (quantum-classical correspondence). Also the value of gravitational constant is invariant under the scalings of Planck constant since one has $G \propto g_K^2 R^2$, R radius of CP_2 for $n_a = 1$.

3. This predicts automatically arbitrarily large values of effective Planck constant n_a/n_b and they correspond to coverings of CP_2 points by large number of $M^4_\pm$ points which can have large distance and have precisely correlated behavior due to the G_a symmetry. One can assign preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible in terms of iterated square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter seem to correspond to these special values of Planck constants. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

4. These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_{2n}, E_6, E_8 are possible so that TGD seems to be able to mimic these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

5. Later it turned out that $H \rightarrow H/G_a \times G_b$ picture does not really allow to understand the fractionization of spin and em charge. As matter fact, just the opposite of fractionization

occurs if one requires $G_a \subset G_b$ invariance of the physical states since the units of the quantum numbers become multiples of n_a *resp.* n_b.

This led to the realization that for a given choice of quantization axes the replacements $M^4 \rightarrow \hat{M}^2 = M^4 \backslash M^2$ and $CP_2 \rightarrow \hat{CP}_2 = CP_2 \backslash S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2, imply that the first homotopy groups of resulting spaces correspond to integers. A hierarchy of covering spaces of $\hat{M}^4$ and $\hat{CP}_2$ labelled by the reduced homotopy groups Z_{n_a} and Z_{n_b} emerges. These covering spaces are naturally extendible to coverings with fiber given by $G_a \times G_b$. One can denote formally this extension by $\hat{M}^4 \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$. The geometric interpretation is that M^2 *resp.* S^2 is replaced by its orbit under G_a *resp.* G_b so that Cartesian product is not in question. This leads naturally to a fractionization of orbital angular momentum and other "orbital" quantum numbers.

The most general picture assumes that for a given choice of quantization axes the generalized imbedding space is the union of $M^2 \times S^2$ common to all factors of the imbedding space and coverings $(\hat{M} \hat{\times} G_a) \times (\hat{CP}_2 \hat{\times} G_b)$, the factor spaces $\hat{M}^4/G_a \times \hat{CP}_2/G_b$, plus the hybrids $(\hat{M}^4 \hat{\times} G_b) \times \hat{CP}_2/G_b$ and $\hat{M}^4/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ resulting as products of covering and factor spaces.

For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labelled by a subset of discrete subgroups of SU(2).

1.2 The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant. The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if one assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of iterated square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 3 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 6 per cent in solar system to be compared with the accepted cosmological value of 4 per cent.

Gravitational Planck constant $\hbar_{gr}$ can be interpreted as effective Planck constant $\hbar_{eff} = (n_a/n_b)\hbar_0$ so that the Planck constant associated with $M^4_\pm$ degrees of freedom (rather than CP_2 degrees of freedom as in the original wrong picture) must be very large in this kind of situation.

If so, its huge value implies that also the von Neumann inclusions associated with $M^4_\pm$ degrees of freedom are involved meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

There is however strong objection based on the observation that the radius of CP_2 would become gigantic. Surprisingly, this need not have any dramatic implications as will be found. It is also quite possible that the biomolecules subgroups of rotation group as symmetries could correspond to $n_a > 1$. For instance, the tedrahedral and icosahedral molecular structures appearing in water would correspond to E_6 with $n_a = 3$ and E_8 with $n_a = 5$. Note that $n_a = 5$ is minimal value of n_a allowing universal topological quantum computation.

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The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)}+1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

2 Basic ideas

The idea that $\hbar$ is dynamical and can have arbitrarily large values is about one and half year old as a write this. A lot of progress has occurred during the last year but I have not yet been able to seriously pose the question whether and how TGD could predict the values of the Planck constant. In the following a proposal for how TGD predicts the value spectrum of $\hbar$ as one aspect of quantum criticality is discussed and number theoretical arguments are used to make a guess about the spectrum of $\hbar$.

2.1 Hints for the existence of large $\hbar$ phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [D6]. This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [D6]. Any interaction, if sufficiently strong, can lead to this kind of phase.

2. Living matter could represent a basic example of large $\hbar$ phase. Even ordinary condensed matter could be "partially dark" in many-sheeted space-time and this could resolve the age old mystery of why water is transparent [J6].

3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [73]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [D5]. The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-canonical gluons [F4, F5]- something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.

4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [J6].

2.2 Quantum coherent dark matter and $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED: $e = \sqrt{\alpha/4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the modified Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in the configuration space degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant $M^4_\pm$ and CP_2 metrics are dynamical. The first guess is that the ratio of scaling factors for the $M^4_\pm$ and CP_2 metrics corresponds to the scaling of effective Planck constant. If one has $\hbar(M^4_\pm) = n_b\hbar_0$ *resp.* $\hbar(CP_2) = n_a\hbar_0$ and n_a^2 *resp.* n_b^2 scales $M^4_\pm$ *resp.* CP_2 metric then the value of Kähler action depends on the ratio n_a/n_b and $\hbar_{eff} = n_a/n_b\hbar_0$ would naturally appear in Schrödinger equation. Since G_b fold covering of $M^4_\pm$ allows fractionization of the angular momentum projection m to m/n_b, this implies a fractionization of angular momenta given by $L_z = \hbar(M^4_\pm)\times m/n_b = (n_a/n_b)m\hbar_0$ and anyonic systems could thus correspond to $n_b > n_a$. Similar fractionization of color charges and em charge are also possible.

The ratio $\lambda(M^4)/\lambda(CP_2)$ of the scaling factors of metrics could be interpreted as coding for radiative corrections to Kähler function and thus space-time physics since Kähler would depend directly on $\hbar_{eff}/\hbar_0$. Even in the case that the radiative corrections to the configuration space functional integral vanish, as suggested by quantum criticality, they would be actually taken into account. The overall scaling of H metric would not however matter as far as the classical dynamics of single space-time sheet is considered. The space-time sheets with different values of scaling constants are however expected to have common points and thus interact and in this manner also the over all scaling becomes relevant.

This kind of dynamics is not consistent with the original view about imbedding space as something completely un-dynamical. The resolution of the problem came from the realization that the fundamental structure is the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD emerges including dynamical scales for the imbedding space metric.

2.3 The phase transition changing the value of Planck constant as a transition to non-perturbatice phase

2.3.1 A phase transition increasing $\hbar$ as a Bose-Einstein condensation type process

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The simplest situation is that conformally confined block of $\mathcal{N}$ particles with identical quantum numbers is formed Compton length scaled up so that $\hbar$ is scaled up by factor $\mathcal{N}$. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result.

The necessity of large $\hbar$ phases (or rather large $\hbar_{eff}$) phases) has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact.

Also the Coulombic contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

2.3.2 The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician.

A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$ and predicts the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule of thumb has now been applied with success the interpretation of hadronic mass calculations and to build models for systems like atomic nucleus and high T_c superconductor and seems to work. Of course, the criterion for transition is primitively formulated and the understanding what really happens in the transition to large $\hbar$ phase behaving like dark matter.

2.4 Planck constant as a scaling factor of metric and possible values of Planck constant

2.4.1 Scaling of Planck constant and scalings of $M^4_\pm$ and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. Minimal substitution does not make sense in CP_2 degrees of freedom

The first crucial observation is that the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a manner that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it. This however means that overall scaling of covariant $M^4_\pm$ metric by factor n_b^2 is equivalent to the scaling of $\hbar^2$ by $1/n_b^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of $M^4_\pm$ and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

Assume that the Dirac operator in $M^4 \times CP_2$ has the following structure.

1. Covariant $M^4_\pm$ *resp.* metric is proportional to $\hbar(CP_2)^2$ *resp.* $\hbar(M^4)^2$.

2. $M^4_\pm$ part is proportional to $\hbar(M^4_\pm)$ and CP_2 part to $\hbar(CP_2)$.

   This implies that $M^4_\pm$ *resp.* CP_2 part of Dirac operator is proportional to $\hbar(M^4_\pm)/\hbar(CP_2)$ *resp.* $\hbar(CP_2)/\hbar(M^4_\pm)$. One can obviously introduce the notion of effective Planck constant $\hbar_{eff}/\hbar_0 = \hbar(M^4_\pm)/\hbar(CP_2)$.

Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^ik_i$ rather than four-momentum squared $p^2 = p^ip_i$ in $M^4_\pm$ degrees of freedom and its analog in CP_2 degrees of freedom. The values of k^2 are proportional to $1/\lambda^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, modified Dirac operator, and Kähler action which carry dynamical information about the ratio $\hbar_{eff}/\hbar_0$.

3. Objection

The fact is that the symmetry for the scalings of the metrics of $M^4_\pm$ *resp.* CP_2 metrics by n_b^2 *resp.* n_a^2 is an ad hoc assumption dictated by unconscious appeal to symmetry. Symmetry is certainly natural for the dual description of space-time surfaces as surfaces in hyper-octonionic space $HO = M^8$ but need not make sense for the description in terms of H. Furthermore, the projective character of CP_2 suggests that one can think that C^3 metric is indeed scaled but that scaling disappears in the projective identification of points belonging to the complex rays of C^3 as same point of CP_2.

The n_a^2 scaling of CP_2 metric is also mathematically questionable since the isometric identification of CP_2 factors for sectors of H with same G_b means that the identified points r_1 and r_2 have same distance from the origin of CP_2: $n_{a_1}d(r_1) = n_{a_2}d(r_2)(r$ is the $U(2)$ invariant radial coordinate). This implies that the CP_2 with $n_{a_1} < n_{a_2}$ is contained as genuine subset on CP_2 with n_{a_2} and that inclusion means kind of blow up of included CP_2 to an open set. It is not clear whether the inclusion extends to a well-define global inclusion.

What is fortunate that the option with universal CP_2 metric does not affect the predictions of existing applications of Planck constant hierarchy at the level of Schrödinger equation. In Kähler action however only n_a makes itself visible so that $n_a = n_b$ situation is not equivalent with $n_a = n_b = 1$ situation in this case.

3. Quantum classical correspondence and the values of $\hbar(M^4_\pm)$

Quantum classical correspondence suggests that the values of n_a should be represented also at space-time level. The variational principle making space-time sheets counterparts of Bohr orbits indeed implies the quantization of Kähler magnetic flux and the quantum need not be the standard flux quantum. The generalized quantization condition would be $\int BdS = n_a\hbar_0$ and in principle it is possible to deduce the values of n_a from the classical theory. The flux integral does not involve the induced metric so that there is no explicit dependence on n_b. The flux integral involves however sum over "sheets" of the covering of $M^4_\pm$ so that the value of flux is $n(G_b)$-fold so that the quantization condition requires scaling of $\int BdS$ for single sheet by n_a/n_b factor corresponding to the value of the effective Planck constant appearing in Schrödinger equation.

2.4.2 The behavior of angular momentum under the scalings of Planck constants

The assumption that mass squared is invariant in the scalings of Planck constants and resulting scaling of $M^4_\pm$ metric follows from the invariance of the mass scale in p-adic mass calculations. This is clear from the fact that massless wave equation $(p^2 - \nabla^2(CP_2))\Psi = 0$ defines the CP_2 contribution to the mass squared and does not contain n_a and n_b explicitly.

1. The invariance of angular momentum under scalings of Planck constants would not be consistent with fractionization of angular momenta in anyonic physics. The scaling in radial degrees of freedom suggest a scaling in angular degrees of freedom which would mean that the scaling of CP_2 Planck constant by n_b inducing n_b-fold scaling of $M^4_\pm$ distances also corresponds to an n_b-fold multiple covering of $M^4_\pm$ by CP_2 points. Assuming this, one finds

angular momentum scales as $L_z = m\hbar_0 \rightarrow (n_a/n_b)m\hbar_0$ since m is fractionized to m/n_b by the presence of G_b covering of $M^4_\pm$. Only for $n_a = n_b$ invariance is obtained. Similar argument applies in color and electro-weak degrees of freedom. The scaling of quantum numbers is $q \rightarrow n_b q$ if CP_2 metric is not scaled.

2. The scaling of the $M^4_\pm$ (CP_2) metric implies that the $M^4_\pm$ size of a given space-time sheet increases by a factor n_b. This is visible as a concrete increase from the point of view of space-time sheets with ordinary values of Planck constant. Note that Compton lengths $\hbar(M^4_\pm)/m$ scale as n_a and this is what makes possible transition to a macroscopic quantum phase.

3. Biologically especially interesting G_a coverings correspond to $n_a = \lambda^k$-fold coverings with $\lambda = 2^{11}$ associated with magnetic flux quanta [M3]. For this sequence $\hbar_{eff}$ increases so that macroscopic quantum phase could result. In this case the unit of angular momentum can become rather large. Note also also that cyclotron energy scales up and exceed thermal energy.

4. One could also understand the approach to quantum chaos as a period doubling type process. The powers $\lambda = 2^k$ are allowed as values of $n_b = \lambda^k$ if one assumes that n_b integers corresponding to Fermat polygons. $\lambda = 2$ is the simplest possibility. Note that h_{eff} decreases in this process. Period 2^k-folding would correspond to the emergence of classical chaos at space-time level by a step-wise process in which the step $\hbar(CP_2) \rightarrow 2^k\hbar(CP_2)$. As $\hbar(CP_2)$ increases by a factor λ, the space-time sheet representing an orbit of particle closing after one turn transforms to an orbit closing only after λ turns. Note that the volume of space-time sheet remains finite only if the orbit closes after finite number of turns. The step $k \rightarrow k+1$ would correspond to a local fractal operation making each sheet of the λ^k sheeted surface λ-sheeted so that λ^{k+1} sheeted surface would result. Instead of period doubling one would have period λ-folding with the value of λ depending on p-adic prime $p \simeq 2^k$.

2.4.3 Why $M^4_\pm$ and CP_2 Planck constants should be integer multiples of $\hbar_0$?

Suppose that the structure in question correspond to an $N(G_b)$-fold covering of $M^4_\pm$ by a symmetry group $G_b \subset SU(2) \subset SU(3)$ acting on CP_2 coordinates so that there are $N(G_b)$ points per each point of $M^4_\pm$. The basic observation is that $n_b \times 2\pi$ rotation, where n_b is the order of maximal cyclic group of G, is needed to bring the particle to the original CP_2 position since the $N(G_b)$-fold covering is analogous to a Riemann surface. This means that angular momentum eigen states $exp(im\phi)$ are replaced with $exp(im\phi/n_b)$.

This is consistent with the quantization of angular momentum and its conservation in the phase transition to non-perturbative phase since the scaling $n_a \rightarrow n_a/n_b$ of the unit of angular momentum occurs. For a cyclic group one would have $n_b = N(G_b)$ but for the dihedral group D_n involving also reflections one would have $N = N(G_b)/2$. One has $n_b = 3$ for tedrahedral and $n_b = 5$ for icosahedral group.

One can however consider also the mirror symmetric situation and in this case one would have covering of CP_2 point by $M^4_\pm$ points with a symmetry which could be some subgroup $G_a \subset SL(2,C)$. This would lead to a scaling of Planck constant appearing in the Lie algebra of color group and this scaling need not be same as for the Lie algebra of Poincare group. Color wave functions $exp(im\phi)$ are replaced with $exp(im\phi/n_a)$ in case of n_a fold covering and the units of color and electro-weak charges are multiplied by n_b.

The formation of these stable multiple coverings could be seen as an analog for a transition in chaos via a process in which a closed Bohr orbit regarded as a particle itself becomes an orbit closing only after m turns. TGD predicts a hierarchy of higher level zero energy states representing S-matrix of lower level as entanglement coefficients. Particles identified as "tracks" of particles at orbits closing after m turns might serve as space-time correlates for this kind of states. There is a direct connection with the fractional quantum numbers, anyon physics and quantum groups. Of course the stability of these coverings is far from obvious. Whether or not the coverings are exact or approximate, it however seems that the basic mathematics of infinite-dimensional Clifford algebras provides an elegant description for them.

2.4.4 The quantization of Planck constants from Jones inclusions

From the beginning the strong gut feeling has been that the allowed values of $\hbar$ are expressible in terms of Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$ related to Jones inclusion hierarchies of hyperfinite factors of type II_1, which correspond to von Neumann algebra naturally associated with configuration space spinors. The proposed formulas were however incredibly clumsy as compared to the final formulas $\hbar(M^4_\pm)/\hbar_0 = n_a$ and $\hbar(CP_2)/\hbar_0 = n_b$.

Consider the inclusion $N \subset M$ of these factors as von Neumann algebras. A deep result is that one can express M as $N : M$-dimensional module over N with fractal dimension $N : M = B_n$. $\sqrt{B_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{B_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of SU(2) defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to the correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tedrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2, C)$) acting as symmetry of space-time sheet in CP_2 ($M^4_\pm$) degrees of freedom. It predicts arbitrarily large $M^4_\pm$ and CP_2 Planck constants in the case of A_n and D_{2n}.

The model provides a concrete view about the transition to non-perturbative phase, justifies the identification of space-time sheets as n_b-fold coverings of $M^4_\pm$ and n_a-fold coverings of CP_2 , and predicts that cyclic and dihedral groups which correspond to polygons constructible using only ruler and compass should correspond to systems especially abundant in Nature. An analog of p-adic length scale hypothesis emerges raising powers of 2 and Fermat primes in special position. The value $\hbar(M^4)/\hbar_0 = n_a = 2^{11}$ and its 2^{11k} multiples for which the physics of living matter provides evidence correspond to these special values of n_a. Occam's razor leaves only this option under serious consideration.

2.4.5 Kähler function codes for a perturbative expansion in powers of $\hbar(M^4_\pm)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of $M^4_\pm$ *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant $M^4_\pm$ by n_b^2/n_a^2 consistent with scalings of M^4 *resp.* CP_2 metrics by n_b^2 *resp.* n_a^2 followed by overall scaling by $1/n_a^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure are avoided.

The first implication of this picture is that the modified Dirac operator determined by the induced metric and spinor structure depends on n_a/n_b in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define configuration space spinor structure and metric depends on the value of n_a/n_b. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of $\hbar_{eff}/\hbar_0 = \hbar(M^4)/\hbar(CP_2)$.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over configuration space defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact. This certainly cannot be the case always: consider only the photon-photon scattering. This paradox can be also regarded as an objection against the proposal that generalized Feynman diagrams are equivalent with tree diagrams or more generally, that each diagram is equivalent with a minimal loopy diagram allowing homologically non-trivial imbedding with non-intersecting lines to a higher genus Riemann surface.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant configuration space metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of $\hbar(M^4_\pm)/\hbar(CP_2)$. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semiclassical" approximation, which might be actually exact, automatically would give a full expansion in powers of $\hbar(M^4_\pm)/\hbar(CP_2)$. This is in a sharp contrast to the usual quantization approach.

2.5 Further ideas related to the quantization of Planck constant

In the following further ideas related to the quantization of Planck constant are discussed. These ideas were originally in central role in attempts to understand the quantization.

2.5.1 The identification of the value of the parameter v_0 in terms of Kähler coupling strength

The quantization of the gravitational Planck constants $\hbar_{gr} = GMm/v_0$ involves the parameter $v_0 \sim 2^{-11}$, which has has actually dimension of velocity unless on puts $c = 1$, and its harmonics and sub-harmonics appear in the scaling of $\hbar_{gr}$. v_0 corresponds to the velocity of distant stars in the model of galactic dark matter. The natural original belief was that this parameter would be fundamental for the understanding of the quantization of $\hbar$. The condition $\hbar_{gr}(M^4)/\hbar_0 = n_a/n_b$ must be however regarded as distinct from the quantization of v_0. Combined with the variation of v_0 implies conditions on the product Mm of the masses. These conditions are however rather mild without additional constraints on $N(M^4)$. Despite the fact that v_0 is not fundamental parameter in quantization of $\hbar$ it deserves to repeat basic argument allowing to identify v_0.

TGD allows to identify this parameter as

$$
\begin{aligned}
v_0 &= 2\sqrt{TG} = \sqrt{\frac{2\pi}{g_K^2}}\sqrt{\frac{G}{R^2}} \ , \\
T &= \frac{1}{8\alpha_K}\frac{\hbar_0}{R^2} \ . \qquad (1)
\end{aligned}
$$

Here T is the string tension of cosmic strings, R denotes the "radius" of CP_2 ($2R$ is the radius of geodesic sphere of CP_2). g_K^2 is Kähler coupling strength assumed to be renormalization group invariant and analogous to critical temperature. Note that this expression could be also regarded as a formula for gravitational constant in terms of fundamental parameters of TGD including v_0 for which TGD predicts the value $v_0 = 2^{-11}$. I have proposed the following formla for α_K for the value $\hbar_0$ of Planck constant

$$
\alpha_K = k\frac{1}{log(p)+log(K)} \ ,
$$

$$K = \frac{R^2}{\hbar_0 G} = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23 = 223,092,870 \ ,$$
$$k \simeq \pi/4 \ . \qquad (2)$$

Equivalence Principle requires that $\hbar$ in the formula for K corresponds to the value $\hbar_0 \equiv 1$ [D6] so that gravitational coupling constant is invariant also with respect to coupling constant evolution associated with Planck constants. One can define "dynamical" Planck length as

$$L_d = \sqrt{G\hbar_{gr}} = \sqrt{M_2/M_1}\frac{1}{v_0}GM_1 \ .$$

The order of magnitude is not too far from Schwartschild radius.

Number theoretic constraints are expected to pose strong constraints on the value of k. The most realistic scenario corresponds to renormalization group invariance of g_K^2. Since this predicts that gravitational constant scales as L_p^2 as a function of p-adic length scale, one must assume that gravitons correspond to Mersenne prime M_{127}, the largest Mersenne which does not correspond super-astrophysical length scale. This hypothesis relates the evolution of color coupling strength to that for electro-weak coupling strength. $v_0 = 2^{-11}$ is predicted to be a fundamental constant. The mean experimental value of $1/v_0$ is $1/v_0 = 2174$ with an accuracy of 1 per cent.

RG invariance of G would imply a discrete version of a typical logarithmic evolution of U(1) coupling constant strength as a function of length scale is in question and for $k = 127$ (M_{127}) the value of α_K is very near to fine structure constant $\alpha \simeq 1/137$. This evolution would be however unrealistic at short distances.

Nottale has argued that also sub-harmonics and harmonics of v_0 must be accepted. In TGD framework this harmonics correspond to different value of gravitational Planck constant and from the general expression $\hbar_{gr} = 2GM_1M_2v_0 = n_a/n_b$ it is clear that one could multiply and divide $\hbar_{gr}$ by integers, in particular multiplication (division) by Fermat primes is possible if they do not appear (appear) in n_a (n_b).

The challenge is to understand why v_0 appears in the basic formula expressing the change of $\hbar$ in the transition increasing the value of $\hbar$. It would seem that $\hbar$ characterizes the magnetic flux tubes (join along boundaries bonds) connecting the interacting systems and serving as space-time correlates for the interaction giving rise to bound state.

The value of v_0 deduced for cosmic strings does not make sense in astrophysical or condensed matter context, where cosmic strings are replaced with magnetic flux tubes. v_0 remains invariant in this scaling down if R^2 is replaced by the p-adic length scale L_p^2 apart from a multiplicative factor in the formulas for G and T so that the product TG remains invariant. $T_m \propto 1/L_p^2$ characterizes the magnetic energy density of the magnetic flux tube and $G_m \rightarrow L_p^2$ is identifiable as a "strong" gravitational coupling strength characterizing the interactions of magnetic flux tubes behaving like string like objects.

2.5.2 Quantum criticality and the spectrum of $\hbar$

The original idea was that number theoretical vision combined quantum criticality could allow to determine the allowed values of Planck constant, or in the new formulation the values of $\lambda(M^4)/\lambda(CP_2)$. It has however become clear that $\lambda(M^4)/\lambda(CP_2)$ is determined by different arguments and quantum criticality could fix the dependence of the Kähler coupling strength on $\lambda(M^4)/\lambda(CP_2)$ rather than determining the values of g_K^2.

Number theoretical vision has led to the proposal that the exponent of Kähler function is expressible as a Dirac determinant for the modified Dirac operator. The condition of quantum criticality would fix that the value of g_K^2, defined as the analog of critical temperature, as a function of $\hbar(M^4_\pm)/\hbar(CP_2)$.

There is an entire hierarchy of algebraic extensions of rationals for which one calculate the Dirac determinant and thus Kähler function. The restriction to an algebraic extension of rationals does not require the restriction of the modified Dirac operator to the set of rationals or their algebraic extensions. The only thing that is required is that the subset of allowed eigenvalues of the modified Dirac operator belongs to the extension considered. It is quite possible that the value of Dirac determinant is finite since the number theoretic restriction might be satisfied only by a finite number of eigenvalues. Hence nature would also take care of regularization of Dirac determinants by using the number theoretic hierarchy. In this manner one can in principle calculate a Dirac determinant for each extension as a function of $\lambda(M^4)/\lambda(CP_2)$ and g_K and fix latter from the quantum criticality.

The fact that quantum phase $q = exp(i\pi/n)$ is algebraic number and exists p-adically only in some minimal algebraic extension of given p-adic number field, suggests that the values of $q(M^4)$ and $q(CP_2)$ might characterize algebraic extensions of p-adic numbers by complex phases so that also $\lambda(M^4)$ and $\lambda(CP_2)$ would depend on the algebraic extension. If so, g_K^2 can depend on the algebraic extension of rationals involved via $\lambda(M^4)/\lambda(CP_2)$. Recall that the original idea was that the requirement that g_K^2 does not depend on algebraic extension, could fix the value of $\lambda(M^4)/\lambda(CP_2)$ for the extension. This might be possible but there is no deep reason to require this.

2.5.3 Renormalization group flow associated with phase resolution and Jones inclusions

The basic philosophical idea behind renormalization group approach is thinning of degrees of freedom when the scale of resolution is reduced. One can also consider renormalization group evolution associated with angle/phase resolution and quantum group phases associated with Beraha numbers defining Jones indices $\mathcal{M} : \mathcal{N}$ are excellent candidates for defining angular resolution in some sense [E9, C7, E10].

In p-adic TGD these angular resolutions would have very concrete interpretation since only algebraic extensions obtained by introducing phases $exp(i\pi/n)$ make it possible to speak about phases in p-adic sense (in cognitive sense). For a finite-dimensional extension of p-adic numbers the number of existing angles is thus always finite. Each Jones inclusion would define algebraic extension of p-adic giving rise to definite angular resolution defined by $\Delta\phi = \pi/n$ and at the limit $n \rightarrow \infty$ the resolution would become ideal.

1. The change $n \rightarrow n-1$ of Jones inclusion could be seen as thinning of degrees of freedom associated with angular resolution leading from resolution $\Delta\phi = \pi/n$ to $\Delta\phi = \pi/(n-1)$. For instance, for $n = 3$ one would obtain a minimal angular resolution with quantum phase equal to $exp(i\pi/3)$. Only three angles values would be discernible p-adically. These phases would correspond naturally to the phases assignable to the center of SU(3) whose Dynkin diagram indeed corresponds to $n = 3$ inclusion. Color $SU(3)$ would be the most rigid or minimal symmetry and would not reduce to $SU(2)$. Color degrees of freedom would correspond to the color rotational rigid body degrees of freedom of a topologically condensed space-time sheet.

2. Contrary to the original guess it seems that the minimum angular resolution cannot correspond to vacuum extremals: this is also consistent with the assumption that non-perturbative phase is in question. Since $SU(2)$ generates homologically non-trivial geodesic spheres of CP_2, G invariance and the requirement that space-time sheet defines a smooth $N(G)$-fold cover of $M^4_\pm$ probably imply non-vacuum extremal property so that CP_2 projection would have CP_2 dimension $D(CP_2) \geq 2$.

3. Since angular resolution becomes poorer in $n \rightarrow n-1$ transition and the dimension $\mathcal{M}:\mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module is reduced in the transition, it seems natural to assign angular resolution to the dynamics to these $\mathcal{M}:\mathcal{N}$ degrees of freedom.

4. The natural physical interpretation for the angular resolution would be in terms of multiple cover of $M^4_\pm$ by a subgroup $G \subset SU/2)$. One could perhaps say that $2\pi/n$ rotation in CP_2 would correspond to 2π rotation in $M^4_\pm$ and would be thus have a very concrete representation.

3 Jones inclusions and dynamical Planck constant

3.1 Basic ideas

The anyonic arguments for the quantization of Planck constant suggest that one can assign separate scalings of Planck constant to $M^4_\pm$ CP_2 degrees of freedom and that these scalings in turn reflect as scalings of $M^4_\pm$ and CP_2 metrics. This is definitely not in accordance with the original TGD vision based on uniqueness of imbedding space but makes sense if space-time and imbedding space are emergent concepts as the hierarchy of number theoretical von Neumann algebra inclusions indeed suggests. Indeed, the scaling factors of $M^4_\pm$ and CP_2 metric remain non-fixed by the general uniqueness arguments since Cartesian product is in question.

3.1.1 Jones inclusions defined by subgroups of $SL(2,C) \times SU(2)$

Jones inclusions with $\mathcal{M}:\mathcal{N} < 4$ have representation as $R^G_0 \subset R^G$ with G a discrete subgroup of $SU(2)$. The localization of Clifford algebra means that octonionic Clifford algebra elements appear as coefficients of powers series in HO coordinate represented as a complexified quaternionic matrix having with determinant equal to the Minkowskian norm squared in HO. $SU(2)$ can be interpreted as acting in E^4 as rotations in $HO = M^4 \times E^4$ decomposition. On quantum spinors the action corresponds to double cover of G.

A more general choice for G would be as a discrete subgroup $G_a \times G_b \subset SL(2,C) \times SU(2) \times SU(2)$. Poincare invariance suggests that the subgroup of $SL(2,C)$ reduces either to a discrete subgroup of $SU(2)$ and in the case that the rotation are genuinely 3-dimensional (E^6, E^8), the only possible interpretation would be as isotropy group of a particle at rest. When the group acts on plane as in case of A_n and D_{2n}, it could be also assigned to a massless particle.

If the group involves boosts it contains an infinite number of elements and it is not clear whether this kind of situation is physically sensible. In this case Jones inclusion could be interpreted as an inclusion for the tensor product of G invariant algebras associated with $M^4_\pm$ and CP_2 degrees of freedom and one would have $\mathcal{M}:\mathcal{N} = \mathcal{M}:\mathcal{N}(G_a) \times \mathcal{M}:\mathcal{N}(G_b)$. Since the index increases as the order of G increases one has reasons to expect that in the case of $G_a = SL(2,C)$ $N_a = \infty$ implies larger $\mathcal{M}:\mathcal{N}(G_a) > 4$.

A possible interpretation is that the values $\mathcal{M}:\mathcal{N} < 4$ are analogous to bound state energies so that a discrete rotation group acting in the relative rotational degrees of freedom can act as a symmetry group whereas the values $\mathcal{M}:\mathcal{N} > 4$ are analogous to ionized states for which particles are almost freely moving with respect to each other with a constant velocity.

When one restricts the coefficients to G-invariant elements of Clifford algebra the Clifford field is G-invariant under the natural action of G. This allows two interpretations. Either the Clifford field is G invariant or that the Clifford field is defined in orbifold $M^4_\pm/G_a \times E^4/G_b$. $M^4_\pm/G_a$ is obtained by replacing hyperboloid H_a $(t^2 - x^2 - y^2 - z^2 = a^2)$ with H_a/G_a. These spaces have been considered as cosmological models having 3-space with finite volume [D5] (also a lattice like structure could be in question).

3.1.2 The quantum phases associated with sub-groups of $SU(2)$

It is natural to identify quantum phase as that defined by the maximal cyclic subgroup for finite subgroups of $SU(2)$ and infinite subgroups of $SL(2,C)$. Before continuing a brief summary about quantum phases associated with finite subgroups of $SU(2)$ is in order. E_6 corresponds to $N = 24$ and $n = 3$ and E_8 to icosahedron with $N = 120$, $n = 5$ and Golden mean and the minimal value of n making possible universal topological quantum computer [E9].

D_n and A_n have orders $2n$ and $n+1$ and act as symmetry groups of n-polygon and have n-element cyclic group as a maximal cyclic subgroup. For double covers the orders are twice this. Thus A_n *resp.* D_{2n} correspond to $q = exp(i\pi/n)$ *resp.* $q = exp(i\pi/2n)$. Note that the restriction $n \geq 3$ means geometrically that only non-trivial polygons are allowed.

3.1.3 Representation of Jones inclusions as singular bundle structures at the level of imbedding space

Since the imbedding space seems to emerge from the number theoretical von Neumann inclusions in TGD, the natural question is whether Jones inclusions could have as space-time correlates singular bundle structures defined by groups G_a and G_b associated with M^4 and CP_2 degrees of freedom.

1. The different local Clifford algebras and corresponding imbedding spaces must be "glued together". $HO = M^8$ defines a singular bundle structure with $HO/G_a \times G_b$ playing the role of base space and $G_a \times G_b$ that of a generic fiber. The inclusion of orbifold $HO/G_a \times G_b$ to HO by gauge fixing assigns to $G_a \times G_b$ coset a single arbitrarily chosen point of $G_a \times G_b$ and thus of HO. The glued imbedding spaces are equivalent as pseudo Riemann manifolds and only the singular bundle structure distinguishes between them. A similar picture makes sense for $M^4 \times CP_2/G_a \times G_b$ regarded as a base space for a singular bundle with $G_a \times G_b$ as a generic fiber. Bundle projections obviously define a dual geometric representation for Jones inclusion.

2. By the previous anyonic arguments one can assign Planck constants to both $M^4_\pm$ and E^4 (CP_2). The invariance of the angular momentum in the transition $exp(im\phi) \rightarrow q = exp(im\phi/n_b)$, where n_b corresponds to the maximal cyclic sub-group of G_b, would suggest that $M^4_\pm$ covariant metric scales as n_b^2 and $\hbar(M^4_\pm) = n_b\hbar_0$. However, the formula consistent with the model for planetary orbits and genuine fractionization of angular momentum requires $\hbar(M^4_\pm) = n_a\hbar_0$.

   Similar argument implies $\hbar(E^4) = \hbar(CP_2) = n_b\hbar_0$. E^4 metric scales naturally as n_a^2 but in the case of CP_2 projective character does not favor scaling and the scaling indeed leads to difficulties as one tries to glue together different variants of CP_2 isometrically (non-isometric gluing would be however possible for CP_2 factors). Neither does the number theoretic interpretation of CP_2 favor the scaling of CP_2 metric. In the case of $M^4_\pm$ factors the non-compactness of $M^4_\pm$ metric makes isometric gluing along $M^4_\pm$ factors possible by the identification $n_{a_1}s_{a_1} = n_{a_2}s_{a_2}$, where s_{a_i} denotes the appropriate light-cone proper time.

   The way out of the problem is the invariance of Kähler action under overall scaling of H metric by $1/n_a^2$ so that the net scaling factor of M^4 covariant metric is $(n_b/n_a)^2$ with CP_2 metric remaining invariant. Effective Planck constant can be regarded as only a conversion factor and scaling of Planck constants has a purely geometric interpretation.

3. Quantum classical correspondence suggests that Planck constants appear at the level of the classical dynamics. The first thing to notice is that Kähler action does not depend on the overall scaling of H metric. If also CP_2 metric is scaled up the classical physics as defined by the extremals of Kähler action depends only on the ratio n_a/n_b so that would have symmetry $(n_a, n_b) \rightarrow k(n_a, n_b)$. This is a non-trivial prediction since bundle coverings are

different. Note however that the distinction between coverings related by scaling is visible in the dynamics involving other levels of dark matter hierarchy. The dependence of Kähler action on n_a/n_b could be interpreted in terms of radiative corrections coded to the Kähler function so that the vanishing of higher order radiative corrections on the functional integral over 3-surfaces around maxima of Kähler function does not lead to a conflict with the fact that radiative corrections must be non-vanishing.

4. In the case of $M^4_\pm$ the orbifold singularity is not only the tip of the light-cone as one might first think. For all groups G_a except E_6 and E_8 the singularity is the time-like plane corresponding to a radial ray through origin defining the quantization axis of angular momentum and intersecting light-cone boundary along a preferred light-like ray. For E_6 and E_8 (tedrahedral and icosahedral symmetries) the singularity is time-like line and in this case there are several alternative identifications of the quantization axes as axis around which the maximal cyclic subgroup acts as rotations.

5. From foregoing it should be obvious that Jones inclusions represented in this manner would relate very closely to the selection of quantization axes and provide a geometric representation for this selection at the level of space-time and configuration space. The existence of the preferred direction of quantization at a given level of dark matter level should have observable consequences. For instance, in cosmology this could mean a breaking of perfect rotational symmetry at dark matter space-time sheets. The interpretation would be as a quantum effect in cosmological length scales. An interesting question is whether the observed asymmetry of cosmic microwave background could have interpretation as a quantum effect in cosmological length and time scales.

3.1.4 About G-invariance of configuration space spinors

Consider now in more detail the question what $G = G_a \times G_b$-invariance of spinors of world of classical worlds (as opposed to spinor fields) could mean.

1. Suppose that partonic 2-surfaces are invariant under G so that one has effectively $H = H/G$.

2. Pose G-invariance condition on the allowed combinations of fermionic oscillator operators whose transformation properties under G are determined by those for the eigen modes of the modified Dirac operator. Spinor modes would naturally transform according to irreducible representations of G. Only the products of fermionic oscillator operators satisfying G-invariance condition would be allowed for $\mathcal{N} \subset \mathcal{M}$. The interpretation in terms of quantum measurement theory would suggest that the action of G-invariant fermionic operators on a given state creates states not distinguishable from the original one. ADE correspondence would suggests that the Clifford algebra elements not invariant under G can be organized to the representations of product of ADE Lie groups corresponding to G_a and G_b.

3. G has a natural action on the modes of Ψ as spinor rotations given by

$$\Psi(z) = D(g)\Psi(g^{-1}(z)) \ .$$

and spinor modes should transform irreducibly under G under this action.

4. Suppose that the generalized eigen modes of the modified Dirac operator are proportional to a function of form $f(r,z) = p^{i\zeta^{-1}(z)g(r)}$, where z represents the projection of the point of partonic 2-surface to the geodesic sphere of CP_2 and corresponds to the projective complex coordinate of S^2 such that $z = 0$ and $z = 1$ corresponds to orbifold points invariant under entire G_b for all other subgroups of $SU(2) \subset SU(3)$ except tedrahedral and icosahedral

groups. In this case only maximal cyclic subgroup would leave the orbifold points consisting of vertices of tedrahedron *resp.* icosahedron invariant.

5. The definition of number theoretic braids is not G_b invariant orbifold points since $z = \zeta(\sum_k n_k s_k)$, where one has $\zeta(s_k = 1/2 + iy_k) = 0$, $y_k > 0$ and $n_k \geq 0$ (recall the conjectured number theoretical universality of ζ), correspond to the points at which $f(r,z)$ reduces to an algebraic number if p^{iy_k} are assumed to be algebraic numbers. The reason is that $g_b(\zeta(\sum_k n_k s_k))$ is not expected to be of the general form $\zeta(\sum_k n_k s_k)$. The exception is formed by the zeros and poles of ζ corresponding to quantum criticality for the phase transition changing the value of CP_2 Planck constant.

6. One can consider also the possibility that both the geodesic sphere $S^2_\pm$ associated with $\delta M^4_\pm$ and CP_2 geodesic sphere S^2 contributes to the generalized eigen modes so that one would have

$$\Psi \propto p^{i(\zeta^{-1}(z))+\zeta^{-1}(w))} ,$$

with $z \in S^2$ and $w \in S^2_\pm$. Only the points $(z,w) \in \{0,\infty\} \times \{0,\infty\}$ could contribute to the definition of S-matrix at quantum criticality.

3.2 A further generalization of the notion of imbedding space?

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

3.3 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M}:\mathcal{N}<4$. Stringy phase would by previous arguments correspond to $\mathcal{M}:\mathcal{N}=4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD}\times \hat{CP}_2$ implying that surfaces in $CD\times S^2$ and $(M^2\cap CD)\times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a}\times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a}\times Z_{n_b}$. One can also consider extension by replacing $M^2\cap CD$ and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD}\hat{\times}G_a$ *resp.* $\hat{CP}_2\hat{\times}G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2\cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2\cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a\times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{CD}/G_a\times(\hat{CP}_2\hat{\times}G_b)$ and $(\hat{CD}\hat{\times}G_a)\times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $(M^2\cap CD)\times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2\times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2\cap CD)\times S^2$ makes sense only for surfaces $X^1\times D^2\subset(M^2\cap CD)\times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2\cap CD)\times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2\cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunneling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from

sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

3.4 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \rightarrow r(X)/n$ for factor space and $r(X) \rightarrow nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

3.5 A simple model of fractional quantum Hall effect

The generalization of the imbedding space suggests that it could possible to understand fractional quantum Hall effect [49] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\sigma = \nu \times \frac{e^2}{h} ,$$
$$\nu = \frac{n}{m} . \quad (3)$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [49].

The model of Laughlin [48] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [50]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of imbedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [51]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [50]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given

sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise.

3.5.1 What is the role of dimensions?

Could the dimensions of M^4 and CP_2 and the dimensions of spaces defined by the choice of the quantization axes play a fundamental role in the construction from the constraint that the fundamental group is non-trivial?

1. Suppose that the sub-manifold in question is geodesic sub-manifold containing the orbits of its points under Cartan subgroup defining quantization axes. A stronger assumption would be that the orbit of maximal compact subgroup is in question.

2. For M^{2n} Cartan group contains translations in time direction with orbit M^1 and Cartan subgroup of $SO(2n-1)$ and would be M^n so that $\hat{M}^{2n}$ would have a trivial fundamental group for $n > 2$. Same result applies in massless case for which one has $SO(1,1) \times SO(2n-2)$ acts as Cartan subgroup. The orbit under maximal compact subgroup would not be in question.

3. For CP_2 homologically non-trivial geodesic sphere CP_1 contains orbits of the Cartan subgroup. For $CP_n = SU(n+1)/SU(n) \times U(1)$ having real dimension $2n$ the sub-manifold CP_{n-1} contains orbits of the Cartan subgroup and defines a sub-manifold with codimension 2 so that the dimensional restriction does not appear.

4. For spheres $S^{n-1} = SO(n)/SO(n-1)$ the dimension is $n-1$ and orbit of $SO(n-1)$ of point left fixed by Cartan subgroup $SO(2) \times ..$ would for $n = 2$ consist of two points and S_{n-2} in more general case. Again co-dimension 2 condition would be satisfied.

3.5.2 What about holes of the configuration space?

One can raise analogous questions at the level of configuration space geometry. Vacuum extremals correspond to Lagrangian sub-manifolds $Y^2 \subset CP_2$ with vanishing induced Kähler form. They correspond to singularities of the configuration space ("world of classical worlds") and configuration space spinor fields should vanish for the vacuum extremals. Effectively this would mean a hole in configuration space, and the question is whether this hole could also naturally lead to the introduction of covering spaces and factor spaces of the configuration spaces. How much information about the general structure of the theory just this kind of decomposition might allow to deduce?

This kind of singularities are infinite-dimensional variants of those discussed in catastrophe theory and this suggests that their understanding might be crucial.

3.6 Dark rules

I have done a considerable amount of trials and errors in order to identify the basic rules allowing to understand what it means to be dark matter is and what happens in the phase transition to dark matter. It is good to try to summarize the basic rules of p-adic and dark physics allowing to avoid obvious contradictions.

3.6.1 The notion of field body

The notion of "field body" implied by topological field quantization is essential. There would be em, $Z0$, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

The p-adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

3.6.2 Critical comment

The proposed scenario could be criticized because subgroups of $SU(2)$ are in a preferred position. The Jones inclusions considered correspond to quantum spinor representations of various quantum groups $SU(2)_q$, $q = exp(i2\pi/n)$. This explains the result $\mathcal{M} : \mathcal{N} \leq 4$. These representations are certainly in preferred role as far as configuration space spinor field are considered but it is possible to assign a hierarchy of Jones inclusions labelled by quantum phase q with arbitrary representation of an arbitrary compact Lie group. These inclusions would be analogous to discrete states in the continuum $\mathcal{M} : \mathcal{N} > 4$ (see Appendix for details).

One could argue that there should be universality of some kind so that only those discrete groups which appear as subgroups of any compact Lie group are assigned with the sectors of the extended imbedding space. If one considers only non-commutative Lie groups, universality allows all discrete subgroups of $SU(2)$. If also Abelian compact Lie groups, and thus also $U(1)$, are allowed, then only the spaces $H/Z_{n_a} \times G_b$, where G_b is abelian subgroup of $SU(3)$ could be glued together to form the extended imbedding space.

Concerning the group G_b the working hypothesis was $G_b \subset U(1)_I \subset SU(2) \subset SU(3)$. The reason for this restriction was the erratic belief that discrete subgroups of arbitrary Lie group cannot be associated with the inclusions of hyper-finite factors. This is possible. These inclusions have $\mathcal{M} : \mathcal{N} > 4$ and are analogous to bounds states in continuum (Appendix).

A physically attractive possibility is that $G_a \times G_b$ leaves the choice of quantization axes invariant. This would leave only Abelian groups into consideration and drop D_{2n}, E_6, and E_8. It is quite possible that only these groups define sectors of the generalized imbedding space.

This means that $G_b = Z_{n_1} \times Z_{n_2} \subset U(1)_I \times U(1)_Y \subset SU(2) \times U(1)_Y$ and even more general subgroups of $SU(3)$ (if non-commutativity is allowed) are a priori possible. Since the inclusions are characterized by single quantum phase $q = exp(i2\pi/n)$ in the case of compact Lie groups (Appendix), it seems that one must have $n_1 = n_2 \equiv n_b$ guaranteing the uniqueness of the identification of the Planck constant. Note that once the CP_2 coordinates transforming linearly under $U(2)$ are fixed, also $U(1)_Y$ is fixed so that only $U(1)_I$ can be chosen freely. For $G_a \times G_b = Z_{n_a} \times Z_{n_b} \times Z_{n_b}$ partonic 2-surface would belong to $M^2 \times CP_2/U(1) \times U(1)$, where M^2 is spanned by the quantization axis of angular momentum and the time axis defining the rest system.

The products of groups Z_n are also number theoretically in a very special position since they relate naturally to the finite cyclic extensions and also to the maximal Abelian extension of rationals. With this restriction on $G_a \times G_b$ one can consider the hypothesis that elementary particles correspond are maximally quantum critical systems left invariant by all groups $G_a \times G_b$ respecting a given choice of quantization axis and implying that darkness is associated only to field bodies and Planck constant becomes characterizer of interactions rather than elementary particles themselves.

A category theoretic picture in which elementary particles correspond to objects and field bodies to morphisms would suggest itself at least in metaphoric sense. The arrows in this category would be bi-directional. The notion of infinite prime involving endless second quantization of an arithmetic quantum field theory with many particle states of the previous level becoming particles at the next level in one-one correspondence with the hierarchy of space-time sheets suggests that there is an infinite hierarchy of "particles" such that the particle identified as a category at a given level of the hierarchy becomes object of the category defining particle at the next level.

3.6.3 What dark variant of elementary particle could mean?

It is not at all clear what the notion of dark variant of elementary particle or of larger structures could mean.

1. Are only field bodies dark?

One variety of dark particle is obtained by making some of the field bodies dark by increasing the value of Planck constant. This hypothesis could be replaced with the stronger assumption that elementary particles are maximally quantum critical systems so that they are same irrespective of the value of the Planck constant. Elementary particles would be represented by partonic 2-surfaces, which belong to the universal orbifold singularities remaining invariant by all groups $G_a \times G_b$ for a given choice of quantization axes. If $G_a \times G_b$ is assumed to leave invariant the choice of the quantization axes, it must be of the form $Z_{n_a} \times G_b \subset SO(3) \times SU(3)$. For $G_b = Z_{n_b} \times Z_{n_b}$ partonic

2-surface would belong to $M^2 \times CP_2/U(1) \times U(1)$, where M^2 is spanned by the quantization axis of angular momentum and the time axis defining the rest system.

A different manner to say this is that the CP_2 type extremal representing particle would suffer multiple topological condensation at its field bodies so that there would be no separate "particle space-time sheet".

Darkness would be restricted to particle interactions. The value of the Planck constant would be assigned to a particular interaction between systems rather than system itself. This conforms with the original finding that gravitational Planck constant satisfies $\hbar = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$. Since each interaction can give rise to a hierarchy dark phases, a rich variety of partially dark phases is predicted. The standard assumption that dark matter is visible only via gravitational interactions would mean that gravitational field body would not be dark for this particular dark matter.

Complex combinations of dark field bodies become possible and the dream is that one could understand various phases of matter in terms of these combinations. All phase transitions, including the familiar liquid-gas and solid-liquid phase transitions, could have a unified description in terms of dark phase transition for an appropriate field body. At mathematical level Jones inclusions would provide this description.

The book metaphor for the interactions at space-time level is very useful in this framework. Elementary particles correspond to ordinary value of Planck constant analogous to the ordinary sheets of a book and the field bodies mediating their interactions are the same space-time sheet or at dark sheets of the book.

2. Connection with quantum criticality

The assumption that elementary particles are maximally quantum critical in the proposed sense would give a precise content for the notion of quantum criticality at elementary particle level. For a given choice of quantization axes partonic 2-surfaces associated with elementary particles would live in the 4-D intersection $M^2 \times CP_2/U(1) \times U(1)$ of copies of H.

The so called factorizing quantum field theories exist only in M^2, and S-matrix is almost trivial in momentum degrees of freedom involving only permutation of momenta. This point is discussed in [C2], where I consider the possibility that U-matrix between zero energy states (not the same as S-matrix defined as time-like entanglement coefficients between positive and negative energy parts of zero energy state) could reduce to a tensor product of factorizing S-matrices.

With appropriate restrictions on the rapidities of incoming and outgoing particles the S-matrix would be algebraic and define U-matrix also for p-adic-to-real transitions assignable to the realization of intensions as actions. In this context, the almost-triviality would turn to a blessing implying a precise correspondence between intentions and actions (in the resolution considered). Intentions would be naturally transformed to actions at complete quantum criticality since this implies best possible "grasp on situation" at 4-D space-time level (CP_2 type extremals have random light-like curve as M^4 projection).

3. Can also elementary particles be dark?

Also dark elementary particles themselves rather than only the flux quanta could correspond to dark space-time sheet defining multiple coverings of $H/G_a \times G_b$. This would mean giving up the maximal quantum criticality hypothesis in the case of elementary particles. These sheets would be exact copies of each other. If single sheet of the covering contains topologically condensed space-time sheet, also other sheets contain its exact copy.

The question is whether these copies of space-time sheet defining classical identical systems can carry different fermionic quantum numbers or only identical fermionic quantum numbers so that the dark particle would be exotic many-fermion system allowing an apparent violation of statistics (N fermions in the same state).

Even if one allows varying number of fermions in the same state with respect to a basic copy of sheet, one ends up with the notion of N-atom in which nuclei would be ordinary but electrons would reside at the sheets of the covering. The question is whether symbolic representations essential for understanding of living matter could emerge already at molecular level via the formation of N-atoms.

3.6.4 Microscopic model for dark elementary particles

The construction of a model for the detection of gravitational radiation assuming that gravitons correspond to a gigantic gravitational constant was carried out in [D6]. One can say that dark gravitons are Bose-Einstein condensates of ordinary gravitons. This suggests that Bose-Einstein condensates of some kind could accompany and perhaps even characterize also the dark variants of ordinary elementary particles.

1. Higgs boson Bose-Einstein condensate as characterized of Planck constant

The following picture is the simplest I have been able to imagine hitherto.

1. Suppose that darkness corresponds to the darkness of the field bodies (em, Z^0,W,...) of the elementary particle so that the elementary particle proper is not affected in the transition to large $\hbar$ phase. This stimulates the idea that some Bose-Einstein condensate associated with the field body provides a microscopic description for the darkness and that one can relate the value of $\hbar$ to the properties of Bose-Einstein condensate.

2. Since the spin of the particle is not affected in the transition, it would seem that the bosons in question are Lorentz scalars. Hence a Bose-Einstein condensate of Higgs suggests itself as the relevant structure. Higgs would have a double role since the coherent state of Higgs bosons associated with the field body would be responsible for or at least closely relate to the contribution to the mass of fermion identified usually in terms of a coupling to Higgs. The ground state would correspond to a coherent state annihilated by the new annihilation operators unitarily related to the original ones. Bose-Einstein condensate would be obtained as a many-Higgs state obtaining by applying these creation operators and would not be an eigen state of particle number in the old basis.

3. As a rule, quantum classical correspondence is a good guideline. Suppose that the field body corresponds to a pair of positive and negative energy MEs connected by wormhole contacts representing the bosons forming the Bose-Einstein condensate. This structure could be more or less universal. In the general case MEs carry light-like gauge currents and light-like Einstein tensor. These currents can also vanish and should do so for the ground state. MEs could carry both coherent states of gauge bosons and gravitons but would not be present in the ground state. The CP_2 part of the trace of second fundamental form transforming as $SO(4)$ vector and doublet with respect to the groups $SU(2)_L$ and $SU(2)_R$, is the only possible candidate for the classical Higgs field. The Fourier spectrum of CP_2 coordinates has only light-like longitudinal momenta so that four-momenta are slightly tachyonic for non-vanishing transverse momenta. This state of facts might be a space-time correlate for the tachyonic character of Higgs.

4. The quantum numbers of the particle should not be affected in the transition changing the value of Planck constant. The simplest explanation is that Higgs bosons have a vanishing net energy. This is possible since in the case of bosons the two wormhole throats have different sign of energy. Indeed, if the energies, spins, and em charges of fermion and antifermion at wormhole throats are of opposite sign, one is left with a coherent state of zero energy Higgs particles as a microscopic description for constant value of Higgs field.

5. How do the properties of the Bose-Einstein condensate of Higgs relate to the value of Planck constant? MEs should remain invariant under the discrete groups Z_{n_a} and Z_{n_b} and the bosons at the sheets of the multiple covering should be in identical state. The number $n_a \times n_b$ of zero energy Higgs bosons in the Bose-Einstein condensate would characterize the darkness at microscopic level.

2. How this affects the view about particle massivation?

This scenario would allow to add some details to the general picture about particle massivation reducing to p-adic thermodynamics plus Higgs mechanism, both of them having description in terms of conformal weight.

1. The mass squared equals to the p-adic thermal average of the conformal weight. There are two contributions to this thermal average. One from the p-adic thermodynamics for super conformal representations, and one from the thermal average related to the spectrum of generalized eigenvalues λ of the modified Dirac operator D. Higgs expectation value appears in the role of a mass term in the Dirac equation just like λ in the modified Dirac equation. For the zero modes of D λ vanishes.

2. There are good motivations to believe that λ is expressible as a superposition of zeros of Riemann zeta or some more general zeta function. The problem is that λ is complex. Since Dirac operator is essentially the square root of d'Alembertian (mass squared operator), the natural interpretation of λ would be as a complex "square root" of the conformal weight.

Remark: The earlier interpretation of λ as a complex conformal weight looks rather stupid in light of this observation.

This encourages to consider the interpretation in terms of vacuum expectation of the square root of Virasoro generator, that is generators G of super Virasoro algebra, or something analogous. The super generators G of the super-conformal algebra carry fermion number in TGD framework where Majorana condition does not make sense physically. The modified Dirac operators for the two possible choices $t_\pm$ of the light-like vector appearing in the eigenvalue equation $D\Psi = \lambda t^k_\pm \Gamma_k \Psi$ could however define a bosonic algebra resembling super-conformal algebra. In fact, the operators $a_\pm = \lambda t^k_\pm \Gamma_k$ are nilpotent and anti-commute to λ so that the minimal super-algebra would be 3-dimensional.

The p-adic thermal expectation values of contractions of $t^k_- \gamma_k D_+$ and $t^k_+ \gamma_k D_-$ should co-incide with the vacuum expectations of Higgs and its conjugate. Note that D_+ and D_- would be same operator but with different definition of the generalized eigenvalue and hermitian conjugation would map these two kinds of eigen modes to each other. The real contribution to the mass squared would thus come naturally as $\langle |\lambda|^2 \rangle$. Of course, $\langle H \rangle = \langle \lambda \rangle$ is only a hypothesis encouraged by the internal consistency of the physical picture, not a proven mathematical fact.

3. Questions

This leaves still some questions.

1. Does the p-adic thermal expectation $\langle \lambda \rangle$ dictate $\langle H \rangle$ or vice versa? Physically it would be rather natural that the presence of a coherent state of Higgs wormhole contacts induces the mixing of the eigen modes of D. On the other hand, the quantization of the p-adic temperature T_p suggests that Higgs vacuum expectation is dictated by T_p.

2. Also the phase of $\langle \lambda \rangle$ should have physical meaning. Could the interpretation of the imaginary part of $\langle \lambda \rangle$ make possible the description of dissipation at the fundamental level?

3. Is p-adic thermodynamics consistent with the quantal description as a coherent state? The approach based on p-adic variants of finite temperature QFTs associate with the legs of generalized Feynman diagrams might resolve this question neatly since thermodynamical states would be genuine quantum states in this approach made possible by zero energy ontology.

3.7 Modified view about mechanism giving rise to large values of Planck constant

This picture differs radically from the earlier ad hoc hypothesis for the dependence of Planck constant on n characterizing quantum phase since Planck constant increases without bound with n in the recent case. The new scenario is consistent with the speculations about dark matter as a phase with a large value of Planck constant and with the basic hypothesis used in quantum biological modelling. It does not however allow the dependence of Planck constant on p-adic prime which has been proposed earlier.

3.7.1 Manifold-to-orbifold transition as a transition to a non-perturbative phase

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1 Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of orbifold $M^4_\pm/G_a$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$.

2. TGD based view about color confinement following from quantum classical correspondence is that states of arbitrary representations of the color group with a vanishing color hypercharge and isospin are possible and that a symmetry breaking to $SU(2) \times U(1)$ occurs. This symmetry reduction is consistent with the effective orbifold structure of CP_2.

3. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

Exactly the same mechanism would give rise to what I have called N-atoms, in particular N-hydrogen atoms suggested to play key model in the chemistry of living matter [J6].

$n = 3$ anyonic statistics would give rise to para statistics mimicking QCD color at space-time level. The folding of space-time sheet without outer boundary making it impossible for quarks to escape the hadron would in turn serve as space-time correlate for color confinement. The singular points of space-time sheet fixed under G_b would play a role hadron horizon analogous to black hole horizon.

3.7.2 Tree like structure of the extended imbedding space

Two imbedding spaces with different scalings factors of metrics are glued directly together only if either $M^4_\pm$ or CP_2 scaling factor is same and only along $M^4_\pm$ or CP_2. This gives a kind of evolutionary tree (actually in rather precise sense as the quantum model for evolutionary leaps as phase transitions increasing $\hbar(M^4_\pm)$ (that is n_a) demonstrates [M3]!). In this tree vertices represent given $M^4_\pm$ (CP_2) and lines represent CP_2:s ($M^4_\pm$:s) with different values of $\hbar(CP_2)$ ($\hbar(M^4)$) emanating from it much like lines from from a vertex of Feynman diagram.

1. In the phase transition between different $\hbar(M^4_\pm)$:s the projection of the 3-surface to $M^4_\pm$ becomes single point so that a cross section of CP_2 type extremal representing elementary particle is in question. Elementary particles could thus leak between different $M^4_\pm$:s easily and this could occur in large $\hbar(M^4_\pm)$ phases in living matter and perhaps even in quantum Hall effect. Wormhole contacts which have point-like $M^4_\pm$ projection would allow topological condensation of space-time sheets with given $\hbar(M^4_\pm)$ at those with different $\hbar(M^4_\pm)$ in accordance with the heuristic picture.

2. In the phase transition different between CP_2:s the CP_2 projection of 3-surface becomes point so that the transition can occur in regions of space-time sheet with 1-D CP_2 projection. The regions of a connected space-time surface corresponding to different values of $\hbar(CP_2)$ can be glued together. For instance, the gluing could take place along surface $X^3 = S^2 \times T$ (T corresponds time axis) analogous to black hole horizon. CP_2 projection would be single point at the surface. The contribution from the radial dependence of CP_2 coordinates to the induced metric giving $ds^2 = ds^2(X^3)+g_{rr}dr^2$ at X^3 implies a radial gravitational acceleration and one can say that a gravitational flux is transferred between different imbedding spaces.

Planetary Bohr orbitology predicting that only 6 per cent of matter in solar system is visible suggests that star and planetary interiors are regions with a large value of CP_2 Planck constant and that only a small fraction of the gravitational flux flows along space-time sheets carrying visible matter. In the approximation that visible matter corresponds to layer of thickness ΔR at the outer surface of constant density star or planet of radius R, one obtains the estimate $\Delta R = .12R$ for the thickness of this layer: convective zone corresponds to $\Delta R = .3R$. For Earth one would have $\Delta R \sim 70$ km which corresponds to the maximal thickness of the crust. Also flux tubes connecting ordinary matter carrying gravitational flux leaving space-time sheet with a given $\hbar(CP_2)$ at three-dimensional regions and returning back at the second end are possible. These flux tubes could mediate dark gravitational force also between objects consisting of ordinary matter.

Concerning the mathematical description of this process, the selection of origin of $M^4_\pm$ or CP_2 as a preferred point is somewhat disturbing. In the case of $M^4_\pm$ the problem disappears since configuration space is union over the configuration spaces associated with future and past light cones of $M^4_\pm$: $CH = CH^+ \cup CH^-$, $CH^\pm = \cup_{m\in M^4} CH^\pm_m$. In the case of CP_2 the same interpretation is necessary in order to not lose $SU(3)$ invariance so that one would have $CH^\pm = \cup_{h\in H} CH^\pm_h$. A somewhat analogous but simpler book like structure results in the fusion of different p-adic variants of H along common rationals (and perhaps also common algebraics in the extensions).

3.7.3 Generalization of the p-adic length scale hypothesis

The evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [H8].

This condition could be interpreted as a kind of resonance condition guaranteing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tedrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5 and and thus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean

There is evidence for an icosahedral clustering in water [82]. Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by $M^4_\pm$ points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of $M^4_\pm$ points by CP_2 could be involved.

It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

3.7.4 Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

2. Nottale [81] has suggested that also the harmonics and subharmonics of λ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [D6]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in

solar system [D6] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet) n_{F,b}(Earth)/n_{F,a}(Earth) n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9 \times 5 \times 17$	$2^8 \times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14} \times 3 \times 5 \times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^{4}\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 1. The table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k \prod_k F_{n_k}$, where $F_i = 2^{2^i} + 1$, $i = 0, 1, 2, 3, 4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254} \times 5 \times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [M3]. For $v_0 = 4.6 \times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1 + \epsilon)$ of the total mass for Earth gives

$$1 + \epsilon = \frac{17}{16} \quad (4)$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1 - \epsilon) \simeq 4.6 \times 10^{-4}$ equals with $v_0(eff) = 1/(2^7 \times F_2) = 4.5956 \times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This suggests that gravitational Bohr rules relate to

velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of $M^4_\pm$ seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of $M^4_\pm$, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

3.7.5 About the interpretation of the parameter v_0

The formula for the gravitational Planck constant contains the parameter $v_0/c = 2^{-11}$. This velocity defines the rotation velocities of distant stars around galaxies. The presence of a parameter with dimensions of velocity should carry some important information about the geometry of dark matter space-time sheets.

Velocity like parameters appear also in other contexts. There is evidence for the Tifft's quantization of cosmic redshifts in multiples of $v_0/c = 2.68 \times 10^{-5}/3$: also other units of quantization have been proposed but they are multiples of v_0 [90].

The strange behavior of graphene includes high conductivity with conduction electrons behaving like massless particles with light velocity replaced with $v_0/c = 1/300$. The TGD inspired model [J1] explains the high conductivity as being due to the Planck constant $\hbar(M^4) = 6\hbar_0$ increasing the delocalization length scale of electron pairs associated with hexagonal rings of mono-atomic graphene layer by a factor 6 and thus making possible overlap of electron orbitals. This explains also the anomalous conductivity of DNA containing 5- and 6-cycles [J1].

1. Is dark matter warped?

The reduced light velocity could be due to the warping of the space-time sheet associated with dark electrons. TGD predicts besides gravitational red-shift a non-gravitational red-shift due to the warping of space-time sheets possible because space-time is 4-surface rather than abstract 4-manifold. A simple example of everyday life is the warping of a paper sheet: it bends but is not stretched, which means that the induced metric remains flat although one of its component scales (distance becomes longer along direction of bending). For instance, empty Minkowski space represented canonically as a surface of $M^4 \times CP_2$ with constant CP_2 coordinates can become periodically warped in time direction because of the bending in CP_2 direction. As a consequence, the distance in time direction shortens and effective light-velocity decreases when determined from the comparison of the time taken for signal to propagate from A to B along warped space-time sheet with propagation time along a non-warped space-time sheet.

The simplest warped imbedding defined by the map $M^4 \rightarrow S^1$, S^1 a geodesic circle of CP_2. Let the angle coordinate of S^1 depend linearly on time: $\Phi = \omega t$. g_{tt} component of metric becomes $1 - R^2\omega^2$ so that the light velocity is reduced to $v_0/c = \sqrt{1 - R^2\omega^2}$. No gravitational field is present.

The fact that M^4 Planck constant $n_a\hbar_0$ defines the scaling factor n_a^2 of CP_2 metric could explain why dark matter resides around strongly warped imbeddings of M^4. The quantization of the scaling factor of CP_2 by $R^2 \rightarrow n_a^2 R^2$ implies that the initial small warping in the time direction given by $g_{tt} = 1-\epsilon$, $\epsilon = R^2\omega^2$, will be amplified to $g_{tt} = 1-n_a^2\epsilon$ if ω is not affected in the transition to dark matter phase. $n_a = 6$ in the case of graphene would give $1 - x \simeq 1 - 1/36$ so that only a one per cent reduction of light velocity is enough to explain the strong reduction of light velocity for dark matter.

2. Is c/v_0 quantized in terms of ruler and compass rationals?

The known cases suggests that c/v_0 is always a rational number expressible as a ratio of integers associated with n-polygons constructible using only ruler and compass.

1. $c/v_0 = 300$ would explain graphene. The nearest rational satisfying the ruler and compass constraint would be $q = 5 \times 2^{10}/17 \simeq 301.18$.

2. If dark matter space-time sheets are warped with $c_0/v = 2^{11}$ one can understand Nottale's quantization for the radii of the inner planets. For dark matter space-time sheets associated with outer planets one would have $c/v_0 = 5 \times 2^{11}$.

3. If Tifft's red-shifts relate to the warping of dark matter space-time sheets, warping would correspond to $v_0/c = 2.68 \times 10^{-5}/3$. $c/v_0 = 2^5 \times 17 \times 257/5$ holds true with an error smaller than .1 per cent.

3. Tifft's quantization and cosmic quantum coherence

An explanation for Tifft's quantization in terms of Jones inclusions could be that the subgroup G of Lorentz group defining the inclusion consists of boosts defined by multiples $\eta = n\eta_0$ of the hyperbolic angle $\eta_0 \simeq v_0/c$. This would give $v/c = sinh(n\eta_0) \simeq nv_0/c$. Thus the dark matter systems around which visible matter is condensed would be exact copies of each other in cosmic length scales since G would be an exact symmetry. The property of being an exact copy applies of course only in single level in the dark matter hierarchy. This would mean a delocalization of elementary particles in cosmological length scales made possible by the huge values of Planck constant. A precise cosmic analog for the delocalization of electron pairs in benzene ring would be in question.

Why then η_0 should be quantized as ruler and compass rationals? In the case of Planck constants the quantum phases $q = exp(im\pi/n_F)$ are number theoretically simple for n_F a ruler and compass integer. If the boost $exp(\eta)$ is represented as a unitary phase $exp(im\eta)$ at the level of discretely delocalized dark matter wave functions, the quantization $\eta_0 = n/n_F$ would give rise to number theoretically simple phases. Note that this quantization is more general than $\eta_0 = n_{F,1}/n_{F,2}$.

3.7.6 Comparison with TGD based model of quantum biology

The TGD based model for quantum biology relies on a hierarchy of $M^4_\pm$ Planck constants coming as powers of integer $n_a \simeq 2^{11}$. This hierarchy has an interpretation in terms of a hierarchy of multifurcations in which space-time sheets suffer n_a-folding (note the analogy with period doubling sequence): this multifurcation indeed occurs since each CP_2 point corresponds in general to $b(G_a)$ $M^4_\pm$ points except at singular points where folding occurs. This hierarchy can be understood number theoretically. For A_n all Fermat polygons $n = n_F$ are possible. For D_{2n} n_F must be even. For $n_F = 2^3 F_3$, $F_3 = 2 \times 257$ both A_n and D_{2n} are possible and would give $\lambda \simeq 2^{11}$. The higher powers of λ would correspond approximately to $n_F = 2^{3+11k} F_3$. Of course also much more general values of λ are possible and it is not clear why just the powers of $\lambda \simeq 2^{11}$ are possible unless one accepts the period λ-folding argument. What makes the prediction very concrete is that the group G_a associated with this covering acts in macroscopic and even astrophysical length scales. The huge value n_a implies that either the cyclic group A_{n_a} or dihedral group D_{n_a} is in question. Planar structures with discrete rotational symmetry, in particular Fermat polygons, suggest themselves at the level of dark matter.

3.7.7 Summary

Although the model for the quantization of Planck constant is not completely free of ad hoc elements, the situation is improved dramatically as compared to the earlier attempts to understand how the large values of Planck constant could emerge. Most importantly, the model makes very strong predictions consistent with experimental indications. What is also nice is that the celebrated and mysterious McKay correspondence between ADE diagrams and finite subgroups of $SU(2)$ finds a direct physical interpretation and is connected directly with manifold-orbifold transition as a general mechanism for the transition to non-perturbative phase. The model also suggests that the inclusions associated with subgroups $G \subset SL(2, C)$ could allow to understand the inclusions with $\mathcal{M} : \mathcal{N} > 4$.

3.8 From naive formulas to conceptualization

I have spent a considerable amount of time on various sidetracks in attempts to understand what the quantization of Planck constant does really mean. As usual, the understanding has emerged by unconscious processing rather than by a direct attack.

3.8.1 Naive approach based on formulas

The whole business started from the naive generalization of various formulas for quantized energies by replacing Planck constant with is scaled value. It seems that this approach does not lead to wrong predictions, and is indeed fully supported by the basic applications of the theory. Mention only the quantization of cyclotron energies crucial for the biological applications, the quantization of hydrogen atom, etc... The necessity for conceptualization emerges when one asks what else the theory predicts besides the simple zoomed up versions of various systems.

3.8.2 The geometric view about the quantization of Planck constant

After the naive approach based on simple substitutions came the attempt to conceptualize by visualizing geometrically what dark atoms could look like, and the description in terms of $N(G_a) \times N(G_b)$-fold covering $H \rightarrow H/G_a \times G_b$ emerged.

Especially confusing was the question whether one should assign Planck constant to particles or to their interactions or both. It is now clear that one can assign Planck constant to both the "personal" field bodies assignable to particles and to their interactions ("relative" or interaction field bodies), and that each interaction can correspond to both kinds of field bodies. Planck constant for the relative field bodies depends on the quantum numbers of both particles as it does in the case of gravitation. The Planck constant assignable to the particle's "personal" field body makes possible generalizations like the notion of N-atom.

Each sheet of the "personal" field body corresponds to one particular Compton length characterizing one particular interaction and electromagnetic interaction would define the ordinary

Compton length. The original picture was that topological condensation of CP_2 type vacuum extremal occurs at space-time sheet with size of Compton length identified usually with particle. In the new picture this space-time sheet can be identified as electromagnetic field body.

Elementary particles have light-like partonic 3-surfaces as space-time correlates. If these 3-surfaces are fully quantum critical, they belong to the intersection of all spaces $H/G_a \times G_b$ with fixed quantization axes. This space is just the 4-D subspace $M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. Partonic 2-surfaces are in general non-critical and one can assign to them a definite value of Planck constant.

There are two geodesic spheres in CP_2. Which one should choose or are both possible?

1. For the homologically non-trivial one corresponding to cosmic strings, the isometry group is $SU(2) \subset SU(3)$. The homologically trivial one S^2 corresponds to vacuum extremals and has isometry group $SO(3) \subset SU(3)$. The natural question is which one should choose. At quantum criticality the value of Planck constant is undetermined. The vacuum extremal would be a natural choice from the point of view of quantum criticality since in this case the value of Planck constant does not matter at all and one would obtain a direct connection with the vacuum degeneracy.

   One can of course ask whether all surfaces $M^2 \times Y^2$, Y^2 Lagrangian sub-manifold of CP_2 defining vacuum sectors of the theory should be allowed. The answer seems to be "No" since in the generic case $SO(3)$ does not act as H-isometries of Y^2. If one allows these sub-manifolds or even sub-manifolds of form $M^4 \times Y^2$ to appear as intersection of fractally scaled up variants, one must replace Cartan algebra as algebra associated with $SO(3)$ subgroup of canonical transformations of CP_2 mapping Y^2 to itself (if this kind of algebra exists).

2. The choice of the homologically non-trivial geodesic sphere as a quantum critical sub-manifold would conform with the previous guess that $\mathcal{M} : \mathcal{N} = 4$ corresponds to cosmic strings. It is however questionable whether the ill-definedness of the Planck constant is consistent with the non-vacuum extremal property of cosmic strings unless one assumes that for partonic 3-surfaces $X^3 \subset M^2 \times S^2$ the effective degrees of freedom reduce to mere topological ones.

3.8.3 Fractionization of quantum numbers and the hierarchy of Planck constants

The original generalization of the notion of imbedding space to a union of the factor spaces $\hat{H}/G_a \times G_b$ discussed in the section "General ideas about dark matter" does not allow charge fractionization whereas the covering spaces $\hat{H}\hat{\times}(G_a \times G_b)$ allow a fractionization in a natural manner. Also hybrid cases are obtained corresponding $(\hat{M}^4 \hat{\times} G_a) \times (\hat{CP}_2/G_b)$ and $(\hat{M}^4/G_a) \times (\hat{CP}_2 \hat{\times} G_b)$. The simplest assumption is that Planck constant is a homomorphism from the lattice like structure of groups with product of groups defined to be the group generated by the groups.

1. $\hat{H}/G_a \times G_b$ option

The safest and indeed natural assumption motivated by Jones inclusions is that physical states in sector $H/G_a \times G_b$ are $G_a \times G_b$ invariant meaning a discrete analog of color confinement. This alone excludes fractionization and actually implies just the opposite of it.

1. For states with vanishing fermionic quantum numbers $G_a \times G_b$ invariance means that wave functions live in the base space $H/G_a \times G_b$. For instance, L_z would be a multiple of n_a defining the order of maximal cyclic subgroup of G_a. Analogous conclusion would hold true for color quantum numbers.

2. Just as in the case of ordinary spin fermionic quantum numbers (spin, electro-weak spin) necessarily correspond to the covering group of the isometry group since a state with a half-odd integer spin does not remain invariant under the subgroups of the rotation group. In

particular, states with odd fermion number cannot be $G_a \times G_b$ invariant. For even fermion numbers it is possible to have many-particle states for which individual particles transform non-trivially under orbital $G_a \times G_b$ if total $G_a \times G_b$ quantum numbers in spin like degrees of freedom compensate for the orbital quantum numbers (for instance, total spin is multiple of n_a). Hence the group algebra of $G_a \times G_b$ would characterize the states in orbital degrees of freedom as indeed assumed. The earlier picture would be correct apart from the lacking assumption about overall $G_a \times G_b$ invariance.

3. The construction of these states could be carried out just like the construction of ordinary $G_a \times G_b$ invariant states in H so that the mathematical treatment of the situation involves no mystics elements. Since $G_a \times G_b$ is actually assigned with a sector $M^4_\pm \times CP_2$ with fixed quantization axes and preferred point of H, one has center of mass degrees of freedom for the position of tip of $M^4_\pm$ and a preferred point of CP_2. This gives new degrees of freedom and one would have a rich spectrum of N-electrons, N-nucleons, N-atoms, etc.... behaving effectively as elementary particles. For example, one interesting question is whether 2-electrons could be interpreted as Cooper pairs of particular kind This would require either $s_z = 0, l_z = 0$ or $s_z = 1, l_z = mn_a - 1$, $m = 0, 1, 2...$ For instance, for $n_a = 3$ (the minimal value of n_a) one could have $s_z = l, l_z = 2$ with $J_z = 3$. One can also ask whether some high spin nuclei could correspond to N-nuclei.

4. This picture is quite predictive. For instance, in the case of gravitational interactions it would mean that the spin angular momentum of an astrophysical system is a multiple of "personal" gravitational Planck constant GM^2/v_0. The value of v_0 could be deduced from this condition and is expected to be a negative power of 2. In the same manner the relative angular momentum of planet-Sun system would be a multiple of GMm/v_0 using the gravitational Planck constant as a unit. This is a strong prediction but reduces to the Bohr quantization rule for circular orbits.

2. $\hat{H}\hat{\times}(G_a \times G_b)$ option

For this option the units of orbital angular momentum and color hyper charge and isospin are naturally scaled down by the factor n_i. In the case of spin and electro-weak spin this kind of scaling would require a covering group of Abelian Cartan group. Since the first homotopy group of SU(2) vanishes there are no coverings of $SU(2)$ in the ordinary sense of the word but quantum version of $SU(2)$ is an excellent candidate for the counterpart of the covering. Also quantum variants of other Lie groups are highly suggestive on basis of ADE correspondence.

There does not seem to be any absolute need for assuming $G_a \times G_b$ singletness. If so, there would be asymmetry between coverings and factor spaces bringing in mind confined and de-confined phases. Since coverings *resp.* factor spaces are labelled by N^{11}-valued lattice momenta *resp.* their negatives, this asymmetry would be analogous to time reversal asymmetry. Note however that all components of lattice momenta are either positive or negative and that this fits nicely with the interpretation of p-adic integers as naturals and "super-naturals". An intriguing question is whether there might be some connection with M-theory and its 4-D compactifications (dropping reflection group one obtains 7-D lattice).

3. Implications of the new picture

This picture has several important implications.

1. There is a symmetry between CP_2 and M^4 so that for a given value of Planck constant one obtains factor space with divisor group $G_a \times G_b$ and covering space with homotopy group $G_a \times G_b$. For large values of Planck constant the large Z_n symmetry acts in M^4 factor *resp.* CP_2 factor for these two options. Therefore the large Z_n symmetry in M^4 degrees of

freedom, which can be challenged in some of the applications, could be replaced with large Z^n symmetry in CP_2 degrees of freedom emerging rather naturally.

2. For a large value of Planck constant it is possible to obtain a relatively small dark matter symmetry group in M^4 factor and also the small genuinely 3-dimensional symmetry groups (tedrahedral, octahedral, icosahedral groups) can act in M^4 factor as symmetries of dark matter. Hence the groups appearing as symmetries of molecular physics (aromatic rings, DNA,...) could be identified as symmetries of dark electron pairs. These symmetries appear also in longer length scales (snow flakes and even astrophysical structures). In earlier picture one had to assume symmetry breaking at the level of visible matter. In particular, these structures could give rise to fractal hierarchy of Planck constants coming as powers of 2. In particular, $\hbar = n_a \times 2^{11k}$ is favored the model of EEG [M3].

3. The notion of N-atom generalizes. The original model predicted large electronic charges suggesting instability plus large Z_n symmetry in M^4 degrees of freedom (identified as a symmetry of field body). For instance, in the case of DNA double helix this kind of large rotational symmetry is questionable. Same applies to astrophysical systems with a gigantic value of gravitational Planck constant. The change of the roles of M^4 and CP_2 and charge fractionization would resolve these problems. This would provide a support for the idea that the electronic or protonic hot spots of catalyst and substrate correspond to fractional charges summing up to a unit charge. This framework could provide a proper realization for the original vision that symbolic level of dynamics and sex emerge already at the molecular level with sequences of catalyst sites representing "words" and their conjugates (opposite molecular sexes).

3.9 The content of McKay correspondence in TGD framework

The possibility to assign Dynkin diagrams with the inclusions of II_1 algebras is highly suggestive concerning possible physical interpretations. The basic findings are following.

1. For $\beta = \mathcal{M} : \mathcal{N} < 4$ Dynkin diagrams code for the inclusions and correspond to simply laced Lie algebras. $SU(2)$, D_{2n+1}, and E_7 are excluded.

2. Extended ADE Dynkin diagrams coding for simply laced ADE Kac Moody algebras appear at $\beta = 4$. Also $SU(2)$ Kac Moody algebra appears.

3.9.1 Does TGD give rise to ADE hierarchy of gauge theories

The first question is whether any finite subgroup $G \subset SU(2)$ acting in CP_2 degrees of freedom could somehow give rise to multiplets of the corresponding gauge group having interactions described by a gauge theory. Orbifold picture suggests that might be the case.

1. The "sheets" for the space-time sheet forming an $N(G)$-fold cover of $M^4_\pm$ are in one-one correspondence with group G. This degeneracy gives rise to additional states and these states correspond to the group algebra having basis given by group characters $\chi(g)$. One obtains irreducible representations of G with degeneracies given by their dimensions. Altogether one obtains $N(G)$ states in this manner. In the case of $A(n)$ the number of these states is $n + 1$, the number of the states of the fundamental representation of $SU(n + 1)$. In the same manner, for D_{2n} the number of these states equals to the number of states in the fundamental representation of D_{2n}. It seems that the rule is quite general. Thus these representations would in the case of fermions give the states of the fundamental representation of the corresponding gauge group.

2. From fermion and antifermion states one can construct in a similar manner pairs giving $N(G)^2$ states defining in the case of $A(n)$ n^2-1-dimensional gauge boson multiplet plus singlet. Also other groups must give boson multiplet plus possible other multiplets. For instance, for $D(4)$ the number of states is 64 and boson multiplet is 8-dimensional so that many other spin 1 states result.

3. These findings give hopes that the orbifold multiplets could be modelled by a gauge theory based on corresponding gauge group. What is nice that this huge hierarchy of gauge theories is associated with dark matter so that the predictivity and falsifiability are not lost unlike in M-theory.

3.9.2 Does one obtain also a hierarchy of conformal theories with ADE Kac Moody symmetry?

Consider next the question Kac Moody interactions correspond to extended ADE diagrams are possible.

1. In this case the notion of orbifold seems to break down since the symmetry related points form a continuum $SU(2)$ and space-time surface would become 6-dimensional if the M^4 projection is 4-dimensional. If one takes space-time as something which emerges, one could take this possibility half seriously. A more natural natural possibility is that M^4 projection is 2-dimensional geodesic sphere in which case one would have string like objects so that conformal field theory with Kac-Moody algebra would emerge naturally.

2. The new degrees of freedom would define 2-dimensional continuum and it would not be completely surprising if conformal field theory based on ADE Kac Moody algebra could describe the situation. One possibility is that these continua for different inclusions correspond to $SU(2)$ decompose to an $N(G)$-fold covers of S^2/G orbifold so that also now groups G would be involved with the Jones inclusions, which might provide a hint about how to construct them. S^2/G would play the role of stringy world sheet for the conformal field theory in question. This effective re-arrangement of the topology S^2 might be due to the fact that conformal fields possess G symmetry which effectively groups points of S^2 to $n(G)$-multiplets. The localized representations of the Lie group corresponding to G would correspond to the multiplets obtained from the representations of group algebra of G as in previous case.

3. The formula for the scaling factor of $M^4_\pm$ metric would give infinite scaling factor if one identifies the scaling factor as maximal order of cyclic subgroup of $SU(2)$. As a matter fact there is no finite cyclic subgroup of this kind. The solution to the problem would be identification of the scaling factor as the order of the maximal cyclic subgroup of G so that the scaling factors would be same for the two situations related by McKay correspondence.

3.9.3 Generalization to $M^4_\pm$ degrees of freedom

One can ask whether the proposed picture generalizes formally also the case of $M^4_\pm$.

1. In this case quantum groups would correspond to discrete subgroups $G \subset SL(2,C)$. Kac Moody group would correspond to G-Kac Moody algebra made local with respect to $SL(2,C)$ orbit in $M^4_\pm$ divided by G. These orbits are 3-dimensional hyperboloids H_a with a constant value of light cone proper time a so that the division by G gives fundamental domain H_a/G with a finite 3-volume.

2. The 4-dimensionality of space-time would require 1-dimensional CP_2 projection. Vacuum extremals of Kähler action would be in question. Robertson-Walker metric have 1-dimensional

CP_2 projection and carry non-vanishing density of gravitational mass so that in this sense the theory would be non-trivial. G would label different lattice like cosmologies defined by tesselations with fundamental domain H_a/G.

3. The multiplets of G would correspond to collections of points, one from each cells of the lattice like structure. Macroscopic quantum coherence would be realized in cosmological scales. If one takes seriously the vision about the role of short distance p-adic physics as a generator of long range correlations of the real physics reflected as p-adic fractality, this idea does not look so weird anymore.

   Complexified modular group $SL(2, Z + iZ)$ and its subgroups are interesting as far as p-adicization is considered. The principal congruence subgroups $\Gamma(N)$ of $SL(2, Z + iZ)$ which are unit matrices modulo N define normal subgroups of the complex modular group and are especially interesting candidates for groups $G \subset SL(2, C)$. The group $\Gamma(N = p^k)$ labelling fundamental domains of the tesselation $H_a/\Gamma(N = p^k)$ defines a mathematically attractive candidate for a point set associated with the intersections of p-adic space-time sheets with real space-time sheets. Also analogous groups for algebraic extensions of Z are interesting.

   The simplest discrete subgroup of SL(2,C) with infinite number of elements would corresponds to powers of boost to single direction and correspond at the non-relativistic limit to multiples of basic velocity. This could also give rise to quantization of cosmic recession velocities. There is evidence for the quantization of cosmic recession velocities (for a model in which single object produces quantized redshifts see [D4]) and it is interesting to see whether they could be interpreted in terms of the lattice like periodicity in cosmological length scales implied by the effective reduction of physics to M^4_+/G_n. In [74] the values $z = 2.63, 3.45, 4.47$ of cosmic red shift are listed. These correspond to recession velocities $v = (z^2 - 1)/(z^2 + 1)$ are (0.75,0.85,0.90). The corresponding hyperbolic angles are given by $\eta = acosh(1/(1 - v^2))$ and the values of η are $(1.46, 1.92, 2.39)$. The differences $\eta(2) - \eta(1) = .466$ and $\eta(3) - \eta(2) = .467$ are same within experimental uncertainties. One has however $\eta(n)/(\eta(2) - \eta(1)) = (3.13, 4.13, 5.13)$ instead of $(3, 4, 5)$. A possible interpretation is in terms of the velocity of the observer with respect to the frame in which quantization of η happens.

3.9.4 Quantitative support for the interpretation

A more detailed analysis of the situation gives support for the proposed vision.

1. A given value of quantum group deformation parameter $q = exp(i\pi/n)$ makes sense for any Lie algebra but now a preferred Lie-algebra is assigned to a given value of quantum deformation parameter. At the limit $\beta = 4$ when quantum deformation parameter becomes trivial, the gauge symmetry is replaced by Kac Moody symmetry.

2. The prediction is that Kac-Moody central extension parameter should vanish for $\beta < 4$. There is an intriguing relationship to formula for the quantum phase q_{KM} associated with (possibly trivial) Kac-Moody central extension and the phase defined by ADE diagram

$$\begin{array}{ll} q_{KM} = exp(i\phi) \ , & \phi_1 = \frac{\pi}{k+h^v} \ , \\ q_{Jones} = exp(i\phi) \ , & \phi = \frac{\pi}{h} \end{array}$$

   In the first formula sum of Kac-Moody central extension parameter k and dual Coxeter number h^v appears whereas Coxeter number h appears in the second formula. Internal consistency requires

$$k + h^v = h \ . \tag{5}$$

It is easy see that the dual Coxeter number h^v and Coxeter number h given by $h = (dim(g) - r)/r$, where r is the dimension of Cartan algebra of g, are identical for ADE algebras so that the Kac-Moody central extension parameter k must indeed vanish. For $SO(2n+1)$, $Sp(n)$, G_2, and F_4 the condition $h = h^v$ does not hold true but one has $h(n) = 2n = h^v + 1$ for $SO(2n+1)$, $h(n) = 2n = 2(h^v - 1)$ for $Sp(n)$, $h = 6 = h^v + 2$ for G_2, and $h = 12 = h^v + 3$ for F_4.

What is intriguing that G_2, which seems to play a fundamental role in the dual formulation of quantum TGD based on the identification of space-times as surfaces in hyper-octonionic space M^8 [E2] is not allowed. As a matter fact, $G_2 \rightarrow SU(3)$ reduction occurs also in the dual formulation based on $G_2/SU(3)$ coset model and is required by the separate conservation of quark and lepton numbers predicted by TGD. ADE groups would be associated with the interaction between space-time sheets rather than entire dynamics and need not have anything to do with the Kac-Moody algebra associated with color and electro-weak interactions appearing in the construction of physical states [F2].

3. There seems to be a concrete connection with conformal field theories. This connection would allow to understand the emergence of quantum groups appearing naturally in these theories. Quite generally, the conformal central extension parameter for unitary Virasoro representations resulting by Sugawara construction from Kac Moody representations satisfies either of the conditions

$$\begin{aligned} c &\geq \frac{kdim(g)}{k+h^v} + 1 \ , \\ c &= \frac{kdim(g)}{k+h^v} + 1 - \frac{6}{(h-1)h} \ . \end{aligned} \tag{6}$$

For $k = 0$, which should be interesting for $\beta < 4$, the second formula reduces to

$$c = 1 - \frac{6}{(h-1)h} \ . \tag{7}$$

The formula gives the values of c for minimal conformal field theories with finite number of conformal fields and real conformal weights. Indeed, h in this formula seems to correspond to the same h as appearing in the expression $\beta \equiv \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h)$.

$\beta = 3, h = 6$ corresponds to three-state Potts model with $c = 4/5$ which should thus have a gauge group for which Coxeter number is 6: the group should be either $SU(6)$ or $SO(8)$. Two-state Potts model, that is Ising model with $\beta = 2, h = 4$ would correspond to $c = 1/2$ and to a gauge group $SU(4)$ or $SO(4)$. For $h = 3$ ("one-state Potts model") with group $SU(3)$ one would have $c = 0$ and vanishing conformal anomaly so that conformal degrees of freedom would become pure gauge degrees of freedom.

These observations give support for the following picture.

1. Quite generally, the number of states of the generalized β-state Potts model has an interpretation as the dimension $\beta = \mathcal{M} : \mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module. Besides the models with integer

number of states there is an infinite number of models for which the number of states is not an integer. The conditions $c \leq 1$ guaranteing real conformal weights and $\beta \leq 4$ correspond to each other for these models.

2. $\beta > 4$ Potts models would be formally obtained by allowing h to be imaginary in the defining formula for $\mathcal{M} : \mathcal{N}$. In this case c would be however complex so that the theory would not be unitary.

3. For minimal models with $(\beta < 4, c < 1)$ Kac-Moody central extension parameter is vanishing so that Kac Moody algebra indeed acts like gauge symmetries and gauge symmetries would be in question. $(\beta = 4, c = 1)$ would define a "four-state Potts model" with infinite-dimensional unitary group acting as a gauge group. On the other hand, the appearance of extended ADE Dynkin diagrams suggests strongly that this limit is not realized but that $\beta = \mathcal{M} : \mathcal{N} = 4$ corresponds to $k = 1$ conformal field theory allowing Kac Moody symmetries for any ADE group, which as simply-laced groups allows vertex operator construction. The appearance of $kdim(g)/(k+g)$ in the more general formula would thus code the Kac Moody group whereas for $\beta < 4$ ADE diagram codes for the preferred gauge group characterizing the minimal CFT.

4. The possibility that any ADE gauge group or Kac-Moody group can characterize the interaction between space-time sheets conforms with the idea about Universe as a Topological Quantum Computer able to simulate any conceivable quantum dynamics. Of course, one cannot exclude the possibility that only electro-weak and color symmetries are realized in this manner.

3.9.5 G_a as a symmetry group of magnetic body and McKay correspondence

The group $G_a \subset SU(2) \subset SL(2,C)$ means exact rotational symmetry realized in terms of $M^4_{\pm}$ coverings of CP_2. The 5 and 6-cycles in biochemistry (sugars, DNA,....) are excellent candidates for these symmetries. For very large values of Planck constant, say for the values $\hbar(M^4_{\pm})/\hbar(CP_2) = GMm/v_0 = (n_a/n_b)\hbar_0$, $v_0 = 2^{-11}$, required by the model for planetary orbits as Bohr orbits [D6], G_a is huge and corresponds to either Z_{n_a} or in the case of even value of n_a to the group generated by Z_n and reflection acting on plane and containing $2n_a$ elements.

The notion of magnetic body seems to provide the only conceivable candidate for a geometric object possessing G_a as symmetries. In the first approximation the magnetic field associated with a dark matter system is expected to be modellable as a dipole field having rotational symmetry around the dipole axis. Topological quantization means that this field decomposes into flux tube like structures related by the rotations of Z_n or D_{2n}. Dark particles would have wave functions delocalized to this set of these flux quanta and span group algebra of G_a. Magnetic flux quanta are indeed assumed to mediate gravitational interactions in the TGD based model for the quantization of radii of planetary orbits and this explains the dependence of $\hbar_{gr}$ on the masses of planet and central object [D6].

For the model of dark matter hierarchy appearing in the model of living matter one has $n_a = 2^{11k}$, $k = 1, 2, 3, .., 7$ for cyclotron time scales below life cycle for a magnetic field $B_d = .2$ Gauss at $k = 4$ level of hierarchy (the field strength is fixed by the model for the effects of ELF em fields on vertebrate brain at harmonics of cyclotron frequencies of biologically important ions [M3]). Note that B_d scales as 2^{-11k} from the requirement that cyclotron energy is constant.

ADE correspondence between subgroups of SU(2) and Lie groups in ADE hierarchy encourages to consider the possibility that TGD could mimic ADE hierarchy of gauge theories. In the case of G_a this would mean that many fermion states constructed from single fermion states, which are in one-one correspondence with the elements of G_a group algebra, would define multiplets of the gauge group corresponding to the Dynkin diagram characterizing G_a: for instance, $SU(n_a)$ in the case of Z_{n_a}. Fermion multiplet would contain n_a states and gauge boson multiplet $n_a^2 - 1$ states.

This would provide enormous information processing capacity since for $n_a = 2^{11k}$ fermion multiplet would code exactly $11k$ bits of information. Magnetic body could represent binary information using the many-particle states belonging to the representations of say $SU(n_a)$ at its flux tubes.

3.10 Jones inclusions, the large N limit of $SU(N)$ gauge theories and AdS/CFT correspondence

The framework based on Jones inclusions has an obvious resemblance with larger N limit of $SU(N)$ gauge theories and also with the celebrated AdS/CFT correspondence [69] so that a more detailed comparison is in order.

3.10.1 Large N limit of gauge theories and series of Jones inclusions

The large N limit of $SU(N)$ gauge field theories has as definite resemblance with the series of Jones inclusions with the integer $n \geq 3$ characterizing the quantum phase $q = exp(i\pi/n)$ and the order of the maximal cyclic subgroup of the subgroup of $SU(2)$ defining the inclusion. Recall that all ADE groups except D_{2n+1} and E_7 are allowed (SU(2) is excluded since it would correspond to $n = 2$).

The limiting procedure keeps the value of g^2N fixed. Rather remarkably, this is equivalent with keeping αN constant but assuming $\hbar$ to scale as $n = N$. Thus the quantization of Planck constants would provide a physical laboratory for the testing of large N limit.

The observation suggesting a description of YM theories in terms of closed strings is that Feynman diagrams can be interpreted as being imbedded at closed 2-surfaces of minimal genus guaranteing that the internal lines meet except in vertices. The contribution of genus g diagrams is proportional to N^{g-1} at the large N limit. The interpretation in terms of closed partonic 2-surfaces is highly suggestive and the N^{g-1} should come from the multiple covering property of CP_2 by N M^4-points (or vice versa) with the finite subgroup of $G \subset SU(2)$ defining the Jones inclusion and acting as symmetries of the surface.

3.10.2 Analogy between stacks of branes and multiple coverings of M^4 and CP_2

An important aspect of AdS/CFT dualities is a prediction of an infinite hierarchy of gauge groups, which as such is as interesting as the claimed dualities. The prediction relies on the notion Dp-branes. Dp-branes are $p+1$-dimensional surfaces of the target space at which the ends of open strings can end. In the simplest situation one considers N parallel p-branes at the limit when the distances between branes characterized by an expectation value of Higgs fields approach zero to obtain what is called N-stack of branes. There are N^2 different strings connecting the branes and the heuristic idea is that they correspond to gauge bosons of $U(N)$ gauge theory. Note that the requirement that AdS/CFT dualities exist forces the introduction of branes and the optimistic interpretation is that a non-perturbative effect of still unknown M-theory is in question. In the limit of an ideal stack one assumes that $U(N)$ gauge theory at the brane representing the stack is obtained. The branes must also carry a p-form defining gauge potential for a closed $p+1$-form. This Ramond charge is quantized and its value equals to N.

Consider now the group $G_a \times G_b \subset SL(2,C) \times SU(2) \subset SU(3)$ defining double Jones inclusion and implying the scalings $\hbar(M^4) \rightarrow n(G_b)\hbar(M^4)$ and $\hbar(CP_2) \rightarrow n(G_a)\hbar(CP_2)$. These space-time surfaces define $n(G_a)$-fold multiple coverings of CP_2 and $n(G_b)$-fold multiple coverings of M^4. In CP_2 degrees of freedom the collection of G_b-related partonic 2-surfaces (/3-surfaces/4-surfaces) is highly analogous to the stack of branes. In M^4 degrees of freedom the stack of copies of surface typically correspond to along a circle (A_n, D_{2n} or at vertices of tedrahedron or isosahedron.

In TGD framework the interpretation strings are not needed to define gauge fields. The group algebra of G realized as discrete plane waves at G-orbit gives rise to representations of G. The hypothesis supported by few examples is that these additional degrees of freedom allow to construct multiplets of the gauge group assignable to the ADE diagram characterizing the inclusion.

3.10.3 AdS/CFT duality

AdS/CFT duality is a further aspect of the brane construction. The dual description of the situation is in terms of a string theory in a background in which N-brane acts as a macroscopic object giving rise to a black-hole like object in (say) 10-dimensional target space. This background has the form $AdS_5 \times X_5$, where AdS_5 is 5-dimensional hyperboloid of M^6 and thus allows $SO(4,2)$ as isometries. X_5 is compact constant curvature space. S_5 gives rise to $N = 4$ SUSY in M^4 with M^4 interpreted as a brane. The first support for the dualities comes from the symmetries: for instance, the $N = 4$ super-symmetrized isometries of $AdS_5 \times S^5$ are same as the symmetries of 4-dimensional $N = 4$ SUSY for $p = 3$ branes. N-branes can be used as models for black holes in target space and black-hole entropy can be calculated using either target space picture or conformal field theory at brane and the results turn out be the same.

Does the TGD equivalent of this duality exists in some sense?

1. As far as partonic 2-surfaces identified as 1-branes are considered, conformal field theory description is trivially true. In TGD framework the analog of Ramond charges are the integers n_a and n_b characterizing the multipliticies of the maximal Abelian subgroups having clear topological meaning. This conforms with the observation that large N limit of the gauge field theories can be formulated in terms of closed surfaces at which the Feynman diagrams are imbedded without self crossings. It seems that the integers n_a and n_b characterizing the Jones inclusion naturally take the role of Ramond charge: this does not of course exclude the possibility they can be expressed as fluxes at space-time level as will be indeed found.

2. Conformal field theory description can be generalized in the sense that one replaces the $n(G_a) \times n(G_b)$ partonic surfaces with single one and describes the new states as primary fields arranged into representations of the ADE group in question. This would mean that the standard model gauge group extends by additional factor which is however non-trivially related to it.

3. If one can accept the idea that the conformal field theory description for partons gives rise to M^4 gauge theory as an approximate description, it is not too difficult to imagine that also ADE hierarchy of gauge theories results as a description of the exotic states. One can say that CFT in p-brane is replaced now with CFT on partonic 2-surface (1-brane) analogous to a closed string.

4. In the minimal interpretation there is no need to add strings connecting the branches of the double covering of the partonic 2-surface whose function is essentially that of making possible gauge bosons as fermion anti-fermion pairs. One could of course imagine gauge fluxes as counterparts of strings but just the fact that G-invariance dictates the configurations completely forces to question this kind of dynamics.

5. There is no reason to expect the emergence of $N = 4$ super-symmetric field theory in M^4 as in the case of super-string models. The reasons should be already obvious: super-conformal generators G anticommute to L_0 proportional to mass squared rather than four-momentum and the spectrum extended by $G_a \times G_b$ degeneracy contains more states.

One can of course ask whether higher values of p could make sense in TGD framework.

1. It seems that the light-like orbits of the partonic 2-surfaces defining 2-branes do not bring in anything new since the generalized conformal invariance makes it possible the restriction to a 2-dimensional cross section of the light like causal determinant.

2. The idea of regarding space-time surface X^4 as a 3-brane in H in which some kind of conformal field theory is defined is in conflict with the basis ideas of TGD. The role of X^4 interior is to provide classical correlates for quantum dynamics to make possible quantum measurement theory and also introduce correlations between partonic 2-surfaces even in the case that partonic conformal dynamics reduces to a topological string theory. It is quantum classical correspondence which corresponds to this duality.

3.10.4 What is the counterpart of the Ramond charge in TGD?

The condition that there exist a p-form defining $p+1$-gauge field with p-charge equal to n_a or n_b is a rather stringent additional condition also in TGD framework. For $n < \infty$ this kind of charge is defined by Jones inclusion and represented topologically so that Ramond charge is not needed in $n < \infty$ case. By the earlier arguments one must however be able to assign integers n_a and n_b also to $G = SU(2)$ inclusions with Kac-Moody algebra characterized by an extended ADE diagram with the phases $q_i = exp(i\pi/n_i)$ relating to the monodromy of the theory. Since Jones inclusion does not define in this case the value of $n < \infty$ in any obvious manner, the counterpart of the Ramond charge is needed.

1. For partonic 2-surfaces ordinary gauge potential would define this form and the condition would state that magnetic flux equals to n so that the anyonic partonic two-surfaces would be homologically non-trivial in CP_2 degrees of freedom. String ends would define basic example of this situation. This would be the case also in M^4_+ degrees of freedom: the partonic 2-surface would essentially wind n_a times around the tip of $\delta M^4_\pm$ and the gauge field in question would be monopole magnetic field in $\delta M^4_\pm$. This kind of situation need not correspond to anything cosmological since future and past light-cones appear in the basic definition of the scattering amplitudes.

2. For $p = 3$ Chern-Simons action for the induced CP_2 Kähler form associated with the partonic 2-surface indeed defines this kind of charge. Ramond charge should be simply N. CP_2 type extremals or their small deformations satisfy this constraint and are indeed very natural in elementary particle physics context but too restrictive in a more general context.

Note that the light-like orbits of non-deformed CP_2 extremals have light-like random curve as an M^4 projection and the conformal symmetries of M^4 obviously respect light-likeness property. Hence $SO(4,2)$ symmetry characterizing AdS$_5$/CFT is not excluded but would be broken by p-adic thermodynamics and by TGD based Higgs mechanism involving the identification of inertial momentum as average value of non-conserved gravitational momentum parallel to the light-like zitterbewegung orbit.

3.10.5 Can one speak about black hole like structures in TGD framework?

For AdS/CFT correspondence there is also a dynamical coupling to the target space metric. The coupling to H-metric is present also now since the overall scalings of the M^4 *resp.* CP_2 metrics by n_b *resp.* by n_a are involved. This applies to when multiple covering is used explicitly. In the description in which one replaces the multiple covering by ordinary $M^4 \times CP_2$, the metric suffers a genuine change and something analogous to the black-hole type metrics encountered in AsS/CFT correspondence might be encountered.

Consider as an example an n_a-fold covering of CP_2 points by M^4 points (ADE diagram A_{n_a-1}). The n-fold covering means only $n2\pi$ rotation for the phase angle ψ of CP_2 complex coordinate leads to the original point. The replacement $\psi \rightarrow \psi/n_a$ gives rise to what would look like ordinary $M^4 \times CP_2$ but with a modified CP_2 metric. The metric components containing ψ as index are scaled down by $1/n_a$ or $1/n_a^2$. Notice that Ψ effectively disappears from the dynamics at the large n_a limit.

If one uses an effective description in which covering is eliminated the metric is indeed affected at the level of imbedding space black hole like structures at the level of dynamic space might make emerge also in TGD framework at large N limit since the masses of the objects in question become large and CP_2 metric is scaled by N so that CP_2 has very large size at this limit. This need not lead to any inconsistencies if these phases are interpreted as dark matter. At the elementary particle level p-adic thermodynamics predicts that p-adic entropy is proportional to thermal mass squared which implies elementary particle black-hole analogy.

3.10.6 Other dualities

Also quantum classical correspondence defines in a loose sense a duality justifying the basic assumptions of quantum measurement theory. The light-like orbits of 2-D partons are characterized by a generalization of ordinary 2-D conformal invariance so that CFT part of the duality would be very natural. The dynamical target space would be replaced with the space-time surface X^4 with a dynamical metric providing classical correlates for the quantum dynamics at partonic 2-surfaces. The duality in this sense cannot be however exact since classical dynamics cannot fully represent quantum dynamics.

Classical description is not expected to be unique. The basic condition on space-time surfaces assignable to a given configuration of partonic 2-surfaces associated with the surface X_V^3 defining S-matrix element are posed by quantum classical correspondence. Both hyper-quaternionic and co-hyper-quaternionic space-time surfaces are acceptable and this would define a fundamental duality.

A concrete example about this HQ-coHQ duality would be the equivalence of space-time descriptions using 4-D CP_2 type extremals and 4-D string like objects connecting them. If one restricts to CP_2 type extremals and string like objects of from $X^2 \times Y^2$, the target space reduces effectively to M^4 and the dynamical degrees of freedom correspond in both cases to transversal M^4 degrees of freedom. Note that for CP_2 type extremals the conditions stating that random light-likeness of the M^4 projection of the CP_2 type extremal are equivalent to Virasoro conditions. CP_2 type extremals could be identified as co-HQ surfaces whereas stringlike objects would correspond to HQ aspect of the duality.

HQ-coHQ provides dual classical descriptions of same phenomena. Particle massivation would be a basic example. Higgs mechanism in a gauge theory description based on CP_2 type extremals would rely on zitterbewegung implying that the average value of gravitational mass identified as inertial mass is non-vanishing and is discussed already. Higgs field would be assigned to the wormhole contacts. The dual description for the massivation would be in terms of string tension and mass squared would be proportional to the distance between G-related points of CP_2.

These observations would suggest that also a super-conformal algebra containing $SL(2,R) \times SU(2)_L \times U(1)$ or its compact version exists and corresponds to a trivial inclusion. This is indeed the case [68]. The so called large $N = 4$ super-conformal algebra contains energy momentum current, 2+2 super generators G, $SU(2) \times SU(2) \times U(1)$ Kac-Moody algebra (both SU(2) and SL(2,R) could be interpreted as acting on M^4 spin degrees of freedom, and 2 spin 1/2 fermionic currents having interpretation in terms of right handed neutrinos corresponding to two H-chiralities. Interestingly, the scalar generator is now missing.

3.11 Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

The commutation relations for $M_{2,q}(C)$ matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \ , \tag{8}$$

read as

$$\begin{array}{ll} ab = qba \ , & ac = qac \ , \quad bd = qdb \ , cd = qdc \ , \\ [ad,da] = (q-q^{-1})bc \ , & bc = cb \ . \end{array} \tag{9}$$

These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a,a^\dagger] = [b,b^\dagger] = [c,c^\dagger] = [d,d^\dagger] = 1 \ . \tag{10}$$

The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{aligned} O|phys\rangle &= 0 \ , \\ O &\in \{a-a^\dagger, d-d^\dagger, b-c^\dagger, c-b^\dagger\} \ . \end{aligned} \tag{11}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. This is the case and means also that the operators representing M^4 coordinates commute and it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

$M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})bc$ does not vanish. The observation that M^4 coordinates can be regarded as eigenvalues of commuting observables proves that quantum $M^4_\pm$ and its orbifold description are equivalent.

What about M^8: does it have analogous description? The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units.

Hyper-octonionic units indeed have anticommutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

Introduce the coefficients of E^4 gamma matrices having interpretation as quaterionic units as

$$\begin{array}{ll} a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\ a_1 = x(b+c) \ , & a_2 = x(ib-c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

The selections of commutative and quaternionic sub-algebras of octonion space are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of octonions the selection of quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned} [a_0, a_3] &= -i(q-q^{-1})(a_1^2+a_2^2) \ , \\ [a_i, a_j] &= 0 \ , \ i,j \neq 0,3 \ , \\ a_0 a_i &= q a_i a_0 \ , \ i \neq 0,3 \ , \\ a_3 a_i &= q a_i a_3 \ , \ i \neq 0,3 \ . \end{aligned} \tag{12}$$

Checking that M^8 indeed corresponds to commutative subspace defined by the eigenvalues of operators is straightforward.

The argument generalizes easily to other dimensions $D \geq 4$ but now quaternionic and octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and imbedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Thus the special role of classical number fields and uniqueness of space-time and imbedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. It is possible to construct the quantal variants of the coset spaces $M^4 \times E^4/G_a \times G_b$ by simply posing restrictions on the of eigen states of the commuting coordinate operators. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible.

4 Has dark matter been observed?

In this section two examples about anomalies perhaps having interpretation in terms of quantized Planck constant are discussed.

4.1 Optical rotation of a laser beam in a magnetic field

The group of G. Cantatore has reported an optical rotation of a laser beam in a magnetic field [83]. The experimental arrangement involves a magnetic field of strength $B = 5$ Tesla. Laser beam travels 22000 times forth and back in a direction orthogonal to the magnetic field travelling 1 m during each pass through the magnet. The wavelength of the laser light is 1064 nm. A rotation of $(3.9 \pm .5) \times 10^{-12}$ rad/pass is observed.

A possible interpretation for the rotation would be that the component of photon having polarization parallel to the magnetic field mixes with QCD axion, one of the many candidates for dark matter. The mass of the axion would be about 1 meV. Mixing would imply a reduction of the corresponding polarization component and thus in the generic case induce a rotation of the polarization direction. Note that the laser beam could partially transform to axions, travel through a non-transparent wall, and appear again as ordinary photons.

The disturbing finding is that the rate for the rotation is by a factor 2.8×10^4 higher than predicted. This would have catastrophic astrophysical implications since stars would rapidly lose their energy via axion radiation.

4.1.1 Could the optical rotation be caused by a pion of a scaled down copy of ordinary QCD

The motivation for introducing axion was the large CP breaking predicted by the standard QCD. No experimental evidence has been found has been found for this breaking. The idea is to introduce a new broken U(1) gauge symmetry such that is arranged to cancel the CP violating terms predicted by QCD. Because axions interact very weakly with the ordinary matter they have been also identified as candidates for dark matter particles.

In TGD framework there is special reason to expect large CP violation analogous to that in QCD although one cannot completely exclude it. Axions are however definitely excluded. TGD predicts a hierarchy of scaled up variants of QCD and entire standard model plus their dark variants corresponding to some preferred p-adic length scales, and these scaled up variants play a key role in TGD based view about nuclear strong force [F8, F9], in the explanation of the anomalous production of e^+e^- pairs in heavy nucleus collisions near Coulomb wall [F7], high T_c superconductivity [J1, J2, J3] and also in the TGD based model of living matter [M3]. Therefore a natural question is whether the particle in question could be a pion of some scaled down variant of QCD having similar coupling to electromagnetic field. Also dark variants of this pion could be considered.

What raises optimism is that the Compton length of the scaled down quarks is of the same order as cyclotron wavelength of electron in the magnetic field in question. For the ordinary value of Planck constant this option however predicts quite too high mixing rate. This suggests that dark matter has been indeed observed in the sense that the pion corresponds to a large value of Planck constant. Here the encouraging observation is that the ratio λ_c/λ of wavelength of cyclotron photon and laser photon is $n = 2^{11}$, which corresponds to the lowest level of the biological dark matter hierarchy with levels characterized the value $\hbar(M^4_\pm) = 2^{11k}\hbar_0$, $k = 1, 2, ...$

The most plausible model is following.

1. Suppose that the photon transforms first to a dark cyclotron photon associated with electron at the lowest $n = 2^{11}$ level of the biological dark matter hierarchy. Suppose that the coupling of laser photon to dark photon can be modelled as a coefficient of the usual amplitude apart from a numerical factor of order one equal to $\alpha_{em}(n) \propto 1/n$.

2. Suppose that the coupling $g_{\pi NN}$ for the scaled down hadrons is proportional to $\alpha_s^4(n) \propto 1/n^4$ as suggested by a simple model for what happens for the nucleon and pion at quark level in the emission of pion.

Under these assumptions one can understand why only an exotic pion with mass of 1 meV couples to laser photons with wavelength $\lambda = 1$ μm in magnetic field $B = 5$ Tesla. The general prediction is that λ_c/λ must correspond to preferred values of n characterizing Fermat polygons constructible using only ruler and compass, and that the rate for the rotation of polarization depends on photon frequency and magnetic field strength in a manner not explained by the model based on the photon-axion mixing.

4.1.2 Scaled up variant of PCAC

Consider first briefly the scaled up variant of partially conserved axial current hypothesis (PCAC).

1. The mass of the particle would be around 1 meV. If a scaled down ordinary pion is in question, the mass ratio $m_\pi/m_A \simeq 140\times 10^9 \sim 2^{37}$ suggests that the space-time sheet associated with gluons of this QCD is related by p-adic scale in question corresponds to $k=107+2\times 37=181$, which is prime and corresponds to p-adic length scale $L(181)=.327$ mm. The predicted pion mass from exact scaling would be 1.1 meV. This pion does not couple to ordinary quarks and therefore this coupling does not affect astrophysics at the level of visible matter. The parameter $\Lambda_{QCD,181}$ would be obtained by the scaling $\Lambda_{QCD}(181)=2^{-37}\Lambda_{QCD}(107)$.

2. The interaction of pion and photons is fixed completely by the anomaly of axial current [66]

$$\langle 0|A_\mu^j(x)|\pi^k\rangle = i\delta^{jk}p_\mu f_\pi exp^{-ip\cdot x} \quad . \tag{13}$$

   Here $f_\pi \simeq 93$ MeV characterizes the matrix element of axial current between vacuum and single-pion state and thus the decay rate of pion.

   The form of the interaction is exactly the same as in the case of axion and given by the interaction Lagrangian

$$\begin{aligned} L &= k_{em}\pi F\wedge F \quad , \\ k_{em} &= \frac{e^2}{32\pi^2 f_\pi} \quad . \end{aligned} \tag{14}$$

   The detailed arguments leading to the expression for k_{em} can be found in [66].

3. Axial current anomaly implies that the divergence of the axial current is proportional to the pion field. Writing the most general form for the matrix element of the axial current between nucleon states, this gives a relationship between pion-nucleon coupling $g_{\pi NN}$ and pion decay rate f_π:

$$\begin{aligned} \frac{g_A(0)}{f_\pi} &= \frac{g_{\pi NN}}{m_N} \quad , \\ g_A(0) &= \frac{G_A}{G_V} \quad . \end{aligned} \tag{15}$$

   One has $m_N=.94$ GeV, $g_{\pi NN}^2/4\pi = 14.6$. $g_A(0)=G_A/G_V=1.22$ is the ratio of axial and vectorial weak couplings for the fermion at zero momentum transfer. Te relationship follows from the conservation of axial current between nucleon and states that the coefficient of the term $q^\mu\overline{u}\gamma_5 u$ in the axial current matrix element between two nucleon states has a pole corresponding to the exchange of approximately massless pion. This formula generalizes trivially for the scaled up variants of QCD. The photon-axion mixing rate is proportional to $1/m_N$, where m_N is the mass of the exotic nucleon.

4.1.3 Comparison with the axion model

Let us compare the predictions of this model with the predictions of the axion model.

1. Axion-photon interaction Lagrangian has exactly the same form as $\pi^0\gamma\gamma$ interaction Lagrangian. The parameter f_a for the axion satisfies the condition

$$f_a \simeq \frac{\Lambda_{QCD}^2}{m_a} . \tag{16}$$

   Here one has $m_a \simeq 1$ meV and $\Lambda_{QCD} \simeq .2$ GeV.

2. From the fact that the rate is by a factor $r = 2.8 \times 10^4$ higher than the rate expected for QCD axion with mass $m_a \simeq 1$ meV one can deduce that the mass scale of the exotic u and d quarks. The condition that the two decay rates differ by the factor $R = 2.8 \times 10^4$ reads as

$$\frac{g_{A,e}(0)}{g_{\pi_e N_e N_e}} \times m_{N_e} = \frac{1}{\sqrt{R}} \frac{\Lambda_{QCD}^2}{m_a} , \tag{17}$$

   where the right hand side refers to the exotic nucleon and pion. The parameter $g_{A,e}$ can be assumed to be near to one.

   Suppose first that exotic pion is not dark and that $g_{\pi_e N_e N_e} = g_{\pi NN}$ holds true. The small mass of axion implies that the right hand side is about 2.4×10^5 GeV so that m_{N_e} should be by a factor about $3.2 \times 10^6 \sim 2^{22}$ larger than m_N and corresponding quarks would roughly correspond to $k \sim 73$. This is in in contradiction with what one would expect. Basically the large decay constant of exotic pion $\propto 1/m_N$ is in conflict with the very small decay constant of axion proportional to $\propto m_a/\Lambda^2$.

Consider now various options which could cure the problem.

Option I: The first dark matter option option is that one has $\hbar = n\hbar_0$ and $g_{\pi_e N_e N_e}$ is by a factor $1/n^k \simeq 2^{-60} \simeq 10^{-18}$ smaller than $g_{\pi NN}$. The factor comes from the overall reduction factor $3.2 \times 10^6 \sim 2^{22}$ of $1/f_\pi$ and from the fact that nucleon mass scale should be reduced roughly by a factor $\sim 2^{-37}$ (just like pion mass scale).

This could be understood if the pion exchange involves the emission of k virtual gluons implying $g_{\pi_e N_e N_e} \propto \alpha_s^k \propto 1/n^k$. One virtual gluon would decay to pion and two additional exchanges are necessary since all three valence quarks of nucleon must interact: hence $k = 3$ is the minimal option. One can also argue that the quarks resulting in the decay of virtual gluon must exchange at least one gluon to become a pion. This would give $1/n^4$ behavior giving the estimate $n = 2^{15}$ assuming $g_{\pi_e N_e N_e} = g\alpha_s^4$, with g having no dependence on α_s. The higher powers of α_s in the expansion of $g_{\pi NN}$ are important for ordinary hadrons physics but small for its dark variants so that the estimate is just a rough order of magnitude estimate if even that.

Option II: One can consider also the possibility that the space-time sheet of the magnetic field is dark so that the disappearance of photons from the laser beam involves a transformation to a dark photon followed by a transformation to a dark neutral pion in the magnetic field used. This would mean that the amplitude for the process would involve an additional dimensionless factor $g_{\gamma\gamma_d} \propto \alpha_{em} \propto 1/\hbar$. This would predict $n \simeq 2^{53}$ and values of this order of magnitude are possible in the model of living matter [M3]. The smallness of this amplitude could explain the discrepancy. This option is however not very plausible.

Option III: The third option would be a combination of the first two so that the vertex would contain the factor $g_{\gamma\gamma_d} g_{\pi_e N_e N_e} = \alpha_{em} g_{\pi NN} n^{-1-k}$. For $k = 4$ one would have $n^5 \sim 2^{53}$ suggesting

$n = 2^{11}$ corresponding to the lowest level in the hierarchy of preferred scaling factors $n = 2^{k11}$ of $\hbar = n\hbar_0$ in living matter. If laser photons are dark photons themselves then $g_{\pi NN} = k\alpha_s^5$ would give the same prediction. Note that the presence of higher powers of α_s in the expansion of $g_{\pi NN}$ could affect these conclusions.

4.1.4 Transformation of laser photons to dark cyclotron photons to exotic pions as the basic mechanism

The cyclotron wave length of electron in a magnetic field of 5 Tesla equals to $\lambda_c = 2$ mm and one has $\lambda_c/\lambda = 2^{11}$. This intriguing finding suggests that λ_c corresponds to the wavelength of dark variant of laser photon at $k = 1$ level of this hierarchy. One can therefore ask whether the basic mechanism is the transformation of the laser photon to a dark cyclotron photon with $\hbar = 2^{11}\hbar_0$ and its mixing with the $k = 181$ exotic pion.

This would predict that the effect is sensitive to the ratio λ_c/λ which should be near $n = 2^{11}$, or to a more general preferred value of n. The preferred values for the scaling factors n of $\hbar$ correspond to n-polygons constructible using ruler and compass. The values of n in question are given by $n_F = 2^k \prod_i F_{s_i}$, where the Fermat primes $F_s = 2^{2^s} + 1$ appearing in the product are distinct. The lowest Fermat primes are $3, 5, 17, 257, 2^{16} + 1$. In the model of living matter the especially favored values of $\hbar$ come as powers 2^{k11}.

Can one understand the mass scale of the exotic pion?

The model predicts preferred values for the ratio λ_c/λ and the experiments correspond to the lowest value of this ratio for biological dark matter hierarchy. In order to be taken seriously the model should also tell why just the scaled up variant of QCD with $m_\pi \simeq 1$ meV is involved.

Also this could relate somehow to the properties of the magnetic field. The frequency associated with the cyclotron photons emitted by electron in the magnetic field is $f = eB/m_e$ and for $B = 5$ Tesla the corresponding wave length is $\lambda_c = 2$ mm to be compared with $L(181) = .327$ mm. As already noticed, $\lambda_c = 2^{11}\lambda$, where $2^{11}\lambda$ is the wavelength of the dark variant of laser photon. Hence it is natural to assume that λ_c corresponds to an characteristic p-adic length scale for the exotic QCD in question.

The p-adic length scale $L(113)$ of u and d quarks is related by a factor 8 to gluon length scale $L(107)$. This would predict that exotic u and d quark correspond to $L(187) = 2.6$ mm to be compared with $\lambda_c = 2$ mm. Hence the latter scale might relate to the p-adic length scales characterizing the Compton lengths of exotic u and d quarks. The prediction would be that the mixing rate depends on magnetic field changing in a discontinuous manner for critical values of the magnetic field.

Summary

To sum up, the assumption that laser photons couple to a dark variant of an exotic pion at the first level of the biological dark matter hierarchy explains the rotation of the polarization direction if one accepts the proposed proportionality $g_{\pi NN} \propto \alpha_s^4 \propto 1/\hbar^4$ and that the transformation of the ordinary laser photon to dark photon can be modelled by a coefficient $k\alpha_{em} \propto 1/\hbar$. The model explains also why dark variants of other exotic pions are not produced.

4.2 Do nuclear reaction rates depend on environment?

Claus Rolfs and his group have found experimental evidence for the dependence of the rates of nuclear reactions on the condensed matter environment [86]. For instance, the rates for the reactions ^{50}V(p,n)^{50}Cr and ^{176}Lu(p,n) are fastest in conductors. The model explaining the findings has been tested for elements covering a large portion of the periodic table.

4.2.1 Debye screening of nuclear charge by electrons as an explanation for the findings

The proposed theoretical explanation [86]is that conduction electrons screen the nuclear charge or equivalently that incoming proton gets additional acceleration in the attractive Coulomb field of electrons so that the effective collision energy increases so that reaction rates below Coulomb wall increase since the thickness of the Coulomb barrier is reduced.

The resulting Debye radius

$$R_D = 69\sqrt{\frac{T}{n_{eff}\rho_a}} , \quad (18)$$

where ρ_a is the density of atoms per cubic meter and T is measured in Kelvins. R_D is of order .01 Angstroms for $T = 373$ K for $n_{eff} = 1$, $a = 10^{-10}$ m. The theoretical model [84, 85] predicts that the cross section below Coulomb barrier for $X(p,n)$ collisions is enhanced by the factor

$$f(E) = \frac{E}{E+U_e}exp(\frac{\pi\eta U_e}{E}) . \quad (19)$$

E is center of mass energy and η so called Sommerfeld parameter and

$$U_e \equiv U_D = 2.09\times 10^{-11}(Z(Z+1))^{1/2}\times (\frac{n_{eff}\rho_a}{T})^{1/2} \ eV \quad (20)$$

is the screening energy defined as the Coulomb interaction energy of electron cloud responsible for Debye screening and projectile nucleus. The idea is that at R_D nuclear charge is nearly completely screened so that the energy of projectile is $E + U_e$ at this radius which means effectively higher collision energy.

The experimental findings from the study of 52 metals support the expression for the screening factor across the periodic table.

1. The linear dependence of U_e on Z and $T^{-1/2}$ dependence on temperature conforms with the prediction. Also the predicted dependence on energy has been tested [86].

2. The value of the effective number n_{eff} of screening electrons deduced from the experimental data is consistent with $n_{eff}(Hall)$ deduced from quantum Hall effect.

The model suggests that also the decay rates of nuclei, say beta and alpha decay rates, could be affected by electron screening. There is already preliminary evidence for the reduction of beta decay rate of ^{22}Na β decay rate in Pd [87], metal which is utilized also in cold fusion experiments. This might have quite far reaching technological implications. For instance, the artificial reduction of half-lives of the radioactive nuclei could allow an effective treatment of radio-active wastes. An interesting question is whether screening effect could explain cold fusion [89] and sono-fusion [88]: I have proposed a different model for cold fusion based on large $\hbar$ in [F8].

4.2.2 Could quantization of Planck constant explain why Debye model works?

The basic objection against the Debye model is that the thermodynamical treatment of electrons as classical particles below the atomic radius is in conflict with the basic assumptions of atomic physics. On the other hand, it is not trivial to invent models reproducing the predictions of the Debye model so that it makes sense to ask whether the quantization of Planck constant predicted by TGD could explain why Debye model works.

TGD predicts that Planck constant is quantized in integer multiples: $\hbar = n\hbar_0$, where $\hbar_0$ is the minimal value of Planck constant identified tentatively as the ordinary Planck constant. The preferred values for the scaling factors n of $\hbar$ correspond to n-polygons constructible using ruler and compass. The values of n in question are given by $n_F = 2^k \prod_i F_{s_i}$, where the Fermat primes $F_s = 2^{2^s} + 1$ appearing in the product are distinct. The lowest Fermat primes are $3, 5, 17, 257, 2^{16} + 1$. In the model of living matter the especially favored values of $\hbar$ come as powers 2^{k11} [M3, J6].

It is not quite obvious that ordinary nuclear physics and atomic physics should correspond to the minimum value $\hbar_0$ of Planck constant. The predictions for the favored values of n are not affected if one has $\hbar(stand) = 2^k\hbar_0$, $k \geq 0$. The non-perturbative character of strong force suggests that the Planck constant for nuclear physics is not actually the minimal one [F8]. As a matter fact, TGD based model for nucleus implies that its "color magnetic body" has size of order electron Compton length. Also valence quarks inside hadrons have been proposed to correspond to non-minimal value of Planck constant since color confinement is definitely a non-perturbative effect. Since the lowest order classical predictions for the scattering cross sections in perturbative phase do not depend on the value of the Planck constant one can consider the testing of this issue is not trivial in the case of nuclear physics where perturbative approach does not really work.

Suppose that one has $n = n_0 = 2^{k_0} > 1$ for nuclei so that their quantum sizes are of order electron Compton length or perhaps even larger. One could even consider the possibility that both nuclei and atomic electrons correspond to $n = n_0$, and that conduction electrons can make a transition to a state with $n_1 < n_0$. This transition could actually explain how the electron conductivity is reduced to a finite value. In this state electrons would have Compton length scaled down by a factor n_0/n_1.

For instance, if one has $n_0 = 2^{11k_0}$ as suggested by the model for quantum biology [M3] and by the TGD based explanation of the claimed detection of dark matter [83], the Compton length $L_e = 2.4 \times 10^{-12}$ m for electron would reduce in the transition $k_0 \rightarrow k_0 - 1$ to $L_e = 2^{-11}L_e \simeq 1.17$ fm, which is rather near to the proton Compton length since one has $m_p/m_e \simeq .94 \times 2^{11}$. It is not too difficult to believe that electrons in this state could behave like classical particles with respect to their interaction with nuclei and atoms so that Debye model would work.

The basic objection against this model is that anyonic atoms should allow more states that ordinary atoms since very space-time sheet can carry up to n electrons with identical quantum numbers in conventional sense. This should have been seen.

4.2.3 Electron screening and Trojan horse mechanism

An alternative mechanism is based on Trojan horse mechanism suggested as a basic mechanism of cold fusion [F8]. The idea is that projectile nucleus enters the region of the target nucleus along a larger space-time sheet and in this manner avoids the Coulomb wall. The nuclear reaction itself occurs conventionally. In conductors the space-time sheet of conduction electrons is a natural candidate for the larger space-time sheet.

At conduction electron space-time sheet there is a constant charged density consisting of n_{eff} electrons in the atomic volume $V = 1/n_a$. This creates harmonic oscillator potential in which incoming proton accelerates towards origin. The interaction energy at radius r is given by

$$V(r) = \alpha n_{eff}\frac{r^2}{2a^3} , \qquad (21)$$

where a is atomic radius.

The proton ends up to this space-time sheet by a thermal kick compensating the harmonic oscillator energy. This occurs below with a high probability below radius R for which the thermal energy $E = T/2$ of electron corresponds to the energy in the harmonic oscillator potential. This gives the condition

$$R = \sqrt{\frac{Ta}{n_{eff}\alpha}}a \ . \tag{22}$$

This condition is exactly of the same form as the condition given by Debye model for electron screening but has a completely different physical interpretation.

Since the proton need not travel through the nuclear Coulomb potential, it effectively gains the energy

$$E_e = Z\frac{\alpha}{R} = \frac{Z\alpha^{3/2}}{a}\sqrt{\frac{n_{eff}}{Ta}} \ . \tag{23}$$

which would be otherwise lost in the repulsive nuclear Coulomb potential. Note that the contribution of the thermal energy to E_e is neglected. The dependence on the parameters involved is exactly the same as in the case of Debye model. For $T = 373$ K in the ^{176}Lu experiment and $n_{eff}(Lu) = 2.2 \pm 1.2$, and $a = a_0 = .52 \times 10^{-10}$ m (Bohr radius of hydrogen as estimate for atomic radius), one has $E_e = 28.0$ keV to be compared with $U_e = 21 \pm 6$ keV of [86] ($a = 10^{-10}$m corresponds to 1.24×10^4 eV and 1 K to 10^{-4} eV). A slightly larger atomic radius allows to achieve consistency. The value of $\hbar$ does not play any role in this model since the considerations are purely classical.

An interesting question is what the model says about the decay rates of nuclei in conductors. For instance, if the proton from the decaying nucleus can enter directly to the space-time sheet of the conduction electrons, the Coulomb wall corresponds to the Coulomb interaction energy of proton with conduction electrons at atomic radius and is equal to $\alpha n_{eff}/a$ so that the decay rate should be enhanced.

5 Appendix

5.1 About inclusions of hyper-finite factors of type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [71]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [71] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [71]. For any amenable group G the the inclusion is also unique apart from outer automorphism [70].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [71]. The construction of Jones leads to a atandard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M} .... \otimes_{\mathcal{N}} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [71].

1. Jones inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n)|n \geq 3\} \cap [4, \infty)$ [71]. They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i-j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [71]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$ define orbifold coverings of $H_\pm = M^4_\pm \times CP_2 \to H_\pm / G_a \times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [71] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2, C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2, C)$ is effectively replaced with the orbifold $M(2, C)/G$, with G acting as automoprhisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \to H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

5.2 Generalization from $SU(2)$ to arbitrary compact group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [72] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \ . \tag{24}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the imbedding space in such a manner that only cyclic group Z_n appears in the covering of $M^4 \to M^4/G_a$ or $CP_2 \to CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the imbedding space. In the case of SU(2) the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi

statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the imbedding space level so that Z_3 statistics would be in question.

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