Does TGD Allow Quantum Field Theory Limits?

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Contents

Abstract

This chapter contains besides genuinely new material also old material which has become more or less obsolete with the advances occurred in the construction of quantum TGD described in the chapters "Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD" and "Construction of Quantum Theory". The conclusions from this work are following.

a) Quantum TGD can be formulated as a quantum field theory using modified Dirac action. This formulation is formally a free field theory for fermions and thus free of divergence difficulties. It satisfies the quantum gravitational holography principle and relies heavily on super-canonical and super Kac-Moody super algebras associated with light like 7-D and 3-D causal determinants which allow a promising matter to understand the classical non-determinism of Kähler action.

b) A completely new element is 7–3 duality and closely related effective 2-dimensionality, basically due to the conformal invariance related to the light like 3-D causal determinants. Effective 2-dimensionality leads to Feynman rules differing dramatically from those of perturbative quantum field theories. The particles on the internal lines are not labelled by four-momenta, and can be said to be on mass shell in the sense that the spinors in the propagator lines correspond to generalized eigen modes of the modified Dirac operator whereas the solutions of the modified Dirac equation represent $N = 4$ gauge super symmetries. Vertices are associated with 7-D CDs. Loop summations are absent and diagrams with loops are equivalent with tree diagrams. Also the finiteness of the S-matrix is manifest. Coupling constant evolution is replaced by a discrete coupling constant evolution induced by the infinite hierarchy of p-adic length scales each giving rise to a value of Kähler coupling strength analogous to a critical temperature.

The beauty and elegance of this approach makes the whole idea of perturbative QFT limit more or less obsolete, and it would seem that the notion of field theory limit should be replaced with a truncation of the full theory by posing restrictions on the energies of the interacting particles. p-Adic length scale hierarchy defines in a very natural manner this kind of truncation hierarchy.

The construction of the various limits of TGD should be based on following observations.

a) One can construct a whole family of limits. Both real and p-adic limits are possible and latter might provide an extremely simple and effective calculational tool. The classification of the particles according to the dimension D of the CP_2 projection of the particle like 3-surface implies that there are several limits. The most interesting $D = 4$ case corresponds to the identification of CP_2 type extremals as fundamental particles, whereas $D < 4$ extremals having at least one large spatial

dimension, most probably serve as templates for topological condensation of CP_2 type extremals.

b) The limit of TGD should involve the effective elimination of configuration space degrees of freedom and their description phenomenologically. In particular, quark color is treated as spin like quantum numbers. Various coupling constants and propagators are inputs of limit and are predicted by the S-matrix constructed using the new Feynman rules. The 2-D surfaces X^2 identified as the intersections of light like 3-D and 7-D causal determinants are TGD counterparts of partons, and by the effective 2-dimensionality code all relevant data about quantum TGD. These surfaces are idealized as point like objects: partons become quarks and gluons. What is lost in this process is the possibility to describe bound states of partons from the first principles: QCD provides a basic example about this.

c) The only elegant manner to define the limits of TGD are as theories in Minkowski space. The information about the geometry of space-time surface is included implicitly in the process giving the coupling constants.

d) The notion of propagator in M^4 makes sense as an approximate concept if S-matrix elements can be expressed as tree diagrams using Feynman rules of effective field theory in M^4. The construction of the scalar propagator as a partition function in super-canonical algebra suggests that the propagator might have a universal form.

For completeness also the older approach based on the Yang-Mills Dirac (YMD) action for induced gauge fields modified by adding a quantum part and defined for maximum of Kähler function (absolute minimum of Kähler action) is discussed. This involves a generalization of the induction procedure so that it applies also the quantum fields. Propagators are put in by hand and propagator poles correspond to the masses of the particles predicted by the p-adic mass calculations. The only sensible interpretation of YMD action is as effective action giving rise only to tree diagrams: this of course conforms with the new view about Feynman diagrams. In principle Poincare invariance produces no problems. The description of family replication phenomenon and quark color require somewhat tricky constructions.

1 Introduction

This chapter represents old material part of which has become more or less obsolete with the advances occurred in the construction of quantum TGD described in the chapters [B4, C2, C5]. The conclusion from this work is that quantum TGD can be formulated in terms of effectively 2-dimensional conformal field theory using modified Dirac action. This formulation is formally a free field theory for fermions and thus free of divergence difficulties. It satisfies the quantum gravitational holography principle and relies heavily on super-canonical and super Kac-Moody super algebras.

A completely new element is effective 2-dimensionality which leads to Feynman diagrams differing dramatically from those of perturbative quantum field theories. The particles on the internal lines are not labelled by four-momenta and can be said to be on mass shell in the sense that the spinors in propagator lines correspond to generalized eigen modes of the modified Dirac operator. Therefore loop summations are absent and diagrams with loops are equivalent with tree diagrams. Also the finiteness of the S-matrix is manifest.

The beauty and elegance of this approach makes the whole idea of perturbative QFT limit more or less obsolete, and it would seem that the notion of field theory limit should be replaced with a truncation of the full theory by posing restrictions on the energies of the interacting particles. p-Adic length scale hierarchy defines in a very natural manner this kind of truncation hierarchy. Despite this I feel it reasonable to consider also the earlier ideas related to QFT limit besides the discussion of what field theory limit could mean and in what sense it might exist.

1.1 What kind of limits of TGD one can consider?

There is a large variety of limits of TGD that one can consider. It is not however at all obvious that this limits deserve the attribute 'QFT'.

a) One can consider ultrahigh energy QFT limit in which all states of super-conformal representations are treated as point like particles and maxima of Kähler action for fixed values of zero modes are included. This means that the 2-surfaces X^2 representing intersections of 3-D and 7-D light like causal determinants (CDs) are approximated as points. Basic dynamical variables are fermionic oscillator operators generating Super-Kac-Moody algebra and super-canonical algebras.

b) Various limits are obtained by including only the massless states which remain light in p-adic massivation in given mass scale. p-Adic thermodynamics and p-adic Higgs mechanism are needed to derive the values of particle masses which are regarded as fixed parameters in the construction of the limit. The full theory is needed to predict the values of coupling constants and relatively satisfactory picture about how the coupling constants are determined already exists: also the model of S-matrix based on CP_2 type extremals could be used in order to derive the values of various coupling constants [C2].

c) One must distinguish between real and various p-adic limits of TGD. p-Adic limits of TGD could be seen as a model for cognitive physics and thus models for models of real physics. The successes of the p-adic mass calculations suggest that p-adic limits of TGD could provide simplified description of the full theory.

d) The limits of TGD also depend on which kind of objects are taken as point-like particles: the number $0 \leq D \leq 4$ of the compactified dimensions classifies various limits. $D = 4$ corresponds to the simplest limit in which CP_2 type extremals are the basic objects. This is certainly the most interesting limit. Although the mass spectrum is universal, these objects seem to correspond to different particle species. All $D < 4$ objects have at least one large space-like dimension, and it is quite possible that they serve as templates for the topological condensation of CP_2 type extremals so that the primary degrees of freedom are masked. For instance, massless extremals are $D = 0$ objects and expected to carry Bose-Einstein condensates of photons and generate coherent light. The construction $D < 4$ limits involves the construction of the fundamental S-matrix in order to fix the basic parameters: for instance, for cosmic strings ($D = 2$) one expects string model type S-matrix to be an excellent approximation. In principle also the transformations of particles with different values of D to each other are possible and a quantum model for these transformations is needed. In the following the considerations are restricted to $D = 4$ case.

For a long time the reason for why the dimension of CP_2 projection emerges in the characterization of the limit remained more or less a mystery. Much later the construction of extremals of Kähler action demonstrated that the dimension of CP_2 projection characterizes the asymptotic behavior of space-time sheets representing self-organization patterns having vanishing Lorentz Kähler 4-force and selected by dissipation. The variational principle selecting preferred extremals of Kähler action as analogs of Bohr orbits could realize the second law of thermodynamics at the space-time level. This is indeed possible by the non-determinism of Kähler action. What is new is that also geometric time reversals of these patterns are possible and are realized for space-time sheets with negative time orientation. Phase conjugate photons and more generally, phase conjugates of any elementary particles, correspond to this geometric time reversal.

1.2 Should the limits of TGD be defined in M^4 or X^4?

The reduction of generalized Feynman diagrams to tree diagrams and the absence of virtual particles and loop summations makes the Feynman rules of TGD quite different from those of standard quantum field theories. For the same reason the generalized Feynman rules are to some extend analogous to those usually assigned with so called effective action. Running coupling constants are not possible nor needed since discrete p-adic coupling constant evolution mimics the coupling constant flow. The basic implication is that any field theory limit must be based on effective action so that one can allow also non-renormalizability.

The values of the coupling constants can be understood only when configuration space degrees of freedom are taken into account. For instance, the construction of $BF\overline{F}$ vertex demonstrates this when B is spin one boson [C2]. The implication is that coupling constants as functions of p-adic length scale must be taken as predictions of fundamental quantum TGD.

Making the questionable assumption that TGD allows some kind of effective field theory as a low energy limit, 7–3 duality provides more or less unique answer to the question whether the limit should be defined in Minkowski space or at space-time surface. By 7–3 duality the construction of S-matrix leads to construction of S-matrix in terms of generalized diagrams and this procedure defines also n-points functions in M^4. The most elegant formulation for effective field theory is in M^4 and would simply result by posing the appropriate constraints on particles appearing as incoming and outgoing states. This approach is attractive also from the point of view of symmetries.

One might also criticize this approach since one loses totally the concept of the classical space-time crucial for various applications of the TGD based space-time concept. For instance, it seems difficult to understand the interaction with classical fields. The defence against this criticism is that the generalized Feynman diagrams carry all relevant data about classical fields but this data is crunched invisible at the level of n-point functions and is visible via general properties like the values of coupling constants depending on the p-adic length scale of space-time sheet.

This does not exclude approximate effective field theories at space-time surfaces defined by a general coordinate invariant action based on induced gauge fields and metric so that the interaction with classical fields becomes part of the theory. It deserves to be noticed that the notion of classical field differs from that in usual gauge theories since one must distinguish between order parameters of coherent states and genuine classical fields induced from the geometry of the imbedding space. In TGD framework Einstein's equations are more like equations of state for the gravitational mass density [D5] so that Einstein-Hilbert action cannot be regarded as a part of effective action of quantum field theory. This action contains also second derivatives of imbedding space coordinates. Indeed, YM action for induced gauge fields plus quantum parts would be enough to define graviton emission vertices. Propagators and particle masses must be put in by hand. Unfortunately, this kind of theory is rather awkward due to the artificial realization of Poincare invariance. The further difficulty is that for $X^4 = M^4$ the YM action treating quantum fields as perturbations of classical fields vanishes identically and the theory is trivial in the simplest possible situation that one can consider.

1.3 How to treat classical and p-adic non-determinisms in QFT limit?

The classical non-determinism of the Kähler action implies that the number of the real degenerate absolute minima associated with a given space-like 3-surface can be very large, in fact infinite as in the case of CP_2 type extremals. This degeneracy gives rise to spin-glass type behavior already in case of CP_2 type extremals in the sense that one must calculate averages over the scattering rates predicted by QFT:s with varying basic coupling parameters and propagators.

In p-adic context p-adic pseudo constants imply an additional cognitive degeneracy which should *not* be confused with the classical non-determinism. Contrary to the original expectations, the ultra-metricities related to the spin-glass behavior and p-adicity are in principle of different origin. p-Adic degeneracy is due to the possibility to choose the parameters labelling different branches of the roots of the quaternionic polynomials to be p-adic pseudo constants so that the space-time surface is a union of pieces associated with various roots.

The obvious question is how should one treat the real degeneracy.

a) Is real degeneracy of the 4-surface associated with virtual particle an additional degeneracy enhancing the propagator in resonant manner? This would mean superposition of transition amplitudes and a sum analogous to path integral would result. This option gives rise to divergent amplitudes and must be given up. Also the generalization of the notion of classical determinism to allow 3-surfaces which are unions of space-like 3-surfaces with time like separations with the property that they fix the solution of field equations uniquely, excludes this option.

b) Should one to assign zero modes a normalized wave functional? This is in principle possible and leads to quantum spin glass structure. One obtains well defined transition amplitudes. It seems that one must apply this procedure if one is interested on how the transition probabilities depend on zero modes.

c) The third, and the simplest, possibility is that localization occurs in the zero modes in each quantum jump and means that quantum measurements are local at the level of the zero modes sector of the configuration space. The localization in zero modes means that one must select single space-time-surface to calculate QFT limit. This non-uniqueness is due to the spin-glass nature of TGD Universe implying that the laws of physics are to some extend a result of a generalized Darwinian selection. A statistical averaging over the scattering rates yielded by degenerate space-time surfaces might be needed for practical purposes. This kind of averaging is very much like statistical averaging of thermodynamics for spin-glass systems.

d) Path integral formalism suggests that the sum over Feynman graphs with lines replaced with CP_2 type extremals could define the QFT limit. The resulting Feynman diagrammatics would be very similar to string diagrammatics. What is especially nice is that that the random zitterbewegung motion with light velocity makes CP_2 type extremal massive and makes thus possible, not only massless states, but also massive on-mass-shell states as well as virtual off-mass-shell states. On mass-shell mass spectrum dictated by conformal invariance. There would be a summation over all M^4_+ positions of the vertices associated with the Feynman diagrams.

However, since zitterbewegung degrees of freedom represent zero modes, one might argue that the localization occurs in these degrees of freedom. This would be in accordance with spin glass nature of TGD Universe in classical sense and with the conformal invariance associated with the zitterbewegung. 7–3 duality implying effective 2-dimensionality forces the same interpretation since the space-time surfaces for which the intersections X^2 of light like 3-D CDs with 7-D CDs are same (including tangent spaces at X^2) are equivalent with respect to the metric of configuration space. It is not quite clear whether one should regard these degrees of freedom as macroscopic zero mode degrees of freedom or as gauge degrees of freedom. One implication is is that in the case of the high energy limit defined by CP_2 type extremals there is no need to sum over the random light like orbits of CP_2 type extremals.

It turns out that the exponent of Kähler action, which is essentially the exponent of the volume of the CP_2 type extremal appearing as a propagator line, is sensitive to the selection of the arbitrary function u of CP_2 coordinates associated with the light like zitterbewegung orbits $m^k = f^k(u)$, and that this forces averaging over scattering rates.

The development of the vacuum expectation value of Higgs field could correspond in TGD framework to the generation of non-vanishing charge

Q_J in the complement of $u(2)$ subalgebra in $su(3)$ algebra defined by the conserved charges associated with the variation of the modified Dirac action with respect to the induced Kähler form. An interesting possibility is that the non-vanishing of these non-diagonal charges correlates directly with zitterbewegung.

It turns out that the inertial four-momentum could be identified as the average of the non-conserved gravitational four-momentum of a topologically condensed CP_2 type extremal [?]. Higgs bosons are in turn identifiable as a wormhole contacts with the property that the two light-like causal determinants associated with it carry the quantum numbers of right (left) handed fermion and left (right) handed antifermion. This picture is consistent with p-adic thermodynamics description of particle massivation requiring also the contribution of Higgs bosons besides purely thermodynamical contribution assignable to the primary topological condensation.

1.4 Localization in zero modes

The occurrence of localization in all zero modes except in center of mass degrees of freedom and modular degrees of freedom means an enormous simplification of the theory and makes the notion of QFT limit sensible. All QFT limits treat space-time as an arena of dynamics although averaging of the scattering rates might be involved and would reflect spin-glass behavior. These space-time surfaces belong to the reduced configuration space CH_{red}, whose points correspond to the interacting 4-surfaces $X^4(\cup_i Y_i^3)$- Spin glass analogy means that scattering rates must be averaged over a probability distribution in CH_{red}.

a) As a consequence of the approximate canonical invariance spoiled only by the classical gravitation, one expects several maxima of the Kähler function associated with given values of the zero modes. Generalization of the notion of classical determinism implies that quantum jump involves selection among these maxima so that degeneracy in fiber degrees of freedom is effectively removed. Macroscopic quantum jumps (say those associated with volitional act) might naturally correspond to the selection of this kind of maximum. This means the effective elimination of the configuration space integral in the fiber degrees of freedom in which the configuration space metric is nontrivial by replacing the functional integral by a perturbative expansion around a maximum of the Kähler function with respect to the fiber degrees of freedom.

b) The construction of the S-matrix suggests that localization in conformal degrees of freedom occurs in the case of CP_2 type extremals, and means that one particular zitterbewegung orbit for which the quantized inertial four-momentum for the modified Dirac equation identified as gravitational four-momentum is in the average direction of cm motion, is selected. This is not in conflict with the conformal invariance and implies quantum counterpart of the spin glass property in the sense that averaging over reaction rates associated with the S-matrices labelled by an arbitrary function of CP_2 coordinates appearing in the definition of CP_2 type extremal is needed.

c) The modular degrees of freedom associated with boundary components of particle like 3-surfaces do not correspond to zero modes. p-Adic mass calculations rely strongly on the notion of elementary particle vacuum functional. Since localization does not occur, one can phenomenologically describe elementary particle vacuum functionals as spin like degrees of freedom in the point like limit of the theory.

There are two possibilities according to whether one identifies the modular contribution to the mass squared as a thermodynamical average or as a quantum mechanical contribution to the vacuum weight of Super Virasoro representation. The calculation of the elementary particle and hadron masses forces to assume that the origin is quantum mechanical and that modular degrees of freedom contribute Dirac operator type term to super generators G: unfortunately this operator is not known yet.

d) In p-adic context there is an additional degeneracy due to p-adic non-determinism but also this degeneracy is removed by similar argument meaning that also imagined worlds behave classically.

e) Fiber degrees of freedom, in particular color, must be described as spin like degrees of freedom at the QFT limit. Kac-Moody algebra and center of mass motion in CP_2 degrees of freedom correspond to separate color degrees of freedom but at the QFT limit the microscopic color structure is not visible.

1.5 Connection between Fock space and topological descriptions of the many particle states

The connection between Fock space- and topological descriptions of particle states and particle reactions is fundamental for the interpretation of the theory. The fact that Feynman diagrams with lines thickened to four-manifolds provide the topological counterpart of of QFT Feynman diagrams in TGD, allows to understand the connection between Fock states and particle like 3-surfaces. QFT interaction vertices are only a phenomenological manner to describe underlying topological vertices for an S-matrix defined by free Hamiltonian associated with the modified Dirac action. One beautiful prediction is the absence of the divergences coming from interaction vertices.

7–3 duality implying effective 2-dimensionality leads to a very concrete picture where partons correspond to the 2-dimensional intersections of 3-D light like CDs and 7-D CDs and particles to 3-surfaces. Partons can carry arbitrarily high fermion and anti-fermion numbers but in practice only the states with fermion numbers zero and one are expected to be stable against rapid decay. The formation of bound states has as space-time correlate formation of join along boundaries bonds so that the outer boundaries of 3-surfaces fuse together. These aspects of quantum TGD have no QFT counterpart so that there is no hope of QFT based description of bound state formation nor simultaneous description of the parton and particle aspects of hadrons.

2 About the low energy limit of TGD defined in M^4

It has become clear that the connection of TGD with quantum field theories and the possible QFT limit of TGD is probably not what naive expectations first suggested. The obvious question is whether any QFT counterpart for TGD exists as a low energy limit: the answer to this question seems to be negative for reasons which should be clear from previous chapters. For instance, the equivalence of generalized loop diagrams with tree diagrams should correspond to the vanishing of loops in the possibly existing QFT limit of TGD and no QFT with the required symmetries seems to exist. The second key question is how to understand the relationship between TGD and QFT.

2.1 Is QFT limit possible at all?

One can invent several arguments in favor that the low energy limit of TGD is very probably not any quantum field theory of a standard kind. For instance, $N = 4$ super conformal gauge symmetry for leptons and quarks does not give rise to $N = 1$ global super-symmetry, not even broken one.

The dependence of S-matrix on background classical fields is predicted but the dependence seems to be much more delicate than one might have expected in QFT framework. Indeed, the micro-locality of QFT limit defined by Eistein Yang-Mills action with induced gauge fields and metric replaced by their quantum corrected versions would lead to horrible divergences and the failure of Poincare and color symmetries. The interpretation of YM action as an effective action giving only tree diagrams is of course possible but not very attractive. In this case the quantum part of the induced gauge field could be however interpreted as a gauge field in H and allow to achieve Poincare invariance. Particle propagators would be identified as free propagators in H and particle masses would be but in as ad hoc parameters.

Second point is that the low energy limit of the space-time S-matrix is not quite enough to give a satisfactory approximation to the real physics. The description of color degrees of freedom necessitates configuration color partial waves and thus configuration space level. This applies also to the Poincare quantum numbers. Besides color rotational and Lorentz degrees of freedom for space-time sheets, also translational degrees of freedom for $X^7_\pm$ must be allowed: the positions of the dips of $X^7_\pm$ would correspond to the arguments of Green's functions of quantum field theory.

Modular degrees of freedom, in particular the size, of X^2_i might be also necessary and indeed suggested by the structure of N-point functions for composite particles of a free quantum field theory [19]. In TGD framework these composites would correspond to X^2_i predicted to have a composite structure in the sense that configuration space super algebras assign to them many-particle state space. The inclusion of only light particles and the description of color and modular degrees of freedom as spin like degrees of freedom would lead to a theory which would have the closest possible resemblance with quantum field theory.

The most one can hope seems to be following. The approximate decomposition of the S-matrix elements to propagators and vertices as functions of incoming and outgoing 4-momenta and other quantum numbers and the assignment of interacting fields with the propagator poles could give rise to an approximate description of the theory as a quantum field theory using effective action. As demonstrated in the earlier chapters, "anyonic shydrodynamics" at light like 3-D CDs predicts the masses of the particles appearing as complex poles of S-matrix. The mimicry of the p-adic coupling constant evolution using full Feynman diagrammatics allowing loops and regularization, does not seem plausible.

2.2 How could one understand the relationship between TGD and quantum field theories?

Concerning the relation between TGD and QFT approach, a hint comes from the fact that the Feynman diagrams of quantum field theories quite generally can be classified by the minimal genus g of the Riemann surface at which they are imbeddable without crossings. The ends of the incoming and outgoing lines obviously correspond to punctures of these Riemann surfaces.

In [19] N-point functions of local composites of free fields are studied. It is demonstrated that the Feynman diagrams of genus g can be expressed as integrals of amplitudes over the moduli space associated with a Riemann surface of genus g and possessing n holes. The argument involves the representation of propagators using Schwinger parameters (proper time τ along the propagator line) and combination of all diagrams with a common skeleton to single diagram: this means that propagator lines connecting two vertices and continuously deformable to each other are lumped together. The counting of the non-redundant Schwinger parameters leads to the identification of these parameters as $D = 6g-6+3n$ moduli associated with a sphere with g handles and n holes. The sizes of the holes define an n-dimensional space R_+^n factoring out and leaving the moduli space of punctures with dimension $D = 6g-6+2n$ (for $(g = 0, n \geq 3)$ the moduli space of punctures is trivial since 3 points of sphere can be mapped to any other 3 points by a Möbius transformation). What remains open is whether the a amplitudes depending on the moduli could be expressed in terms of correlation functions of some conformal field theory perhaps related to string model. This leads the conjecture of [19] that some kind of a closed string model allows to re-organize the perturbative expansion of also interacting quantum field theories. Irrespective of whether this is the case, the finding suggests ideas about how the low energy limit of TGD relates to the existing quantum field theories.

It is interesting to translate this result to TGD framework. 7–3 duality implies effective two-dimensionality and hence the generalized tree diagrams are all that is needed to compute the S-matrix. Loops in the ordinary sense should thus vanish. Tree diagrams have $g = 0$ and are imbeddable to plane or sphere. The counterpart for the Riemann surface would be δM^4_+. The generalized Feynman diagram connecting incoming particles at $X^7_+ = \delta M^4_+ \times CP_2$ and outgoing particles at $X^7_- = \delta M^4_- \times CP_2$ would be mapped to X^7_+ via the identification of X^7_+ and X^7_- mapping the particles to X^7_+ projected further down to δM^4_+. The lines of the generalized Feynman diagram would be mapped to a tree graph connecting the incoming and outgoing particles at X^7_+.

That this surface is metrically a 2-D sphere, is consistent with the prediction of effective 2-dimensionality implying that loop corrections vanish. A good guess is that the presence of light like dimension is essential in making it possible to have non-trivial Feynman graph expansion with vanishing loops.

In TGD context punctures defined by the incoming and outgoing particles would correspond to the surfaces X^2 resulting as intersections of the light like 3-D CDs and 7-D $\delta M^4_\pm \times CP_2$. The projections of X^2_i define 2-D light like surfaces in δM^4_+ so that also the moduli characterizing the sizes of the holes are present. When X^2 is "small", its size preserving projection along a suitably chosen light ray to $r_M = constant$ sphere with a large radius is a puncture whose angular size can be arbitrarily small unless X^2 is homologically non-trivial and surrounds the dip of the light cone. The moduli space of punctures is $(n-3)2$-dimensional. 3 punctures necessary for having a non-trivial braid are needed.

The value of the Kähler coupling strength α_K would dictate the values of various coupling constants. The low energy limits associated with various p-adic length scales are able to produce an effective coupling constant evolution since α_K depends on p-adic length scale defining the length scale resolution, and possibly also on phase resolution. In p-adic context the phase resolution corresponds to an algebraic extension allowing to express the phase factors $exp(i\pi/n)$ as p-adic numbers in the extension. Note that the angles defined by the quantization of angular momentum via $cos(\phi) = m/\sqrt{j(j+1)}$ requires only quadratic extension.

The latter coupling constant evolution would mean the proportionality of $1/\hbar$, and thus of $\alpha_K = g_K^2/4\pi\hbar$, on the ratio $log(B_n)/log(B(\infty)$, where $B_n = 4cos^2(\pi/n)$, $n \geq 3$, are so called Beraha numbers appearing in braid and knot theories and quantum group theory. $1/\hbar$ would be thus dynamical, and the rule would be that when the charges of interacting system make coupling so large that perturbative approach fails, the value of $1/\hbar$ is reduced [D6]. The extreme situation corresponds to $n = 3$ allowing the resolution of only three angles coming as multiples of $2\pi/3$.

The recent findings [18, D6] that gravitational physics of astrophysical systems could be understood using Schrödinger equation with a gigantic value of $\hbar$ for the evolution of dark matter, could be understood if gravitational interaction between gravitational masses larger than Planck mass induces a perturbative correction $\Delta(1/\hbar) = v_0/GMm$ to the value of $1/\hbar(n)$ which is extremely small for $n > 3$ but in the case of $1/\hbar(3) = 0$ determines $1/\hbar$ completely. $v_0 \simeq 10^{-4}$ is a small parameter proportional to the ratio of Planck length and CP_2 length.

3 Construction of S-matrix at high energy limit

The construction of S-matrix for CP_2 extremals can be formulated rather precisely and one can even speak about well defined Feynman rules. The

QFT limit associated with this S-matrix certainly represents the most interesting QFT limit of the theory.

3.1 S-matrix at short length scale limit

The approximate construction of S-matrix at the length scales, where particles propagate as CP_2 type extremals is discussed in the chapter [C2]. This construction serves as the basic framework for the understanding of the QFT limit.

3.2 Basic properties of CP_2 type extremals

CP_2 type extremal has the following explicit representation

$$m^k = f^k(u(s^k)) \ , \quad m_{kl}\frac{df^k}{du}\frac{df^l}{du} = 0 \ . \tag{1}$$

The function $u(s^k)$ is an arbitrary function of CP_2 coordinates and serves effectively as a time parameter in CP_2 defining a slicing of CP_2 to time=constant sections. The functions f^k are arbitrary apart from the restriction coming from the light likeness. When one expands the functions f^k to Fourier series with respect to the parameter u, light likeness conditions reduce to classical Virasoro conditions $L_n = 0$.

It is possible to write the expression for m^k in a physically more transparent form by separating the center of mass motion and by introducing p-adic length scale L_p as a normalization factor.

$$\frac{m^k}{L_p} = p^k u + \sum_n \frac{1}{\sqrt{n}} a_n^k exp(i2\pi nu) + c.c. \ . \tag{2}$$

The first term corresponds to the center of mass term responsible for rectilinear motion along geodesic line and second term corresponds to the zitterbewegung motion. p^k serves as effective classical momentum: what however has significance is whether p^k is time like, light like, or space-like. Conformal invariance corresponds to the freedom to replace u with a new 'time parameter' $f(u)$.

The physically most natural representation of u is as a function $f(U)$ of the fractional volume U for a 4-dimensional sub-manifold of CP_2 spanned by the 3-surfaces $X^3(U=0)$ and $X^3(U)$:

$$u = f(U) \ , \quad U = \frac{V(s^k)}{V(CP_2)} = \frac{S_K(u)}{S_K(CP_2)} \ . \tag{3}$$

The range of the values for U is bounded from above: $U \leq V_{max}/V(CP_2)$ and the value $U = 1$ is possible only if CP_2 type extremal begins and ends as a point. U represents also Kähler action using the value of the Kähler action for CP_2 as a unit.

The requirement that CP_2 type extremal extends over an infinite time and spatial scale implies the requirement

$$f(U_{max}) = \infty \ . \tag{4}$$

For $f(U_{max}) < \infty$ CP_2 type extremal can exist only in a finite temporal and spatial interval for finite values of 'momentum ' components p^k. This suggest a precise geometric distinction between real and virtual particles: virtual particles correspond to the functions $f(U_{max}) < \infty$ in contrast to the incoming and outgoing particles for which one has $f(U_{max}) = \infty$. This hypothesis, although it looks like an ad hoc assumption, is at least worth of studying.

The mere requirement that virtual CP_2 type extremal extends over a temporal or spatial distance of order $L > L_p$ implies that for $L < L_p$ the value of U is smaller than one. Kähler action, which is given by

$$S_K(X^4) = U \times S_K(CP_2) \ , \tag{5}$$

remains small for distances much smaller than L. For $f(U_{max}) = \infty$ this is even more true. This has an important implication: below a certain length scale the exponential of the Kähler action associated with the internal line of a Feynman diagram does not give rise to a suppression factor whereas above some characteristic length L and time scale there is an exponential suppression of the propagator by the factor $exp(-S_K(CP_2))$ practically hindering the propagation over distances larger than this length scale.

The presence of the exponential obviously introduces an effective infrared cutoff: this cutoff is prediction of the fundamental theory rather than ad hoc input as in quantum field theories. Of course, infrared cutoff results also from the condition $f(U_{max}) < \infty$. Physically the infrared cutoff results from the topological condensation of the CP_2 type extremals to larger space-time sheets. These could correspond to massless extremals (MEs). p-Adic length scale L_p is an excellent candidate for the cutoff length scale in the directions transversal to ME.

The suppression factor coming from the exponent of the Kähler action implies a distance dependent renormalization of the propagators. In the long length scale limit the suppression factor approaches to a constant value

$$exp\left[-\frac{V_{max}}{V(CP_2)}S_K(CP_2)\right] \ ,$$

and can be absorbed to the coupling constant so that the dependence on the maximal length of the internal lines can be interpreted as an effective coupling constant evolution. For instance, the smallness of the gravitational constant could be understood as follows. Since gravitons propagate over macroscopic distances, the virtual CP_2 type extremals develops a full Kähler action and there is huge suppression factor reducing the value of the gravitational coupling to its observed value: at short length scales the values of the gravitational coupling approaches to $G_{short} = L_p^2$ which means strong gravitation for momentum transfers $Q^2 > 1/L_p^2$. The values of V_{max} and thus those of the suppression factor can vary: only at the limit when CP_2 extremal has point-like contact with the lines it joins together, one has $V_{max} = V(CP_2)$. If the boundary component characterizing elementary particle family belongs to CP_2 type extremal (it could be associated with a larger space-time sheet), CP_2 type extremal contains a hole: also this reduces the maximal volume of the CP_2 extremal.

3.3 Feynman diagrams with lines thickened to CP_2 type extremals

CP_2 type extremals are just what the on-mass-shell and off-mass shell particles of string models are expected to be.

a) The variation of the modified Dirac operator with respect to the imbedding space coordinates implies Euler-Lagrange equations for the Kähler action and this in turn implies that massless Dirac equation is satisfied. Quaternion- analyticity allows to write the solutions of the modified Dirac equation explicitly and the requirement that the supercharges associated with $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$ generate super-Kac-Moody algebra, fixes the anti-commutation relations of the fermionic super charges which come in two varieties corresponding to the supercharges associated with the conserved fermion numbers and isometry charges. The super-Kac-Moody algebra in question gives rise to the physical states satisfying Super Virasoro conditions. Mass squared is quantized for the representations of Super Virasoro. There is degeneracy caused by the cm degrees of freedom forcing to introduce plane waves and color partial waves. Note that the degeneracy in CP_2 degrees of freedom is present because CP_2 type extremal is not the entire CP_2. Genus-generation correspondence requires the presence of 3-dimensional boundary either inside CP_2 type extremal or on the space-time sheet at which CP_2 type extremal is condensed at.

b) Kähler action results as a c-number term from the normal ordering of the modified Dirac action and appears in the exponent of the modified Dirac action defining the vacuum functional of the theory. The exponent of the Kähler action for a piece of CP_2 type extremal defined by the line of the Feynman diagram appears as a factor in each internal line of the Feynman diagram.

c) For CP_2 type extremals the spectrum of the conserved momenta is continuous. The reason is that the random motion with light velocity can be regarded as a superposition of classical random zitterbegung motion and an average motion along time or space-like geodesic line. This means that mass squared operator associated with the CP_2 type extremal is continuous and CP_2 type extremals can represent virtual particles appearing in the internal lines of Feynman diagrams.

d) The condition that the orbit is light like random curve reduces to classical Virasoro conditions and the mass squared of the particle corresponds classical to the 'momentum ' squared associated with the zitterbewegung motion.

3.4 Feynman rules

The heuristic view about Feynman rules (there are certainly delicacies involved not taken into account in this simplistic discussion) is following.

a) There is a sum over all possible Feynman graphs with CP_2 type extremals appearing as lines. This means integration over the positions of the vertices characterized by points of $M^4_+ \times CP_2$ corresponding to cm degrees of freedom. One must assign to each external particle a plane wave in M^4_+ degrees of freedom and color partial wave in CP_2 center of mass degrees of freedom.

b) To each 3-vertex of the Feynman graph one assigns a Glebsch-Gordan coefficient $V(a,b,c)$ for the tensor product of the incoming super-Kac-Moody representations besides the factor taking care of the conservation of quantum numbers, in particular four-momentum and color and electro-weak quantum numbers.

The lines of the Feynman diagram contain two factors: the exponent of Kähler action and translation operator along line.

a) The time development operator of wave mechanics is replaced with the unitary translation operator along the line connecting the two vertices P_1 and P_2. Translation operator is expressible as the exponent of the conserved four-momentum associated with the modified Dirac operator. The momentum operator is in the direction of the propagator line automatically. By using an eigen state basis of four-momenta, translation operator along the line connecting P_1 and P_2 can be expressed as

$$U(P_1,P_2) = exp(iP_k\Delta m^k) \ , \quad \Delta m^k = m_2^k - m_1^k \ . \tag{6}$$

Rather remarkably, the contribution of the time development operator in the dynamics trivializes totally and there is no need to construct explicit representation of the momentum generators.

b) In order to get the propagator pole correctly it is necessary to assign with the propagator line the factor

$$I = \frac{1}{L_0 + i\epsilon} \ ,$$

where L_0 is the representation of the Virasoro generator representing scaling in the Super-Kac-Moody algebra defined by $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. In string models the propagator factor follows from the Hamiltonian time development operator defined by L_0. In present case propagator-factor should result from the vertex operators. The vertices at the ends of the Ramond type propagator line should be proportional to $1/G_0$ resp. $1/G_0^\dagger$. When the internal line corresponds to N-S-representation, the vertices at the ends of the propagator line should be proportional to the inverses of the super-generator $G^{\pm 1/2}$ resp. $G^{\mp 1/2\dagger}$.

c) Internal lines contain also an exponential suppression factor $f(V)$ given by the exponent of the Kähler function for the piece of CP_2 type external defined by the line. This factor is given by

$$F(V) \ = \ exp(-S_K(X^4)) = exp\left[-\frac{V}{V(CP_2)}S_K(CP_2)\right] \ . \tag{7}$$

X^4 is the four-dimensional sub-manifold of CP_2 having as its boundaries the 3-surfaces $X^3(U=0)$ and $X^3(U = V/V_{CP_2})$. The latter form follows from the fact that Kähler action density is constant for CP_2 type extremals so that Kähler action is proportional to the volume of X^4. All functions U for which the internal line defines the same CP_2 volume give rise to the same Kähler action. In accordance with the conformal invariance, there is no explicit dependence on the zitterbewegung orbit.

The presence of the plane wave factors implies that the integration over the vertex positions multiplies the stringy propagator $1/(L_0 + i\epsilon)$ with an infrared suppression factor given by the Fourier transform of $F(V)$ which on basis of Lorentz invariance is only a function of invariant line length of M^4_+ (V and invariant line-length are alternative parameters for the internal line).

Scattering amplitude is obviously very sensitive to this factor and since the suppression factor determines the momentum dependence of the propagators, one can say that the laws of physics depend on the distribution for the functions $u(s^k)$ sensitively. This distribution is in turn constrained by the requirement that CP_2 type extremals have suffered topological condensation on larger space-time surfaces.

3.5 Fundamental coupling constants as Glebsch-Gordan coefficients

The Glebsch-Gordan coefficients associated with the quaternion-conformal super-Kac-Moody algebra $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$ should be determined by a construction analogous to the vertex operator construction encountered in string models. In present case also a dramatically simpler approximative treatment suggests itself.

3.5.1 Vertex operator construction

The construction of the vertex operators could proceed roughly as follows.

a) If one requires that CP_2 type extremals form smooth surfaces one must assume that the vertex regions are deformed so that the vertex represents topological sum of two CP_2 type extremals. This means that vertex region has higher than 1-dimensional M^4_+ projection and is presumably non-vacuum classically. A simple analogy is that of gluing a cylindrical tube to another cylindrical tube smoothly. In principle there are three functions $U = V/V(CP_2)$ involved: denote them by U_i, $i = 1,2,3$. U_2 and U_3 are associated with the outgoing CP_2 type extremals and have value $U_i = 0$ at the vertex.

b) Since only 3-vertices are involved one can visualize the situation as flows associated with two incoming lines combining to single flow along the outgoing line. The CP_2 'time' coordinate $U(s^k)$ serves as the time parameter for the flow. One can continue the flow lines of the incoming flows such that they intersect the outgoing 3-surface $X^3(U = 0, out)$ surface. Thus it seems possible to divide the outgoing 3-surface $X^3(U = 0, out)$ to two parts $X^3_1(out)$ and $X^3_2(out)$ such that flow lines of the flows U_i, $i = 1,2$ from two external legs $X^3_i(U, in)$, $i = 1,2$ enter these regions.

c) This inspires the hypothesis that the fermionic quantum states associated with the two incoming lines are constructible using the oscillator operators constructed from the fermion fields of $X^3(U=0,out)$ restricted to the region $X^3_i(in)$, $i=1,2$. This would allow to express the fermionic state at $X^3(U=0,out)$ using the fermionic oscillator operators associated with the outgoing line and one would obtain a superposition of various states restricted by the conservation of basic quantum numbers.

d) Coupling constants $V(a,b,c)$ are not genuine constants of Nature since they are parameterized by arbitrary functions $U(s^k)$ associated with the incoming and outgoing lines. The dependence on these functions is expected to be very weak. This dependence is present irrespective of whether a complete localization occurs in zero modes or whether wave functionals are possible in zero modes.

The general construction is clearly akin to the construction of vertex operators in string models. For string models the fusion of incoming strings defines splitting of outgoing string to two parts and essentially similar relationship between incoming and outgoing oscillator operators results. In present case the situation is complicated by the fact that the fusing objects are 3-dimensional sub-manifolds of CP_2 rather than strings propagating in some higher-dimensional Minkowski space. On the other hand, the dynamics of the basic objects is almost trivial since CP_2 geometry is not affect at all by the warped imbedding. In any case, the vertex operator is in principle functional of the incoming and outgoing 3-surfaces X^3_i.

3.5.2 Simplified model for the vertices

One can construct a simplified model giving a good idea about what for the vertex operators look like.

a) Idealize the projection of the vertex region to a point in M^4_+ so that the CP_2 type extremals are not deformed in any manner in the vertex region. To get a minimally non-singular surface one must assume that the functions u_i for the CP_2 type extremals define same 3-surface X^3 at the vertex. This means that the conditions $U_1 = constant$ for the incoming line and the conditions $U_2 = 0$ and $U_3 = 0$ for the outgoing lines define same 3-surface. This means that the three 'time-coordinates' U_i have same 3-surface as a common time=constant slice. What this condition means geometrically is that CP_2 type extremal branches: Y-shaped 1-dimensional surface is the homological equivalent of the resulting surface. In fact, the branching means that the situation is effectively 1-dimensional just as it is quantum field theories. Although this surface is singular it might provide a realistic idealization for the construction of vertices.

b) The picture suggests the possibility that, apart from creation or annihilation of fermion pairs, the Fock state representing the incoming particle simply splits into a product of the Fock states associated with the outgoing lines. This assumption is analogous to Zweig rule and would trivialize the vertex construction. If this approximation is sensible, vertices would be simply Fock space inner products between the initial state and the state created by the product of the operators creating final final states. QFT limit suggests that the operators creating the states are analogous to the products of quantum fields $\psi(x)$ at same point x, say $x = 0$. This requires that operators can be constructed as products of the operators which are sums of positive energy creation operators for fermion and negative energy annihilation operator for anti-fermion. This would perhaps make it possible to have nontrivial vertices since annihilation and creation of fermion pairs becomes possible in the vertex provided that the annihilating fermions belong to different lines: this is essentially what Zweig rule states. For 'Zweig option' fermionic statistics implies that same fermionic oscillator operator cannot occur in each line. It is not clear whether the vertices for the emission of graviton can be non-vanishing in this approximation. For photon graviton vertex the total number of oscillator operators involved is just the minimal one to allow graviton emission. If this picture is correct, the effective values of various coupling constants at QFT limit should be determined by the average value of the exponential of the Kähler action associated with the propagator lines representing the particles.

c) Besides the conservation of various quantum numbers 'Zweig rule' suggests the conservation of the vacuum weight h_{vac}. This conservation law could be an excellent approximation quite generally. The conservation of h_{vac} eliminates very large number of the vertices involving exotic particles and gives strong constraints on the vacuum weights of the observed particles. For instance, in the emission of neutral gauge bosons vacuum weight is conserved. This means that Z^0, photon, gluon, and graviton must correspond to particles having a vanishing vacuum weight. Furthermore, the differences of the vacuum weights for the fermions inside electro-weak doublets must differ by the vacuum weight of W boson.

d) One must somehow take into account the fact that the fermions inside CP_2 type extremals move in different directions. The momentum directions of the incoming state and outgoing states are related by a rotation. This rotation corresponds to a unitary operator U_i, $i = 2, 3$ represented as an exponent of the angular momentum operator associated with the modified Dirac action. Therefore a natural idea is to perform the transformation

$a \to U_i a U_i^{-1}$ for the fermionic oscillator operators of the incoming state.

e) The requirement that the vertices involve smooth topological sum of CP_2 type extremals implies that vertex regions cannot be vacua in a finite region surrounding the vertex point. Therefore it is not possible to have vertices which are too close to each other so that the sizes of the loops have lower bound, which saves from ultraviolet divergences. It is quite probable that the loop diagrams using the vertex operators obtained by allowing singular vertices give rise to ultraviolet divergences unless one introduces the ultraviolet cutoff by hand.

3.6 S-matrix at QFT limit

The replacement of particle like topological inhomogenities with point-like particles means the loss of all relevant geometric and topological information since interacting and free space-time surfaces become identical. The basic question is how to code the information about the restriction of the S-matrix to the massless sector of the theory to an information characterizing the QFT.

At practical level symmetry arguments is expected to lead to YM type action having $P \times SU(3) \times U(2)_{ew}$ as quaternion-conformal gauge group of symmetries. The corresponding Hamiltonian dictates the S-matrix as time evolution operator. The construction of S-matrix in terms of the time-ordered correlation functions using effective action approach suggests the interpretation of the Feynman diagrams of YM theory as QFT counterparts for the topological sums of CP_2 type extremals. Only tree diagrams are included.

3.6.1 What are the input parameters of the QFT limit?

The exponential factor $exp(-S_K(X^4))$ defined by the Kähler action is exponentially sensitive to the volume of the internal line and thus to the length L of the propagator line connecting the two vertices. Thus the propagator in the momentum representation involves the Fourier transform of $exp(-S_K(X^4))$ which is functional of the function U.

The behavior of the propagator as a function of the momentum transfer depend on the selection of the function U associated with vacuum extremals and it seems that one cannot avoid spin-glass type averaging over QFT limits associated with various choices of U. Various coupling constants are proportional to the Glebsch-Gordan coefficients $V(a,b,c)$ for the tensor product of Super-Kac-Moody representations for two incoming lines. Besides propagators with highly nontrivial infrared behavior depending on U, these coupling constants, are the basic inputs of the QFT limit. Spin glass averaging over the allowed functions U is certainly necessary also for for the results of QFT limit.

3.6.2 Can one avoid infrared suppression and how the values of the coupling constants are determined?

CP_2 type extremals of infinite duration ($u = f(U) \rightarrow \infty$ at the limit $U \rightarrow U_{max}$) can appear as incoming and outgoing states since wave function normalization (division of the propagator factors of external lines away) compensates the suppression factors coming from the external legs of the Feynman diagrams. In case of internal lines the situation is different. An interesting question is whether the exponential IR suppression could be mildened by some mechanism and whether the mildened IR suppression could in fact determine the values of the effective coupling constant strengths as proportional to the suppression factors associated with the propagator lines emerging from the vertex. If virtual CP_2 type extremals have finite length ($f(U)$ is finite for all values of U), there is always also absolute length scale cutoff involved with the interactions induced by them. This cutoff could explain color confinement and imply deviations from QED at large distances.

3.6.3 Could CP_2 type extremals have a small volume?

CP_2 type extremals could have a volume which is only a small fraction of the full volume of the entire CP_2 type extremal: the exponents of Kähler action for the virtual particles in the vertex would thus define the values of the effective coupling constant strengths.

a) This would be the case if generation-genus correspondence is due to the holes inside CP_2 type extremals and if CP_2 type extremals have considerable volume at the moment of absorption and emission. On the other hand, the volumes of the virtual CP_2 type extremals should be essentially the same irrespective of the genus of the hole since the couplings of the fermionic generations to photons are in an excellent precision the same. If the hole gives rise to a large reduction of the volume of CP_2 type extremal, it is difficult to understand why the reduction factor would not depend on the topology of the hole. The safest conclusion is that the hole should give rise to a negligibly small reduction of volume.

b) The simplified model for the emission of CP_2 extremal assumes that the 3-surfaces associated with the incoming and outgoing particles are identical at the vertex. Since one of these particles is the incoming particle, it is natural to assume that these 3-surfaces are far from point-like so that a considerable reduction of the volume would automatically occur. One can also consider the possibility that CP_2 volume increases rapidly to its asymptotic value so that the incoming surfaces at the vertices are always in the asymptotic region and have volume near the maximal volume. This implies that the values of the effective coupling constants are determined by the the averages volumes of the CP_2 type extremals.

c) Gravitons could differ from other particles basically because the size of the gravitonic 3-surface at the moment of emission is very small. This could be understood if the vertex for the emission of graviton vanishes in the approximation in which vertex represents singular manifold homologically equivalent with a 3-vertex of QFT. This is quite possible and in accordance with the standard physical intuition that quantum field theory description of graviton is not possible but requires genuinely higher-dimensional vertices.

4 What the low energy QFT limits of TGD in X^4 might look like if they exist?

There are good reasons to be skeptic about quantum field theory limit of TGD define in space-time surface. It would be however not wise to not see what this kind of limit would look like and whether it can cope with various challenges posed by new views such as the new understanding of color and family replication phenomenon.

4.1 Basic approaches

A priori, one can consider several approaches to the construction of the low energy QFT limits of TGD corresponding to Kaluza-Klein type theory, sigma model type theory and a hybrid of these two approaches: the last approach turns out to be the most promising one.

4.1.1 Sigma-model approach

The definition of the QFT limit as a quantum field theory on the effective, quantum average space-time surface, determined as a maximum of Kähler function, leads to a well defined concept of classical space-time consistent with basic quantum TGD. The inner product of configuration space spinors contains phase factor and the exponent $exp(iS_{eff}(X^4_i(X^3)))$ of the classical effective action could appear as a phase factor in matrix element distinguishing between different quaternion-conformal deformations. On the other hand, the symmetry of the effective action under these deformations suggests that S_{eff} is quaternion-conformal invariant and thus constant.

The integral over the degenerate space-time surfaces related by quaternion-conformal deformations is assumed to mimic the effects of the path integral giving rise to Feynman diagrams identifiable as QFT counterparts for Feynman diagrams with lines thickened to particle-like 4-manifolds. YM action plus Dirac action for the induced spinors, with color index added in case of quarks, seems to provide a natural manner to define the theory.

This approach has however some severe problems, the worst of them being the phenomenological description of the particle massivation. It has become clear that particle massivation in TGD framework involves both p-adic thermal massivation which is an excellent approximation in case of fermions, and TGD version of Higgs mechanism which is an excellent approximation in the bosonic sector. To describe the thermal contribution to the masses one could introduce besides dynamical Higgs field also a phenomenological Higgs field expressible in terms of CP_2 gamma matrices. This is indeed formally possible by introducing a Higgs field of unit norm with couplings to fermions put in by hand to reproduce fermion masses correctly. Ordinary Higgs field would be presumably enough to save the unitarity and would only induce only mass shifts in case of fermions. The beautiful general covariance at the level of the imbedding space is however lost via the addition of these phenomenological couplings. Also the many-sheeted nature of the space-time and the presence of space-time boundaries leads to grave technical difficulties in this approach.

4.1.2 Hybrid of Kaluza-Klein and 4-dimensional QFT

One can consider also a hybrid of the previous approaches based on the quantum generalization of the induction procedure. This approach seems to be the most promising one.

a) The states described by the configuration space spinor field reduce at the point like limit to a Kaluza-Klein type fields in H. This suggests that one should replace the metric and gauge potentials of the imbedding space H (rather than those of space-time surface) with their quantized counterparts by adding to them the quantum fields describing electro-weak and color gauge potentials and quantized gravitational field understood as quantum fields in H reducing to quantum fields in M^4 in the low energy limit. Also H-gamma matrices should be quantized and their quantization induces the quantization of H-metric. The generalization is possible by regarding the

H-vielbein as a fundamental variable, which is replaced with the sum of the classical and quantum term so that the quantum deformation of H-metric (actually M^4-metric) contains also a term quadratic in the quantum part of the vielbein defined by the standard formulas. The quantization induces also a quantum term also to the M^4 spinor connection. The quantization need to be done only once unlike in the previous case and one avoids the horrible technical problems caused by the many-sheeted space-time. Note that quark color must be described as a spin index rather than color partial wave.

b) The induction procedure is generalized: instead of inducing the classical metric and spinor connection of H, the quantized metric and spinor connection are induced using the same basic formulas as in the classical case. No troubles from the normal ordering are encountered since H-coordinates are regarded as classical variables. Also gluon field, having only M^4 components, is induced by simply projecting it to the space-time surface.

c) Yang-Mills-Dirac action, defined in terms of the induced fields on the absolute minimum of Kähler action and maximum of Kähler funciton, defines the low energy QFT. The bosonic fields appearing in the action are superpositions of the induced field plus c-number quantum terms. Spinor fields are Grassmann algebra valued fields. Of utmost importance is that these fields are basically fields in M^4 and have Fourier expansions in terms of M^4 momenta with masses predicted by quantum TGD and p-adic thermodynamics. There are no problems with Poincare invariance: polarization vectors, momenta, etc. are associated with M^4 rather than X^4. The approach makes also possible to avoid the difficulties related to the many-sheeted nature of space-time. For instance, there is no need to consider the problem how to relate the quantum fields of two space-time sheets connected by wormholes to each other or how to define propagators in a topologically nontrivial space-time. Space-time boundaries pose no special problems. Theory is formally defined even well defined even for string and membrane like objects.

d) The YM action is regarded as an effective action so that no functional integral is carried and only tree diagrams are taken into account. Propagators do not result from the kinetic term of the action, which would give rise to massless poles but from the time ordered products for the M^4 quantum fields assumed to be those of free fields. The entire YMD action is therefore regarded as an interaction term unlike in the usual approach. Hence there is no need to reproduce particle masses using formal Higgs fields.

4.2 Induction procedure at quantum level

In the definition of QFT limit based on the generalization of the induction procedure, all quantum fields are basically free quantum fields in H, which at the low energy limit reduce to quantum fields in M^4 with color quantum numbers described as phenomenological spin degrees of freedom. The Fourier expansion of these quantum fields is given a priori and corresponds to the mass predicted by the quantum TGD and p-adic mass calculations. At the calculational level this means that propagators are free field propagators of M^4 QFT. Hence there is no need to reproduce thermal contributions to particle masses in QFT using artificial Higgs fields (genuine Higgs field causing the massivation at the fundamental level). Also Poincare invariance is achieved in an elegant manner. A further nice feature is that one can circumvent all the problems related to the nontrivial space-time topology in the quantization.

Just as in the classical theory, space-time quantum fields are induced fields and hence defined as projections of quantum fields of H to space-time. Electro-weak gauge bosons correspond to quantum term in H-spinor connection in M^4 degrees of freedom. The gamma matrices of H are also quantized. Graviton is identified as the quantum contribution to M^4 vielbein rather than M^4 metric. The quantum contribution to the M^4 metric is defined by the standard formula for the metric in terms of the vielbein. The components of the gluon field

$$g_A = g_A(c) + g_A(q) \ ,$$

are defined as space-time projections of the H-vector field defined as a sum of the Killing vector $g_A(c) = j_A$ having only CP_2 components of the color isometry and gluon field $g_A(q)$ having only M^4 components. The contribution of the classical gluon field is necessary in order to understand the concept of the color flux tube. Also fermion fields are regarded effectively as fields in M^4 at the low energy limit of the theory. Clearly gluon field can be regarded as a quantized Killing vector field having also M^4 components.

The fact that gluon and graviton have very similar role in quantum TGD, raises the possibility that gluons actually contribute to the quantized metric of H. A suggestion as how this should be described formally, is given by the identity

$$s^{kl} = j_A^k j^{Al} \ ,$$

for the CP_2 metric in terms of the Killing vector fields of the color group.

This identity allows the representation

$$e_A^k(c) = e_{Al}(c) j_B^l j^{Bk} \ .$$

for the vielbein of CP_2. This suggests a generalization of CP_2 vielbein to

$$e_A = e_{Al}(c) g_B^l g^B = e_A(c) + e_{Al}(c) j_B^l(c) j g_A(q) \ . \qquad (8)$$

having also M^4 components. In this manner the quantized metric contains an additional $M^4 - CP_2$ cross terms proportional to quantum gluon field as well as M^4 term quadratic in quantum gluon field. Since M^4 vielbein corresponds to translation generators one can conclude that the quantum contribution to H metric can be expressed in terms of the quantized isometry generators of H.

Second interesting question is whether one could understand the quantum parts of the electro-weak gauge potentials as resulting from the quantum part of the CP_2 vielbein having M^4 components. If this where the case, bosonic fields would result from the quantum deformation of H-metric. The deformation of CP_2 vielbein indeed introduces to the spinor connection of H a term expressible in terms of gluon field contracted with the color isometry generators but this term cannot be identified as electro-weak gauge potential (also the couplings of this term are of the order of the gravitational coupling) so that an additional term is needed.

There are two objections against this vision.

a) If one regards quantum parts as quantum fields in H the commutators and anticommutators are 7-D delta functions and restricted to space-time surface the normal orderings give rise to horrible singularities. This problem can be avoided if the action is regarded only as an effective action. One could try to avoid the difficulty is to assume that quantum fields are constant in CP_2 degrees of freedom and color degrees of freedom so that commutators give 3-D delta functions.

b) One could worry about the large mass renormalization corrections caused by the kinetic terms associated with the quantum parts of the induced H-fields. Since the coefficients of the parts depend on the space-time point, one might argue that they do not give rise to genuine mass corrections.

4.3 The general form of the effective action

The need to obtain physical interaction vertices forces the action for the quantum field theory in question to be the low energy action for the Yang

Mills Dirac (YMD) type theory defined by quantum TGD. This effective action is estimate for maxima of Kähler function and thus absolute minima of Kähler action. The presence of curvature scalar is not necessary nor even possible since Einstein's equations have interpretation as kind of equation of state in TGD framework. Super Virasoro generator L_0 generating the time evolution in quantum TGD is indeed analogous to Yang Mills Dirac Hamiltonian. The effective action should decompose as

$$S_{eff} = S_B + S_F \ , \tag{9}$$

where S_B and S_F are the fermionic and bosonic parts of the action. Bosonic action is expected to decompose to YM action describing electro-weak gauge bosons and gluons plus curvature scalar term describing gravitons. In general, the induced bosonic fields are sums of the classical and quantum contributions. This is also true for the induced gamma matrices.

4.4 Description of bosons

The spectrum of the light particles is determined by the quaternion-conformal super-Kac-Moody algebra associated with $P\times SU(3)\times U(2)_{ew}$, where P denotes Poincare group, and by p-adic thermodynamics telling which massless particle remain light in given mass scale. Neither the identification of the light particle spectrum nor a detailed description of effective action is attempted here and the consideration is restricted to the description of most important terms present in the effective action.

The Yang Mills fields appearing in the low energy effective action are expected to correspond to the induced gauge fields so that electro-weak and color gauge fields are the only fields appearing in the effective action at low energies. The bosonic part of the action decomposes therefore as

$$S_B = S_{ew} + S_c + S_{gr} \ , \tag{10}$$

to electro-weak, color and gravitational parts respectively. S_{ew} and S_c are YM actions for electro-weak and color interactions with coupling constants, which quantum TGD in principle predicts as a function of p-adic length scale $L(p)$.

The induced gauge potentials can be decomposed to the sum of the quantum and classical parts:

$$A = A_q + A_{cl} \ . \tag{11}$$

The presence of the genuinely classical part, determined by H-geometry, in gauge potential is an effect characteristic to TGD and leads to interesting predictions. For instance, space-time surface can carry non-vanishing vacuum gauge currents and electromagnetic gauge currents can give rise to coherent states of photons.

The metric tensor appearing in the action is regarded as the induced metric associated with H- metric containing quantum part and YM action gives rise to the standard coupling of matter to gravitons. Graviton contributes only to the M^4 metric and one can write the M^4 metric as a sum of the classical and quantum part

$$g_H = g_H + g_q \ . \tag{12}$$

Minkowski coordinates are the preferred coordinates used to defined g. Quantum part of the induced metric, is by definition treated as a small perturbation in the genuinely classical background metric. Einstein's equations are regarded as equations for the density of gravitational mass identified as the difference of inertial mass densities of positive and negative energy matter.

4.5 Description of the fermions

The fermionic term S_F contains the contribution of all light families in the energy scale considered. The concept of induced spinor field generalized to quantum context is very attractive. The description of the fermionic degrees of freedom is however not quite straightforward.

4.5.1 How to define quantum counterparts of the induced gamma matrices?

Induced gamma matrices can be defined as space-time projections of the quantized H-gamma matrices. The problem is to define these objects. The low energy limit only the M^4 metric contains quantum part and the problem reduces to the quantization of the M^4 gamma matrices. The formal definition of the quantized gamma matrices reads as

$$\begin{aligned}\gamma_k &= \gamma_k(c) + \gamma_k(q) \ , \\ \{\gamma_k, \gamma_l\} &= 2m_{kl} \ , \end{aligned} \tag{13}$$

with quantum correction γ_q defined so that the anti-commutator of the induced gamma matrices gives M^4 metric with a quantum correction.

An elegant manner to get rid of possible difficulties related to divergences and normal ordering problems is to identify graviton basically as the quantum deformation of the vielbein so that the quantum term in M^4 metric is *defined* by the anti-commutator of the quantized gamma matrices. The quantization of the gamma matrices in turn reduces to the quantization of the vielbein, when γ_k is expressed in terms of the ordinary gamma matrices:

$$\begin{aligned}\gamma_k &= e_k^A \gamma_A \ , \\ e_k^A &= e_k^A(c) + e_k^A(q) \ . \end{aligned} \tag{14}$$

Here γ_A are the ordinary flat space gamma matrices. The quantized metric reads as

$$m_{kl} = e_k^A e_l^A = m_{kl}(c) + e_k^A(c) e_l^A(q) + e_k^A(q) e_l^A(c) + e_k^A(q) e_l^A(q) \ . \tag{15}$$

4.5.2 How to describe quark color phenomenologically?

Color is not a spin like quantum number at the level of the induced spinor fields and at the level of the configuration space spinors it corresponds to a color partial wave associated with boundary component plus Super Kac Moody excitation in the interior degrees of freedom.

If one describes fermion states as H-spinor fields one should associate the color quantum numbers of the state to a color partial wave unless one is willing to introduce color as a phenomenological color index. The first approach however leads to problems.

a) If one associates to a fermionic state H-spinor and requires it to be an eigen state of CP_2 spinor Laplacian, one ends up with difficulties with color since color partial waves have wrong correlation between color and electro-weak quantum numbers. The only possibility seems to be the association of the desired color partial wave to a state with given electro-weak quantum numbers by hand.

b) It is not clear whether this procedure is consistent with the properties of the CP_2 spinor structure also it seems difficult to understand how one could get the conservation of the color quantum numbers since the restriction of the state to a space-time surface means a total localization in CP_2 degrees of freedom and the overlaps of the colored states, orthogonal at the level of the configuration space, become non-vanishing.

These arguments strongly suggest the use of of H-spinors as the counterparts of the quantum states with color described completely phenomenologically as an additional spin degree of freedom.

4.6 QFT description of family replication phenomenon

TGD provides topological explanation for the family replication phenomenon [F1]. Particle families correspond to various topologies for boundary component of 3-surface carrying elementary particle quantum numbers. Also gauge bosons are predicted to have infinite number of families. TGD gives general mass formula for particle families [F2]) and only three lowest generations are expected to be light [F1]). For the possible physical implications of massive higher generations of gluons and electro-weak bosons see [F5].

The problem is that family degeneracy for gauge bosons is not necessarily consistent with gauge invariance. First, one should somehow eliminate time like polarizations for the vector fields representing higher generation bosons and the only known manner to achieve this is via a gauge fixing. Some kind of extended gauge invariance suggests strongly itself but standard type extension of the gauge group is out of question.

It turns out that consistent description of generation degeneracy is possible and given gauge theory with family replication in M^4 is formally equivalent with same gauge theory without family replication but defined in $M^4 \times S^1$ and this is essentially due to the infinite number of particle generations for both fermions and bosons.

4.6.1 Family replication phenomenon in gauge theory context

To avoid unessential complications the argument is described first in real QFT context and neglecting the topological mixing of the particle families.

1. $M^4 \rightarrow M^4 \times S^1$ trick

The fact that the genus of boundary component is effectively conserved discrete quantum number suggests how to achieve this. Genus corresponds effectively to a discrete momentum in additional dimension so that the QFT describing the family degeneracy is perhaps *formally* identical with same QFT without family replication but defined in $M^4 \times S^1$, where S^1 is circle with some radius R. Lowest fermion genera and ordinary gauge bosons correspond to the modes of quantum fields with are constant in S^1 and have momentum $g = 0$. Higher genera would correspond to $g > 0$ modes $exp(ig\phi)$ in S^1.

The action of the theory is essentially Standard Model action in 5 dimensions (leaving from the consideration color exotic states now). The action consists of pieces (color and electro-weak gauge interactions) of following general form

$$\begin{aligned} S &= \int (L_{YM} + L_{matter}) d^4x \frac{d\phi}{2\pi} \ , \\ L_{YM} &= \frac{1}{16\pi g^2}((F_{\mu\nu}F^{\mu\nu} + 2F_{\mu\phi}F^{\mu\phi}) \ , \\ L_D &= \bar{\Psi}(\gamma^\mu D_\mu + \gamma^\phi D_\phi)\Psi \ . \end{aligned} \tag{16}$$

The metric of S^1 is $g_{\phi\phi} = R^2$. YM part of the action has been decomposed into standard four-dimensional YM action plus the term corresponding to the new components $F_{\mu\phi}$ of gauge field. $F_{\alpha\beta}$ is obtained from gauge potential (A_μ, A_ϕ) in standard manner. A_ϕ is has interpretation as a formal Higgs field in 4-dimensional context. Fermion fields have no additional components since the number of components of fermion fields is same in dimensions $D = 2n$ and $D = 2n + 1$. The gamma matrix field γ_ϕ can be chosen to be proportional to the CP_2 gamma matrix Γ representing constant non-dynamical 'Higgs' field defined by the CP_2 part of the second fundamental form.

2. Elimination of 'Higgs' field part of the action

If TGD picture is correct it should be possible to eliminate the Higgs field part of YM action associated with genus-degeneracy and this is indeed the case. Of course, the couplings to the ordinary Higgs remain. In Landau gauge

$$A_\phi = 0 \ . \tag{17}$$

The Higgs field associated with genus-degeneracy indeed disappears completely and the field equations for YM fied read as

$$\begin{aligned}(D_\nu D^\nu + \frac{1}{R^2}\partial_\phi\partial_\phi)A^\mu + D^\mu(D^\nu A_\nu) &= J^\mu \ , \\ \partial_\phi(D^\nu A_\nu) &= 0 \ . \end{aligned} \quad (18)$$

J^μ denotes the source term coming from Dirac part of the action.

It is instructive to write field equations separately for each mode $g \neq 0$.

$$\begin{aligned}(D_\nu D^\nu A^\mu)(g) - m^2(g)A^\mu(g) &= J^\mu \ , \\ D^\nu A_\nu(g) &= 0 \ , \\ m^2(g) &= \frac{g^2}{R^2} \ . \end{aligned} \quad (19)$$

Here the covariant derivative terms contain nonlinear couplings with $g_1 \neq g$ modes of gauge boson fields not written explicitly. The field equations for $g \neq 0$ modes are field equations for massive vector field with mass $m^2(g) = g^2/R^2$ and additional condition eliminates time like polarization. Higgs field $A_\phi(g)$ has transformed to the longitudinal polarization of massive YM field $A_\mu(g)$. The gauge condition remains invariant under arbitrary gauge transformations not depending on S^1 coordinate. This freedom allows to pose in standard manner gauge condition to the gauge field component with $g = 0$ and vanishing mass and standard YM field equations for $g = 0$ gauge field component result.

Also for fermions one obtains the desired results. Different fermion families are coupled together via the gauge potentials $A(g)_\mu$ and gauge coupling is same for all modes (renormalization corrections probably preserve the couplings as identical). The Dirac equation for the mode g contains $\gamma^\phi D_\phi$ as mass term and the mass is just $m(g) = g/R$. Note that the mass formula is for fermions and bosons is necessarily identical and un-physical.

3. The problems of the approach

The first problematic feature of the proposed scenario is that the introduction of the gamma matrix γ_ϕ brings in a definite ad hoc-ness. The reproduction of the particle masses is also an obvious shortcoming of the approach. The simple mass formula $m^2(g) = g^2/R^2$ is quite different as compared to the mass formula excluding electro-weak corrections and containing only modular contribution to mass coming from the boundary component

$$m^2 \ = \ f^2(g) = 3 \cdot 2^{g-1}(2^g+1)g\frac{1}{L_p^2} \ , \ g < 3 \ , \quad (20)$$

L_p is the p-adic length scale determining the mass scale. For $g > 2$ masses should be of order Planck mass but the mass formula is not known. A possible resolution of this difficulty in *real context* would be following. The mass operator $m_{op} = D/R = \frac{i\partial_\phi}{R}$ acts as a multiplication operator in plane wave basis $exp(ig\phi)$ for S^1 and it formally possible to replace D/R with Hermitian multiplication operator giving the required mass formula

$$\begin{aligned} m_{op} = \frac{D}{R} \quad &\rightarrow \quad f(|D|)\frac{D}{|D|} \ , \\ D \quad &= \quad i\partial_\phi \ . \end{aligned} \tag{21}$$

The only consequence of this replacement at the level of Feynman rules is to replace masses with their physical values. At the level of YM fields one replaces the partial differential operator ∂_ϕ^2 by m_{op}^2. A possible interpretation is as a redefinition of gauge fields and spinor fields in Landau gauge via the linear but nonlocal transformation defined by the differential equation

$$m_{op}\Phi_{new} \quad \equiv \quad \frac{i}{R}\partial_\phi\Phi_{old} \ . \tag{22}$$

Transformation reduces to purely algebraic form in plane wave basis and writing the action in plane wave basis one finds that the desired field equations are obtained.

A further problem is related to the physical interpretation of $g < 0$ states. The number of handles of the boundary component is nonnegative but one must associate to it sign factor in order to allow reactions such as $\gamma \rightarrow \mu^+\mu^-$, decay $g(0) \rightarrow g(1) + g(1)$ of gluon to higher generation gluon pair, virtual processes $e \rightarrow \mu + Z(1)$ and $\mu \rightarrow e + Z(1)$,etc. These processes can be geometrically understood as process in which $g > 0$ boundary component emits $g = 0$ boundary component and possibly also changes its direction of propagation in time. For instance, in $e \rightarrow \mu + Z(1)$ $Z(1)$ proceeding backwards in time emits e and changes to μ proceeding forwards in time. This means that depending on situations same particle behaves as $g > 0$ plane wave or as $g < 0$ plane wave. One cannot however allow both $g > 0$ and $g < 0$ states as physically different modes. This requires that both fermions and bosons correspond to the real plane wave basis

$$f_g(\phi) \quad = \quad \frac{1}{\sqrt{\pi}}cos(g\phi) \ , \tag{23}$$

rather than complex plane waves $exp(ig\phi)$. This complicates the Feynman rules somewhat but does not bring anything essentially new.

4.6.2 Family replication phenomenon for graviton and super partners

The formal extension of M^4 QFT to $M^4 \times S^1$ applies also to the description of the family replication phenomenon for super partners and graviton.

One can formally extend the super-symmetrized theory to $M^4 \times S^1$ since the spinors of $4+1$ dimensional space are identical with the spinors of 4-dimensional space and Majorana (Weyl) condition essential for super symmetry can be realized. The thermal and modular contribution to mass makes higher generations massive and break super symmetry.

The formal treatment of the family replication for graviton in the ordinary field theory context seems also possible by replacing background space-time with $M^4 \times S^1$ just as in the case of bosons and fermions. The physically natural coordinate conditions read as $g_{\mu\phi} = 0$ and $g_{\phi\phi} = R^2$ and are completely analogous to Landau gauge $A_\phi = 0$ for gauge bosons. For $g \neq 0$ the conditions imply that on mass shell modes of the metric deformation satisfy the usual harmonic gauge condition $\partial_\nu h^{\mu\nu}(g) = 0$, $g \neq 0$ in lowest order and linearized field equations for $h_{\mu\nu}(g \neq 0)$ contain the desired mass term. For $g = 0$ one can pose these conditions as gauge conditions in standard manner.

4.7 Features of the QFT limit characteristic to TGD

The presence of the classical terms in the YM action defining low energy QFT limit, leads to some predictions characteristic for TGD approach.

a) The many-sheeted space-time concept is a spectacular prediction, which might explain the formation of the structures purely topologically. A closely related prediction are wormholes feeding gravitational and electro-weak fluxes between space-time sheets. The wormholes feeding the gauge fluxes must be light. p-Adic length scale hypothesis suggests that their masses for a space-time sheet corresponding the p-adic prime p is or order $1/L(p)$, $L(p) = \sqrt{p}l$, $l \simeq 1.37 \times 10^4 \sqrt{G}$. This would mean that in the length scales below $L \sim 1/T$ the BE condensates of wormholes are possible and stable in the room temperature corresponding to $L \sim 10^{-4}$ meters.

b) The presence of a genuinely classical term in the gauge potentials and gravitational field imply the generation of coherent states of photons and gravitons since classical gauge currents as well as non-trivial vacuum Einstein tensor are possible in TGD. For instance, the so called massless extremals, for which the gauge currents and Einstein tensor are light like, could serve as sources of coherent photons and gravitons. The source term generating coherent gravitons would be the energy momentum tensor of the classical Kähler field with vacuum Einstein tensor subtracted.

c) The basic objection against TGD is that the set of the space-time metrics imbeddable in $H = M^4_+ x \times CP_2$ is extremely restricted. Reissner-Nordstrm̈ metric is imbeddable but already the imbedding of a rotating black-whole is very probably impossible. The generation of coherent states of gravitons suggests how to circumvent the counter argument. The expression for the quantized induced metric contains besides the classical part also the quantized part and coherent states should be eigen states of the negative energy part of the quantized metric. Therefore quantum expectation of the quantized gravitational field for coherent states of gravitons contains additional part making possible more general effective space-time metrics. Einstein's equations should be satisfied in the sense that the expectation value should satisfy Einstein's equations for the quantized energy momentum tensor from which the contribution coming from the Einstein tensor of the space-time surface, identifiable as vacuum contribution, is subtracted. Vacuum contribution is unique since space-time surfaces are absolute minima of Kähler action.

4.8 About coupling constants

It is possible to understand the value of the gravitational constant by a simple dimensional argument based on the properties of the CP_2 type extremals and p-adic length scale hypothesis [**?**, E5]. This argument also fixed the dependence of the Kähler coupling strength on the p-adic length scale completely: in electron length scale Kähler coupling strength was assumed to be in a good approximation equal to the fine structure constant since purely electromagnetic classical fields, believed to dominate in electron length scale, are proportional to Kähler field so that bosonic YM action is essentially Kähler action in these length scales:

$$\alpha_K(m_e) \simeq \alpha_{em} \ . \tag{24}$$

Recall that the order of magnitude estimate for the gravitational constant in terms of the p-adic length scale $L_p = \sqrt{p}l$, $l \simeq 1.36 \times 10^4 \sqrt{G}$ and of the suppression factor given by the Kähler action exponential, is obtained by studying the diagram representing an exchange of a graviton and is given by

$$G \simeq exp(-2S(CP_2)/\alpha_K) L_p^2 \ ,$$

$$L_p = \sqrt{p}l \ . \tag{25}$$

As found, $\alpha_K(M_{127})$ is very nearly the fine structure constant at m_e corresponding to $p = 2^{127}-1$, gives G correctly. In the absence of the suppression factor one would have strong gravitation. The requirement that G does not depend on the p-adic length scale leads to the evolution of α_K as a function of the p-adic length scale.

The values of the remaining coupling strengths are predicted by quantum TGD and appear as input parameters in the low energy QFT limit of TGD. Obviously these coupling strengths are proportional to the Kähler coupling strength α_K.

5 Classical part of YM action

The basic idea of the approach is that absolute minima of Kähler action define the background defining various classical YM and gravitational fields. Quantum theory is obtained by adding to these fields quantum corrections and performing perturbation theory.

5.1 The field equations for coherent states

Classical field equations define the classical space-time serving and coherent state serving as a background for the low energy QFT. In a given quantum state the vacuum expectation value of the YM action contains state dependent terms giving rise to source terms in classical field equations. In the following only the field equations in vacuum are considered.

The field equations can be written in the form

$$S^k = S_1^k + S_2^k + S_3^k = 0 \ . \tag{26}$$

The first term is given by the expression

$$\begin{aligned} S_1^k &= F^\alpha h^k_{,\alpha} \ , \\ F^\alpha &= \sum_a Tr(J_a^\beta F^\alpha_{\beta,a}) \ , \\ J_a^\alpha &= \frac{1}{g_a^2} D_\beta F_a^{\alpha\beta} \ . \end{aligned} \tag{27}$$

Here J_a is just the standard YM current (the corresponding YM couplings are included in the definition of J_a^α) and a labels various subgroups of the total gauge group. The term F^α can be interpreted as a sum of the force densities associated with the electro-weak and color interactions.

The second term of the field equations is given by

$$\begin{aligned} S_2^k &= T^{\alpha\beta}H^k_{\alpha\beta} \ , \\ T^{\alpha\beta} &= T^{\alpha\beta}_{YM} \ . \end{aligned} \tag{28}$$

The third term in the decomposition is given by

$$S_3^k = \sum_a Tr(J^\alpha F^k_l)h^l_{,\alpha} \ . \tag{29}$$

The quantum versions of classical field equations are obtained by taking expectations between coherent states. In this manner the complex parameters characterizing coherent states are in principle fixed.

5.2 The detailed structure of the classical YM action

In the following the general structure of the classical YM action will be considered.

5.2.1 Electro-weak term at the symmetry limit

The $U(1)$ coupling of the spinor field to Kähler potential of CP_2 is necessary to obtain an acceptable spinor structure [16]. This coupling also fixes electromagnetic charges and the triality of $SU(3)$ representations for the solutions of Dirac equation in CP_2 [16]. The allowed couplings are given by odd integers. $n = -1$ corresponds to leptons and $n = 3$ corresponds to quarks.

The vielbein curvature of CP_2 is given by the expression

$$\begin{aligned} R_{kl} &= \frac{1}{2}R_{klAB}\Sigma^{AB} \ , \\ \Sigma^{AB} &= \frac{i}{4}[\Gamma^A,\Gamma^B] \ . \end{aligned} \tag{30}$$

where R_{klAB} denotes the components of curvature tensor. $R_{\alpha\beta AB}$ denotes the projection of this tensor to X^4.

The contribution of vielbein term to the electro-weak action is given by the expression

$$
\begin{aligned}
L_{VB} &= (r/16g^2)R_{\alpha\beta AB}R^{\alpha\beta AB} \ , \\
r &= \frac{1}{8} \ .
\end{aligned} \qquad (31)
$$

The term differs by a factor of $r = 1/8$ from the standard definition of gauge action. The reason is that for sigma-matrices of H have normalization, which differs by a factor 8 from the normalization of the standard Lie-algebra generators

$$
Tr(\Sigma^{AB}\Sigma^{AB}) = \frac{r}{2} = 4 \ . \qquad (32)
$$

To obtain a definition of coupling constant comparable with the standard definition one must therefore include the factor $r = 1/8$.

The expression for the $U(1)$ part of the spinor curvature is

$$
F^1_{\alpha\beta} = (n_+ Id_+ + n_- Id_-)J_{\alpha\beta} \ . \qquad (33)
$$

where n_+ and n_- denote the couplings of Kähler potential to quarks and leptons respectively and $J_{\alpha\beta}$ is the induced Kähler form. $Id_\pm$ is the projector to the space of the spinors with chirality $e = \pm 1$.

The expression for the $U(1)$ part of the electro-weak action at the symmetry limit is given by the expression

$$
\begin{aligned}
L_{U(1)} &= \frac{r}{4g^2}Tr(F^1_{\alpha\beta}F_1^{\alpha\beta}) \\
&= r(n_+^2 + n_-^2)\frac{1}{g^2}J_{\alpha\beta}J^{\alpha\beta} \ .
\end{aligned} \qquad (34)
$$

Again the factor $r = 1/8$ is necessary since the unit matrix of gamma matrix algebra has trace, which is 8 times larger than the trace associated with standard $U(1)$ generator.

5.2.2 Electro-weak symmetry breaking term

Electro-weak symmetry breaking term, which is proportional to Kähler action and results from the deviation of electro-weak $U(1)$ coupling from its value at symmetry limit reads as

$$L_{Br} = \frac{r}{4}\Delta(n_+^2 + n_-^2)J_{\alpha\beta}J^{\alpha\beta} \quad . \tag{35}$$

The expression for Weinberg angle can be deduced by requiring that the action contains no $Z-\gamma$ type cross terms and is given by [see Appendix]

$$\begin{aligned} p &\equiv sin^2(\theta_W) = \frac{9}{(28+g^2\Delta)} \ , \\ g^2\Delta &= \frac{9}{p} - 28 \ , \end{aligned} \tag{36}$$

where Δ parameterizes electro-weak symmetry breaking. The value of the Weinberg angle at symmetry limit is $p = 9/28$ and differs somewhat from $p = 3/8$ of a typical GUT [17]. $p = \frac{9}{28}$ is also obtained in quantum TGD from the requirement that Z^0 and photon states are orthogonal to each other.

5.2.3 Gluon term

Gluonic gauge potentials are proportional to the X^4 projections of Killing vectors of $SU(3)$ isometries,

$$g_\alpha^A = kj_\alpha^A \quad . \tag{37}$$

The curvature form of the classical gluon field is given by

$$g_{\alpha\beta}^A = kH^A J_{\alpha\beta} \ , \tag{38}$$

where H^A denotes the Hamiltonian of the color isometry. Since the identity

$$\sum_A H^A H_A = 1 \ , \tag{39}$$

holds for the Hamiltonians of color isometries gluon term itself is proportional to Kähler action

$$L_{gluon} = \frac{r}{4g_s^2}kJ_{\alpha\beta}J^{\alpha\beta} \quad . \tag{40}$$

The normalization of the gluon field itself is a matter of convention: the ratio of the gluonic term to the electro-weak term contains the physics. To fix the ratio consider the following argument. Colored point like fermions correspond in TGD approach to color partial waves. CP_2 allows neither ordinary color representations with non-zero triality nor standard spinor structure. By coupling the Kähler potential to the spinor field one however obtains representations for the Abelian extension of the color group [appendix,Pope].

In the spirit of this picture it is natural to assume lepton quark symmetry at high energies: electro-weak action for lepton is identical to electro-weak plus color interaction for quark.

$$L_{ew,l} = L_{ew,q} + L_{color} \ . \quad (41)$$

This assumption indeed makes sense only at very high energies: at low energies assumption leads to wrong value for strong coupling. If this assumption is made the gluon term in action is given by the expression

$$\begin{aligned} L_{gluon} &= \frac{rk}{4g_s^2} J_{\alpha\beta} J^{\alpha\beta} \ , \\ \frac{k}{4g_s^2} &= (n_+^2 - n_-^2)\frac{1}{g^2} = \frac{8}{g^2} \ . \end{aligned} \quad (42)$$

where one has used $n_+ = 3$ and $n_- = 1$. A natural manner to fix the normalization factor k of the gluon field is to require that gluon coupling constant is equal to the electro-weak coupling at very high energies. This is a pure convention: all the physics is contained in the hypothesis described above.

5.2.4 Explicit form of the curvature scalar

Although the inclusion of curvature scalar to the effective action does not make sense in TGD framework, the explicit form of EYM action deserves to be discussed. The gravitational part of the EYM action decomposes contains curvature scalar term and volume term. The issue of cosmological constant Λ is discussed in the sequel. The gravitational part of the EYM action would be given by

$$L_{gr} = \frac{1}{16\pi G} R + \Lambda \ . \quad (43)$$

In the induced metric curvature scalar decomposes into two terms

$$R \equiv R_1 + R_2 \quad = \quad [R_{\alpha\beta\gamma\delta} + h_{kl}(H^k_{\alpha\beta}H^l_{\gamma\delta} - H^k_{\alpha\gamma}H^l_{\beta\delta})]g^{\alpha\beta}g^{\gamma\delta} \quad . \tag{44}$$

The first term results from the curvature of CP_2 geometry contains only first order derivatives. The second term is quadratic in the second fundamental form $H^k_{\alpha\beta} = D_\beta h^k_{,\alpha}$ and is expressible in terms of the covariant derivatives of induced gamma matrices:

$$R_2 \quad = \quad \frac{1}{16}Tr(D_\alpha\Gamma^\alpha D_\beta\Gamma^\beta - D_\alpha\Gamma^\beta D_\beta\Gamma^\alpha) \quad . \tag{45}$$

This term resembles closely what might be imagined to be a kinetic term for the induced gamma matrices regarded as dynamical variables.

5.3 Some useful data

In the following some basic data about CP_2 geometry are listed. The components of the vielbein curvature of CP_2 are given by

$$\begin{array}{ll} R_{0101} = -R_{0123} = 1 \ , & R_{0202} = -R_{0231} = 1 \ , \\ R_{2301} = -R_{2323} = -1 \ , & R_{3102} = -R_{3131} = -1 \ , \\ R_{0303} = 2R_{0312} = 4 \ , & 2R_{1203} = R_{1212} = 4 \ . \end{array} \tag{46}$$

The components of the Kähler form are given by

$$J_{12} \quad = \quad J_{03} = 2 \quad . \tag{47}$$

The volume of CP_2 is given by $V(CP_2) = \pi^2 R^4/2$, where R is the so called CP_2 radius defined by the condition that the length of CP_2 geodesic is $L = \pi R$. The area of CP_2 geodesic sphere is given by $A = \pi R^2$.

The value of the Kähler action for CP_2 is given by the expression

$$S \quad = \quad -\frac{\pi}{8\alpha_K} \quad . \tag{48}$$

A representative for the geodesic sphere of type I carrying vanishing Kähler form is given by the condition

$$\xi^1 \quad = \quad \bar{\xi}^2 \quad . \tag{49}$$

The condition

$$\xi^1 \quad = \quad \xi^2 \ . \tag{50}$$

defines a representative for the geodesic sphere of type II.

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