Construction of Quantum Theory: Symmetries

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Contents

Abstract

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

1.1 Physics as infinite-dimensional Kähler geometry

a) The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

b) Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-canonical algebra.

Super-canonical degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-canonical quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

Besides super-canonical symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and

plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

c) Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

1.2 p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only superconformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

1.3. Hierarchy of Planck constants and dark matter hierarchy

The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

This picture also leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. The outcome is an explicit formula for the Dirac determinant consistent with the vacuum degeneracy of Kähler action and its finiteness and algebraic number property required by p-adicization by number theoretic universality.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. One can conclude that the hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

1.4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

a) There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

b) HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

2. The construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

2.1 Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2.2 Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

2.3. Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this symmetry should fix possible M-matrices to a very high degree.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

1 Introduction

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

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   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-canonical algebra.

   Super-canonical degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-canonical quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   Besides super-canonical symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. The assumption that the commutator algebra of these super-canonical and super Kac-Moody algebras annihilates physical states gives rise to Super Virasoro conditions which could be regarded as analogs of configuration space Dirac equation.

   Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

1.1.2 p-Adic physics as physics of cognition and intentionality

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic imbedding spaces are glued together along rational imbedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range. p-Adic space-time sheets with same prime should have many common rational points with the real space-time and be easily transformable to the real space-time sheet in quantum jump representing intention-to-action transformation. The concrete model for the transformation of intention to action leads to a series of highly non-trivial number theoretical conjectures assuming that the extensions of p-adics involved are finite-dimensional and can contain also transcendentals.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labelled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of configuration space spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [E3]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and imbedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

1.1.3 Hierarchy of Planck constants and dark matter hierarchy

The work with hyper-finite factors of type II_1 (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [A9]. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

This framework also leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of a generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. The outcome is an explicit formula for the Dirac determinant consistent with the vacuum degeneracy of Kähler action and its finiteness and algebraic number property required by p-adicization requiring number theoretic universality. The zeta function associated with the eigenvalues (rather than Riemann Zeta as believed originally) in turn defines the super-canonical conformal weights as its zeros so that a highly coherent picture result.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

1.1.4 Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞. The groups G should relate closely to finite groups defining inclusions of HFFs.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

3. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

1.2 The construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

1.2.1 Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

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Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

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The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this symmetry should fix possible M-matrices to a very high degree.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$ and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

1.2.4 Generalization of Feynman diagrams

An essential difference between TGD and string models is the replacement of stringy diagrams with generalized Feynman diagrams obtained by gluing 3-D light-like surfaces (instead of lines) together at their ends represented as partonic 2-surfaces. This makes the construction of vertices very simple. The notion of number theoretic braid in turn implies discretization having also interpretation in terms of non-commutativity due to finite measurement resolution replacing anti-commutativity along stringy curves with anti-commutativity at points of braids. Braids can replicate at vertices which suggests an interpretation in terms of topological quantum computation combined with non-faithful copying and communication of information. The analogs of stringy diagrams have quite different interpretation in TGD: for instance, photons travelling via two different paths in double slit experiment are represented in terms of stringy branching of the photonic 2-surface.

1.2.5 Symplectic variant of QFT as basic building block of construction

The latest discovery related to the construction of M-matrix was the realization that a symplectic variant of conformal field theories might be a further key element in the concrete construction of n-point functions and M-matrix in zero energy ontology. Although I have known super-canonical (super-symplectic) symmetries to be fundamental symmetries of quantum TGD for almost two decades, I failed for some reason to realize the existence of symplectic QFT, and discovered it while trying to understand quite different problem - the fluctuations of cosmic microwave background! The symplectic contribution to the n-point function satisfies fusion rules and involves only variables which are symplectic invariants constructed using geodesic polygons assignable to the sub-polygons of n-polygon defined by the arguments of n-point function. Fusion rules lead to a concrete recursive formula for n-point functions and M-matrix in contrast to the iterative construction of n-point functions used in perturbative QFT.

1.3 Some general predictions of quantum TGD

TGD is consistent with the symmetries of the standard model by construction although there are definite deviations from the symmetries of standard model. TGD however predicts also a lot of new physics. Below just some examples of the predictions of TGD.

1. Fractal hierarchies meaning the existence of scaled variants of standard model physics is the basic prediction of quantum TGD. p-Adic length scale hypothesis predicts the possibility that elementary particles can have scaled variants with mass scales related by power of $\sqrt{2}$. Dark matter hierarchy predicts the existence of infinite number of scaled variants with same mass spectrum with quantum scales like Compton length scaling like $\hbar$.

2. TGD predicts that standard model fermions and gauge bosons differ topologically in a profound manner. Fermions correspond to light-like wormhole throats associated with topologically condensed CP_2 type extremals whereas gauge bosons correspond to fermion-antifermion states associated with the throats of wormhole contacts connecting two space-time sheets with opposite time orientation. The implication is that Higgs vacuum expectation value cannot contribute to fermion mass: this conforms with the results of p-adic mass calculations. Super-canonical quanta give dominating contribution to most hadron masses. These degrees of freedom correspond to those of hadronic string and should not reduce to QCD. They are also crucial for TGD variants of black holes.

3. The most fascinating applications of zero energy ontology are to quantum biology and TGD inspired theory of consciousness. Basic new element are negative energy photons making possible communications to the direction of geometric past. Here also dark matter hierarchy is involved in an essential manner.

4. In cosmology the mere imbeddability required for Robertson-Walker cosmology implies that critical and over-critical cosmologies are almost unique and characterized by their finite duration. The cosmological expansion is accelerating for them and there is no need to assume cosmological constant. Macroscopic quantum coherence of dark matter in astrophysical scales is a dramatic prediction and allows also to assign periods of accelerating expansion to quantum phase transition changing the value of gravitational Planck constant. The dark matter parts of astrophysical systems are predicted to be quantum systems.

5. The notion of generalized imbedding space and intriguing findings about inclusions of HFFs suggests that the physics of TGD Universe is universal

in the sense that it is possible to engineer a system able to mimic the physics of any consistent gauge theory. Kind of analog of Turing machine would be in question.

1.4 Relationship to super-strings and M-theory

There are arguments suggesting that the almost topological conformal field theory associated with quantum TGD has maximal $N = 4$ super-conformal symmetry with the inherent gauge group $SU(2) \times U(2)$ identified in terms of rotations and electro-weak symmetries acting on imbedding space spinors. The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2+2+2+2$ structure of the Cartan algebra, which together with the notions of the configuration space and generalized coset representation formed from super Kac-Moody and super-canonical algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches would be due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

2 Symmetries

Besides isometries of the imbedding space, the most important symmetries of TGD Universe are general coordinate invariance fixing completely the general mathematical structure of the theory and super-canonical and super Kac-Moody conformal symmetries allowing to understand the quantum number spectrum of physical states.

2.1 General Coordinate Invariance and Poincare invariance

The following considerations related to general coordinate invariance became somewhat obsolete after the emergence of the TGD as a generalized number theory vision which led to the realization that thanks to the non-determinism of Kähler action the dynamics nicely factors into cosmological and local dynamics.

General coordinate invariance raises heavy technical difficulties in the quantization of General Relativity. Similar technical difficulties are created by ordinary gauge invariance in gauge theories and by conformal invariance in string theories. This problem is encountered also in TGD approach. The integration over all possible $Diff^4$ equivalent 3-surfaces X^3 at the orbit of $X^4(Y^3)$ such that Y^3 belongs to light cone boundary is a poorly defined procedure. Nothing however forbids 'gauge fixing' by restricting configuration space integration to the lightcone boundary. In fact, configuration space integration *defined* in this manner is certainly a $Diff^4$ invariant procedure.

Classical non-determinism however destroys this naive picture. The huge vacuum degeneracy of the Kähler action suggests that one can have large numbers of degenerate absolute minima of the Kähler action obtained by gluing vacuum extremals to non-vacuum space-time surfaces and by performing a slight deformation. For a given Y^3 there are in general several absolute minimum 4-surfaces $X^4(Y^3)$ going through Y^3: typically $X^4(Y^3)$ is expected to suffer multi-furcations at some time values. This degeneracy forces the generalization of the concept of 3-surface by allowing also 3-surfaces, which are unions of Y^3 and minimal number of space like 3-surfaces X_i^3 at a particular space-time surface $X^4(Y^3)$ fixing this particular 4-surface uniquely. The mutual separations of X_i^3 and Y^3 are in general time like and in TGD inspired theory of consciousness these 3-surfaces provide a geometric model of thought as an N-snapshot simulation of the classical time development.

As far as configuration space integration is considered, *discrete* classical non-determinism of the Kähler action would not pose any problems of principle. Since the value of the Kähler function is same for all degenerate space-time surfaces $X^4(Y^3)$, the functional integration over the association sequences would reduce to a summation over the degenerate branches of space-time surfaces so that one can still reduce configuration space integration to the light cone boundary.

The description in terms of association sequences fails for CP_2 type extremals which are essentially four-dimensional objects and the attempts to reduce the configuration space integration to the light cone boundary become unpractical. In fact, very natural separation of local physics from light cone boundary occurs and by the crossing symmetry argument one can deduce S-matrix as the overlaps of the vacuum vacuum functional with zero energy states. Of course, this is kind of reduction is not even desirable unless one is interested in quantum cosmology.

The possibility of negative Poincare energies inspires the hypothesis that the total quantum numbers and classical conserved quantities of the Universe vanish. This view is consistent with experimental facts if gravitational energy is defined as a difference of Poincare energies of positive and negative energy matter. Space-time surface consists of pairs of positive and negative energy space-time sheets created at some moment from vacuum and branching at that moment. This allows to select X^3 uniquely and define $X^4(X^3)$ as the absolute minimum of Kähler action in the set of 4-surfaces going through X^3. These space-time sheets should also define uniquely the light like 7-surface $X_l^3 \times CP_2$, most naturally as the "earliest" surface of this kind. Note that this means that it become possible to assign a unique value of geometric time to the space-time sheet. This is of special importance in TGD inspired theory of consciousness since it makes possible to understand the connection between subjective experienced time (quantum jump sequence) and geometric time. As far as fractal Russian doll cosmology predicted by TGD is considered M^4 and M^4_+ options are more or less equivalent.

For M^4 option all Poincare transforms of surfaces $X^3_l \times CP_2$ are possible. It seems that the most natural identification for $X^3_l \subset M^4$ is as unions of arbitrary numbers of future and past directed light cone boundaries so that configuration space would decompose into a union of sectors labelled by the positions for the dips of the light cone boundaries. The exact Poincare invariance of the theory realized also at configuration space level means an enormous simplification and the earlier proposal for the definition of Diff^4 invariant M^4 translations becomes un-necessary. This definition can be however found from Appendix of [B3].

2.2 Super-symmetry at the space-time level

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X^3_l. Gauge symmetries are in question when the intersections of X^3_l with 7-D CDs X^7 are not changed. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced configuration space consisting of the light like 3-D CDs. The conformal symmetries in question imply a further degeneracy of the configuration space metric and effective metric 2-dimensionality of 3-surfaces as a consequence. These conformal symmetries are accompanied by $N = 4$ local super conformal symmetries defined by the solutions of the induced spinor fields.

If the notion of number theoretic spontaneous compactification [E2] makes sense, space-time surfaces can be also regarded as hyper-quaternionic 4-surfaces in M^8 having hyper-octonionic structure. Hyper-quaternionic and hyper-octonionic variants of ordinary conformal symmetries would act as hidden symmetries not manifest in $M^4 \times CP_2$ picture.

The solutions of the modified Dirac equation $D\Psi = 0$, define the modes which do not contribute to the Dirac determinant of the modified Dirac operator in terms of which the vacuum functional assumed to correspond to the exponent of the Kähler action is defined. Thus they define gauge super-symmetries. Usually D selects the physical helicities by the requirement that it annihilates physical states: now the situation is just the opposite. D^2 annihilates the generalized eigen states both at space-like and light like 3-surfaces. Hence the roles of the physical and non-physical helicities are switched. It is the generalized eigen modes of D with non-vanishing eigenvalues λ, which code for the physics whereas the solutions of the modified Dirac equation define super gauge symmetries.

At the space-like 3-surfaces associated with 7-D CDs the spinor harmonics of the configuration space satisfy the $M^4 \times CP_2$ counterpart of the massless Dirac equation so that non-physical helicities are eliminated in the standard sense at the imbedding space level. The righthanded neutrino does not generate an $N = 1$ space-time super-symmetry contrary to the long held belief.

2.3 Super-symmetry at the level of configuration space

The gamma matrices of the configuration space are defined as matrix elements of properly chosen operators between right-handed neutrino and second quantized induced spinor field at space-like boundaries X^3. These generators define the fermionic generators of what I call super-canonical algebra. The right handed neutrino can be replaced with any spinor harmonic of the imbedding space to obtain an extended super-algebra, which can be used to construct the physical states.

The requirement that super-generators vanish for the vacuum extremals requires that the modified Dirac operator D_+ or the inverse of D_- appearing in the matrix element of the "Hermitian conjugate" $S^- = (S^+)^\dagger$ of the super charge S^+. Here $\pm$ refers to the negative and positive energy space-time sheets meeting at X^3 or to the two maximally deterministic space-time regions separated by the causal determinant. The operators D_+ and D_-^{-1} are restricted to the spinor modes not annihilated by $D_\pm$. The super-generator generated by the covariantly constant right handed neutrino vanishes identically: a more rigorous argument showing that $N = 1$ global super symmetry is indeed absent.

If the configuration space decomposes into a union of sectors labelled by unions of light cones having dips at arbitrary points of M^4, the spinor harmonics can be assumed to define plane waves in M^4 and even possess well-defined four-momenta and mass squared values. Same applies to the super-canonical generators defined by their commutators. This means that the generators of the super-canonical algebra generated in this manner would possess well defined four-momenta and thus their action would change the mass of the state. Space-time super-symmetries would be absent. Similar argument applies to the Kac Moody algebras associated with the light like 3-D CDs if super-canonical Super Kac-Moody algebras provide dual representations of quantum states.

If the gist of these admittedly heuristic arguments is correct, they force to modify drastically the existing view about space-time super-symmetries. The problem how to break super-symmetry disappears since there is no space-time symmetry be broken down. Super-symmetries are realized as a spectrum generating algebra rather than symmetries in the standard sense.

2.3.1 Super-canonical algebra

Super-canonical algebras are most relevant for the construction of the configuration space geometry. The original belief was that they predict only cosmological effects seems to be incorrect since the new view about energy and non-determinism of Kähler action allow surfaces $X^3_l \times CP_2$ such that X^3_l is a light-like 3-surface of M^4_+ (union of boundaries of future and past directed light cones) as causal determinants.

The number theoretically inspired hypothesis is that the conformal weights of the super-canonical generators expressible in terms of zeros of Riemann Zeta (and perhaps also of polyzetas in case of bound states) take the role of complex coordinate of complex plane and super-canonical generators labelled by $SO(3) \times SU(3)$ quantum numbers take the role of primary fields on which super Kac-Moody conformal transformations act as gauge transformations. This would mean that super-conformal field theories would appear directly in the construction of vertices and propagators and that super-canonical algebra brings in additional completely new element. A more detailed discussion of this action can be found in [C7, B4].

It has taken a lot of effort to find a correct interpretation of the super-canonical algebra. It is now however clear that the super-canonical algebra decomposes to two sub-algebras corresponding to Ramond and NS type super generators which both come as two variants carrying quark *resp.* lepton number. N-S type generators super-symmetrize the function algebra of the configuration space whereas Ramond type generators super-symmetrize the Poisson algebra. The anti-commutators of both leptonic and quark like Ramond type super generators give components of the configuration space Kähler metric. The dichotomies Ramond-NS, SUSY-kappa, and Poisson algebra-function algebra are equivalent.

The light like surfaces $X_l^3 \subset M^4$ appearing in the definition of 7-D CDs are metrically 2-dimensional. Same applies to the light like 3-D CDs X_l^3, and to 3-surfaces in general by the degeneracy of the configuration space metric in the set of space-time sheets for which 3-D light like CDs and their tangent spaces coincide at $X^2 = X_l^3 \times X^7$. All relevant data about configuration space geometry is contained by the 2-D surfaces X^2.

This gives hopes of constructing unitary representations for a suitable Abelian extension of super-canonical group. The extension is not however ordinary central extension but the symplectic extension obtained by coupling the isometry generators of this group to a suitable multiple of the Kähler potential of the configuration space. This extension is *not* equivalent with the ordinary Kac Moody extension since the anomaly term is not proportional to the integer n labelling the generators. Furthermore, the global $U(1)$ acting as a central extension is replaced by a δM_+^4-local $U(1)$ defined by the function algebra of the light cone boundary. The central extension term is defined uniquely at the maximum of Kähler function by requiring that configuration space Hamiltonians vanish at this point.

1. Are super-canonical conformal weights expressible in terms of seros of Riemann zeta and polyzetas?

If the exponents p^{iy_k} for the zeros $z = 1/2 + iy_k$ of Zeta are algebraic numbers, linear combinations of zeros of zeta or at least $s = 1/2 + \sum_k n_k y_k$ are good candidates for radial conformal weights of super-canonical algebra. The reason is that r^s is algebraic number for r rational.

Riemann poly-zetas $\zeta_n(\Delta_1, ..., \Delta_n)$ could allow to generalize the notion of binding energy to that of binding conformal weight. In this case zeros form a continuum so that the set of points $(\Delta_1, ..., \Delta_n) = \zeta_n^{-1}(z = \xi^1/\xi^2)$ forms a n-1 complex dimensional surface in C^n. Completely symmetrized polyzetas are expressible using products of Riemann Zetas for arguments which are sums of arguments for polyzeta. If Δ_i are linear combinations of zeros of Zeta, polyzeta involves Riemann Zeta only for arguments which are sums of zeros of ζ. Symmetrized polyzeta is non-vanishing when Δ_i are non-trivial zeros of Zeta but vanishes for trivial zeros at $\Delta_i = -2n_i$. Also the zeros of symmetrized polyzeta would have interpretation in terms of quantum criticality.

An interesting question is whether ζ_n has a discrete subset of zeros for which p^{Δ_i} is algebraic number for all primes p and Δ_i. This could be the case. For instance, suitable linear combinations of zeros of ζ define zeros of polyzeta. For instance, $(a,b) = (s_1, s_1 - s_2)$ for any pair of zeros of zeta is zero of $P_2(a,b) = \zeta(a)\zeta(b) - \zeta(a+b)$ whereas $(a_1,a_2,a_3) = (s_1, s_2 - s_1, \overline{s}_2 - s_1)$ defines a zero of

$$\begin{aligned} &P_3(a_1,a_2,a_3) = 2\zeta(a_1+a_2+a_3) + \zeta(a_1)P_2(a_2,a_3) + \zeta(a_2)P_2(a_3,a_1) \\ &+\zeta(a_3)P_2(a_1,a_2) - 2\zeta(a_1)\zeta(a_2)\zeta(a_3) \end{aligned}$$

for any pair (s_1, s_2) of zeros of ζ.

The conditions state that all P_m:s, $m < n$ in the decomposition of P_n vanish separately. Besides this one a_k, say $a_1 = s_1$ must correspond to a zero of ζ. Same is true for the sum σa_k and sub-sums involving a_1. The number of conditions increases rapidly as n increases. In the case of P_4 the three triplets (a_1, a_i, a_j) must be of same form as $n = 3$ case and this allows only the trivial solution with say $a_4 = 0$. Thus it would seem that only $n = 2$ and $n = 3$ allow non-trivial solutions for which bound state conformal weights are expressible in terms of differences of zeros of Riemann ζ. What is nice that the linear combinations of these conformal multi-weights give total conformal weights which are linear combinations of zeros of zeta.

The special role of 2- and 3-parton states brings unavoidably in mind mesons and baryons and the fact that hadrons containing larger number of valence quarks have not yet been identified experimentally.

2. In what sense conformal confinement is realized?

The net values of conformal weights are real for physical states in the zero energy ontology if the imaginary part of super-canonical conformal weight is a conserved quantum number. If conformal confinement holds true at single particle level then physical particles would have vanishing conformal weights. In particular, ordinary baryons and mesons have real conformal weights and could not therefore correspond to the 2- and 3-parton bound states having same spectrum of net conformal weights as partons.

One must however take the notion of conformal confinement very critically. The point is that the one-dimensional logarithmic plane waves $x^{1/2+iy}$ have unitary inner product with respect to the scaling invariant inner product defined by the integration measure dx/x. For this inner product, the real part of the conformal weight should be 1/2 as it indeed is for the solutions of the conditions.

If this interpretation is correct, then hadrons would represent states with non-vanishing imaginary part of super-canonical conformal weight.

If one accepts complex conformal weights one must have some physical interpretation for them. The identification of conjugation of zeros of zeta as charge conjugation does not look promising since it would not leave neutral pion invariant. Of course, critical configurations with real conformal weight are possible at least formally and would correspond to trivial zeros $s_2 = -2n$ of ζ but s_1 arbitrary zero. These configuration would not however define logarithmic plane waves.

Laser physics might come in rescue here. So called phase conjugate photons are known to behave differently from photons. I have already proposed that all particles possess phase conjugates in TGD Universe. Phase conjugation is identified as reversal of time arrow mapping positive energy particles to negative energy particles. At space-time level this would mean an assignment of time orientation to space-time sheet. This is consistent with the fact that energy momentum complex consists of vector currents rather than forming a tensor. The implication is that in S-matrix positive energy particles travelling towards geometric future are not equivalent with negative energy particle travelling towards geometric past. This is essential for the notions like remote metabolism and time mirror mechanism.

The precise definition of phase conjugation at quantum level has remained obscure. The identification of phase conjugation as conjugation for the zeros of Zeta looks however very natural. This would suggest that the imaginary part of complex conformal weight defines an additional momentum like quantum number for elementary particles so that for complex conformal weights one could assign to the particle a definite time orientation. This would give precise meaning for the arrow of time in geometric sense at fundamental level.

Number theoretical considerations suggest that for a given algebraic extension of p-adic numbers only a finite number of zeros of Zeta are involved. This would mean that elementary particles can have linear combinations $\sum_k n_k y_k$ of imaginary parts of zeros of zeta as an additional quantum number conserved in reaction vertices. If one allows also trivial zeros as conformal weights there must be correlation between SO(3) quantum numbers of δM^4_+ Hamiltonians and real conformal weight to guarantee orthonormalization [B2]. The possibility that ordinary elementary particles correspond to real conformal weights cannot be excluded.

According to the considerations of [C7], super-canonical conformal weights which are zeros of zeta would characterize quantum critical states able to decay to states which correspond to different values of Planck constant. Geometrically this corresponds to leakage between different sectors of imbedding space obtained by gluing together imbedding spaces corresponding to different values of Planck constant.

2.3.2 Super Kac-Moody algebra associated with 3-D causal determinants

3-D light like CDs are metrically 2-dimensional and give rise to a conformal symmetry in degrees of freedom transversal to the light like coordinate. The bosonic generators of the conformal Kac Moody algebra correspond to the deformations of the 2-surface X^2 defining a particular light-like elementary particle horizon or light-like boundary of a space-time sheet X^3_l. If X^3 is surface of M^4_+, X^2 is analogous to a light source generating an expanding light front and in this case spherical topology is the only stable topology. The conformal structure of 2-surface defines the local function algebra multiplying the generators of the imbedding space isometries defining deformations of this algebra. Virasoro algebra corresponds to the deformations, which map X^2 to itself or move it along the 3-dimensional light like surface X^3_l defined by it. The analogs in case of $\delta M^4_+ = S^2 \times R_+$ are conformal symmetries of S^2 and radial conformal transformations. Fermionic super charges contribute in $M^4 \times SO(3,1) \times SU(3)$ degrees of freedom and one can define Kac Moody generators as bilinears of the fermionic generators (essentially sigma matrices of configuration space).

Also the super Kac-Moody algebra decomposes to two sub-algebras corresponding to super generators carrying lepton and quark number. Now Ramond-NS dichotomy corresponds to Poisson-function algebra dichotomy restricted to the Hamiltonians representing isometries of $\delta M^4_+ \times CP_2$ localized with respect to X^2 complex coordinate. 7–3 duality suggests that the anti-commutators of quark generators give configuration space Kähler metric but in different coordinate system. 7–3 duality would thus correspond to a coordinate transformation relating preferred coordinates of the configuration space. Quantum measurement theory and explicit construction of quantum states [F2] suggest a more plausible interpretation: Kac-Moody algebra parameterizes zero modes identifiable as classical degrees of freedom and in 1-1 correspondence with quantal degrees of freedom contributing to the configuration space metric. Quantum entanglement would induce the "coordinate map" between the two degrees of freedom.

Both the vector fields of color and $M^4 \times SO(3,1)$ local transformations contribute if all light like 7-surfaces $X^3_l \times CP_2 \subset H$ are allowed (electroweak symmetries have fermionic realization). The restriction of X^3_l to the unions of $\delta M^4_\pm$ means that configuration space decomposes into union of M^4 translated copies so that only local $M^4 \times SU(3)$ acts as bosonic Kac Moody symmetries.

Since $SU(3)$ does not actually correspond to a symmetry algebra now and since $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection, one could argue that the $U(2)$ generators of the bosonic $SU(3)$ Kac-Moody algebra should be identified as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than composite of fermions. Whether also fermionic $SU(3)$ generators should be interpreted in the same manner is an open question.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surfaces X^2 correspond to the intersection $X_l^3 \times X^7$ of 3-D and 7-D Cds. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

By quantum classical correspondence one expects that Virasoro algebra associated with super Kac-Moody algebra acts on the conformal weights of the super-canonical representations as conformal transformations and the generators of the super-canonical algebra can be regarded

2.3.3 Do NS and Ramond representation for the super Kac-Moody algebra combine to a representation of single larger algebra?

In TGD context NS and Ramond representations belong automatically to a larger algebraic structure defined by the quark like and leptonic oscillator operators. One could even wonder whether one could combine these structures to a larger algebra. Indeed, in [B4] it was noticed that one could combine NS and Ramond type Super Virasoros to single larger Super Virasoro related by the index scaling $n \rightarrow 2n$ to Ramond type Super Virasoro. The motivation for this operation was the possibility to formulate elegantly the vertices of the Yang Mills type quantum field theory for Super Virasoro representations.

The extension is in practice almost trivial and means the introduction of new Kac Moody and Virasoro generators with half odd integer conformal weight. These generators are introduced automatically for the dynamical Virasoro for which these generators are quadratic in the fermionic generators by allowing the appearance of both half odd integer and integer fermionic generators in the definition of Kac Moody generators and Super Virasoro generators. Half-odd integer bosonic generators carry quantum numbers of leptoquark and do not have interpretation as geometric transformations. Half odd integer generators would create states having non-vanishing lepton and quark numbers and would not contribute to the p-adic thermodynamics. This kind of scenario works only provided the configuration space gamma matrices with integer and half odd integer indices anti-commute. This is certainly the case by quarks $\leftrightarrow NS$ and leptons $\leftrightarrow$ Ramond correspondences. One can also construct the ground states using *both* $n = 0$ Ramond type fermionic generators and $n = 1/2$ leptonic generators as well as linear and local bilinear fermionic invariants having $n = 0$. This is in fact all what might be needed to construct vertex operators elegantly. Hamiltonian evolution for L_0 in turn is expected to define S-matrix (L_0 contains automatically interaction terms analogous to those associated with Yang Mills action). Already the p-adic mass calculations suggested that NS and Ramond type conditions are satisfied separately and this is indeed implied by the fact that the corresponding super-generators carry different fermion numbers.

In p-adic thermodynamics one must pose the additional condition that the total fermion number associated with the operators G_n and $G_n^\dagger$ creating thermal excitations from the ground state vanishes separately in each sector. This implies automatically the chirality condition of p-adic mass calculations stating that in each sector physical states contain an even number of super generators G_n. There are reasons to expect that partition functions, the crucial element of the p-adic mass calculations, are not changed. The reason is that the representation for the generalized algebra provides also a representation for the ordinary Super Virasoro. It is obtained by defining Hermitian Super Virasoro generator $\hat{G}$ as $\hat{G} = G + G^\dagger$. L_n:s are same for these two kinds of representations. In [F2] it is shown that the partition functions indeed remain unchanged if in each sector of the Super Virasoro only fermion number zero operators create $n > 0$ excitations.

One immediate prediction of the modified Super Virasoro, is the existence of elementary particles with fermion number higher than one. This result means the extension of Super Virasoro structure to the level of many fermion states. These states are probably highly unstable.

2.4 Comparison with string models

2.4.1 Super conformal symmetries in TGD and string theory

Super-conformal symmetries are fundamental symmetries of super string models, and TGD provides a realization of these symmetries in case of 3-dimensional objects. What came as a surprise that there is handful of super-conformal symmetries. 7-D causal determinants $\delta M^4_\pm \times CP_2$ allow both radial and transversal super-conformal symmetries. Same applies to the light like 3-D CDs X^3_l and this boils down to the degeneracy of the configuration space metric in the sense that two CDs X^3_l with same intersection with 7-D CDs are metrically equivalent. This in turn implies the effective 2-dimensionality of also space-like 3-surfaces.

Also the interior of space-time surface seems to allow super-conformal symmetries acting as super gauge symmetries of the modified Dirac action. The most important implication Quaternion conformal invariance and its Abelian version would be the corresponding super-symmetries perhaps realized only for certain representative 4-surfaces in the gauge equivalence classes of 4-surfaces having X^3_l having given intersections with 7-D CDs.

That any solution of the modified Dirac operator defines a super symmetry and it is generalized eigen states of the modified Dirac operator which correspond to physical states. This means a profound deviation from both string models and quantum field theories since the notion of virtual particle disappears from the conceptual arsenal. Perhaps the most important implication is that for generalized Feynman rules diagrams with loops are equivalent with tree diagrams and there is no sense in making loop summations. Generalized Feynman diagrams are more akin to braid diagrams than ordinary Feynman diagrams. This means enormous simplification of the theory.

Super-canonical symmetry is something totally new and the generalization of coset construction motivated by the commutatitivity of super Kac Moody and super-canonical Virasoro algebras leads to the requirement that it is the differences of super-canonical and super Kac-Moody Virasoro generators which annihilate physical states. By 7-3 duality the conformal central charge is vanishing. In string model the requirement that the central charge vanishes leads to the critical dimension.

Effective 2-dimensionality means that super-conformal field theory having $N = 4$ complex supersymmetries as gauge symmetries can be used to construct the vertices of the theory. The surfaces X^2 are analogous to orbits of closed strings. On the other hand, 2-dimensional objects are fundamental rather than 1-dimensional objects so that the conformal invariance in TGD is more akin to that of 2-D critical statistical systems than that of strings. The interpretation of $N = 4$ complex super-symmetries as pure gauge super symmetries means decisive different between TGD and string models reflecting as the absence of sparticles.

2.4.2 The super generators G are not Hermitian in TGD!

An important difference between TGD based and the usual Super Virasoro representations is that the Super Virasoro generator G cannot Hermitian in TGD. The reason is that configuration space gamma matrices possess a well defined fermion number. The hermiticity of the configuration space gamma matrices Γ and of the Super Virasoro current G could be achieved by posing Majorana conditions on the second quantized H-spinors. Majorana conditions can be however realized only for space-time dimension $D\, mod\, 8 = 2$ so that super string type approach does not work in TGD context. This kind of conditions would also lead to the non-conservation of baryon and lepton numbers.

An analogous situation is encountered in super-symmetric quantum mechanics, where the general situation corresponds to super symmetric operators S, $S^\dagger$, whose anti-commutator is Hamiltonian: $\{S, S^\dagger\} = H$. One can define a simpler system by considering a Hermitian operator $S_0 = S + S^\dagger$ satisfying $S_0^2 = H$: this relation is completely analogous to the ordinary Super Virasoro relation $GG = L$. On basis of this observation it is clear that one should replace ordinary Super Virasoro structure $GG = L$ with $GG^\dagger = L$ in TGD context.

It took a long time to realize the trivial fact that $N = 2$ super-symmetry is the standard physics counterpart for TGD super symmetry. $N = 2$ super-symmetry indeed involves the doubling of super generators and super generators carry $U(1)$ charge having an interpretation as fermion number in recent context. The so called short representations of $N = 2$ super-symmetry algebra can be regarded as representations of $N = 1$ super-symmetry algebra.

Configuration space gamma matrix Γ_n, $n > 0$ corresponds to an operator creating fermion whereas Γ_n, $n < 0$ annihilates antifermion. For the Hermitian conjugate $\Gamma_n^\dagger$ the roles of fermion and antifermion are interchanged. Only the anti-commutators of gamma matrices and their Hermitian conjugates are non-vanishing. The dynamical Kac Moody type generators are Hermitian and are constructed as bilinears of the gamma matrices and their Hermitian conjugates and, just like conserved currents of the ordinary quantum theory, contain parts proportional to $a^\dagger a$, $b^\dagger b$, $a^\dagger b^\dagger$ and ab (a and b refer to fermionic and antifermionic oscillator operators). The commutators between Kac Moody generators and Kac Moody generators and gamma matrices remain as such.

For a given value of m G_n, $n > 0$ creates fermions whereas G_n, $n < 0$ annihilates antifermions. Analogous result holds for $G_n^\dagger$. Virasoro generators remain Hermitian and decompose just like Kac Moody generators do. Thus the usual anti-commutation relations for the super Virasoro generators must be replaced with anti-commutations between G_m and $G_n^\dagger$ and one has

$$\begin{array}{c} \{G_m, G_n^\dagger\} = 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \ , \\ \{G_m, G_n\} = 0 \ , \\ \{G_m^\dagger, G_n^\dagger\} = 0 \ . \end{array} \tag{1}$$

The commutators of type $[L_m, L_n]$ are not changed. Same applies to the purely kinematical commutators between L_n and $G_m/G_m^\dagger$.

The Super Virasoro conditions satisfied by the physical states are as before in case of L_n whereas the conditions for G_n are doubled to those of G_n, $n < 0$ and $G_n^\dagger$, $n > 0$.

3 Does the modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

3.1 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced. In particular, the following problems are discussed.

1. Try to guess general formula for the spectrum of the modified Dirac operator and for super-canonical conformal weights by assuming that the eigenvalues are expressible in terms of the data assignable to the two kinds of of number theoretical braids and that the product of vacuum functional expressible as exponent of Kahler function and of the exponent of Chern-Simons action is identifiable as Dirac determinant expressible as product of M^4 and CP_2 parts. Since Kähler function is isometry invariant only the Dirac determinant defined by M^4 braid can contribute to it. Chern-Simons action is not isometry invariant and can be identified as the Dirac determinant associated with CP_2 braid.

2. Try to understand whether the zeta functions involved can be identified as Riemann Zeta or some zeta coding geometric data about partonic 2-surface. Try to understand whether the assignment of a fixed prime p to

a partonic 2-surface implies that the zeta function is actually an analog for basic building block of Riemann Zeta.

3.1.1 Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z,\overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or a more general principle selecting preferred extremals as Bohr orbits [E2].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

3.1.2 Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k_\alpha} L_K \ . \end{aligned} \tag{2}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R}\Gamma^k T^{\alpha}_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{3}$$

having a vanishing covariant divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T^{\alpha}_l \Gamma^l \Psi \tag{4}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R}\Gamma^k T^{\alpha}_l \Gamma^l D_\alpha \Psi \ . \tag{5}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_\alpha \Psi &= 0 \ , \\ \hat{\Gamma}^{\alpha} &= T^{\alpha}_l \Gamma^l \ . \end{aligned} \tag{6}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^{\alpha} D_\alpha \Psi \ . \tag{7}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^{\mu} = 0 \tag{8}$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

3.1.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective n metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T^k_\mu T_{\nu k} \ . \qquad (9)$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

3.1.4 Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [E2]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X^3_l, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the canonical currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-canonical algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

3.2 The association of the modified Dirac action to Chern-Simons action and explicit realization of super-conformal symmetries

Super Kac-Moody symmetries should correspond to solutions of modified Dirac equation which are in some sense holomorphic. The discussion below is based on the same general ideas but differs radically from the previous picture at the level of details. The additional assumption inspired by the considerations of this section is that the action associated with the partonic 3-surfaces is non-singular and therefore Chern-Simons action for the induced Kähler gauge potential.

This means that TGD is at the fundamental level almost-topological QFT: only the light-likeness of the partonic 3-surfaces brings in the induced metric and gravitational and gauge interactions and induces the breaking of scale and super-conformal invariance. The resulting theory possesses the expected super Kac-Moody and super-canonical symmetries albeit in a more general form than suggested by the considerations of this section. A connection of the spectrum of the modified Dirac operator with the zeros or Riemann Zeta is suggestive and provides support for the earlier number theoretic speculations concerning the spectrum of super-canonical conformal weights. One can safely say, that if this formulation is correct, TGD could not differ less from a physically trivial theory.

3.2.1 Zero modes and generalized eigen modes of the modified Dirac action

Consider net the zero modes and generalized eigen modes for the modified Dirac operator.

1. The modified gamma matrices appearing in the modified Dirac equation are expressible in terms of the Lagrangian density L assignable to the light-like partonic 3-surface X^43_l as

$$\hat{\Gamma}^{\alpha} = \frac{\partial L}{\partial_{\alpha} h^k}\Gamma_k \ , \tag{10}$$

where Γ_k denotes gamma matrices of imbedding space. The modified Dirac operator is defined as

$$D = \hat{\Gamma}^{\alpha} D_{\alpha} \ , \tag{11}$$

where D_{α} is the covariant derivative defined by the induced spinor connection. Modified gamma matrices satisfy the condition

$$D_{\alpha}\hat{\Gamma}^{\alpha} = 0 \tag{12}$$

if the field equations associated with L are satisfied. This guarantees that one indeed obtains the analog of the massless Dirac equation. Zero modes of the modified Dirac equation should define the conformal supersymmetries.

2. The generalized eigenvalues and eigen solutions of the modified Dirac operator are defined as

$$\begin{aligned} D\Psi &= \lambda N \Psi \ , \\ N &= n^k \Gamma_k \ . \end{aligned} \tag{13}$$

Here n^k denotes a light-like vector which must satisfy the integrability condition

$$\left[D, n^k \Gamma_k\right] = 0 \ . \tag{14}$$

if the analog $D^2\Psi = 0$ for the square of massless Dirac equation is to hold true. n^k should be determined by the field equations associated with L somehow and commutativity condition could fix n more or less uniquely.

If the commutativity condition holds true then any generalized eigen mode Ψ_{λ} gives rise to a zero mode as $\Psi = N\Psi_{\lambda}$. One can add to a given non-zero mode any superposition of zero modes without affecting the generalized eigen mode property.

The commutativity condition can be satisfied if the tangent space at each point of X^4 contains preferred plane M^2 guaranteing $HO-H$ duality and having interpretation as a preferred plane of non-physical polarizations. In this case n can be chosen to be constant light-like vector in M^2.

3. The hypothesis is that Kähler function is expressible in terms of the Dirac determinant of the modified Dirac operator defined as the product of the generalized eigenvalues. The Dirac determinant must carry information about the interior of the space-time surface determined as preferred extremal of Kähler action or (as the hypothesis goes) as hyper-quaternionic or co-hyper-quaternionic 4-surface of M^8 defining unique 4-surface of $M^4 \times CP_2$. The assumption that X^3_L is light-like brings in an implicit dependence on the induced metric. The simplest but non-necessary assumption is that n^k is a light-like vector field tangential to X^3_l so that the knowledge of X^3_l fixes completely the dynamics.

4. If the action associated with the partonic light-like 3-surfaces contains induced metric, the field equations become singular and ill-defined unless one defines the field equations at X^3_l via a limiting procedure and poses additional conditions on the behavior of Ψ at X^3_l. Situation changes if the action does not contain the induced metric. The classical field equations are indeed well-defined at light-like partonic 3-surfaces for Chern-Simons action for the induced Kähler gauge potential

$$L = L_{C-S} \quad = \quad k\epsilon^{\alpha\beta\gamma}J_{\alpha\beta}A_\gamma \ . \tag{15}$$

   One obtains the analog of WZW model with gauge field replaced with the induced Kähler form. This action does not depend on the induced metric explicitly so that in this sense a topological field theory results. There is no dependence on M^4 gamma matrices so that local Lorentz transformations act as super-conformal symmetries of both classical field equations and modified Dirac equation and $SL(2,C)$ defines the analog of the $SU(2)$ Kac-Moody algebra for $N=4$ SCA.

The facts that the induced metric is light-like for X^3_l, that the modified Dirac equation contains information about this and therefore about induced metric, and that Dirac determinant is the product of the non-vanishing eigen values of the modified Dirac operator, imply the failure of topological field theory property at the level of Kähler function identified as the logarithm of the Dirac determinant.

A more complicated option would be that the modified Dirac action contains also interior term corresponding to the Kähler action. This alternative would break super-conformal symmetries explicitly and almost-topological QFT property would be lost. This option is not consistent with the idea that quantum-classical correspondence relates the partonic dynamics at X^3_l with the classical dynamics in the interior of space-time providing first principle justification for the basic assumptions of the quantum measurement theory.

The classical field equations defined by L_{C-S} read as

$$D_\mu \frac{\partial L_{C-S}}{\partial_\mu h^k} \quad = \quad 0 \ ,$$

$$\frac{\partial L_{C-S}}{\partial_\mu h^k} = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k)\right] \quad . \tag{16}$$

From the explicit form of equations it is obvious that the most general solution corresponds to a X_l^3 with at most 2-dimensional CP_2 projection.

Although C-S action vanishes, the color isometry currents are in general non-vanishing. One can assign currents also to super-Kac Moody and super-canonical transformations using standard formulas and the possibility that the corresponding charges define configuration space Hamiltonians and their super-counterparts must be considered seriously.

Suppose that the CP_2 projection is 2-dimensional and not a Lagrange manifold. One can introduce coordinates for which the coordinates for X^2 are same as those for CP_2 projection. For instance, complex coordinates $(z, \overline{z})$ of a geodesic sphere could be used as local coordinates for X^2. One can also assign one M^4 coordinate, call it r, with M^4 projection X^1 of X_l^3. Locally this coordinate can be taken to be one of the standard M^4 coordinates. The remaining five H-coordinates can be expressed in terms of $(r, z, \overline{z})$ and light-likeness condition boils down to the vanishing of the metric determinant:

$$det(g_3) = 0 \ . \tag{17}$$

All diffeomorphisms of H respecting the light-likeness condition are symmetries of the solution ansatz.

Consider some special cases serve as examples.

1. The simplest situation results when X_l^4 is of form $X^1 \times X^2$, where X^1 is light-like random curve in M^4 as for CP_2 type vacuum extremals. In this case light-likeness boils down to Virasoro conditions with real parameter r playing the role analogous to that of a complex coordinate: this conformal symmetry is dynamical and must be distinguished from conformal symmetries assignable to X^2. A plausible guess is that light-likeness condition quite generally reduces to the classical Virasoro conditions.

2. A solution in which CP_2 projection is dynamical is obtained by assuming that for a given value of M^4 time coordinate CP_2- and M^4- projections are one-dimensional curves. For instance, CP_2 projection could be the circle $\Theta = \Theta(m^0 \equiv t)$ whereas M^4 projection could be the circle $\rho = \sqrt{x^2+y^2} = \rho(m^0)$. Light-likeness condition reduces to the condition $g_{tt} = 1 - R^2\partial_t\Theta^2 - \partial_t\rho^2 = 0$.

3.2.2 Classical field equations for the modified Dirac equation defined by Chern-Simons action

The modified Dirac operator is given by

$$D = \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k D_\mu$$

$$
\begin{aligned}
&= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\
\hat{\epsilon}^{\alpha\beta\gamma} &= \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ .
\end{aligned} \tag{18}
$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric. The operator is non-trivial only for 3-surfaces for which CP_2 projection is 2-dimensional non-Lagrangian sub-manifold. The modified Dirac operator reduces to a one-dimensional Dirac operator

$$
D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{19}
$$

The solutions of the modified Dirac equation are obtained as spinors which are covariantly constant with respect to the coordinate r:

$$
D_r\Psi = 0 \ . \tag{20}
$$

Non-vanishing spinors $\Psi_1 = \partial_r\Psi$ satisfying $\Gamma_r\Psi_1 = 0$ are not possible. Ψ defines super-symmetry for the generalized eigen modes if the additional condition

$$
\Psi = N\Psi_0 \tag{21}
$$

is satisfied. The interpretation as super-conformal symmetries makes sense if the Fourier coefficients of zero modes and their conjugates are anticommuting Grassmann numbers. The zero modes which are not of this form do not generate super-conformal symmetries and might correspond to massless particles. TGD based vision about Higgs mechanism suggest the interpretation of n^k as a non-conserved gravitational four-momentum whose time average defines inertial four-momentum of parton. The sum of the partonic four-momenta would be identified as the classical four-momentum associated with the interior of the space-time sheet.

The covariant derivatives D_α involve only CP_2 spinor connection and the metric induced from CP_2. D_r involves CP_2 spinor connection unless X^3_l is of form $X^1 \times X^2 \subset M^4 \times CP_2$. The eigen modes of D correspond to the solutions of

$$
D\Psi = \lambda N\Psi \tag{22}
$$

The first guess is that $N = n^k\gamma_k$ corresponds to the tangential light-like vector $n^k = \Phi\partial_r h^k$ where Φ is a normalization factor which can depend on position.

The obvious objection is that with this assumption it is difficult to understand how Dirac determinant can correspond to an absolute extremum of Kähler action for 4-D space-time sheet containing partonic 3-surfaces as causal determinants ($\sqrt{g_4} = 0$). However, if one can select a unique M^4 time coordinate, say as that associated with the rest system for the average four-momentum defined as Chern-Simons Noether charge, then one can assign to n^k a unique dual obtained by changing the sign of its spatial components. The condition that this vector is tangential to the 4-D space-time sheet would provide information about the space-time sheet and bring in 4-dimensionality. At this stage one must however leave the question about the choice of n^k open.

One should be able to fix Φ apart from overall normalization. First of all, the requirement that zero modes defines super symmetries implies the condition $[D, n^k\Gamma_k]\Psi = 0$ for zero modes. This condition boils down to the requirement

$$D_r(\Phi\partial_r h^k\Gamma_k)\Psi \quad = \quad 0 \ . \tag{23}$$

This in turn boils down to a condition

$$D_r\partial_r h^k + \frac{\partial_r\Phi}{\Phi}\partial_r h^k \quad = \quad 0 \ . \tag{24}$$

These conditions in turn guarantee that the condition

$$D_r(h_{kl}\partial_r h^k\partial_r h^l) \quad = \quad 0 \tag{25}$$

implied by the light-likeness condition are satisfied. Since Φ is determined apart from a multiplicative constant from the light-likeness condition the system is internally consistent. The conditions above are not general coordinate invariant so that the coordinate r must correspond to a physically preferred coordinate perhaps defined by the conditions above.

One can express the eigenvalue equation in the form

$$\begin{aligned}
\partial_r\Psi &= \lambda O\Psi \ , \\
O &= (\hat{\Gamma}^r)^{-1}N \ , \\
(\hat{\Gamma}^r)^{-1} &= \frac{\hat{\Gamma}^r}{a^k a^l h_{kl}} \ , \ \hat{\Gamma}^r \equiv a^k\Gamma_k \ .
\end{aligned} \tag{26}$$

This equation defines a flow with r in the role of a time parameter. The solutions of this equation can be formally expressed as

$$\Psi(r,z,\overline{z}) \quad = \quad Pe^{\lambda\int O(r,z,\overline{z})dr}\Psi_0(z,\overline{z}) \ . \tag{27}$$

Here P denotes the ordered exponential needed because the operators $O(r, z\overline{z})$ need not commute for different values of r.

3.2.3 Can one allow light-like causal determinants with 3-D CP_2 projection?

The standard quantum field theory wisdom would suggest that light-like partonic 3-surfaces which are extremals of the Chern-Simons action correspond only to what stationary phase approximation gives when vacuum functional is the product of exponent of Kähler function resulting from Dirac determinant and an imaginary exponent of Chern-Simons action whose coefficient is proportional to the central charge of Kac-Moody algebras associated with CP_2 degrees of freedom.

One cannot exclude the possibility that 3-D light-like causal determinants might be required by the general consistency of the theory. The identification of the exponent of Kähler function as Dirac determinant remains a viable hypothesis for this option. "Off mass shell" breaking of super-conformal symmetries is implied since modified Dirac equation implies the conservation of super conformal currents only when CP_2 projection is at most 2-dimensional.

3.2.4 Some problems of TGD as almost-topological QFT and their resolution

There are some problems involved with the precise definition of the quantum TGD as an almost-topological QFT at the partonic level and the resolution of these problems leads to an unexpected connection between cosmology and parton level physics.

1. Three problems

The proposed view about partonic dynamics is plagued by three problems.

1. The definition of supercanonical and super-Kac-Moody charges in M^4 degrees of freedom poses a problem. These charges are simply vanishing since M^4 coordinates do not appear in field equations.

2. Classical field equations for the C-S action imply that this action vanishes identically which would suggest that the dynamics does not depend at all on the value of k. The central extension parameter k determines the over-all scaling of the eigenvalues of the modified Dirac operator. $1/k$-scaling occurs for the eigenvalues so that Dirac determinant scales by a finite power k^N if the number N of the allowed eigenvalues is finite for the algebraic extension considered. A constant $Nlog(k)$ is added to the Kähler function and its effect seems to disappear completely in the normalization of states.

3. The general picture about Jones inclusions and the possibility of separate Planck constants in M^4 and CP_2 degrees of freedom suggests a close symmetry between M^4 and CP_2 degrees of freedom at the partonic level. Also in the construction of the geometry for the world of classical worlds the symplectic and Kähler structures of both light-cone boundary and

CP_2 are in a key role. This symmetry should be somehow coded by the Chern-Simons action.

2. A possible resolution of the problems

A possible cure to the above described problems is based on the modification of Kähler gauge potential by adding to it a gradient of a scalar function Φ with respect to M^4 coordinates.

1. This implies that super-canonical and super Kac-Moody charges in M^4 degrees of freedom are non-vanishing.

2. Chern-Simons action is non-vanishing if the induced CP_2 Kähler form is non-vanishing. If the imaginary exponent of C-S action multiplies the vacuum functional, the presence of the central extension parameter k is reflected in the properties of the physical states.

3. The function Φ could code for the value of $k(M^4)$ via a proportionality constant

$$\Phi = \frac{k(M^4)}{k(CP_2)}\Phi_0 \ , \tag{28}$$

   Here $k(CP_2)$ is the central extension parameter multiplying the Chern-Simons action for CP_2 Kähler gauge potential. This tricks does just what is needed since it multiplies the Noether currents and super currents associated with M^4 degrees of freedom with $k(M^4)$ instead of $k(CP_2)$.

The obvious breaking of $U(1)$ gauge invariance looks strange at first but it conforms with the fact that in TGD framework the canonical transformations of CP_2 acting as $U(1)$ gauge symmetries do not give to gauge degeneracy but to spin glass degeneracy since they act as symmetries of only vacuum extremals of Kähler action.

3. How to achieve Lorentz invariance?

Lorentz invariance fixes the form of function Φ uniquely as the following argument demonstrates.

1. Poincare invariance would be broken in any case for a given light-cone in the decomposition $CH = \cup_m CH_m$ of the configuration space to sub-configuration spaces associated with light-cones at various locations of M^4 but since the functions Φ associated with various light cones would be related by a translation, translation invariance would not be lost.

2. The selection of Φ should not break Lorentz invariance. If Φ depends on the Lorentz proper time a only, this is partially achieved. Momentum

currents would be proportional to m^k and become light like at the boundary of the light-cone. This fits very nicely with the interpretation that the matter emanates from the tip of the light cone in Robertson-Walker cosmology.

Lorentz invariance poses even stronger conditions on Φ.

1. Partonic four-momentum defined as Chern-Simons Noether charge is definitely not conserved and must be identified as gravitational four-momentum whose time average corresponds to the conserved inertial four-momentum assignable to the Kähler action [D3, D5]. This identification is very elegant since also gravitational four-momentum is well-defined although not conserved.

2. Lorentz invariance implies that mass squared is constant of motion. Hence it is interesting to look what expression for Φ results if the gravitational mass defined as Noether charge for C-S action is conserved. The components of the four-momentum for Chern-Simons action are given by

$$P^k = \frac{\partial L_{C-S}}{\partial(\partial_\alpha a)} m^{kl}\partial_{m^l} a \quad .$$

   Chern-Simons action is proportional to $A_\alpha = A_a \partial_\alpha a$ so that one has

$$P^k \propto \partial_a \Phi \partial_{m^k} a = \partial_a \Phi \frac{m^k}{a} \quad .$$

   The conservation of gravitational mass gives $\Phi \propto a$. Since CP_2 projection must be 2-dimensional, M^4 projection is 1-dimensional so that mass squared is indeed conserved.

   Thus one could write

$$\Phi = \frac{k(M^4)}{k(CP_2)} x\theta(a)\frac{a}{R} \quad , \tag{29}$$

   where R the radius of geodesic sphere of CP_2 and x a numerical constant which could be fixed by quantum criticality of the theory. Chern-Simons action density does not depend on a for this choice and this independence guarantees that the earlier ansatz satisfies field equations. The presence of the step function $\theta(a)$ tells that Φ is non-vanishing only inside light-cone and gives to the gauge potential delta function term which is non-vanishing only at the light-cone boundary and makes possible massless particles.

3. If M^4 projection is 1-dimensional, only homologically charged partonic 3-surfaces can carry gravitational four-momentum. This is not a problem since M^4 projection can be 2-dimensional in the general case. For CP_2

type extremals, ends of cosmic strings, and wormhole contacts the non-vanishing of homological charge looks natural. For wormhole contacts 3-D CP_2 projection suggests itself and is possible only if one allows also quantum fluctuations around light-like extremals of Chern-Simons action. The interpretation could be that for a vanishing homological charge boundary conditions force X^4 to approach vacuum extremal at partonic 3-surfaces.

This picture does not fit completely with the picture about particle massivation provided by CP_2 type extremals. Massless partons must correspond to 3-surfaces at light-cone boundary in this picture and light-likeness allows only linear motion so that inertial mass defined as average must vanish.

5. Comment about quantum classical correspondence

The proposed general picture allows to define the notion of quantum classical correspondence more precisely. The identification of the time average of the gravitational four-momentum for C-S action as a conserved inertial four-momentum associated with the Kähler action at a given space-time sheet of a finite temporal duration (recall that we work in the zero energy ontology) is the most natural definition of the quantum classical correspondence and generalizes to all charges.

In this framework the identification of gravitational four-momentum currents as those associated with 4-D curvature scalar for the induced metric of X^4 could be seen as a phenomenological manner to approximate partonic gravitational four-momentum currents using macroscopic currents, and the challenge is to demonstrate rigorously that this description emerges from quantum TGD.

For instance, one could require that at a given moment of time the net gravitational four-momentum of $Int(X^4)$ defined by the combination of the Einstein tensor and metric tensor equals to that associated with the partonic 3-surfaces. This identification, if possible at all, would certainly fix the values of the gravitational and cosmological constants and it would not be surprising if cosmological constant would turn out to be non-vanishing.

3.2.5 The eigenvalues of D as complex square roots of conformal weight and connection with Higgs mechanism?

An alternative interpretation for the eigenvalues of D emerges from the TGD based description of particle massivation. The eigenvalues could be interpreted as complex square roots of conformal weights in the sense that $|\lambda|^2$ would have interpretation as a conformal weight. There is of course the possibility of numerical constant of proportionality.

The physical motivation for the interpretation is that λ is in the same role as the mass term in the ordinary Dirac equation and thus indeed square root of mass squared proportional to the conformal weight. The vacuum expectation of Higgs would correspond to that for λ and Higgs contribution to the mass squared would correspond to the p-adic thermodynamical expectation value $\langle|\lambda|^2\rangle$ [A9]. Additional contributions to mass squared would come from super conformal and modular degrees of freedom. The interpretation of the generalized eigenvalue as a Higgs field is also natural because the generalized eigen values of the modified Dirac operator can depend on position.

3.2.6 Super-conformal symmetries

The topological character of the solutions spectrum makes possible the expected and actually even larger conformal symmetries in X^2 degrees of freedom. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the canonical transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the canonical transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

Diffeomorphisms of M^4 respecting the light-likeness condition define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-analytic deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2,C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only. Also electro-weak $U(2)$ acts as X^2-local Kac-Moody algebra of symmetries. Hence all the desired Kac-Moody symmetries are realized.

The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D to a given eigen mode. This defines a symmetry if zero modes satisfy the additional condition $N\Psi = 0$ implied by $\Psi = N\Psi_0$ in turn guaranteed by the already described conditions. These symmetries are super-conformal symmetries with respect to z and $\overline{z}$.

The radial conformal symmetries generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-canonical conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_\pm$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X^3_l carrying information about H metric makes possible these symmetries.

$N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2) \times SU(2) \times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

3.2.7 How the super-conformal symmetries of TGD relate to the conventional ones?

The representation of super-symmetries as an addition of anticommuting zero modes to the second quantized spinor field defined by the superposition of non-zero modes of the modified Dirac equation differs radically from the standard realization based on the replacement of the world sheet or target space coordinates with super-coordinates. Also the fundamental role of the generalized eigen modes of the modified Dirac operator is something new and absolutely essential for the understanding of how super-conformal invariance is broken: the breaking of super-symmetries is indeed the basic problem of the super-string theories.

Since the spinor fields in question are not Majorana spinors the standard super-field formalism cannot work in TGD context. It is however interesting to look to what extent this formalism generalizes and whether it allows some natural modification allowing to formally integrate the notions of the bosonic action and corresponding modified Dirac action.

1. One can consider the formal introduction of super fields by replacing of X^3_l coordinates by super-coordinates requiring the introduction of anticommuting parameters θ and $\overline{\theta}$ transforming as H-spinors of definite chirality, which is not consistent with Majorana condition. Using real coordinates x^α for X^3_l, one would have

   $$x^\alpha \rightarrow X^\alpha = x^\alpha + \overline{\theta}\hat{\Gamma}^\alpha\Psi + \overline{\Psi}\hat{\Gamma}^\alpha\theta \ ,$$

   Super-conformal symmetries would add to θ a zero mode with Grassmann number valued coefficient. The replacement $z^\alpha \rightarrow X^\alpha$ for the arguments of CP_2 and M^4 coordinates would super-symmetrize the field C-S action density. As a matter fact, the super-symmetrization is non-trivial only in radial degree of freedom since only $\hat{\Gamma}^r$ is non-vanishing.

2. Also imbedding space coordinates could be formally replaced with super-fields using a similar recipe and super-symmetries would act on them. The topological character of Chern-Simons action would allow the super-symmetries induced by the translation of θ by an anticommuting zero mode as formal symmetries at the level of the imbedding space. In both cases it is however far from clear whether the formal super-symmetrization has any real physical meaning.

3. The notion of super-surface suggests itself and would mean that imbedding space Θ parameters are functions of single θ parameter assignable with X^3_l. A possible representation of super-part of the imbedding is a generalization of ordinary imbedding in terms of constraints $H_{i)}(h^k) = 0,\ i = 1, 2....$ Symmetries allow only linear functions so that one would have

   $$c^\alpha_{i)}(r, z, \overline{z})\Theta_\alpha = 0 \ .$$

A hyper-plane in the space of theta parameters is obtained. Since only single theta parameter is possible in integral the number of constraints is seven and one obtains the modified Dirac action from the super-space imbedding.

Consider next the basic difficulty and its resolution.

1. The super-conformal symmetries do not generalize to the level of action principle in the standard sense of the word and the reason is the failure of the Majorana property forced by the separate conservation of quark and lepton numbers so that the standard super-space formalism remains empty of physical content.

2. One can however consider the modification of the integration measure $\prod_i d\theta_i d\overline{\theta}_i$ over Grassmann parameters by replacing the product of bilinears with

$$d\overline{\theta}\gamma_1 d\theta d\overline{\theta}\gamma_2 d\theta ...$$

   analogous to the product $dx^1 \wedge dx^2 ...$ (where γ^k would be gamma matrices of the imbedding space) transforming like a pseudoscalar. It seems that the replacement of product with wedge product leads to a trivial theory. This formalism could work for super fields obeying Weyl condition instead of Majorana condition and it would be interesting to find what kind of super-symmetric field theories it would give rise to.

   The requirement that the number of Grassmann parameters given by $2D$ is the number of spinor components of definite chirality (counting also conjugates) given by $2 \times 2^{D/2-1}$ gives critical dimension $D = 8$, which suggest that this kind of quantum field theory might exist. As found, the zero modes which are not of form $\Psi = N\Psi_0$ do not generate super-conformal symmetries in the strict sense of the word and might correspond to light particles. One could ask whether chiral SUSY in $M^4 \times CP_2$ might describe the low energy dynamics of corresponding light parton states. General arguments do not however support space-time super-symmetry.

3. Because of the light-likeness the super-symmetric variant of C-S action should involve the modified gamma matrices $\hat{\Gamma}^\alpha$ instead of the ordinary ones. Since only $\hat{\Gamma}^r$ is non-vanishing for the extremals of C-S action and since super-symmetrization takes place for the light-like coordinate r only, the integration measure must be defined as $d\overline{\theta}\hat{\Gamma}_r d\theta$, with θ perhaps assignable to a fixed covariantly constant right-handed neutrino spinor and $\hat{\Gamma}_r$ the inverse of $\hat{\Gamma}^r$. This action gives rise to the modified Dirac action with the modified gamma matrices emerging naturally from the Taylor expansion of the C-S action in powers of super-coordinate.

3.3 Why the cutoff in the number superconformal weights and modes of D is needed?

Two kinds of cutoffs are necessary in the number theoretic approach involving a hierarchy of algebraic extensions of rationals with increasing algebraic dimension.

3.3.1 Spatial cutoff realized in terms of number theoretical braids

The first cutoff corresponds to a spatial discretization selecting a subset of algebraic points of the partonic 2-surface X^2 as a subset of the points common to the real and p-adic variants of X^2 obeying the same algebraic equations. Almost topological field theory property allows to assume algebraic equations and also quantum criticality and generalization of the imbedding space concept are crucial for achieving the cutoff as a completely inherent property of X^2.

3.3.2 Cutoff in the number of super-canonical conformal weights

It is not quite clear whether the number of radial conformal weights should be finite or not. The assumption HFF property is realized also in configuration space degrees of freedom would motivate finiteness for the number of conformal weights and would effectively replace the world of the classical worlds with a finite-D space. Also super-symmetry suggests the same. Finiteness would be guaranteed if the ζ function involved characterizes partonic 2-surface and is labelled by p-adic prime: this would also guarantee that zeros of ζ are algebraic numbers. If the zeta function in question characterizes the spectrum of modified Dirac operator and the number of eigenvalues is finite then this goal is achieved. In the case of Riemann Zeta one would be forced to use cutoff due related to the algebraic extension of p-adic numbers used and to assume that zeros and even more general arguments are algebraic numbers.

3.3.3 Cutoff in the number of generalized eigenvalues of the modified Dirac operator

Second cutoff corresponds to a cutoff in the number of generalized eigenvalues of the modified Dirac operator and also now almost TQFT provides the needed flexibility.

1. If the generalized eigenvalues are interpreted as Higgs field then the number of eigenvalues is just one and also orthogonality condition for the modes is achieved without posing ad hoc correlations between longitudinal and transversal degrees of freedom.

2. A priori the dependence of the eigenmodes on transversal degrees of freedom of X^2 is arbitrary. This looks strange on basis of experience with quantum field theory and would imply non-stringy anti-commutation relations. Holomorphic dependence however leads to stringy anti-commutations.

3. Anti-commutativity at braid points only would be highly satisfactory since it would allow to avoid delta functions but would require that the transverse degrees of freedom reduce to a finite number of modes. The reduction of this cutoff to inherent properties of X^2 remains to be understood. What is clear is that the number of conformal modes in transversal degrees of freedom corresponds essentially to the number of points in the braid and the precise realization of this cutoff remains to be understood. Since this cutoff relates to finite measurement resolution, the idea that non-commutative S^2_{II} coordinates provides an elegant manner to realize the anti-commutativity at finite number of points.

It is natural to choose the modes to be S^2_{II} partial waves with a well defined color isospin quantum numbers I, I_3. The Abelianity of the color holonomy group of induced spinor connection suggests also color confinement in weak sense meaning vanishing of I_3 and Y for the physical states.

Since cutoff hierarchy must relate closely to the hierarchy of quantum phases, it seems natural to assume that for given value of $q = exp(i2\pi/n_b)$ only the angular momentum values $I \leq n_b$ are allowed. Here n_b is the order of the maximal cyclic subgroup of G_b involved with the Jones inclusion. In the similar manner one can introduce cutoff for S^2 partial waves in $\delta M^4_{\pm}$ as cutoff $l \leq n_b$ for angular momentum. Both cutoffs are needed in the definition of configuration space Hamiltonians and super-Hamiltonians allowing to approximate configuration space with a finite-dimensional space which is obviously in spirit with the hyper-finiteness.

Cutoffs imply that n-point functions are finite and non-trivial since the anticommutators of second quantized induced spinor fields are non-local and delta function singularity is smoothed out. Non-locality implies that vertices are non-trivial and pair creation becomes possible. It is of course essential that the dynamics of the space-time interior induces correlations between different partonic 2-surfaces.

That this picture can give rise to the basic vertices of quantum theory seems clear. For instance, suppose that bosons can be assigned to the fermionic representation of Hamiltonians and fermions by super Hamiltonians. The idea would be that right handed neutrino represents vacuum state to which imbedding space gamma matrices act like creation operators. The vertex for the emission of boson would involve sum of vacuum expectation values for the product of the operators $\overline{\Psi} J_A \Psi(x)$, $\overline{\nu} J_B \Psi(y)$ $\overline{\Psi} J_C \nu(z)$, $J_A = j^k_A \Gamma_k$ with various choices of arguments. Anticommutation relations would give sum over the values of the quantity $\overline{\nu} J_A(x) J_B(y) J_C(z) \nu$ multiplied by "wave functions" coming the modes of Ψ. Summation would be over the discrete set of points of the number theoretical braid. A discretized version of stringy scattering amplitude would be in question.

3.3.4 Attempt to form an overall view

This approach leads to both a hierarchy of discretized theories and continuum theory. Continuum theory indeed seems to be completely well defined and would correspond to string theory with free fermions with $N = 4$ super-conformal symmetry as far vertices are considered.

The interpretation encouraged by Jones inclusion hierarchy is that the limit $n \rightarrow \infty$ for quantum phase $q = exp(i2\pi/n)$ is not equivalent with the exact real theory based on stringy amplitudes defined using 1-D integrals over the inverse image of the image of the critical line. The natural interpretation for the stringy option without discretization could be in terms of Jones inclusions with group $SU(2)$ and classified by extended ADE diagrams relating to the monodromies of the theory. This interpretation would also conform with the full Kac-Moody invariance whereas for quantum version infinite-dimensional symmetries are reduced to finite-dimensional ones. Note that quantum trace should be equivalent with the condition that the trace of the unit matrix is unity for hyper-finite factors of type II_1.

The number theoretic cutoff hierarchy for the allowed zeros of ζ relates closely to the hierarchy of finite-dimensional extensions of p-adic numbers and to the quantum criticality realized in terms of the generalized imbedding space. This hierarchy of extensions defines a hierarchy of number theoretic braids with an increasing number of strands since the number of points in the intersection between real and corresponding p-adic surface increases and does also the number of allowed zeros. Also the hierarchy of finite-dimensional approximations for the inclusions of hyperfinite factors of type II_1 can be visualized in terms of a hierarchy of braid inclusions with increasing number of braids and is described in terms of Temperley-Lieb algebras. This hierarchy of approximate representations of the inclusion means the replacement of the Beraha number $B_n = 4cos^2(\pi/n)$ by a rational number defining the ratio of dimensions of two subsequent finite-dimensional algebras in the hierarchy. Hence the number theoretic braid hierarchy could provide a concrete representation for the hierarchy of approximations for the hyper-finite factors of type II_1 and their Jones inclusions in terms of inclusions of Temperley Lieb algebras assignable to the number theoretic braids. Physics itself would define this sequence of approximations via p-adicization which basically means space-time realization of cognitive representations.

3.4 The spectrum of Dirac operator and radial conformal weights from physical and geometric arguments

The identification of the generalized eigenvalues of the modified Dirac operator as Higgs field suggests the possibility of understanding the spectrum of D purely geometrically by combining physical and geometric constraints.

The standard zeta function associated with the eigenvalues of the modified Dirac action is the best candidate concerning the interpretation of super-canonical conformal weights as zeros of ζ. This ζ should have very concrete geometric and physical interpretation related to the quantum criticality if these eigenvalues have geometric meaning based on geometrization of Higgs field.

Before continuing it its convenient to introduce some notations. Denote the complex coordinate of a point of X^2 w, its $H = M^4 \times CP_2$ coordinates by $h = (m, s)$, and the H coordinates of its $R_+ \times S^2_{II}$ projection by $h_c = (r_+, s_{II})$.

3.4.1 Generalized eigenvalues

The generalized eigenvalue equation defined by the modified Dirac equation is a differential equation involving only the derivative with respect to r. Hence the eigenvalues λ can depend on X^2 coordinate w and on the coordinates of the critical manifold $R_+ \times S^2_{II}$ via the dependence of w these. As a function of $R_+ \times S^2_{II}$ coordinates they would be many-valued functions of these coordinates since several points of X^2 can project at given point of $R_+ \times S^2_{II}$.

The replacement of the ordinary eigenvalues with continuous functions would be a space-time analog for generalized eigenvalues identified as Hermitian operators (or equivalently, their spectra) inspired by the quantum measurement theory based on inclusions of hyper-finite factors of type II_1 [A8]. The replacement of these functions with their values in a discrete set defined by number theoretic braid would in turn be the counterpart for the finite measurement resolution.

The interpretation of eigenvalue as a complex Higgs field gives the most concrete interpretation for the generalized eigenvalues. Of course, only single eigenvalue would be realized in this kind of situation. Also the requirement that different modes are orthogonal with respect to the inner product at the partonic 2-surface allows only single generalized eigenvalue. Hence the modes in transversal degrees of freedom would code for physics as in the usual QFT.

This interpretation does not kill the idea about eigenvalues as inverses of zeta function $\lambda = \zeta^{-1}(z)$, S^2_{II}. The point is that one can regard X^2 as a covering of S^2 and assign different branches of ζ^{-1} to the different sheets of covering. Different branches of $\zeta^{-1}(z)$, call them $\zeta_k^{-1}(z)$, would combine to single function of the coordinate w of X^2. In the case of Riemann zeta the corresponding construction would replaced complex plane with its infinite-fold covering.

3.4.2 General definition of Dirac determinant

The first guess is that Dirac determinant can defined as a product of determinants assignable to the points $z = z_k$ of the number theoretic braids:

$$det(D) = \prod_{z_k)} det(D(z_k)) \ . \tag{30}$$

The determinant $det(D(z))$ at point z of S^2 would be defined as the product of the eigenvalues $\lambda(z)$ at points associated with the number theoretic braids.

$$det(D)(z_k) = \left[\prod_i \zeta_i^{-1}(z_k)\right]^{n(z_k)} , \quad (31)$$

$n(z_k)$ is the number of strands in the number theoretical braid of associated with z_k. Higgs interpretation would imply that only single value of Higgs contributes for a given point of X^2. Dirac determinant must be an algebraic number. This is the case if the total number of points of number theoretic braids involved is finite. It turns out that this guess is quite not general enough: it turns out that actual Dirac determinant must be identified as a ratio of two determinants.

3.4.3 Interpretation of eigenvalues of D as Higgs field

The eigenvalues of the modified Dirac operator have a natural interpretation as Higgs field which vanishes for the unstable extrema of Higgs potential. These unstable extrema correspond naturally to quantum critical points resulting as intersection of M^4 *resp.* CP_2 projection of the partonic 2-surface X^2 with R_+ *resp.* S^2_{II}.

Quantum criticality suggests that the counterpart of Higgs potential could be identified as the modulus square of ζ:

$$V(H(s)) = -|H(s)|^2 . \quad (32)$$

which indeed has the points s with $V(H(s)) = 0$ as extrema which would be unstable in accordance with quantum criticality. The fact that for ordinary Higgs mechanism minima of V are the important ones raises the question whether number theoretic braids might more naturally correspond to the minima of V rather than intersection points with S^2. This turns out to be the case. It will also turn out that the detailed form of Higgs potential does not matter: the only thing that matters is that $|V|$ is monotonically decreasing function of the distance from the critical manifold.

3.4.4 Purely geometric interpretation of Higgs

Geometric interpretation of Higgs field suggests that critical points with vanishing Higgs correspond to the maximally quantum critical manifold $R_+ \times S^2_{II}$. The value of H should be determined once $h(w)$ and $R_+ \times S^2_{II}$ projection $h_c(w)$ are known. $|H|$ should increase with the distance between these points. The question is whether one can assign to a given point pair $(h(w), h_c(w))$ naturally a value of H. The first guess is that value of H is most determined by the shortest piece of the geodesic line connecting the points which is a product of geodesics of δM^4_+ and CP_2.

This guess need not be quite correct. An alternative guess is that M^4 projection is indeed geodesic but that CP_2 projection extremizes its length subject to the constraint that the absolute value of the phase defined by the one-dimensional Kähler action $\int A_\mu dx^\mu$ is minimized: this point will be discussed below.

The value should be in general complex and invariant under the isometries of δH affecting h and h_c. The minimal distance $d(h, h_c)$ between the two points constrained by extremal property of phase would define the first candidate for the modulus of H.

The phase factor should relate close to the Kähler structure of CP_2 and one possibility would be the non-integrable phase factor $U(s, s_{II})$ defined as the integral of the induced Kähler gauge potential along the geodesic line in question. Hence the first guess for the Higgs would be as

$$
\begin{aligned}
H(w) &= d(h, h_c) \times U(s, s_{II}) \ , \\
d(h, h_c) &= \int_h^{h_c} ds \ , \ U(s, s_{II}) = exp\left[i \int_s^{s^1} A_k ds^k \right] \ .
\end{aligned} \tag{33}
$$

This gives rise to a holomorphic function in X^2 the local complex coordinate of X^2 is identified as $w = d(h, h_s)U(s, s_{II})$ so that one would have $H(w) = w$ locally. This view about H would be purely geometric.

One can ask whether one should include to the phase factor also the phase obtained using the Kähler gauge potential associated with S^2_r having expression $(A_\theta, A_\phi) = (k, cos(\theta))$ with k even integer from the requirement that the non-integral phase factor at equator has the same value irrespective of whether it is calculated with respect to North or South pole. For $k = 0$ the contribution would be vanishing. The value of k might correlate directly with the value of quantum phase. The objection against inclusion of this term is that Kähler action defining Kähler function should contain also M^4 part if this term is included. If this inclusion is allowed then internal consistency requires also the extremization of $\int A_\mu dx^\mu$ so that geodesic lines are not allowed.

In each coordinate patch Higgs potential could be simply the quadratic function $V = -w\overline{w}$. Negative sign is required by quantum criticality. As noticed any monotonically increasing function of V works as well since the minima of the potential remain unaffected. Potential could indeed have minima as minimal distance of X^2 point from $R_+ \times S^2_{II}$. Earth's surface with zeros as tops of mountains and bottoms of valleys as minima would be a rather precise visualization of the situation for given value of r_+. Mountains would have a shape of inverted rotationally symmetry parabola in each local coordinate system.

3.4.5 Consistency with the vacuum degeneracy of Kähler action and explicit construction of preferred extremals

An important constraint comes from the condition that the vacuum degeneracy of Kähler action should be understood from the properties of the Dirac determinant. In the case of vacuum extremals Dirac determinant should have unit modulus.

Suppose that the space-time sheet associated with the vacuum parton X^2 is indeed vacuum extremal. This requires that also X_l^3 is a vacuum extremal: in this case Dirac determinant must be real although it need not be equal to unity. The CP_2 projection of the vacuum extremal belongs to some Lagrangian sub-manifold Y^2 of CP_2. For this kind of vacuum partons the ratio of the product of minimal H distances to corresponding $M^4_\pm$ distances must be equal to unity, in other words minima of Higgs potential must belong to the intersection $X^2 \cap S^2_{II}$ or to the intersection $X^2 \cap R_+$ so that distance reduces to M^4 or CP_2 distance and Dirac determinant to a phase factor. Also this phase factor should be trivial.

It seems however difficult to understand how to obtain non-trivial phase in the generic case for all points if the phase is evaluated along geodesic line in CP_2 degrees of freedom. There is however no deep reason to do this and the way out of difficulty could be based on the requirement that the phase defined by the Kähler gauge potential is evaluated along a curve either minimizing the absolute value of the phase modulo 2π.

One must add the condition that curve is not shorter than the geodesic line between points. For a given curve length s_0 the action must contain as a Lagrange multiplier the curve length so that the action using curve length s as a coordinate reads as

$$S = \int A_s ds + \lambda(\int ds - s_0) \ . \tag{34}$$

This gives for the extremum the equation of motion for a charged particle with Kähler charge $Q_K = 1/\lambda$:

$$\begin{aligned} \frac{D^2 s^k}{ds^2} + \frac{1}{\lambda} \times J^k_{\ l} \frac{ds^l}{ds} &= 0 \ , \\ \frac{D^2 m^k}{ds^2} &= 0 \ . \end{aligned} \tag{35}$$

The magnitude of the phase must be further minimized as a function of curve length s.

If the extremum curve in CP_2 consists of two parts, first belonging to X^2_{II} and second to Y^2, the condition is certainly satisfied. Hence if $X^2_{CP_2} \times Y^2$ is not empty, the phases are trivial. In the generic case 2-D sub-manifolds of CP_2 have intersection consisting of discrete points (note again the fundamental role of 4-dimensionality of CP_2). Since S^2_{II} itself is a Lagrangian sub-manifold, it has especially high probably to have intersection points with S^2_{II}. If this is not the case one can argue that X^3_l cannot be vacuum extremal anymore.

Radial conformal invariance of $\delta M^4_\pm$ raises the question whether $\delta M^4_\pm$ geodesics should be defined by allowing $r_M(s)$ to be arbitrary rather than constant. The minimization of $\delta M^4_\pm$ distance would favor geodesics for which $r_M(s)$ decreases as fast as possible so that one has a light-like geodesics going directly to the tip of $\delta M^4_\pm$. Therefore this option does not seem to work.

The construction gives also a concrete idea about how the 4-D space-time sheet $X^4(X^3_l)$ becomes assigned with X^3_l. The point is that the construction extends X^2 to 3-D surface by connecting points of X^2 to points of S^2_{II} using the proposed curves. This process can be carried out in each intersection of X^3_l and M^4_+ shifted to the direction of future. The natural conjecture is that the resulting space-time sheet defines the 4-D preferred extremum of Kähler action.

The most obvious objection is that this construction might not work for cosmic strings of form $X^2 \times S^2_I$, where S^2_I is a homologically non-trivial geodesic sphere of CP_2. In this case X^2 would correspond to string ends, copies of S^2_I at different points of $\delta M^4_\pm$. There seems to be however no real problem. If $S^2_I \cap S^2_{II}$ is not empty, the orbits representing motion in the induced Kähler gauge field could simply define a flow at S^2_I connecting the points of S^2_I to one of the intersection points. Since geodesic manifold is in question one expects that the orbits indeed belong to S^2_I and cosmic string is obtained. Also a flow with several sources and sinks is possible. Situation should be the same for complex 2-sub-manifolds of CP_2. The 3-D character of the resulting surface would be due to the fact that $\delta M^4_\pm$ projections of the orbits are not points. If the second end of the string is at R_+ string and has the same value of r_M coordinate, single string would result. Otherwise one would obtain two strings with second end point at R_+ with the same value of r_M.

3.4.6 About the definition of the Dirac determinant and number theoretic braids

The definition of Dirac determinant should be independent of the choice of complex coordinate for X^2 and local complex coordinate implied by the definition of Higgs is a unique choice for this coordinate. The physical intuition based on Higgs mechanism suggests that apart from normalization factor the Dirac determinant should be defined simply as product of the eigenvalues of D, that is those of Higgs field, associated with the number theoretic braids.

If only single kind of braid is allowed then the minima of Higgs field define the points of the braid very naturally. The points in $R_+ \times S^2_{II}$ cannot contribute to the Dirac determinant since Higgs vanishes at the critical manifold. Note that at S^2_{II} criticality Higgs values become real and the exponent of Kähler action should become equal to one. This is guaranteed if Dirac determinant is normalized by dividing it with the product of $\delta M^4_\pm$ distances of the extrema from R_+. The value of the determinant would equal to one also at the limit $R_+ \times S^2_{II}$.

One would define the Dirac determinant as the product of the values of Higgs field over all minima of local Higgs potential

$$det(D) = \frac{\prod_k H_(w_k)}{\prod_k H_0(w_k)} = \prod_k \frac{w_k}{w^0_k} . \quad (36)$$

Here w^0_k are M^4 distances of extrema from R_+. Equivalently: one can identify the values of Higgs field as dimensionless numbers w_k/w^0_k. The modulus of Higgs field would be the ratio of H and $M^4_\pm$ distances from the critical sub-manifold. The modulus of the Dirac determinant would be the product of the ratios of H and M^4 depths of the valleys.

This definition would be general coordinate invariant and independent of the topology of X^2. It would also introduce a unique conformal structure in X^2 which should be consistent with that defined by the induced metric. Since the construction used relies on the induced metric this looks natural. The number of eigen modes of D would be automatically finite and eigenvalues would have purely geometric interpretation as ratios of distances on one hand and as masses on the other hand. The inverse of CP_2 length defines the natural unit of mass. The determinant is invariant under the scalings of H metric as are also Kähler action and Chern-Simons action. This excludes the possibility that Dirac determinant could also give rise to the exponent of the area of X^2.

Number theoretical constraints require that the numbers w_k are algebraic numbers and this poses some conditions on the allowed partonic 2-surfaces unless one drops from consideration the points which do not belong to the algebraic extension used.

3.4.7 About the detailed definition of number theoretic braids

Consider now the detailed definition of number theoretic braids. One can define a pile X^2_t of cross sections of $X^3_l \cap (\delta M^4_{\pm,t} \times CP_2)$, where $\delta M^4_{\pm,t}$ represents $\delta M^4_\pm$ shifted by t in a preferred time direction defined by M^2. In the same manner one can decompose M^2 to a pile of light-like geodesics $R_{+,t}$ defining the quantization axis of angular momentum. For each value of t one obtains a collection of minima of the "Higgs field" λ_t in 3-dimensional space $R_{+,t} \times S^2_{II}$. The minima define orbits $\gamma(t)$: $(r_{+,i}(t), s_{II}(t))$ in $M^2 \times S^2_{II}$ space.

One can consider braidings (or more generally tangles, two minima can disappear in collision or can be created from vacuum) both in X^3_l and at the level of imbedding space.

1. Braids in X^3_l

A braid in X^3_l is obtained by considering the fate of points of $X^2 t = 0$ in X^3_l and by assigning a braiding to the minima of Higgs field in X^3_l. Also the field lines of Kähler magnetic field or of Kähler gauge potential on X^3_l going through the initial positions of Higgs minima can be considered. Since the construction of the Higgs field involves induced Kähler gauge potential in an essential manner, the braiding defined by the Kähler gauge potential could be consistent with the time evolution for the positions of the minima of Higgs.

Recall that only topological rather than point-wise equivalence of the braids is required. It is not clear how much these definition depend on the coordinates used for X^3_l. For instance, could one trivialize the braid by making a time dependent coordinate change for X^2? This requires that it is possible to define global time coordinate whose coordinate lines correspond to field lines. This is possible only if the flow satisfies additional integrability conditions [D1].

2. Braidings defined by imbedding space projections

One can define braidings also by the projections to the heavenly spheres S^2_{II} of CP_2 and S^2_r of $\delta M^4_\pm$. A linear braid like structure is also obtained by considering the projections of Higgs minima in M^2.

1. The simplest option is the identification of the braid as the projections of the orbits of the minima of Higgs field to S^2_{II} or S^2_r (for various values of t). This seems to be the most elegant choice. One could decompose the braid to sub-braids such that each initial value $r_{+,i}(0)$ would define its own braid in S^2_{II} or S^2_r. Also each point of S^2_{II} or S^2_r could define its own sub-braid.

2. Factoring quantum field theories defined in M^2 [24, 34] suggest a further definition of a braid like structure based on the projections of Higgs minima to M^2. The braid like structure would result from the motion of braid points with different velocities so that they would pass by each other. This kind of pattern with constant velocities of particles describes scattering in factoring quantum field theories defined in M^2. The M^2 velocities of particles would not be constant now. S-matrix is almost trivial inducing only a permutation of the initial state momenta and S-matrix elements are mere phases. The interpretation is that each pass-by process induces a time lag. At the limit when the velocities approach to zero or infinity such that their ratios remain constant, S-matrix reduces to a braiding S-matrix.

The Higgs minima contributing to the elements of S-matrix (or at least U-matrix) should correspond to algebraic points of braids. This suggests that the information about the braids comes from the minima of Higgs in X^3_l rather than X^2_t so that only some values of t at each strand $\gamma(t)$ give rise to physically relevant braid points. The condition that the resulting numbers are algebraic poses restrictions on X^3_l as does also the condition that X^3_l have also p-adic counterparts. This does not of course mean the loss of braids. Note that the discretization allows to assign Dirac determinant and zeta function to any 3-surface X^3_l rather than only those corresponding to the maxima of Kähler function.

3.4.8 The identification of zeta function

The proposed picture supports the identification of the eigenvalues of D in terms of a Higgs fields having purely geometric meaning. It also seems that number theoretic braids must be identified as minima of Higgs potential in X^2. Furthermore, the braiding operation could be defined for all intersections of X^3_l defined by shifts $M^4_\pm$ as orbits of minima of Higgs potential. Second option is braiding by Kähler magnetic flux lines.

The question is how to understand super-canonical conformal weights for which the identification as zeros of a zeta function of some kind is highly suggestive. The natural answer would be that the normalized eigenvalues of D defines this zeta function as

$$\zeta(s) = \sum_k (\frac{H(w_k)}{H_0(w_k)})^{-s} \ . \quad (37)$$

The number of eigenvalues contributing to this function would be finite and $H(w_k)/H_0(w_k)$ should be rational or algebraic at most. ζ function would have a precise meaning consistent with the usual assignment of zeta function to Dirac determinant.

The case of Riemann Zeta inspires the question whether one should allow only the moduli of the eigenvalues in the zeta or allow only real and positive eigenvalues. The moduli of eigenvalues are not smaller than unity as is the case also for Riemann Zeta. Real eigenvalues correspond to vanishing phase and thus vanishing Chern-Simons action and unit eigenvalues to the quantum critical points of S^2_{II}.

The ζ function would directly code the basic geometric properties of X^2 since the moduli of the eigenvalues characterize the depths of the valleys of the landscape defined by X^2 and the associated non-integrable phase factors. The degeneracies of eigenvalues would in turn code for the number of points with same distance from a given zero intersection point.

The zeros of the ζ function in turn define natural candidates for the super-canonical conformal weights and their number would thus be finite in accordance with the idea about inherent cutoff present also in configuration space degrees of freedom. Super-canonical conformal weights would be functionals of X^2. The scaling of λ by a constant depending on p-adic prime factors out from the zeta so that zeros are not affected: this is in accordance with the renormalization group invariance of both super-canonical conformal weights and Dirac determinant.

The zeta function should exist also in p-adic sense. This requires that the numbers λ^s at the points s of S^2_{II} which corresponds to the number theoretic braid are algebraic numbers. The freedom to scale λ could help to achieve this.

The conformal weights defined by the zeros of zeta would be constant. One could however consider also the generalization of the super-canonical conformal weights to functions of S^2_{II} or S^2_r coordinate although this is not necessary and would spoil the simple group theoretical properties of the δH Hamiltonians. The coordinate s appearing as the argument of ζ could be formally identified as S^2_{II} or S^2_r coordinate so that generalized super-canonical conformal weights could be interpreted geometrically as inverses of $\zeta^{-1}(s)$ defined as a function in S^2_{II} or S^2_r.

In this case also the notion of number theoretic braids defined as sets of points for which $X^2_{M^4}$ projection intersects R_+ at same point could make sense for super-canonical conformal weights. This would require that the number for the branches of ζ^{-1} is same as the number of points of braid.

3.4.9 The relationship between λ and Higgs field

The generalized eigenvalue $\lambda(w)$ is only proportional to the vacuum expectation value of Higgs, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). Gauge bosons can have Higgs expectation proportional to λ. The proportionality must be of form $\langle H \rangle \propto \lambda/p^{n/2}$ if gauge boson mass squared is of order $1/p^n$.

3.4.10 Possible objections related to the interpretation of Dirac determinant

Suppose that that Dirac determinant is defined as a product of determinants associated with various points z_k of number theoretical braids and that these determinants are defined as products of corresponding eigenvalues.

Since Dirac determinant is not real and is not invariant under isometries of CP_2 and of $\delta M^4_\pm$, it cannot give only the exponent of Kähler function which is real and $SU(3)\times SO(3,1)$ invariant. The natural guess is that Dirac determinant gives also the Chern-Simons exponential and possible phase factors depending on quantum numbers of parton.

1. The first manner to circumvent this objection is to restrict the consideration to maxima of Kähler function which select preferred light-like 3-surfaces X^3_l. The basic conjecture forced by the number theoretic universality and allowed by TGD based view about coupling constant evolution indeed is that perturbation theory at the level of configuration space can be restricted to the maxima of Kähler function and even more: the radiative corrections given by this perturbative series vanish being already coded by Kähler function having interpretation as analog of effective action.

2. There is also an alternative way out of the difficulty: define the Dirac determinant and zeta function using the minima of the modulus of the generalized Higgs as a function of coordinates of X^3_l so that continuous strands of braids are replaced by a discrete set of points in the generic case.

The fact that general Poincare transformations fail to be symmetries of Dirac determinant is not in conflict with Poincare invariance of Kähler action since preferred extremals of Kähler action are in question and must contain the fixed partonic 2-surfaces at $\delta M^4_\pm$ so that these symmetries are broken by boundary conditions which does not require that the variational principle selecting the preferred extremals breaks these symmetries.

One can exclude the possibility that the exponent of the stringy action defined by the area of X^2 emerges also from the Dirac determinant. The point is that Dirac determinant is invariant under the scalings of H metric whereas the area action is not.

The condition that the number of eigenvalues is finite is most naturally satisfied if generalized ζ coding information about the properties of partonic 2-surface and expressible as a rational function for which the inverse has a finite number of branches is in question.

3.4.11 How unique the construction of Higgs field is?

Is the construction of space-time correlate of Higgs as λ really unique? The replacement of H with its power H^r, $r > 0$, leaves the minima of H invariant as points of X^2 so that number theoretic braid is not affected. As a matter fact, the group of monotonically increasing maps real-analytic maps applied to H leaves number theoretic braids invariant.

The map $H \rightarrow H^r$ scales Kähler function to its r-multiple, which could be interpreted in terms of $1/r$- scaling of the Kähler coupling strength. Also super-canonical conformal weights identified as zeros of ζ are scaled as $h \rightarrow h/r$ and Chern-Simons charge k is replaced with k/r so that at least $r = 1/n$ might be allowed.

One can therefore ask whether the powers of H could define a hierarchy of quantum phases labelled by differen values of k and α_K. The interpretation as separate phases would conform with the idea that D in some sense has entire spectrum of generalized eigenvalues.

3.5 Quantization of the modified Dirac action

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. Stringy picture need not be correct with string being replaced number theoretic braids.

1. The first question is how M^4 and CP_2 braids relate. Since one assumes that the data associated with both braids are independent, it seems necessary to assume anti-commutativity between all points of X^2 belonging to some number theoretic braid.

2. There is no correlation between λ and eigenvalues associated with transverse degrees of freedom as in the case of d'Alember operator. Therefore an infinite number of eigen-modes of D for a given eigenvalue λ can be considered unless one poses some additional conditions. This would mean that one could have anti-commutativity for different points of X^2 and anti-commutators of Ψ and conjugate at same point would be proportional to delta function. This would not conform with the stringy picture.

3. How could one obtain stringy anticommutations? The assumption that modes are holomorphic or antiholomorphic would guarantee this since formally only single coordinate variable would appear in Ψ. Anti-commutativity along string requires that in a given sector of configuration space isometries

commute with the selection of quantization axes for the isometry algebra of the imbedding space. This might be justified by quantum classical correspondence. The unitarity for Yang-Baxter matrices and unitarity of the inner product for the radial modes r^Δ, $\Delta = 1/2 + iy$, is consistent with the stringy option where y would now label those points of R_+ which do not correspond to $z = 0$. String corresponds to the ζ-image of the critical line containing non-trivial zeros of zeta at the geodesic sphere of S^2_r.

4. One could ask whether number theoretic braids might have deeper meaning in terms of anticommutativity. This would be the case if the modes in transversal degrees of freedom reduce to a finite number and are actually labelled by λ. This could be achieved if there is no other dependence on transverse degrees of freedom than that coming through $\lambda(z)$. Anti-commutativity would hold true only at finite number of points and that anti-commutators would be finite in general. This outcome would be very nice.

5. An interesting question is whether the number theoretic braid could be also described by introducing a non-commutativity of the complex coordinate of X^2 provided by S^2_r or S^2_{II}. This should replace anti-commutativity in X^2 with anti-commutativity for different points of the number theoretic braid. The nice outcome would be the finiteness of anti-commutators at same point.

The following is an attempt to formulate this general vision in a more detail manner.

3.5.1 Fermionic anticommutation relations: non-stringy option

The fermionic anti-commutation relations must be consistent with the vacuum degeneracy and with the anti-commutation relations of configuration space gamma matrices defining the matrix elements of configuration space metric between complexified Hamiltonians.

1. The bosonic representation of configuration space Hamiltonians is naturally as Noether charges associated with Chern-Simons action:

$$
\begin{aligned}
H_A &= \int d^2x \pi^0_k j^k_A \ , \\
\pi^\alpha &= \frac{\partial L_{C-S}}{\partial_\alpha h^k} \ .
\end{aligned}
\tag{38}
$$

π^0_k denotes bosonic canonical momentum density. Note that also fermionic dynamics allows definition of Hamiltonians as fermionic charges) and this would give rise to fermionic representation of super-canonical algebra. Same applies to the super Kac-Moody algebra generators which super

Kac-Moody generators realized as X^3-local isometries of the imbedding space.

2. Super Hamiltonians identifiable as contractions of configuration space gamma matrices with Killing vector fields of symplectic transformations in CH can be defined as matrix elements of $j_A^k\Gamma_k$ between $\overline{\nu}_R$ and Ψ:

$$J_A^K\Gamma_K \quad \equiv \quad \Gamma_A = H_{S,A} = \int d^2x\overline{\nu}_R j_A^k\Gamma_k\Psi \quad . \tag{39}$$

$H_{S,A}^{\dagger}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations read as

$$\{\overline{\Psi}(x),\Gamma_k\Psi(y)\} \quad = \quad \pi_k^0 J^{rs}\Sigma_{rs}\delta^2(x,y) \quad . \tag{40}$$

Here J^{rs} denotes the degenerate Kähler form of $\delta M_+^4\times CP_2$. What makes these anti-commutation relations non-stringy is that anti-commutator is proportional to 2-D delta function rather than 1-D delta function at 1-D sub-manifold of X^2 as in the case of conformal field theories. Hence one would have 3-D quantum field theory with one light-like direction.

4. The matrix elements of configuration space metric for the complexified Killing vector fields of symplectic transformations give the elements of configuration space Kähler form and metric as

$$\{\Gamma_A^{\dagger},\Gamma_B\} \quad = \quad iG_{\overline{A},B} = J_{\overline{A},B} = \{\overline{H_A},H_B\} = H_{[\overline{A},B]} \quad . \tag{41}$$

3.5.2 Fermionic anti-commutation relations: stringy option

As already noticed, 2-dimensional delta function in the anti-commutation relations implies that spinor field is 2-D Euclidian free field rather than conformal field. The usual stringy picture would require anti-commutativity only along circle and nonlocal commutators outside this circle.

Also the original argument based on the observation that the points of CP_2 parameterize a large class of solutions of Yang-Baxter equation suggests the stringy option. The subset of commuting Yang-Baxter matrices corresponds to a geodesic sphere S^2 of CP_2 and the subset of unitary Yang-Baxter matrices to a geodesic circle of S^2 identifiable as real line plane compactified to S^2. Physical intuition strongly favors unitarity.

Stringy choice is consistent with the identification of the configuration space Hamiltonians as bosonic Noether charges only if Noether charges correspond to closed but in general not exact 2-forms and thus reduce to integrals of a 1-form over 1-dimensional manifold representing the discontinuity of the associated vector potential. That Noether charges would reduce to cohomology would conform with almost TQFT property. This is indeed the case under conditions which will be idenfied below.

1. The canonical momentum density associated with C-S action has the expression

$$\pi_k \quad = \quad \epsilon_{\alpha\beta 0}(\partial_\beta\left[A_\alpha A_k\right) - \partial_\alpha(A_\beta A_k\right] \quad , \tag{42}$$

   and is thus a closed two-form. Note that the discontinuity of the monopole like vector potential implies that the form in question is not exact.

2. Also the Hamiltonian densities

$$H_A \quad = \quad j_A^k \pi_k = J^{kl}\partial_l H_A \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k) - \partial_\alpha(A_\beta A_k)\right] \tag{43}$$

   should define closed forms

$$H_A \quad = \quad j_A^k \pi_k = \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k J^{kl}\partial_l H_A) - \partial_\alpha(A_\beta A_k \partial_l J^{kl} H_A\right] \ . \tag{44}$$

3. This is not the case in general since the derivatives coming from j_A^k give the term

$$\epsilon_{\alpha\beta 0} A_\alpha A_k J^{kl} D_r(\partial_l H_A)\partial_\beta h^r - A_\beta A_k J^{kl} D_r(\partial_l H_A)\partial_\alpha h^r \ . \tag{45}$$

   which does not vanish unless the condition

$$A_k J^{kl} D_r(\partial_l H_A) = \partial_r \Phi \tag{46}$$

   holds true.

   The condition is equivalent with the vanishing of the Poisson bracket between Hamiltonian and components of the Kähler potential:

$$\partial_k H_A J^{kl} \partial_l A_r \quad = \quad 0 \ . \tag{47}$$

   This poses a restriction on the group of isometries of configuration space. The restriction of Kähler potential to A_r is given by $(A_\theta, A_\phi) = (0, cos(\theta))$ and A_ϕ generates rotations in z-direction. Hence only the Hamiltonians commuting with Kähler gauge potential of $\delta M^4_\pm \times CP_2$ at X^2 would have vanishing color isospin and presumably also vanishing color hyper charge in the case of CP_2 and vanishing net spin in case of δM^4_+.

4. The discontinuity of Φ would result from the topological magnetic monopole character of the Kähler potential A_k in $\delta M^4_\pm \times CP_2$.

5. Quantum classical correspondence suggests that quantum measurement theory is realized at the level of the configuration space and induces a decomposition of the configuration space to a union of sub-configuration spaces corresponding to different choices of quantization axes of angular momentum and color quantum numbers. Hence the interpretation of configuration space isometries in terms of a maximal set of commuting observables would make sense. Of course, also the canonical transformations for which Hamiltonians do not reduce to 1-D integrals act as symmetries although they do not possess super counterparts. They play same role as Lorentz boosts whereas the super-symmetrizable part of the algebra is analogous to the little group of Lorentz group leaving momentum invariant. This means that complete reduction to string model type theory does not occur even at the level of quantum states.

Consider now the basic formulas for the stringy option.

1. Hamiltonians can be expressed as

$$H_A = \int dx A A_k J^{kl} \partial_l H_A \ . \tag{48}$$

   where A denotes the projection of Kähler gauge potential to the 1-dimensional manifold in question.

2. The fermionic super-currents defining super-Hamiltonians and configuration space gamma matrices would be given by

$$J_A^K \Gamma_K \equiv \Gamma_A = H_{S,A} = \int dx \overline{\nu}_R j_A^k \Gamma_k \Psi \ . \tag{49}$$

   $H^\dagger_{S,A}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations would read as

$$\{\overline{\Psi}(x), \Gamma_k \Psi(y)\} = A A_k J^{kl} \partial_l H_A J^{rs} \Sigma_{rs} \delta(x,y) \ . \tag{50}$$

   The general formulas for the matrix elements of the configuration space metric and Kähler form are as for the non-stringy option.

3.5.3 String as the inverse image for image of critical line for zeros of zeta

Number theoretical argument suggests that 1-D dimensional delta function corresponds to the point set for which δM^4_+ projection corresponds to the line of non-trivial zeros for ζ: $z = \zeta(1/2+iy)$ that is intersection of X^2 with R_+. Thus stringy anti-commutation would be along R_+. In CP_2 the discrete set of points along which anticommutations would be given would be subset in S^2_{II}. Anti-commutativity on quantum critical set which corresponds to vacuum extremals would be indeed very natural.

In case of Riemann zeta one must consider also trivial zeros at $x = -2n$, $n = 1, 2....$ These would correspond to the integer powers of r^n for which the definition of inner product is problematic. Note however that for negative powers $-2n$ corresponding to zeros of ζ there are no problems if there is cutoff $r > r_0$.

The number theoretic counterpart of string would be most naturally a curve whose S^2_r projection belongs to the image of the critical line consisting of points $\zeta(1/2+iy)$. This image consist of the real axis of S^2 interpreted as compactified plane since ζ is real at the critical line. Note that in case of Riemann zeta also real axis is mapped to the real line so that it gives nothing new. Also this has a number theoretical justification since the basis $r^{1/2+iy}$, where r could correspond to the light-like coordinate of both $\delta M^4_\pm$ and partonic 3-surface, forms an orthogonal basis with respect to the inner product defined by the scaling invariant integration measure dx/x.

For number theoretical reasons which should be already clear, the values of y would be restricted to $y = \sum_k n_k y_k$ of imaginary parts of zeros of ζ. In the case of partonic 3-surface this would mean that eigenvalues of the modified Dirac operator would be of form $1/2 + i\sum_k n_k y_k$ and the number theoretical cutoff regularizing the Dirac determinant would emerge naturally. The important implication would be that not only q^{iy_k} but also y_k must be algebraic numbers. Note that the zeros of Riemann zeta at this line correspond to quantum criticality against phase transitions changing Planck constant meaning geometrically a leakage between different sectors of the imbedding space.

3.6 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-surfaces taking which would thus seem to take the role of fundamental dynamical objects. Conformal invariance in turn seems to make the 2-D partons 1-D objects and number theoretical braids in turn discretizes strings. And it also seems that the strands of number theoretic braid can in turn be discretized by considering the minima of Higgs potential in 3-D sense.

Somehow these apparently contradictory views should be unifiable in a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales. The notions of measurement resolution and number theoretic braid indeed provide the needed insights in this respect.

3.6.1 Anti-commutations of the induced spinor fields and number theoretical braids

The understanding of the number theoretic braids in terms of Higgs minima and maxima allows to gain a global view about anti-commutations. The coordinate patches inside which Higgs modulus is monotonically increasing function define a division of partonic 2-surfaces $X_t^2 = X_l^3 \cap \delta M^4_{\pm,t}$ to 2-D patches as a function of time coordinate of X_l^3 as light-cone boundary is shifted in preferred time direction defined by the quantum critical sub-manifold $M^2 \times CP_2$. This induces similar division of the light-like 3-surfaces X_l^3 to 3-D patches and there is a close analogy with the dynamics of ordinary 2-D landscape.

In both 2-D and 3-D case one can ask what happens at the common boundaries of the patches. Do the induced spinor fields associated with different patches anti-commute so that they would represent independent dynamical degrees of freedom? This seems to be a natural assumption both in 2-D and 3-D case and correspond to the idea that the basic objects are 2- *resp.* 3-dimensional in the resolution considered but this in a discretized sense due to finite measurement resolution, which is coded by the patch structure of X_l^3. A dimensional hierarchy results with the effective dimension of the basic objects increasing as the resolution scale increases when one proceeds from braids to the level of X_l^3.

If the induced spinor fields associated with different patches anti-commute, patches indeed define independent fermionic degrees of freedom at braid points and one has effective 2-dimensionality in discrete sense. In this picture the fundamental stringy curves for X_t^2 correspond to the boundaries of 2-D patches and anti-commutation relations for the induced spinor fields can be formulated at these curves. Formally the conformal time evolution scaled down the boundaries of these patches. If anti-commutativity holds true at the boundaries of patches for spinor fields of neighboring patches, the patches would indeed represent independent degrees of freedom at stringy level.

The cutoff in transversal degrees of freedom for the induced spinor fields means cutoff $n \leq n_{max}$ for the conformal weight assignable to the holomorphic dependence of the induced spinor field on the complex coordinate. The dropping of higher conformal weights should imply the loss of the anti-commutativity of the induced spinor fields and its conjugate except at the points of the number theoretical braid. Thus the number theoretic braid should code for the value of n_{max}: the naive expectation is that for a given stringy curve the number of braid points equals to n_{max}.

3.6.2 The decomposition into 3-D patches and QFT description of particle reactions at the level of number theoretic braids

What is the physical meaning of the decomposition of 3-D light-like surface to patches? It would be very desirable to keep the picture in which number theoretic braid connects the incoming positive/negative energy state to the partonic 2-surfaces defining reaction vertices. This is not obvious if X_l^3 decomposes into causally independent patches. One can however argue that although each patch can define its own fermion state it has a vanishing net quantum numbers in zero energy ontology, and can be interpreted as an intermediate virtual state for the evolution of incoming/outgoing partonic state.

Another problem - actually only apparent problem - has been whether it is possible to have a generalization of the braid dynamics able to describe particle reactions in terms of the fusion and decay of braid strands. For some strange reason I had not realized that number theoretic braids naturally allow fusion and decay. Indeed, cusp catastrophe is a canonical representation for the fusion process: cusp region contains two minima (plus maximum between them) and the complement of cusp region single minimum. The crucial control parameter of cusp catastrophe corresponds to the time parameter of X_l^3. More concretely, two valleys with a mountain between them fuse to form a single valley as the two real roots of a polynomial become complex conjugate roots. The continuation of light-like surface to slicing of X^4 to light-like 3-surfaces would give the full cusp catastrophe.

In the catastrophe theoretic setting the time parameter of X_l^3 appears as a control variable on which the roots of the polynomial equation defining minimum of Higgs depend: the dependence would be given by a rational function with rational coefficients.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

3.6.3 About 3-D minima of Higgs potential

The dominating contribution to the modulus of the Higgs field comes from $\delta M^4_\pm$ distance to the axis R_+ defining quantization axis. Hence in scales much larger than CP_2 size the geometric picture is quite simple. The orbit for the 2-D minimum of Higgs corresponds to a particle moving in the vicinity of R_+ and minimal distances from R_+ would certainly give a contribution to the Dirac determinant. Of course also the motion in CP_2 degrees of freedom can generate local minima and if this motion is very complex, one expects large number of minima with almost same modulus of eigenvalues coding a lot of information about X^3_l.

It would seem that only the most essential information about surface is coded: the knowledge of minima and maxima of height function indeed provides the most important general coordinate invariant information about landscape. In the rational category where X^3_l can be characterized by a finite set of rational numbers, this might be enough to deduce the representation of the surface.

What if the situation is stationary in the sense that the minimum value of Higgs remains constant for some time interval? Formally the Dirac determinant would become a continuous product having an infinite value. This can be avoided by assuming that the contribution of a continuous range with fixed value of Higgs minimum is given by the contribution of its initial point: this is natural if one thinks the situation information theoretically. Physical intuition suggests that the minima remain constant for the maxima of Kähler function so that the initial partonic 2-surface would determine the entire contribution to the Dirac determinant.

3.6.4 How generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams to incoming and outgoing partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time

scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

3.6.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

3.6.6 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

4 Super-symmetries at space-time and configuration space level

The first difference between TGD and standard conformal field theories and string models is that super-symmetry generators acting as configuration space gamma matrices acting as super generators carry either lepton or quark number. Only the anti-commutators of quark like generators expressible in terms of Hamiltonians H_A of $X^3_l \times CP_2$ can contribute to the super-symmetrization of the Poisson algebra and thus to CH metric via Poisson central extension, whereas leptonic generators, which are proportional to $j^{Ak}\Gamma_k$ can contribute to the super-symmetrization of the function algebra of CH. Quarks correspond to N-S type representations and kappa symmetry of string models whereas leptons correspond to Ramond type representations and ordinary super-symmetry.

Also Super Kac-Moody invariance allows lepton-quark dichotomy. What forces to assign leptons with Ramond representation is that covariantly constant neutrino must correspond to one conformal mode $(z^n, n = 0)$. The p-adic mass calculations [6] carried for more than decade ago led to the same assignment on physical grounds: p-adic mass calculations also forced to include $SO(3,1)$ besides M^4 a tensor factor to super-conformal representations, which in recent context suggests that causal determinants $X^3_l \times CP_2$, $X^3_l \subset M^4$ an arbitrary light like 3-surface rather than just a translate of δM^4_+, must be allowed. Also now the lepton-quark, Ramond-NS and SUSY-kappa dichotomies correspond to one and same dichotomy so that the general structure looks quite satisfactory although it must be admitted that it is based on heuristic guess work.

Second deep difference is the appearance of the zeros of Riemann Zeta as conformal weights of the generating elements of the super-canonical algebra and the expected action of conformal algebra associated with 3-D CDS as a spectral flow in the space of super-canonical conformal weights inducing a mere gauge transformation infinitesimally and a braiding action in topological degrees of freedom.

In this section the relationship of Super Kac-Moody invariance to ordinary super-conformal symmetry and the interaction between Super-Kac Moody and super-canonical symmetries are discussed. For years the role of quaternions and octonions in TGD has been under an active speculation. These aspects are considered in [E2], where the number theoretic equivalent of spontaneous compactification is proposed. The conjecture states that space-time surfaces can be regarded either as 4-surfaces in $M^4 \times CP_2$ or as hyper-quaternionic 4-surfaces in the space $HO = M^8$ possessing hyper-octonionic structure (the attribute 'hyper' means that imaginary units are multiplied by $\sqrt{-1}$ in order to achieve number theoretic norm with Minkowskian signature).

4.1 Super-canonical and Super Kac-Moody symmetries

The proper understanding of super symmetries has turned out to be crucial for the understanding of quantum TGD and it seems that the mis-interpreted super-symmetries are one of the basic reasons for the difficulties of super string models too. At this moment one can fairly say that the construction of the configuration space spinor structure reduces to a purely group theoretical problem of constructing representations for the super generators of the super-canonical algebra of CP_2 localized with respect to $\delta M^4_\pm$ in terms of second quantized induced spinor fields.

4.1.1 Super canonical symmetries

One can imagine two kinds of causal determinants besides $\delta M^4_+ \times CP_2$. In principle all surfaces $X^3_l \times CP_2$, where X^3_l is a light like 3-surface of M^4, could act as effective causal determinants: the reason is that the creation of pairs of positive and negative energy space-time sheets is possible at these surfaces. There are good hopes that the super-canonical and super-conformal symmetries associated with δX^3_l allow to generalize the construction of the configuration space geometry performed at $\delta M^4_+ \times CP_2$. If X^3_l can be restricted to be unions of future and past light cone boundaries, the generalization is more or less trivial: one just forms a union of configuration spaces associated with unions of translates of δM^4_+ and δM^4_-.

As explained in the previous chapter, one can understand how the causal determinants $X^3_l \times CP_2$ emerge from the facts that space-time sheets with negative time orientation carry negative energy and that the most elegant theory results when the net quantum numbers and conserved classical quantities vanish for the entire Universe. Crossing symmetry allows consistency with elementary particle physics and the identification of gravitational 4-momentum as difference of conserved inertial (Poincare) 4-momenta for positive and negative energy matter provides consistency with macroscopic physics.

The emergence of these additional causal determinants means that super-canonical symmetries become microscopic, rather than only cosmological, symmetries commuting with Poincare transformations exactly for $M^4 \times CP_2$ and apart from small quantum gravitational effects for $M^4_+ \times CP_2$. Super-canonical symmetry differs in many respects from Kac-Moody symmetries of particle physics, which in fact correspond to the conformal invariance associated with the modified Dirac action and correspond to the product of Poincare, electro-weak and color groups. It seems that these symmetries are dually related.

4.1.2 Super Kac-Moody symmetries associated with the light like causal determinants

Also the light like 3-surfaces X^3_l of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results. Also the condition $\sqrt{(g_3)} = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction. This conforms with duality since also the 7-D causal determinants $X^3_l \times CP_2$ allow both radial and transversal conformal symmetry.

Good candidate for the counterpart of this symmetry in the interior of space-time surface is hyper-quaternion conformal invariance [E2]. All that is needed for these symmetries to be equivalent that the spaces of super-gauge degrees of freedom defined by them are equivalent. Kac Moody generators and their super counterparts can be associated with the 3-D light like CDs.

If is enough to localize only the H-isometries with respect to X^3_l, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation is not equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them. The following arguments fix the identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X^3_l-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X^3_l-local color transformations on configuration space spinor fields represents local color transformations. If the action of X^3_l-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

4.1.3 The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N = 2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N=2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N=2$ super-conformal representations are modular invariant, and that $N=2$ algebra defines so called chiral ring defining a topological quantum field theory [24], lend a further support for the belief that $N=2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ \ l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ \ m=-l,-l+2,....,l-2,l \ . \end{aligned} \tag{51}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [24].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h=1$ whereas T and G have conformal weights $h=2$ and $h=3/2$.

The experience with $N = 4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h=1/2$ and their super-partners with $h=0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

4.1.4 How could conformal symmetries of light like 3-D CDs act on super-canonical degrees of freedom?

An important challenge is to understand the action of super-conformal symmetries associated with the light like 3-D CDs on super-canonical degrees of freedom. The breakthrough in this respect via the algebraic formulation for the vision about vanishing loop corrections of ordinary Feynman diagrams in terms of equivalence of generalized Feynman diagrams with loops with tree diagrams [C7]. The formulation involves Yang-Baxter equations, braid groups, Hopf algebras, and so called ribbon categories and led to the following vision. The original formulation to be discussed in this sub-subsection is very heuristic and a more quantitative formulation follows in the next subsection.

1. Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of the homologically non-trivial geodesic sphere S^2 of CP_2 corresponds to the super-canonical conformal weights, and corresponds to a discrete set of points at the space-time surface. These points would also label mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. These points would belong to a "time=constant" section of 2-dimensional "space-time", presumably circle, defining physical states of a two-dimensional conformal field theory for which the scaling operator L_0 takes the role of Hamiltonian.

2. One could thus regard super-generators as conformal fields in space-time or complex plane having super-canonical conformal weights as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories [24] would induce by a pull-back a braid group action on the super-canonical conformal weights of configuration space gamma matrices (super generators) and corresponding isometry generators.

At the first sight the explicit realization of super-canonical and Kac Moody generators seems however to be in conflict with this vision. The interaction of the conformal algebra of X^3_l on super-canonical algebra is a pure gauge interaction since the definition of super canonical generators is not changed by the action of conformal transformations of X^3_l. This is however consistent with the assumption that the action defined by the quantum-classical correspondence is also a pure gauge interaction locally. The braiding action would be analogous to the holonomies encountered in the case of non-Abelian gauge fields with a vanishing curvature in spaces possessing non-trivial first homotopy group.

Quantum classical correspondence would allow to map abstract configuration space level to space-time level.

1. The complex argument z of Kac Moody and Virasoro algebra generators $T(z) = \sum T_n z^n$ would be discretized so that it would have values on the set of supercanonical conformal weights corresponding to the space t in the Cartan decomposition $g = t + h$ of the tangent space of the configuration space. These points could be interpreted as punctures of the complex plane restricted to the lines $Re(z) = \pm 1/2$ and positive real axis if zeros of Riemann zeta define the conformal weights.

2. The vacuum expectation values of the enveloping algebra of the super-canonical algebra would reduce to n-point functions of a super-conformal quantum field theory in the complex plane containing infinite number of punctures defined by the super-canonical conformal weights, for which primary fields correspond to the representations of $SO(3)\times SU(3)$. These representations would combine to form infinite-dimensional representations of super-canonical algebra. The presence of the gigantic super-canonical symmetries raises the hope that quantum TGD could be solvable to a very high degree.

3. The Super Virasoro algebra and Super Kac Moody algebra associated with 3-D light like CDs would act as symmetries of this theory and the S-matrix of TGD would involve the n-point functions of this field theory. By 7–3 duality this indeed makes sense. The situation would reduce to that encountered in WZW theory in the sense that one would have space-like 3-surfaces X^3 containing two-dimensional closed surfaces carrying representations of Super Kac-Moody algebra.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors [25] having a close connection with the non-trivial representations of braid group and quantum groups. The sequence of non-trivial zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded an an infinite braid behind the von Neumann algebra [25]. Contrary to the expectations, also trivial zeros seem to be important. The finite braids defined by subsets of zeros, and also superpositions of non-trivial zeros of form $1/2+\sum_i y_i$, could be seen as a hierarchy of completely integrable 1-dimensional spin chains leading to quantum groups and braid groups [23, 24] naturally.

It seems that not only Riemann's zeta but also polyzetas [26, 27, 28, 29] could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically. The physical justification for the hypothesis would be quantum criticality. Indeed, it has been found that the loop corrections of quantum field theory are expressible in terms of polyzetas [30]. If the arguments of polyzetas correspond to conformal weights of particles of many-particle bound state, loop corrections vanish when the super-canonical conformal weights correspond to the zeros of polyzetas including zeta.

4.2 The relationship between super-canonical and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-canonical algebra (SC) acting at light-cone boundary and Super Kac-Moody algebra (SKM) acting on light-like 3-surfaces has remained somewhat enigmatic due to the lack of physical insights. This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP is waiting for an answer. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra -in obvious conflict with the basic wisdom that this generator should annihilate physical states- is lacking. It seems that these three problems could have a common solution.

Before going to describe the proposed solution, some background is necessary. The latest proposal for $SC-SKM$ relationship relies on non-standard and therefore somewhat questionable assumptions.

1. SKM Virasoro algebra (SKMV) and SC Virasoro algebra (SCV) (anti)commute for physical states.

2. SC algebra generates states of negative conformal weight annihilated by SCV generators L_n, $n < 0$, and serving as ground states from which SKM generators create states with non-negative conformal weight.

This picture could make sense for elementary particles. On other hand, the recent model for hadrons [F4] assumes that SC degrees of freedom contribute about 70 per cent to the mass of hadron but at space-time sheet different from those assignable to quarks. The contribution of SC degrees of freedom to the thermal average of the conformal weight would be positive. A contradiction results unless one assumes that there exists also SCV ground states with positive conformal weight annihilated by SCV elements L_n, $n < 0$, but also this seems implausible.

4.2.1 New vision about the relationship between SCV and $SKMV$

Consider now the new vision about the relationship between SCV and $SKMV$.

1. The isometries of H assignable with SKM are also symplectic transformations [B3] (note that I have used the term canonical instead of symplectic previously). Hence might consider the possibility that SKM could be identified as a subalgebra of SC. If this makes sense, a generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of SCV and $SKMV$ elements would annihilate physical states and (anti)commute with $SKMV$. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$.

2. The super-generator G_0 contains the Dirac operator D of H. If the action of SCV and $SKMV$ Dirac operators on physical states are identical then cm of degrees of freedom disappear from the differences $G_0(SCV) - G_0(SKMV)$ and $L_0(SCV) - L_0(SKMV)$. One could interpret the identical action of the Dirac operators as the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to SC (imbedding space level) are equal to the gravitational four-momentum and color quantum numbers assignable to SKM (space-time level). Note that since super-canonical transformations correspond to the isometries of the "world of classical worlds" the assignment of the attribute "inertial" to them is natural.

3. The analog of coset construction applies also to SKM and SC algebras which means that physical states can be thought of as being created by an operator of SKM carrying the conformal weight and by a genuine SC operator with vanishing conformal weight. Therefpre the situation does not reduce to that encountered in super-string models

4. The reader can recognize $SC - SKM$ as a precise formulation for $7-3$ duality discussed in the section *About dualities and conformal symmetries in TGD framework* stating that 3-D light-like causal determinants and 7-D causal determinants $\delta M^4_\pm \times CP_2$ are equivalent.

4.2.2 Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. In physical states the p-adic thermal expectation value of the SKM and SC conformal weights would be non-vanishing and identical and mass squared could be identified to the expectation value of SKM scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SCV - SKMV$.

2. There is consistency with p-adic mass calculations for hadrons [F4] since the non-perturbative SC contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SC is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SC - SKM$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SKM whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SC. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [F5] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. It seems that holonomic contributions that is electro-weak and spin contributions must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. Why mass squared corresponds to the thermal expectation value of the net conformal weight? This option is forced among other things by Lorentz invariance but it is not possible to provide a really satisfactory answer to this question yet. In the coset construction there is no reason to require that the mass squared equals to the integer value conformal weight for SKM algebra. This allows the possibility that mass squared has same value for states with different values of SKM conformal weights appearing in the thermal state and equals to the average of the conformal weight.

   The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

2. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations.

This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \qquad (52)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \qquad (53)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

3. Single particle super-canonical conformal weights can have also imaginary part, call it y. The question is what complex mass squared means physically. Complex conformal weights have been assigned with an inherent time orientation distinguishing positive energy particle from negative energy antiparticle (in particular, phase conjugate photons from ordinary photons). This suggests an interpretation of y in terms of a decay width. p-Adic thermodynamics suggest that y vanishes for states with vanishing conformal weight (mass squared) and that the measured value of y is a p-adic thermal average with non-vanishing contributions from states with mass of order CP_2 mass. This makes sense if y_k are algebraic or perhaps even rational numbers.

   For instance, if a massless state characterized by p-adic prime p has p-adic thermal average $y = psy_k$, where s is the denominator of rational valued $y_k = r/s$, the lowest order contribution to the decay width is proportional to $1/p$ by the basic rules of p-adic mass calculations and the decay rate is of same order of magnitude as mass. If the p-adic thermal average of y is of form $p^n y_k$ for massless state then a decay width of order $\Gamma \sim p^{-(n-1)/2}m$ results. For electron n should be rather large. This argument generalizes trivially to the case in which massless state has vanishing value of y.

4.2.3 Can SKM be lifted to a sub-algebra of SC?

A picture introducing only a generalization of coset construction as a new element, realizing mathematically Equivalence Principle, and justifying p-adic thermodynamics is highly attractive but there is a problem. SKM is defined at light-like 3-surfaces X^3 whereas SC acts at light-cone boundary $\delta H_\pm = \delta M^4_\pm \times CP_2$. One should be able to lift SKM to imbedding space level somehow. Also SC should be lifted to entire H. This problem was the reason why I gave up the idea about coset construction and $SC-SKM$ duality as it appeared for the first time.

A possible solution of the lifting problem comes from the observation making possible a more rigorous formulation of $HO-H$ duality stating that one can regard space-time surfaces either as surfaces in hyper-octonionic space $HO = M^8$ or in $H = M^4 \times CP_2$ [**?**, E2]. Consider first the formulation of $HO-H$ duality.

1. Associativity also in the number theoretical sense becomes the fundamental dynamical principle if $HO-H$ duality holds true [E2]. For a space-time surface $X^4 \subset HO = M^8$ associativity is satisfied at space-time level if the tangent space at each point of X^4 is some hyper-quaternionic sub-space $HQ = M^4 \subset M^8$. Also partonic 2-surfaces at the boundaries of causal diamonds formed by pairs of future and past directed light-cones defining the basic imbedding space correlate of zero energy state in zero energy ontology and light-like 3-surfaces are assumed to belong to $HQ = M^4 \subset HO$.

2. $HO-H$ duality requires something more. If the tangent spaces contain the same preferred commutative and thus hyper-complex plane $HC = M^2$, the tangent spaces of X^4 are parameterized by the points s of CP_2 and $X^4 \subset HO$ can be mapped to $X^4 \subset M^4 \times CP_2$ by assigning to a point of X^4 regarded as point (m, e) of $M^4_0 \times E^4 = M^8$ the point (m, s). Note that one must also fix a preferred global hyper-quaternionic subspace $M^4_0 \subset M^8$ containing M^2 to be not confused with the local tangent planes M^4.

3. The preferred plane M^2 can be interpreted as the plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible.

4. An open question is whether the resulting surface in H is a preferred extremal of Kähler action. This is possible since the tangent spaces at light-like partonic 3-surfaces are fixed to contain M^2 so that the boundary values of the normal derivatives of H coordinates are fixed and field equations fix in the ideal case X^4 uniquely and one obtains space-time surface as the analog of Bohr orbit.

5. The light-like "Higgs term" proportional to $O = \gamma_k t^k$ appearing in the generalized eigenvalue equation for the modified Dirac operator is an essential element of TGD based description of Higgs mechanism. This term can cause complications unless t is a covariantly constant light-like vector. Covariant constancy is achieved if t is constant light-like vector in M^2. The interpretation as a space-time correlate for the light-like 4-momentum assignable to the parton might be considered.

6. Associativity requires that the hyper-octonionic arguments of N-point functions in HO description are restricted to a hyperquaternionic plane $HQ = M^4 \subset HO$ required also by the $HO - H$ correspondence. The intersection $M^4 \cap int(X^4)$ consists of a discrete set of points in the generic case. Partonic 3-surfaces are assumed to be associative and belong to M^4. The set of commutative points at the partonic 2-surface X^2 is discrete in the generic case whereas the intersection $X^3 \cap M^2$ consists of 1-D curves so that the notion of number theoretical braid crucial for the p-adicization of the theory as almost topological QFT is uniquely defined.

7. The preferred plane $M^2 \subset M^4 \subset HO$ can be assigned also to the definition of N-point functions in HO picture. It is not clear whether it must be same as the preferred planes assigned to the partonic 2-surfaces. If not, the interpretation would be that it corresponds to a plane containing the over all cm four-momentum whereas partonic planes M_i^2 would contain the partonic four-momenta. M^2 is expected to change at wormhole contacts having Euclidian signature of the induced metric representing horizons and connecting space-time sheets with Minkowskian signature of the induced metric.

The presence of globally defined plane M^2 and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of SC and SKM to H. At the light-cone boundary the light-like radial coordinate can be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3 one can fix the light-like coordinate varying along the braid strands can be lifted to some hyper-complex coordinate of M^2 defined in the interior of X^4. The total four-momenta and color quantum numbers assignable to the SC and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

4.2.4 Questions about conformal weights

One can pose several non-trivial questions about conformal weights.

1. The negative SKM conformal weights of ground states for elementary particles [F3] remain to be understood in this framework. In the case of light-cone boundary the natural value for ground state conformal weight of a scalar field is $-1/2$ since this implies a complete analogy with a plane wave with respect to the radial light-like coordinate r_M with inner product defined by a scale invariant integration measure dr_M/r_M. If the coset construction works same should hold true for SKM degrees of freedom for a proper choice of the light-like radial coordinate. There are thus good hopes that negative ground state conformal weights could be understood.

2. Further questions relate to the imaginary parts of ground state conformal weights, which can be vanishing in principle. Do the ground state

conformal weights correspond to the zeros of some zeta function- most naturally the zeta function defined by generalized eigenvalues of the modified Dirac operator and satisfying Riemann hypothesis so that ground state conformal weight would have real part -1/2? Do SC and SKM have same spectrum of complex conformal weights as the coset construction suggests? Does the imaginary part of the conformal weight bring in a new degree of freedom having interpretation in terms of space-time correlate for the arrow of time with the generalization of the phase conjugation of laser physics representing the reversal of the arrow of geometric time?

3. The opposite light-cone boundaries of the causal diamond bring in mind the hemispheres of S^2 in ordinary conformal theory. In ordinary conformal theory positive/negative powers of z correspond to these hemispheres. Could it be that the radial conformal weights are of opposite sign and of same magnitude for the positive and negative energy parts of zero energy state?

4.2.5 Further questions

There are still several open questions.

1. Is it possible to define hyper-quaternionic variants of the superconformal algebras in both H and HO or perhaps only in HO. A positive answer to this question would conform with the conjecture that the geometry of "world of classical worlds" allows Hyper-Kähler property in either or both pictures [B3].

2. How this picture relates to what is known about the extremals of field equations [D1] characterized by generalized Hamilton-Jacobi structure bringing in mind the selection of preferred M^2?

3. Is this picture consistent with the views about Equivalence Principle and its possible breaking based on the identification of gravitational four-momentum in terms of Einstein tensor is interesting [D3]?

4.3 Brief summary of super-conformal symmetries in partonic picture

The notion of conformal super-symmetry is very rich and involves several non-trivial aspects, and as the following discussions shows, one could assign the attribute super-conformal to several symmetries. In the following I try to sum up what I see as important. What is new is that it is now possible to tie everything to the fundamental description in terms of the parton level action principle and provide a rigorous justification and precise realization for the claimed super-conformal symmetries.

4.3.1 Super-canonical symmetries

Super-canonical symmetries correspond to the isometries of the configuration space CH (the world of classical worlds) and are induced from the corresponding symmetries of $\delta H_\pm \equiv \delta M^4_\pm \times CP_2$. The explicit representations have been constructed for both 2-D and stringy options. The most stringent option having strong support from various considerations is that single particle conformal weights are of form $1/2 + i\sum_k n_k y_k$, where $s_k = 1/2 + iy_k$ is zero of Riemann zeta. The construction of many particle conformally bound states for poly-zetas leads to the same spectrum for bound states and predicts that only 2- and 3-parton bound states are irreducible. On the other hand, conformal weights are additive for the (anti)commutators of (super)Hamiltonians and gives thus all weights of form $s = n + i\sum_k n_k y_k$.

The interpretation of this picture is not obvious.

1. The first interpretation would be that also other conformal weights are possible but that the commutator and anti-commutator algebras of super-canonical algebra containing conformal weights $Re(s) = k/2$, $k > 1$, represent gauge degrees of freedom. The sub-Virasoro algebra generated by L_n, $n > 0$, would generate these conformal weights which would suggest that L_n, $n > 0$, but not L_0, must annihilate the physical states. The problem is that this makes p-adic thermodynamics impossible.

2. p-Adic mass calculations would suggest that Super Kac-Moody Virasoro (SKMV) generators L_n, $n > 0$, do not correspond to pure gauge degrees of freedom, and a more general interpretation would be that all these conformal weights are possible and represent genuine physical degrees of freedom. The extension of the algebra using the standard assumption $L_{-n} = L_n^\dagger$ would bring in also the conformal weights $Re(s) = -k/2$, $k \geq 1$. p-adic mass calculations would encourage to think that it is super-canonical (SC) generators L_{-n}, $n > 0$, which annihilate tachyonic ground states and stabilize them against tachyonic p-adic thermodynamics. The physical ground state with a vanishing conformal weight would be constructed from this tachyonic ground state and p-adic thermodynamics for $SKMV$ generators L_n, $n > 0$, would apply to it.

3. In the discrete variant of theory required by number theoretic universality all stringy sub-manifolds of X^2 corresponding to the inverse images of $z = \zeta(n/2 + i\sum_k n_k y_k) \in S^2 \subset CP_2$ would be realized so that one would have probability amplitude in the discrete set of these number theoretic strings. $SKMV$ generators L_n and G_r would excite $n > 0$ "shells" in this structure whereas SC generators would generate $n < 0$ shells.

4. Also the trivial zeros $s_n = -2n$, $n > 0$, of Riemann Zeta could correspond to physically interesting conformal weights for the super-canonical algebra (at least). In the region $r \geq r_0$ the function r^{-2n} approaches zero and these powers are square integrable in this region. The orthogonality with other

states could be achieved by arranging things suitably in other degrees of freedom [B2]. Since ζ is real also along real line, the set of even integers $\sum_k n_k s_k$, $n_k \in Z$ is mapped by ζ to the same real line of $S^2 \subset CP_2$ as non-trivial zeros of ζ. p-Adic mass calculations would suggest that states with conformal weight $s_{min} = -2n_{max}$ (at least these) could represent null states annihilated by L_{-n}, $n > 0$.

4.3.2 Bosonic super Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{54}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \tag{55}$$

1. Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{56}$$

This gives

$$\begin{aligned} c_A(x) & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k}\partial_\beta h^l \\ = & \ \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta\xi^\mu \ . \end{aligned} \tag{57}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{58}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \quad . \tag{59}$$

2. A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \quad . \tag{60}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \quad . \tag{61}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3-local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \quad . \tag{62}$$

   The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^{\alpha}\partial_{\alpha}g_{ij} + g_{kj}\partial_i\xi^k + g_{ki}\partial_j\xi^k = \partial_i c_A h_{kl} j^{Ak}\partial_j h^l \ . \tag{63}$$

   These are 3 differential equations for 3 functions ξ^{α} on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

   This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

3. Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^{μ} are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_AJ^A, c_BJ^B]$.

$$\left[c_AJ^A, c_BJ^B\right] = c_Ac_BJ^{[A,B]} + J^A\circ c_BJ^B - J^B\circ c_AJ^A \ , \tag{64}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl}D - M^{kl} \ . \end{array} \quad (65)$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labelled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_\pm$.

Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_\pm$ along this ray defining also $SO(2)$ rotation axis.

4. Hamiltonians

The action of these transformations on Chern-Simons action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_\pm \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

5. Action on spinors

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors. This action is not consistent with the generalized eigenvalue equation unless one restricts it to X^2 at $\delta H_\pm$.

2. Since Kac-Moody generator performs a local spinor rotation and increases the conformal weight by n units, the simplest possibility is that the action of transformation adds to Ψ_λ with $\lambda = 1/2 + i\sum_k n_k y_k$, a term with eigenvalue $\lambda + n$ and having $J^A \Psi_\lambda$ as initial values at X^2. This would make natural the interpretation as a gauge transformation apart from the effects caused by the possible central extension term.

6. How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-canonical algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

4.3.3 Fermionic Kac-Moody algebra in spin and electro-weak degrees of freedom

The action of spin rotations and electro-weak rotations can be identified in terms of the group $SU(2) \times SU(2) \times U(1)$ associated inherently with $N =$

4 super-conformal symmetry. The action on zero modes and eigen modes Ψ is straightforward to write as multiplication on the initial values at X^2 and assuming that λ in the generalized eigenvalue equation is replaced by $\lambda + n$.

Fermionic super-generators correspond naturally to zero modes and eigen modes of the modified Dirac operator labelled by the radial conformal weights $\lambda = 1/2 + i\sum_k n_k yk$ and by the quantum numbers labelling the dependence on transversal degrees of freedom. The real part of the conformal weight would corresponds for $D\Psi = 0$ to ground state conformal weight $h = 0$(Ramond) and to $h = 1/2$ for $\lambda \neq 0$ (N-S). That also bosonic super-canonical Hamiltonians can have half odd integer conformal weight is however in conflict with the intuition that half-odd integer conformal weights correspond to states with odd fermion number.

For Ramond representations the lines $\zeta(Re(s) = n) \subset S^2$, $n \geq 0$, would represent the conformal weights at space-time level and for N-S representations the lines would correspond to $\zeta(Re(s) = n + 1/2) \subset S^2$. If also trivial zeros are possible they would correspond to the lines $\zeta(Re(s) = n - 2k) \subset S^2$, $k = 1, 2, ...$

4.3.4 Radial Super Virasoro algebras

The radial Super Virasoro transformations act on both $\delta H_\pm$ and partonic 3-surface X^3 and are consistent with the freedom to choose the basis of $H_\pm$ Hamiltonians and the eigenmode basis of the modified Dirac operator by a re-scaling the light-like vector (t^k or more plausibly, its dual n^k) appearing in the definition of the generalized eigenvalue equation.

In the partonic sector a possible interpretation is as local diffeomorphisms of X^3. These transformations do not however leave X^3 invariant as a whole, which brings in some delicacies. In the case of $\delta H_\pm$ the tip of the future light-cone remains invariant only for $n \geq 0$ and $r = \infty$ only for $n \leq 0$. These facts could explain why only the generators L_n, $n < 0$ (or $n < 0$ depending on whether positive or negative energy component of zero energy state is in question) annihilate the ground states.

One can assign to the Virasoro algebra of $H_\pm$ Hamiltonians as Noether charges defined by current $\Pi^0_k j^{Ak}$ which reduces to a dual of a closed 2-form in the case of $H_\pm$ because its symplectic form annihilates j^{Ak}. The transformations associated with X^3 correspond to a unique shift of X^2 in the light-like direction by $\delta h^k = r^n \partial_r h^k$ so that the Hamiltonian is well-defined and reduces to a value of a closed 2-form so that the stringy picture emerges.

The corresponding fermionic super Hamiltonians $G_r = \overline{\nu} r^n \Gamma_r \Psi$ anti-commute to these as is easy to see by noticing that the light-like radial gamma matrices Γ_r appear in the combination $\Gamma_r \gamma^0 \Gamma_r = \gamma_0$ in the anti-commutator so that it does not vanish. One can consider also more general fermionic generators obtained by replacing right-handed neutrino spinor with an arbitrary solution of $D\Psi = 0$ which is eigen spinor of $J^{kl}\Sigma_{kl}$ appearing in the fermionic anti-commutation relations. This would give rise to a full $N = 4$ super-conformal symmetry of Ramond type but having infinite degeneracy due to the dependence on transversal coordinates of X^3. If one allows also the solutions of $D\Psi = \lambda\Psi$ one obtains counterparts of N-S type representations with a similar degeneracy.

It must be emphasized that four-momentum does not appear neither in the representations of Super Virasoro generators as it does in string models and this is consistent with the Lorentz invariant identification of mass squared as vacuum expectation value of the net conformal weight. Also the problems with tachyons are avoided. Four-momentum could creep in if one had Sugawara type representation of Super Virasoro generators in terms of Kac-Moody generators which indeed contain also translation generators now. Note also that the stringy conformal weight would be associated with partonic 2-surface, whereas radial conformal weight is associated with its light-like orbit. Furthermore, the origin of the radial super-conformal symmetries is light-likeness rather than stringy character. It is not clear whether it is useful to assign the usual conformal weights with the conformal fields at X^2 and whether the stringy anti-commutation relations for Ψ force this kind of assignment.

4.3.5 Gauge super-symmetries associated with the generalized eigenvalue equation for D

Zero modes which are annihilated by the operator $T = t^k\gamma_k$ or $N = n^k\gamma_k$. t^k (n^k) is the light-like appearing in the generalized eigenvalue equation for the modified Dirac operator. t^k is parallel to X^3 and n^k, which corresponds to the more plausible option, is obtained by changing the direction of the spatial part of t^k in the preferred M^4 coordinate frame associated with the space-time sheet (the rest system or number theoretically determined M^4 time). n^k defines inwards directed tangent vector to the space-time sheet containing X^3. The zero modes of the modified Dirac operator annihilated by T (N) act as super gauge symmetries for the generalized eigen modes of the generalized Dirac operator. They do not depend on r and thus have a vanishing conformal weight.

The freedom to choose the scaling of t^k (n^k) rather freely gives rise to a further symmetry which does not affect the eigenvalue spectrum but modifies the eigen modes. This symmetry is definitely a pure gauge symmetry.

4.3.6 What about ordinary conformal symmetries?

Ordinary conformal symmetries acting on the complex coordinate of X^2 have not yet been discussed. These symmetries involve the dependence on the induced metric through the moduli of characterizing the conformal structure of X^2. Stringy picture would suggest in the case of a spherical topology that the zero modes and eigen modes of Ψ are proportional to z^n at X^2. Only $n \geq 0$ mode would be non-singular at the northern hemisphere and $n \leq 0$ at the southern hemisphere and the eigen modes are non-normalizable.

One cannot glue these modes together at equator unless one assumes the behavior z^n, $n \geq 0$, on the northern hemisphere and $\overline{z}^{-n}$, $n \geq 0$, on the southern hemisphere. The identification $\Psi_+(z) = \Psi_-^\dagger(z)$ ($z \rightarrow \overline{z}$ in Hermitian conjugation) at equator would state that "positive energy" particle at the northern hemisphere corresponds to a negative energy antiparticle at the southern hemisphere. The assumption that energy momentum generators $T_+(z)$ and $T_-(z)$ are related in the same manner at equator gives $L_n = L^\dagger_{-n}$ as required. Second candidate for the basis are spherical harmonics which are eigenstates of $L_0 - \overline{L_0}$ defining angular momentum operator L_z but they do not possess well defined conformal weights.

The radial time evolution for the Kac-Moody generators does not commute with L_0 whereas well-defined radial conformal weights are possible. This would support the view that the conformal weight associated with X^2 degrees of freedom does not contribute to the mass squared. If this picture is correct, L_0 would label different SKM representations and play a role similar to that in conformal field theories for critical systems.

4.3.7 How to interpret the overall sign of conformal weight?

The overall sign of conformal weight can be changed by replacing r with $1/r$ and the region $r > r_0$ with $r < r_0$ of $\delta H_\pm$ or of partonic 3-surface. The earlier idea that the conformal weights associated with the super-conformal algebras assignable to $\delta H_\pm$ and to light-like partonic 3-surfaces have opposite signs would allow to construct representations of super-canonical algebra by constructing a tachyonic ground state using super-canonical generators and its excitations using super Super-Kac Moody generators as in super string models.

There is however an objection against this idea. The partons at $\delta H_\pm$ would have a finite distance from the tip of the light cone at all points where they correspond to non-vacuum extremals, so that the phase transitions changing the value of Planck constant should always occur via vacuum extremals. This would not allow the leakage of Kähler magnetic flux between different sectors of imbedding space. The cautious conclusion is that at least in the super-canonical sector both $r > r_0$ and $r < r_0$ sectors related by the conformal transformation $r \rightarrow 1/r$ must be allowed and correspond to positive and negative values for the radial super-conformal weights.

In zero energy ontology particle reactions correspond to zero energy states which at space-time level carry positive energy particles at the end of world in geometric past and negative energy particles at the end of world in the geometric future. Also conformal weights are of opposite sign so that vanishing of the net conformal weights holds true only for zero energy states in accordance with the spirit of p-adic mass calculations. If the states of geometric past correspond to positive (negative) super Kac-Moody (super-canonical) conformal weights, the scattering could be regarded as a process leading from the region $r > r_0$ at δM^4_+0 to the region $r < r_0$ at δM^4_-. At partonic level the incoming partons would correspond to the region $r < r_0$ and outgoing partons to the region $r > r_0$, which conforms with the idea that the final state can partons can be arbitrary far in the geometric future.

In certain sense this picture would reproduce big ban-big crush picture at the level of super-canonical algebra. $r < r_0$ means that partons can be arbitrarily near to the tip of δM^4_- representing the final singularity whereas $r > r_0$ for δM^4_+ would be the counterpart for big bang.

4.3.8 Absolute extremum property for Kähler action implies dynamical Kac-Moody and super conformal symmetries

The identification of the criterion selecting the preferred extremal of Kähler action defining space-time surface as a counterpart of Bohr orbit has been a long standing challenge. The first guess was that an absolute minimum is in question. The number theoretic theoretic picture, in particular $HO-H$ duality [E2] resolves the problem by assigning to each point of X^4 a preferred plane M^2, which also fixes the boundary conditions for the field equations at light-like partonic 3-surfaces. The still open questions are whether the H images of hyper-quaternionic 4-surfaces of $HO = M^8$ are indeed extremals of Kähler action and whether these preferred extremals satisfy absolute extremum property. Be as it may, the following argument suggests that absolute extremum property gives rise to additional symmetries.

The extremal property for Kähler action with respect to variations of time derivatives of initial values keeping h^k fixed at X^3 implies the existence of an infinite number of conserved charges assignable to the small deformations of the extremum and to H isometries. Also infinite number of local conserved super currents assignable to second variations and to covariantly constant right handed neutrino are implied. The corresponding conserved charges vanish so that the interpretation as dynamical gauge symmetries is appropriate. This result provides strong support that the local extremal property is indeed consistent with the almost-topological QFT property at parton level.

The starting point are field equations for the second variations. If the action contain only derivatives of field variables one obtains for the small deformations δh^k of a given extremal

$$\begin{aligned} \partial_\alpha J_k^\alpha &= 0 \ , \\ J_k^\alpha &= \frac{\partial^2 L}{\partial h_\alpha^k \partial h_\beta^l} \delta h_\beta^l \ , \end{aligned} \tag{66}$$

where h_α^k denotes the partial derivative $\partial_\alpha h^k$. A simple example is the action for massless scalar field in which case conservation law reduces to the conservation of the current defined by the gradient of the scalar field. The addition of mass term spoils this conservation law.

If the action is general coordinate invariant, the field equations read as

$$D_\alpha J^{\alpha,k} = 0 \tag{67}$$

where D_α is now covariant derivative and index raising is achieved using the metric of the imbedding space.

The field equations for the second variation state the vanishing of a covariant divergence and one obtains conserved currents by the contraction this equation with covariantly constant Killing vector fields j_A^k of M^4 translations which means that second variations define the analog of a local gauge algebra in M^4 degrees of freedom.

$$\begin{aligned} \partial_\alpha J_n^{A,\alpha} &= 0 \ , \\ J_n^{A,\alpha} &= J_n^{\alpha,k} j_k^A \ . \end{aligned} \tag{68}$$

Conservation for Killing vector fields reduces to the contraction of a symmetric tensor with $D_k j_l$ which vanishes. The reason is that action depends on induced metric and Kähler form only.

Also covariantly constant right handed neutrino spinors Ψ_R define a collection of conserved super currents associated with small deformations at extremum

$$\begin{aligned} J_n^\alpha &= J_n^{\alpha,k} \gamma_k \Psi_R \ , \\ & . \end{aligned} \tag{69}$$

Second variation gives also a total divergence term which gives contributions at two 3-dimensional ends of the space-time sheet as the difference

$$\begin{aligned} Q_n(X_f^3) - Q_n(X^3) &= 0 \ , \\ Q_n(Y^3) &= \int_{Y^3} d^3x J_n \ , \ \ J_n = J^{tk} h_{kl} \delta h_n^l \ . \end{aligned} \tag{70}$$

The contribution of the fixed end X^3 vanishes. For the extremum with respect to the variations of the time derivatives $\partial_t h^k$ at X^3 the total variation must vanish. This implies that the charges Q_n defined by second variations are identically vanishing

$$Q_n(X_f^3) = \int_{X_f^3} J_n = 0 \ . \tag{71}$$

Since the second end can be chosen arbitrarily, one obtains an infinite number of conditions analogous to the Virasoro conditions. The analogs of unbroken loop group symmetry for H isometries and unbroken local super symmetry generated by right handed neutrino result. Thus extremal property is a necessary condition for the realization of the gauge symmetries present at partonic level also at the level of the space-time surface. The breaking of super-symmetries could perhaps be understood in terms of the breaking of these symmetries for light-like partonic 3-surfaces which are not extremals of Chern-Simons action.

4.4 Large $N = 4$ SCA is the natural option

The arguments below support the view that "large" $N = 4$ SCA is the natural algebra in TGD framework.

4.4.1 How $N = 4$ super-conformal invariance emerges from the parton level formulation of quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects led to the final understanding of super-conformal symmetries and their breaking. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2)\times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra having interpretation in terms of rotations and electro-weak gauge group.

4.4.2 Large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry seems to define the fundamental partonic super-conformal symmetry in TGD framework. In the case of SKM algebra the groups would act on induced spinors with $SU(2)_+$ representing spin rotations and $SU(2)_- \times U(1) = U(2)_{ew}$ electro-weak rotations. In super-canonical sector the action would be geometric: $SU(2)_+$ would act as rotations on light-cone boundary and $U(2)$ as color rotations leaving invariant a preferred CP_2 point.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [42]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$\begin{aligned} k_\pm &\equiv k(SU(2)_\pm) \ , \\ k_1 &\equiv k(U(1)) = k_+ + k_- \ . \end{aligned} \tag{72}$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{73}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k+1 \ , \ k_1 = k+2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{74}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. Central extension would be trivial in rotational degrees of freedom but non-trivial in $U(2)_{ew}$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$. A possible interpretation is in terms of electro-weak symmetry breaking with $k_+ > 0$ signalling for the massivation of electro-weak gauge bosons.

An interpretation consistent with the general vision about the quantization of Planck constants is that k_+ and k_- relate directly to the integers n_a and n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants via the formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$. This would require $k_\pm \geq 1$ for G_i a finite subgroup of $SU(2)$ ("anyonic" phases). In stringy phases with $G_i = SU(2)$ for $i = a$ or $i = b$ or for both, k_i could also vanish so that also $n_i = 2$ corresponding to A_2 ADE diagram and $SU(2)$ Kac-Moody algebra becomes possible. In the super-canonical sector $k_+ = 0$ would mean massless gluons and $k_- = k_1$ that $U(2) \subset SU(3)$ and possibly entire $SU(3)$ represents an unbroken symmetry.

4.4.3 About breaking of large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking could be thought to occur via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2) \times U(1)$ would result. The next step would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

a) $SU(2) \times U(1)$ could correspond to electro-weak gauge group such that rotational degrees of freedom are frozen dynamically by the massivation of the corresponding excitations. This interpretation could apply in stringy phase: for cosmic strings rotational excitations are indeed hyper-massive.

b) The interpretation of $SU(2)$ as spin rotation group and $U(1)$ as electromagnetic gauge group conforms with the general vision about electroweak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free.

The next step in the symmetry breaking sequence would be $N = 2$ SCA with $U(1) \subset SU(2) \times U(2)$ sub-algebra. The interpretation could be as electro-weak symmetry breaking in the stringy sector (cosmic strings) so that $U(1)$ would correspond to em charge or possibly weak isospin.

4.4.4 Relationship to super-strings and M-theory

The (4,4) signature characterizing $N = 4$ SCA topological field theory is not a problem since in TGD framework the target space becomes a fictive concept defined by the Cartan algebra. Both $M^4 \times CP_2$ decomposition of the imbedding space and space-time dimension are crucial for the $2+2+2+2$ structure of the Cartan algebra, which together with the notions of the configuration space and generalized coset representation formed from super Kac-Moody and super-canonical algebras guarantees $N = 4$ super-conformal invariance.

Including the 2 gauge degrees of freedom associated with M^2 factor of $M^4 = M^2 \times E^2$ the critical dimension becomes $D = 10$ and and including the radial degree of light-cone boundary the critical dimension becomes $D = 11$ of M-theory. Hence the fictive target space associated with the vertex operator construction corresponds to a flat background of super-string theory and flat background of M-theory with one light-like direction. From TGD point view the difficulties of these approaches are due to the un-necessary assumption that the fictive target space defined by the Cartan algebra corresponds to the physical imbedding space. The flatness of the fictive target space forces to introduce the notion of spontaneous compactification and dynamical imbedding space and this in turn leads to the notion of landscape.

4.4.5 Questions

A priori one can consider 3 different options concerning the identification of quarks and leptons.

1. Could also quarks define $N = 4$ superconformal symmetry?

One can ask, whether the construction could be extended by allowing H-spinors of opposite chirality to have leptonic quantum numbers so that free quarks would have integer charge. The construction does not work. The direct sum of $N = 4$ SCAs can be realized but $N = 8$ algebra would require SO(7) rotations mixing states with different fermion numbers: for $N = 4$ SCA this is not needed. Furthermore, only $N < 4$ super-conformal algebras allow an associative realization and $N = 8$ non-associative realization discovered first by Englert exists only at the limit when Kac-Moody central extension parameter k becomes infinite (this corresponds to a critical phase formally and $q = 1$ Jones inclusion). This is not enough for the purposes of TGD and number theoretic vision strongly supports "small" $N = 4$ SCA.

2. Integer charged leptons and fractionally charged quarks?

Second option would be leptons and fractionally charged quarks with $N = 4$ SCA in leptonic sector. It is indeed possible to realize both quark and lepton spinors as super generators of super affinized quaternion algebras (a generalization of super-Kac Moody algebras) so that the fundamental spectrum generating algebra is obtained. Quarks with their natural charges can appear only in $n = 3, k = 1$ phase together with fractionally charged leptons. Leptons in this phase would have strong interactions with quarks. The penetration of lepton into hadron would give rise to this kind of situation. Leptons can indeed move in triality 1 states since 3-fold covering of CP_2 points by M^4 points means that 3 full rotations for the phase angle of CP_2 complex coordinate corresponds to single 2π rotation for M^4 point.

Hadron like states would correspond to the lowest possible Jones inclusion characterized by n=3 and the subgroup A_2 (Z_3) of SU(2). The work with quantization of Planck constant had already earlier led to the realization that ADE Dynkin diagrams assignable to Jones inclusions indeed correspond to gauge groups [A8]: in particular, A_2 corresponds to color group SU(3). Infinite hierarchy of hadron like states with $n = 3, 4, 5...$ quarks or leptons is predicted corresponding to the hierarchy of Jones inclusions, and I have already earlier proposed that this hierarchy should be crucial for the understanding of living matter [M3]. For states containing quarks n would be multiple of 3.

One can understand color confinement of quarks as absolute if one accepts the generalization of the notion of imbedding space forced by the quantization of Planck constant. Ordinary gauge bosons come in two varieties depending on whether their couplings are H-vectorial or H-axial. Strong interactions inside hadrons could be also interpreted as H-axial electro-weak interactions which have become strong (presumably because corresponding gauge bosons are massless) as is clear from the fact that arbitrary high n-point functions are non-vanishing in the phases with $q \neq 1$. Already earlier the so called HO-H duality inspired by the number theoretical vision [E2] led to the same proposal but for ordinary electro-weak interactions which can be also imagined in the scenario in which only leptons are fundamental fermions.

3. Quarks as fractionally charged leptons?

For the third option only leptons would appear as free fermions. The dramatic prediction would be that quarks would be fractionally charged leptons. It is however not clear whether proton can decay to positron plus something (recall the original erratic interpretation of positron as proton by Dirac!): lepton number fractionization meaning that baryon consists of three positrons with fermion number 1/3 might allow this. If not, then only the interactions mediated by the exchanges of gauge bosons (vanishing lepton number is essential) between worlds corresponding to different Jones inclusions are possible and proton would be stable.

There are however also objections. In particular, the resulting states are not identical with color partial waves assignable to quarks and the nice predictions of p-adic mass calculations for quark and hadron masses might be lost. Hence the cautious conclusion is that the original scenario with integer charged quarks predicting confinement automatically is the correct one.

4.5 How could exotic Kac-Moody algebras emerge from Jones inclusions?

Also other Kac-Moody algebras than those associated with the basic symmetries of quantum TGD could emerge from Jones inclusions. The interpretation would be the TGD is able to mimic various conformal field theories. The discussion is restricted to Jones inclusions defined by discrete groups acting in CP_2 degrees of freedom in TGD framework but the generalization to the case of M^4 degrees of freedom is straightforward.

4.5.1 $\mathcal{M} : \mathcal{N} = \beta < 4$ case

The first situation corresponds to $\mathcal{M} : \mathcal{N} = \beta < 4$ for which a finite subgroup $G \subset SU(2)_L$ defines Jones inclusion $\mathcal{N}^G \subset \mathcal{M}^G$, with G commuting with the Clifford algebra elements creating physical states. $\mathcal{N}$ corresponds to a subalgebra of the entire infinite-dimensional Clifford algebra Cl for which one 8-D Clifford algebra factor identifiable as Clifford algebra of the imbedding space is replaced with Clifford algebra of M^4.

Each M^4 point corresponds to G orbit in CP_2 and the order of maximal cyclic subgroup of G defines the integer n defining the quantum phase $q = exp(i\pi/n)$. In this case the points in the covering give rise to a representation of G defining multiplets for Kac-Moody group $\hat{G}$ assignable to G via the ADE diagram characterizing G using McKay correspondence. Partonic boundary component defines the Riemann surface in which the conformal field theory with Kac Moody symmetry is defined. The formula $n = k + h_{\hat{G}}$ would determine the value of Kac-Moody central extension parameter k. The singletness of fermionic oscillator operators with respect to G would be compensated by the emergence of representations of G realized in the covering of M^4.

4.5.2 $\mathcal{M} : \mathcal{N} = \beta = 4$ case

Second situation corresponds to $\beta = 4$. In this case the inclusions are classified by extended ADE diagrams assignable to Kac Moody algebras. The interpretation $n = k + h_G$ assigning the quantum phase to $SU(2)$ Kac Moody algebra corresponds to the Jones inclusion $\mathcal{N}^{\hat{G}} \subset \mathcal{M}^{\hat{G}}$ of configuration space spinors for $\hat{G} = SU(2)_L$ with index $\mathcal{M} : \mathcal{N} = 4$ and trivial quantum phase $q = 1$. The Clifford algebra elements in question would be products of fermionic oscillator operators having vanishing $SU(2)_L$ quantum numbers but arbitrary $U(1)_R$ quantum numbers if the identification $\hat{G} = SU(2)_L$ is correct. Thus only right handed fermions carrying homological magnetic charge would be allowed and obviously these fermions must behave like massless particles so that $\beta < 4$ could be interpreted in terms of massivation. The ends of cosmic strings $X^2 \times S^2 \subset M^4 \times CP_2$ would represent an example of this phase having only Abelian electro-weak interactions.

According to the proposal of [A8] the finite subgroup $G \subset SU(2)$ defining the quantum phase emerges from the effective decomposition of the geodesic sphere $S^2 \subset CP_2$ to a lattice having S^2/G as the unit cell. The discrete wave functions in the lattice would give rise to $SU(2)_L \supset G$-multiplets defining the Kac Moody representations and S^2/G would represent the 2-dimensional Riemannsurface in which the conformal theory in question would be defined. Quantum phases would correspond to the holonomy of S^2/G. Therefore the singletness in fermionic degrees of freedom would be compensated by the emergence of G- multiplets in lattice degrees of freedom.

4.6 The M^4 local variants of super conformal algebras

The M^4 local versions of super conformal algebras bring in a completely new mathematical element allowing to unify string model type conformal algebras and conformal algebras appearing in statistical physics so that stringy mass formula can be seen as identification of M^4 conformal weight with ordinary conformal weight instead of interpretation of mass squared as a contribution cancelling the net conformal weight.

The explicit formulas for generators of local Super algebras can be guesses by from those for ordinary conformal algebras. In the defining representation the infinitesimal generators of loop group can be written as

$$T_n^A(z) \quad = \quad T^A z^n \quad , \tag{75}$$

where T^A satisfy the usual Lie algebra commutation relations. Obviously satisfy the commutation relations of Kac-Moody algebra without central extension. The quantal variant of the Kac Moody algebra is obtained via the expansion

$$\begin{aligned} T^A(z) \quad &= \quad \sum_n T_n^A z^n \quad , \\ (T_n^A)^\dagger \quad &= \quad T_{-n}^a \quad , \end{aligned} \tag{76}$$

where T_n^A satisfy the commutation relations of Kac-Moody algebra with central extension.

In the recent case z^n would be replaced with m^N where m is representation of M^4 coordinate as hyper-quaternion which can be represented as 2×2 matrix. The hyper-quaternions m_1 and m_2 do not commute unless they belong to same hyper-complex plane M^2 defining the Minkowskian variant of complex plane. Thus if m_1 and m_2 they are on the same hyper-complex line ($m_1 = \lambda m_2$) this is the case. Complex conjugation for m can be defined as a change of sign for the imaginary part of hyper-quaternion. The commutation and anti-commutation relations for M^4 local variants of super-conformal algebras can be fixed by considering them at hyper-complex planes of M^4.

4.6.1 Hyper-quaternionic variant of super-conformal algebras

Concerning the definition of the hyper-quaternionic variants of super-conformal algebras two options can be considered.

Option I: One could allow only the generators $X_N^{A,n}$ with $N = n$ so that the resulting super conformal algebra restricted to the hyper-quaternionic plane could be seen as a dual for the original algebra at partonic 2-surface obtained by replacing powers of z with powers of m. $N = n$ condition is very attractive since N as a conformal weight in M^4 naturally corresponds to the mass squared eigen value. In super string models mass squared would give a contribution to conformal weight compensating the contributions possible also in Euclidian conformal field theories. Now the duality between the two algebras would give the mass formula.

Option II: One could try to genuinely localize the super-conformal algebra. This would bring in an infinite number of new degrees of freedom. Even the unit matrix Id belonging to the super-conformal algebra should be localized to give a family Id_N of unit operators. In principle new kind of central extensions would become possible and would be completely independent of the original one. This approach does not seem to be promising and does not conform with the physical intuitions. Hence only the first option will be considered.

Since Gamma matrices carry fermion number in TGD framework, one must assume that $N \rightarrow -N$ transformation involves fermionic charge conjugation so that N corresponds to say creation operator for fermion and $-N$ to annihilation operator for antifermion. Therefore one has doubling of the fermionic algebra giving generators $\Gamma_{A,N}$ and $\Gamma^c_{A,N}$. The crucial deviation from string models emerges where hermiticity conditions for Γ field lead to Majorana spinors and critical dimension $D = 10$. This leads to non-hermiticity and doubling of also Super-Virasoro generators G_N. This doubling seems to be completely analogous to that occurring in $N = 2$ super-symmetric theories.

The non-vanishing commutators and anti-commutators are

$$\begin{aligned} \left[T^A_M(\lambda m), T^B_N(m)\right] &= \lambda^M f^{AB}_C T^C_{M+N} m^{M+N} + k\delta_{M+N}\delta_{A,B} \ , \\ \{\Gamma_{A,M}(\lambda m), \Gamma^c_{B,N}(m)\} &= \lambda^M G_{AB}(N)\delta_{M+N} \ . \end{aligned} \tag{77}$$

Here G_{AB} denotes configuration space metric for CH_m.

The local super Kac-Moody field are

$$\begin{aligned} T^A(m) &= \sum_n T^A_N m^N \ , \\ \Gamma^A(m) &= \sum_n \Gamma^A_N m^N \ , \ . \end{aligned}$$

Commutations and anticommutators along the hyper-complex planes $m_1/m_2 = \lambda$ can be calculated. One can always make the choice $|\lambda| < 1$. The central extension term in Kac-Moody algebra commutation relations and the fermionic anti-commutator read as

$$\begin{aligned} \left[T^A(\lambda m), T^B(m)\right]_c &= k\delta_{A,B}\frac{1}{1-\lambda} \ , \\ \{\Gamma_A(\lambda m), \Gamma_B(m)\} &= \sum_M G_{A,B}(M)\lambda^M \ . \end{aligned} \tag{78}$$

At the limit $\lambda \rightarrow 1$ the commutator and anti-commutator diverge. Exactly the same behavior results in the ordinary conformal field theory. The functional form of the commutators in hyper-complex algebra is identical to that obtained in the complex case which suggests that everything from conformal theories in plane generalizes more or less trivially to the recent situation by replacing z with m in some hyper-complex subspace and possibly algebraically continuing to the other values of m.

4.6.2 Comparison with the Super Virasoro conditions of string models

The next question is how the resulting theory relates to the stringy conformal invariance and the conformal invariance of Euclidian conformal theories. In string models the quantization of mass squared results from Virasoro conditions $L_0|phys\rangle = 0$ and mass squared gives a compensating contribution to the conformal weight from degrees of freedom not present in Euclidian conformal field theories. It is essential that all generators L_n annihilate the physical states and this implies that the parameters labelling Super Virasoro representations cancel: $c = 0, h = 0$. This is what makes the dimension of the imbedding space so unique. In TGD framework number theoretical considerations imply the uniqueness.

The situation is now different.

1. Partonic boundary components are Euclidian and there is no room for four-momentum at this level. Since N appears as an exponent of the hyper-quaternionic coordinate m highly analogous to the hyper-complex coordinate for the stringy world sheet, the identification of the mass squared eigenvalues as conformal weights $N = n$ is natural and one would have the formula

$$m^2 = m_0^2 N = m_0^2 n \ . \tag{79}$$

2. There is no need to pose the condition $L_N|phys\rangle = 0$ for $N > 0$. Neither there is any need to assume that Super Virasoro generators L_N create zero norm states. Obviously this would be also inconsistent with mass formula. Therefore both c and h can be non-vanishing and all non-stringy conformal field theories can in principle be associated with partonic boundary components. This is what one wants since these theories should be assignable to the Jones inclusions characterizing the limitations of quantum measurements.

3. Four-momentum does not appear in Super Virasoro generators. In stringy conformal theories L_n involves four-momentum linearly. Four-momentum appears also in super generators G_n in string model approach and forces Majorana condition since the center of mass term for G_0 in Ramond representation is ordinary Dirac operator. In TGD framework the application of stringy conformal invariance would lead to difficulties since G has fermion number and one should modify ordinary Dirac operator so that it has fermion number. The longitudinal degrees of freedom where it is not possible to complexify the gamma matrices and replace them with fermionic oscillator operators are problematic in this respect. The disappearance of four-momentum from Super Virasoro conditions for G_N resolves this difficulty trivially. Notice that in the recent approach hyper-complex plane for purely number theoretic reasons is analogous to the breaking of manifest Lorentz invariance implied by gauge fixing in stringy approach.

4. Ground state conformal weight h for ordinary Super Virasoro representations can be non-vanishing and even negative, this is indeed the case in TGD framework. Also non-integer conformal weights for ground states are

possible and in N-S representation super generators carry half-odd-integer conformal weights. In the recent case this would require the appearance of fractional powers of hyper quaternion m in N-S type super generators and in the generalizations of hyper-quaternion valued correlation functions (if they exist and have some physical meaning). The square roots appearing in N-S generators are given by

$$
\begin{aligned}
\sqrt{m}^0 &= \tfrac{1}{\sqrt{2}}\sqrt{m^0 \pm \sqrt{a^2}} \quad , \quad a^2 = m_{kl}m^k m^l \\
(\sqrt{m})^i &= \tfrac{m^i}{2\sqrt{m^0}} \quad .
\end{aligned}
$$

Square root is well defined inside future light-cone whereas outside the light-cone the square root forces to introduce an additional imaginary unit. This suggests that only integer valued total conformal weights are acceptable for physical states and correlation functions. In conformal field theories single valuedness of the correlation functions forces the conformal blocks to have integer conformal weights. Roots exist for all fractional powers inside light-cone. In a matrix representation for quaternion such that m corresponds to a 2×2 matrix, the roots are obtained by applying the root of the Lorentz transformation as an automorphism to the root of $m^k = m^0 \delta_{k,0}$. Note that only future or past light cone at a given point is possible since the roots of m^0 appearing in correlation functions with fractional conformal weights must be real. A stronger conclusion is that the conformal fields are defined only inside future or past lightcone and it turns out that this interpretation is physically plausible.

4.6.3 p-Adic thermodynamics and the new approach

The recent approach gives also a sound basis for p-adic thermodynamics.

1. Physical states possess now genuine conformal weights and mass squared is the thermal expectation value of the conformal weight so that there are no problems with Lorentz invariance. In stringy approach net conformal weights of the physical states vanish and one must apply p-adic thermodynamics to the four-momentum squared so that a breaking of Lorentz invariance is unavoidable.

2. The super-conformal partition functions crucial for the p-adic mass calculations using p-adic thermodynamics for L_0 are identical with the ordinary ones if super-generators carry fermion number since this means effectively only $N = 2$ super-symmetry.

3. $N \geq 0$ for the conformal weights of physical states is an obvious requirement in the recent framework since $N < 0$ corresponds to field modes which are singular at $m = 0$. This excludes tachyons. This condition is of special importance in TGD framework since super-canonical generators create ground states with arbitrarily large negative conformal weights [E2].

5 Trying to understand $N = 4$ super-conformal symmetry

I ended up with $N = 4$ super-conformal symmetry as being generated by the solutions of the modified Dirac equation for the induced spinor fields. Later I was ended up with this symmetry by considering the general structure of these algebras interpreted in TGD framework. In the following the latter approach is discussed in detail. Needless to say, a lot remains to be understood.

In particular, the realization of the super-conformal symmetry in the quark sector does not seem to be possible if one assumes fractionally charged free quarks unless one somehow modifies the motion of the super-symmetry. The proposed replacement of covariantly constant spinor fields as generators of super-conformal symmetries with solutions of the modified Dirac equation might resolve this problem. Also the breaking of $N = 4$ to $N = 2$ symmetry is suggested by physical arguments and could be understood in this framework. In particular, $N = 2$ super-conformal symmetry could result in the quark sector in this manner.

5.1 $N = 4$ super-conformal symmetry as a basic symmetry of TGD

$N > 0$ super-conformal algebras contain besides super Virasoro generators also other types of generators and this raises the question whether it might be possible to find an algebra coding the basic quantum numbers of the induced spinor fields.

5.1.1 Right-handed neutrinos as source of $N = 4$ super-conformal symmetry in TGD framework

$N = 2$ super-conformal symmetry would correspond in TGD framework to covariantly constant complex right handed neutrino spinors with two spin directions forming a right handed doublet and would be exact and act only in the leptonic sector relating configuration space Hamiltonians and super-Hamiltonians. This algebra extends to the so called small $N = 4$ algebra if one introduces the conjugates of the right handed neutrino spinors. This symmetry is exact if only leptonic chirality is present in theory or if free quarks carry leptonic charges.

There are several variants of $N = 4$ SCAs and they correspond to the Kac-Moody algebras $SU(2)$ (small SCA), $SU(2) \times SU(2) \times U(1)$ (large SCA) and $SU(2) \times U(1)^4$. Rasmussen has found also a fourth variant based on $SU(2) \times U(1)$ Kac-Moody algebra [42]. It seems that only minimal and maximal $N = 4$ SCAs can represent realistic options. The reduction to almost topological string theory in critical phase is probably lost for other than minimal SCA but could result as an appropriate limit for other variants.

5.1.2 Small $N = 4$ SCA ...

Consider the TGD based interpretation of the small $N = 4$ SCA.

1. The group $SU(2)$ associated with the small $N = 4$ SCA and acting as rotations of covariantly constant right-handed neutrino spinors allows also an interpretation as a group SO(3) leaving invariant the sphere S^2 of the light-cone boundary identified as $r_M = m^0$=constant surface defining generalized Kähler and symplectic structures in $\delta M^4_\pm$.

2. The choice of the preferred coordinate system should have a physical justification. The interpretation of $SO(3)$ as the isotropy group of the rest system defined by the total four-momentum assignable to the 3-surface containing partonic 2-surfaces is supported by the quantum classical correspondence. The subgroup $U(1)$ of $SU(2)$ acts naturally as rotations around the axis defined by the light ray from the tip of $M^4_\pm$ orthogonal to S^2. For $c = 0, k = 0$ case these groups define local gauge symmetries. In the more general case local gauge invariance is broken whereas global invariance remains as it should.

   In $M^2 \times E^2$ decomposition E^2 corresponds to the tangent space of S^2 at a given point and M^2 to the plane orthogonal to it. The natural assumption is that the right handed neutrino spinor is annihilated by the momentum space Dirac operator corresponding to the light-like momentum defining $M^2 \times E^2$ decomposition.

1. Why $N = 4$ super-conformal symmetry would be so nice?

What makes this so interesting is that $N = 2$ super-conformal invariance has been claimed to imply the vanishing of all amplitudes with more than 3 external legs for closed critical $N = 2$ strings having $c = 6, k = 1$ which corresponds to $n \to \infty$ limit and $q = 1$ for Jones inclusions [36, 37]. Only the partition function and $2 \leq N \leq 3$ scattering amplitudes would be non-vanishing. The argument of [36] relies on the imbedding of $N = 2$ super-conformal field theory to $N = 4$ topological string theory whereas in [37] the Ward identities for additional unbroken symmetries associated with the chiral ring accompanying $N = 2$ super-symmetry [24] are utilized. In fact, $N = 4$ topological string theory allows also imbeddings of $N = 1$ super strings [36].

The properties of $c = 6$ critical theory allowing only integral valued $U(1)$ charges and fermion numbers would conform nicely with what we know about the perturbative electro-weak physics of leptons and gauge bosons. $c = 1, k = 1$ sector with $N = 2$ super-conformal symmetry would involve genuinely stringy physics since all N-point functions would be non-vanishing and the earlier hypothesis that strong interactions can be identified as electro-weak interactions which have become strong inspired by HO-H duality [E2] could find a concrete realization.

In $c = 6$ phase $N = 2$-vertices the loop corrections coming from the presence of higher lepton genera in amplitude could be interpreted as topological mixing forced by unitarity implying in turn leptonic CKM mixing for leptons. The non-triviality of 3-point amplitudes would in turn be enough to have a stringy description of particle number changing reactions, such as single photon brehmstrahlung. The amplitude for the emission of more than one brehmstrahlung photons from a given lepton would vanish. Obviously the picture would conform with the vision of [C7] based on the idea of generalizing braid diagrams by allowing branching of braids. Obviously the connection with quantum field theory picture would be extremely tight and imbeddability to a topological $N = 4$ quantum field theory could make the theory to a high degree exactly solvable.

2. Objections

There are also several reasons for why one must take the idea about the usefulness of $c = 6$ super-conformal strings from the point of view of TGD with an extreme caution.

1. Stringy diagrams have quite different interpretation in TGD framework. The target space for these theories has dimension four and metric signature (2,2) or (0,4) and the vanishing theorems hold only for $(2, 2)$ signature. In lepton sector one might regard the covariantly constant complex right-handed neutrino spinors as generators of $N = 2$ real super-symmetries but in quark sector there are no super-symmetries.

2. The spectrum looks unrealistic: all degrees of freedom are eliminated by symmetries except single massless scalar field so that one can wonder what is achieved by introducing the extremely heavy computational machinery of string theories. This argument relies on the assumption that time-like modes correspond to negative norm so that the target space reduces effectively to a 2-dimensional Euclidian sub-space E^2 so that only the vibrations in directions orthogonal to the string in E^2 remain. The situation changes if one assigns negative conformal weights and negative energies to the time like excitations. In the generalized coset representation used to construct physical states this is indeed assumed.

3. The central charge has only values $c = 6k$, where k is the central extension parameter of SU(2) algebra [43] so that it seems impossible to realize the genuinely rational values of c which should correspond to the series of Jones inclusions. One manner to circumvent the problem would be the reduction to $N = 2$ super-conformal symmetry.

4. SU(2) Kac-Moody algebra allows to introduce only 2-component spinors naturally whereas super-quaternions allow quantum counterparts of 8-component spinors.

5.1.3 ...or maximal $N = 4$ SCA?

Consider the Kac-Moody algebra $SU(2)\times SU(2)\times U(1)$ associated with the maximal $N = 4$ SCA. Besides Kac-Moody currents it contains 4 spin 1/2 fermions having an identification as quantum counterparts of leptonic spinor fields. The interpretation of the first $SU(2)$ is as rotations of covariantly constant right handed neutrino spinors and rotations leaving invariant the sphere $S^2 \subset \delta M^4_{\pm}$. $U(2)$ has interpretation as electro-weak gauge group and as maximal linearly realized subgroup of $SU(3)$. This algebra acts naturally as symmetries of the 8-component spinors representing super partners of quaternions.

The algebra involves the integer value central extension parameters k_+ and k_- associated with the two SU(2) algebras as parameters. The value of $U(1)$ central extension parameter k is given by $k = k_+ + k_-$. The value of central extension parameter c is given by

$$c = 6k_-\frac{x}{1+x} < 6k_+ \ \ , \ \ x = \frac{k_+}{k_-} \ \ .$$

c can have all non-negative rational values m/n for positive values of $k_\pm$ given by $k_+ = rm, k_- = (6nr-1)m$. Unitarity might pose further restrictions on the values of c. At the limit $k_- = k$, $k_+ \rightarrow \infty$ the algebra reduces to the minimal $N = 4$ SCA with $c = 6k$ since the contributions from the second $SU(2)$ and $U(1)$ to super Virasoro currents vanish at this limit.

5.1.4 What about $N = 4$ SCA with $SU(2)\times U(1)$ Kac-Moody algebra?

Rasmussen [42] has discovered an $N = 4$ super-conformal algebra containing besides Virasoro generators and 4 Super-Virasoro generators $SU(2)\times U(1)$ Kac-Moody algebra and two spin 1/2 fermions and a scalar. In TGD framework it is difficult to interpret physically the scalar.

There are actually two versions about Rasmussen's article [42]: in the first version the author talks about $SU(2) \times U(1)$ Kac-Moody algebra and in the second one about $SL(2) \times U(1)$ Kac-Moody algebra. These variants would correspond in TGD framework to two different Jones inclusions.

1. The first inclusion is defined by $G = SL(2,R) \subset SO(3,1)$ acting on M^4 part of H-spinors (or alternatively, as Lorentz group inducing motions in the plane E^2 orthogonal to a light-like ray from the origin of light-cone M^4_+). Physically the inclusion would mean that Lorentz degrees of freedom are frozen in the physical measurement. This leaves electro-weak group $SU(2)_L \times U(1)$ as the group acting on H-spinors.

2. The second inclusion is defined by the electro-weak group $SU(2)_L$ so that Kac-Moody algebra $SL(2,R) \times U(1)$ remains dynamical.

5.2 The interpretation of the critical dimension $D = 4$ and the objection related to the signature of the space-time metric

The first task is to show that $D = 4$ ($D = 8$) as critical dimension of target space for $N = 2$ ($N = 4$) super-conformal symmetry makes sense in TGD framework and that the signature (2,2) ((4, 4) of the metric of the target space is not a fatal flaw.

5.2.1 Space-time as a target space for partonic string world sheets?

Since partonic 2-surfaces are sub-manifolds of 4-D space-time surface, it would be natural to interpret space-time surface as the target space for $N = 2$ super-conformal string theory so that space-time dimension would find a natural explanation. Different Bohr orbit like solutions of the classical field equations could be the TGD counterpart for the dynamic target space metric of M-theory. Since partonic two-surfaces belong to 3-surface X^3_V, the correlations caused by the vacuum functional would imply non-trivial scattering amplitudes with CP_2 type extremals as pieces of X^3_V providing the correlate for virtual particles. Hence the theory could be physically realistic in TGD framework and would conform with perturbative character for the interactions of leptons. $N = 2$ super-conformal theory would of course not describe everything. This algebra seems to be still too small and the question remains how the functional integral over the configuration space degrees of freedom is carried out. It will be found that $N = 4$ super-conformal algebra results neatly when super Kac-Moody and super-canonical degrees of freedom are combined.

5.2.2 The interpretation of the critical signature

The basic problem with this interpretation is that the signature of the induced metric cannot be (2,2) which is essential for obtaining the cancellation for $N = 2$ SCA imbedded to $N = 4$ SCA with critical dimension $D = 8$ and signature (4,4). When super-generators carry fermion number and do not reduce to ordinary gamma matrices for vanishing conformal weights, there is no need to pose the condition of the metric signature. The (4,4) signature of the target space metric is not so serious limitation as it looks if one is ready to consider the target space appearing in the calculation of N-point functions as a fictive notion.

The resolution of the problems relies on two observations.

1. The super Kac-Moody and super-canonical Cartan algebras have dimension $D = 2$ in both M^4 and CP_2 degrees of freedom giving total effective dimension $D = 8$.

2. The generalized coset construction to be discussed in the sequel allows to assign opposite signatures of metric to super Kac-Moody Cartan algebra and corresponding super-canonical Cartan algebra so that the desired signature (4,4) results. Altogether one has 8-D effective target space with signature (4,4) characterizing $N = 4$ super-conformal topological strings. Hence the number of physical degrees of freedom is $D_{phys} = 8$ as in super-string theory. Including the non-physical M^2 degrees of freedom, one has critical dimension $D = 10$. If also the radial degree of freedom associated with $\delta M^4_\pm$ is taken into account, one obtains $D = 11$ as in M-theory.

5.3 About the interpretation of $N = 2$ SCA and small $N = 4$ SCA

The $N = 2$ super-conformal algebra automatically extends to the so called small $N = 4$ algebra with four super-generators $G_\pm$ and their conjugates [36]. In TGD framework $G_\pm$ degeneracy corresponds to the two spin directions of the covariantly constant right handed neutrinos and the conjugate of $G_\pm$ is obtained by charge conjugation of right handed neutrino. From these generators one can build up a right-handed SU(2) algebra.

Hence the $SU(2)$ Kac-Moody of the small $N = 4$ algebra corresponds to the three imaginary quaternionic units and the $U(1)$ of $N = 2$ algebra to ordinary imaginary unit. Energy momentum tensor T and SU(2) generators would correspond to quaternionic units. $G_\pm$ to their super counterparts and their conjugates would define their "square roots".

5.3.1 The connection between super-conformal algebras and classical division algebras

There are well-known connections with classical number fields and super-conformal algebras.

1. There exists two proposals for a simple super-affinization of the octonionic algebra realized in terms of spin 1/2 super fields obeying expected octonionic anticommutation relations in the fermionic sector. Otherwise the fields behave like like octonionic units. These constructions are discussed in [38, 39].

2. It is known that only $N \leq 4$ super-conformal algebras allow Sugawara construction [38]. For $N = 8$ super-affine octonionic algebra the Sugawara construction does not give a closed algebraic structure except at the limit $k \to \infty$ for the Kac-Moody central charge [39]: this algebra is the non-associative SCA discovered first by Englert *et al* [40]. This limit could be interpreted in terms of a critical conformal field theory. The minimal super-affine quaternionic sub-algebra reduces to a small $N = 4$ SCA and allows Sugawara construction [38]. This limit would correspond to $n \to \infty$ limit for the Jones inclusion and critical value of c corresponding to the almost-topologization of $N = 2$ n-point functions. The problem is that the representations do not exist for finite values of k which are also needed.

   The number theoretical vision supports the view that only quaternionic SCA can be used in the construction of physical states. A stronger conclusion would be that only the quaternionic SCA is possible so that quarks would be fractionally charged leptons in $k = 1$ phase. The topologication of $N = 4$ n-point functions in the critical phase could be consistent with the possibility to describe electro-weak interactions perturbatively since partonic 2-surfaces would still interact classically and these interactions would correspond to exchanges of virtual particles represented by CP_2 type extremals.

5.3.2 Small $N = 4$ SCA as sub-algebra of $N = 8$ SCA in TGD framework?

A possible interpretation of the small $N = 4$ super-conformal algebra would be quaternionic sub-SCA of the non-associative octonionic SCA. The $N = 4$ algebra associated with a fixed fermionic chirality would represent the fermionic counterpart for the restriction to the hyper-quaternionic submanifold of HO and $N = 2$ algebra in the further restriction to commutative sub-manifold of HO so that this algebra would naturally appear at the parton level. Super-affine version of the quaternion algebra can be constructed straightforwardly as a special case of corresponding octonionic algebra [39]. The construction implies 4 fermion spin doublets corresponding and unit quaternion naturally corresponds to right handed neutrino spin doublet. The interpretation is as leptonic spinor fields appearing in Sugawara representation of Super Virasoro algebra.

A possible octonionic generalization of Super Virasoro algebra would involve 4 doublets $G^{i)}_{\pm}$, $i = 1, ..., 4$ of super-generators and their conjugates having interpretation as SO(8) spinor and its its conjugate. $G^{i)}_{\pm}$ and their conjugates $\overline{G}^{i)}_{\pm}$ would anti-commute to SO(8) vector octet having an interpretation as a super-affine algebra defined by the octonionic units: this would conform nicely with SO(8) triality.

One could say that the energy momentum tensor T extends to an octonionic energy momentum tensor T as real component and affine generators as imaginary components: the real part would have conformal weight $h = 2$ and imaginary parts conformal weight $h = 1$ in the proposed constructions reflecting the special role of real numbers. The ordinary gamma matrices appearing in the expression of G in Sugawara construction should be represented by units of complexified octonions to achieve non-associativity. This construction would differ from that of [39] in that G fields would define an SO(8) octet in the proposed construction: HO-H duality would however suggest that these constructions are equivalent.

One can consider two possible interpretations for $G^{i)}_{\pm}$ and corresponding analogs of super Kac-Moody generators in TGD framework.

1. Leptonic right handed neutrino spinors correspond to $G^{i)}_{\pm}$ generating quaternionic units and quark like left-handed neutrino spinors with leptonic charges to the remaining non-associative octonionic units. The interpretation in terms of so called mirror symmetry would be natural. What is is clear the direct sum of $N = 4$ SCAs corresponding to the Kac-Moody group $SU(2) \times SU(2)$ would be exact symmetry if free quarks and leptons carry integer charges. One might however hope of getting also $N = 8$ super-conformal algebra. The problem with this interpretation is that SO(8) transformations would in general mix states with different fermion numbers. The only way out would be the allowance of mixtures of right-handed neutrinos of both chiralities and also of their conjugates which looks an ugly option.

In any case, the well-definedness of the fermion number would require the restriction to $N = 4$ algebra. Obviously this restriction would be a super-symmetric version for the restriction to 4-D quaternionic- or co-quaternionic sub-manifold of H.

2. One can ask whether $G_{\pm}^{i)}$ and their conjugates could be interpreted as components of leptonic H-spinor field. This would give 4 doublets plus their conjugates and mean $N = 16$ super-symmetry by generalizing the interpretation of $N = 4$ super-symmetry. In this case fermion number conservation would not forbid the realization of $SO(8)$ rotations. Super-conformal variant of complexified octonionic algebra obtained by adding a commuting imaginary unit would result. This option cannot be excluded since in TGD framework complexified octonions and quaternions play a key role. The fact that only right handed neutrinos generate associative super-symmetries would mean that the remaining components $G_{\pm}^{i)}$ and their conjugates could be used to construct physical states. $N = 8$ super-symmetry would thus break down to small $N = 4$ symmetry for purely number theoretic reasons and the geometry of CP_2 would reflect this breaking.

   The objection is that the remaining fermion doublets do not allow covariantly constant modes at the level of imbedding space. They could however allow these modes as induced H-spinors in some special cases which is however not enough and this option can be considered only if one accepts breaking of the super-conformal symmetry from beginning. The conclusion is that the $N = 8$ or even $N = 16$ algebra might appear as a spectrum generating algebra allowing elegant coding of the primary fermionic fields of the theory.

5.4 Large $N = 4$ SCA is the natural option

The arguments below support the view that "large" $N = 4$ SCA is the natural algebra in TGD framework.

5.4.1 How $N = 4$ super-conformal invariance emerges from the parton level formulation of quantum TGD?

The discovery of the formulation of TGD as a $N = 4$ almost topological super-conformal QFT with light-like partonic 3-surfaces identified as basic dynamical objects led to the final understanding of super-conformal symmetries and their breaking. $N = 4$ super-conformal algebra corresponds to the maximal algebra with $SU(2) \times U(2)$ Kac-Moody algebra as inherent fermionic Kac-Moody algebra. Concerning the interpretation the first guess would be that $SU(2)_+$ and $SU(2)_-$ correspond to vectorial spinor rotations in M^4 and CP_2 and $U(1)$ to Kähler charge. A more educated guess is that $SU(2)_+$ and $SU(2)_-$ correspond to right and left handed spinorial rotations in M^4 and $U(1)$ to electromagnetic charge.

5.4.2 Large $N = 4$ SCA algebra

Large $N = 4$ super-conformal symmetry with $SU(2)_+ \times SU(2)_- \times U(1)$ inherent Kac-Moody symmetry seems to define the fundamental partonic super-conformal symmetry in TGD framework.

A concise discussion of this symmetry with explicit expressions of commutation and anticommutation relations can be found in [42]. The representations of SCA are characterized by three central extension parameters for Kac-Moody algebras but only two of them are independent and given by

$$\begin{aligned} k_\pm &\equiv k(SU(2)_\pm) \ , \\ k_1 &\equiv k(U(1)) = k_+ + k_- \ . \end{aligned} \tag{80}$$

The central extension parameter c is given as

$$c = \frac{6k_+k_-}{k_+ + k_-} \ . \tag{81}$$

and is rational valued as required.

A much studied $N = 4$ SCA corresponds to the special case

$$\begin{aligned} k_- &= 1 \ , \ k_+ = k + 1 \ , \ k_1 = k + 2 \ , \\ c &= \frac{6(k+1)}{k+2} \ . \end{aligned} \tag{82}$$

$c = 0$ would correspond to $k_+ = 0, k_- = 1, k_1 = 1$. For $k_+ > 0$ one has $k_1 = k_+ + k_- \neq k_+$.

5.4.3 About unitary representations of large $N = 4$ SCA

The unitary representations of large $N = 4$ SCA are briefly discussed in [46]. The representations are labelled by the ground state conformal weigh h, SU(2) spins l_+, l_-, and U(1) charge u. Besides the inherent Kac-Moody algebra there is also "external" Kac-Moody group G involved and corresponds in TGD framework to the canonical algebra associated with $\delta H_\pm = \delta M^4_\pm \times CP_2$.

Unitarity constraints apply completely generally irrespective of G so that one can apply them also in TGD framework. There are two kinds of unitary representations.

1. Generic/long/massive representations which are ge generated from vacuum state as usual. In this case there are no null vectors.

2. Short or massless representations have a null vector. The expression for the conformal weigt h_{short} of the null vector reads in terms of l_+, l_- and k_+, k_- as

$$h_{short} = \frac{1}{k_+ + k_-}(k_- l_+ + k_+ l_- + (l_+ - l_-)^2 + u^2) \ . \qquad (83)$$

Unitarity demands that both short and long representations lie at or above $h \geq h_{short}$ and that spins lie in the range $l_\pm = 0, 1/2, ..., (k_\pm - 1)/2$.

Interesting examples of $N = 4$ SCA are provided by WZW coset models $\mathcal{W} \times U(1)$, where $\mathcal{W}$ is WZW model associated wto a quaternionic (Wolf) space. Examples based on classical groups are $\mathcal{W} = G/H = SU(n)/SU(n-1) \times U(1)$, $SO(n)/SO(n-4) \times SU(2)$, and $Sp(2n)/Sp(2n-2)$. For $n = 3$ first series gives CP_2 whereas second series gives for $n = 4$ $SO(4)/SU(2) = SU(2)$. In this case one has $k_+ = \kappa + 1$, and $k_- = \hat{c}_G$, where κ is the level of the bosonic current algebra for G and $\hat{c}_G$ is its dual Coxeter number.

5.4.4 What is the interpretation of $SO(4) \times U(1)$ in TGD framework?

A priori there are several options concerning the identification of the group $SO(4) \times U(1)$ inherent to the $N = 4$ SCA the representation.

1. The requirement that h_{short} does not depend on spin and M^4 chirality of the fermion allows three alternatives. $SU(2)_+$ and $SU(2)_-$ corresponds to right and left handed spinor rotations in 1) M^4 or 2) CP_2 or 3) to vectorial rotations in M^4 and CP_2. The fact the rotations of right handed covariantly constant neutrino define a remnant of super-conformal symmetry, exclude option 2). $U(1)$ would most naturally correspond to em charge or Kähler charge. Kähler charge is the only possible identification for options 2) and 3).

2. p-Adic mass calculations suggest that the value of $h_{min} \geq h_{short}$ depends on em charge states of fermion. The naive guess would be that one has $h = h_{min}$ and that u should be proportional to electromagnetic charge. Since $U(1)_{em}$ must commute with $SO(4)$, this would leave only the option 1). It however turns out that this idea does not work and that p-adic mass calculations allow to identify h as the conformal weight associated with the color partial wave in the cm degrees of freedom of partonic 2-surface indeed correlating with the electro-weak quantum numbers of the state. This means that in principle any of the alternative internally consistent scenarios is consistent with p-adic mass calculations.

3. In TGD framework the partonic states constructed using second quantized imbedding space spinor fields should define fundamental representations of $N = 4$ SCA. They define $(l_+, l_-) = (1/2, 1/2)$ representation for option 3) and $(l_+, l_-) = (1/2, 0)$ and $(l_+, l_-) = (0, 1/2)$ representations for options 1) and 2). Singlet would correspond to unit operator to which Hamiltonians of Can are proportional. For options 1) and 2) the representation (1/2,1/2) must be constructed as fermion antifermion states and

this would force $(k_+, k_-) = (3,3)$ implying also Higgs and gauge bosons. For option 3) it is not clear how to obtain representations $(0, 1/2)$ and $(1/2, 0)$. This argument would disfavor, if note even exclude, option 3).

4. The general vision about the quantization of Planck constants raises the question how k_+ and k_- relate to the integers n_a and n_b characterizing the values of $M^4_\pm$ and CP_2 Planck constants. The formulas $n_a = k_+ + 2$ and $n_b = k_- + 2$ would look natural for option 3).

   For option 1) the correspondence between n_a and n_b and central extension parameters could be following. Parity symmetry favors the identification $k_+ = k_- = k$ (predicting $c = 3k$). Later additional arguments suggesting this symmetry are developed. If this symmetry holds true generally, $n_a = k + 2$ would be the natural identification. For option 1) the Kac-Moody algebra associated with electro-weak spinor rotations and E^2 translations and rotations and X^2 localized super-canonical algebra have interpretation as external Kac-Moody algebra and the "external" Kac-Moody algebra associated with CP_2 spinor rotations would in turn correspond to $n_b = k(SU(2)_L) + 2$.

5. The conclusion is that options 1) and 2) look the most plausible ones. The lack of constraints from p-Adic mass calculations however allow to consider also the possibility that different internally consistent choices of $SO(4)\times U(1)$ algebra are actually equivalent mathematically so that there would be additional symmetry involved.

5.4.5 Consistency with p-adic mass calculations

The consistency with p-adic mass calculations provides a strong guide line in attempts to interpret $N = 4$ SCA. The basis ideas of p-adic mass calculations are following.

1. Fermionic partons move in color partial waves in their cm degrees of freedom. This gives to conformal weight a vacuum contribution equal to the CP_2 contribution to mass squared. The contribution depends on electro-weak isospin and equals $h_c(U) = 2$ and $h_c(D) = 3$ for quarks and one has $h_c(\nu) = 1$ and $h_c(L) = 2$.

2. The ground state can correspond also to non-negative value of L_0 for SKMV algebra which gives rise to a thermal degeneracy of massless states. p-Adic mass calculations require $(h_{gr}(D), h_{gr}(U)) = (0, -1,)$ and $(h_{gr}(L), h_{gr}(\nu)) = (-1, -2)$ so that the super-canonical operator O_c screening the anomalous color charge has conformal weight $h_c = -3$ for all fermions.

The simplest interpretation is that the free parameter h appearing in the representations of the SCA corresponds to the conformal weight due to the color partial wave so that the correlation with electromagnetic charge would indeed emerge but from the correlation of color partial waves and electro-weak quantum numbers.

The requirement that ground states are null states with respect to the SCV associated with the radial light-like coordinate of $\delta M^4_\pm$ gives an additional consistency condition and $h_c = -3$ should satisfy this condition. p-Adic mass calculations do not pose non-trivial conditions on h for option 1) if one makes the identification $u = Q_{em}$ since one has $h_{short} < 1$ for all values of $k_+ + k_-$. Therefore both options 1) and 2) can be considered.

5.4.6 How the canonical algebra is integrated into this scheme?

The canonical algebra of $\delta H_\pm$ takes the role as "external" Lie algebra G in TGD framework. Indeed the localization of super-canonical algebra with respect to X^2 gives the counterpart of super Kac-Moody algebra associated with G. Therefore the condition that the commutator of SKM and SC algebras annihilates physical states posed in the state construction generalizes the condition that $n > 0$ generators of SKM(G) annihilate the physical states.

1. For option 3) a good guess is that the $CP_2) \times U(1)$ coset representing $SU(3)$ and electro-weak algebra as a gauge algebra extends to the canonical algebra and brings in the electro-weak spinor rotations. The $U(1)$ factor could correspond to Kähler charge.

2. Since the canonical transformations of CP_2 and $\delta M^4_\pm$ containing $SO(3)$ and $SU(3)$ as subgroups integrate to a larger group, the symplectic extension parameter k must be equal to Kac-Moody central extension parameter $k(SU(3)) = k(SO(3))$ which in turn is expected to be equal to k_- if coset representation generalizes.

3. Since only $SU(3)$ is represented as non-trivial holonomies (as $U(2)_{ew}$ rotations) of induced spinor fields, one could argue that bosonic $SO(3)$ generators can be joined to inherent Kac-Moody algebra and extended to canonical algebra. Furthermore, the dual Coxeter number of $SU(3)$ would dictate the value of k_- as $k_- = \hat{c}_{SU(3)} = 3$. The value of central extension parameter would be $c = 9$. This interpretation would give $l_- = 1$ as maximum value of fermionic contribution to the bosonic spin. Electro-weak gauge bosons and Higgs scalar would be included to the fundamental partonic representation. Graviton could correspond to a state for which one one unit of spin originates from SO(3) and one unit from spinorial degrees of freedom as suggested earlier. This might relate closely to the weakness of the gravitational interaction.

5.4.7 About breaking of large $N = 4$ SCA

Partonic formulation predicts that large $N = 4$ SCA is a broken symmetry, and the first guess is that breaking occurs via several steps. First a "small" $N = 4$ SCA with Kac-Moody group $SU(2)_+ \times U(1)$ would result. The next step would lead to $N = 2$ SCA and the final step to $N = 0$ SCA. Several symmetry breaking scenarios are possible.

1. The interpretation of $SU(2)_+$ in terms of right handed spin rotations and $U(1)$ as electromagnetic gauge group conforms with the general vision about electro-weak symmetry breaking in non-stringy phase. The interpretation certainly makes sense for covariantly constant right handed neutrinos for which spin direction is free. For left handed charged electro-weak bosons the action of right-handed spinor rotations is trivial so that the interpretation would make sense also now.

2. The next step in the symmetry breaking sequence would be $N = 2$ SCA with electromagnetic Kac-Moody algebra as inherent Kac-Moody algebra.

5.4.8 Consistency with critical dimension of super-string models and M-theory

Mass squared is identified as the conformal weight of the positive energy component of the state rather than as a contribution to the conformal weight cancelling the total conformal weight. Also the Lorentz invariance of the p-adic thermodynamics requires this. As a consequence, the pseudo 4-momentum p assignable to M^4 super Kac-Moody algebra could be always light-like or even tachyonic.

Super-canonical algebra would generate the negative conformal weight of the ground state required by the p-adic mass calculations and super-Kac Moody algebra would generate the non-negative net conformal weight identified as mass squared. In this interpretation SKM and SC degrees of freedom are independent and correspond to opposite signs for conformal weights.

The construction is consistent with p-adic mass calculations [F2, F3] and the critical dimension of super-string models.

1. Five Super Virasoro sectors are predicted as required by the p-adic mass calculations (the predicted mass spectrum depends only on the number of tensor factors). Super-canonical algebra gives $Can(CP_2)$ and $Can(S^2)$. In SKM sector one has $SU(2)_L$, $U(1)$, local SU(3), SO(2) and E^2 so that 5 sectors indeed result.

2. The Cartan algebras involved of SC is 2-dimensional and that of SKM is 7-dimensional so that 10-dimensional Cartan algebra results. This means that vertex operator construction implies generation of 10-dimensional target space which in super-string framework would be identified as imbedding space. Note however that these dimensions have Euclidian signature unlike in superstring models. SKM algebra allows also the option $SO(3) \times E(3)$ in M^4 degrees of freedom: this would mean that SKM Cartan algebra is 10-dimensional and the whole algebra 11-dimensional.

5.4.9 $N = 4$ super-conformal symmetry and WZW models

One can question the naive idea that the basic structure $G_{int} = SU(2) \times U(2)$ structure of $N = 4$ SCA generalizes as such to the recent framework.

1. $N = 4$ SCA is originally associated with Majorana spinors. $N = 4$ algebra can be transformed from a real form to complex form with 2 complex fermions and their conjugates corresponding to complex H-spinors of definite chirality having spin and weak isospin. At least at formal level the complexification of $N = 4$ SCA algebra seems to make sense and might be interpreted as a direct sum of two $N = 4$ SCAs and complexified quaternions. Central charge would remain $c = 6k_+k_-/(k_+ + k_-)$ if naive complexification works. The fact that Kac-Moody algebra of spinor rotations is $G_{int} = SO(4) \times SO(4) \times U(1)$ is naturally assignable naturally to spinors of H suggests that it represents a natural generalization of $SO(4) \times U(1)$ algebra to inherent Kac-Moody algebra.

2. One might wonder whether the complex form of $N = 4$ algebra could result from $N = 8$ SCA by posing the associativity condition.

3. The article of Gunaydin [41] about the representations of $N = 4$ superconformal algebras realized in terms of Goddard-Kent-Olive construction and using gauged Wess-Zumino-Witten models forces however to question the straightforward translation of results about $N = 4$ SCA to TGD framework and it must be admitted that the situation is something confusing. Of course, there is no deep reason to believe that WZW models are appropriate in TGD framework.

   i) Gauged WZW models are constructed using super-space formalism which is not natural in TGD framework. The coset space $CP_2 \times U(2)$ where $U(2)$, could be identified as sub-algebra of color algebra or possibly as electro-weak algebra provides one such realization. Also the complexifixation of the $N = 4$ algebra is something new.

   ii) The representation involves 5-grading by the values of color isospin for $SU(3)$ and makes sense as a coset space realization for $G/H \times U(1)$ if H is chosen in such a manner that $G/H \times SU(2)$ is quaternionic space. For SU(3) one has $H = U(1)$ identifiable in terms of color hyper charge CP_2 is indeed quaternionic space. For $SU(2)$ 5-grading degenerates since spin 1/2 Lie-algebra generators are absent and H is trivial group. In M^4 degrees of gauged WZW model would be trivial.

   iii) $N = 4$ SCA results as an extension of $N = 2$ SCA using so called Freudenthal triple system. $N = 2$ SCA has realization in terms of $G/H \times U(1)$ gauged WZW theory whereas the extension to $N = 4$ SCA gives $G \times U(1)/H$ gauged WZW model: note that $SU(3) \times U(1)/H$ does not have an obvious interpretation in TGD framework. The Kac-Moody central extension parameters satisfy the constraint $k_+ = k + 1$ and $k_- = \hat{g} - 1$, where k is the central extension parameter for G. For $G = SU(3)$ one obtains $k_- = 1$ and $c = 6(k + 1)/(k + 2)$. $H = U(1)$ corresponding to color hyper-charge and $U(1)$ for $N = 2$ algebra corresponds to color isospin. The group $U(1)$ appearing in $SU(3) \times U(1)$ might be interpreted in terms of fermion number or Kähler charge.

iv) What looks somewhat puzzling is that the generators of second SU(2) algebra carry fermion number $F = 4I_3$. Note however that the sigma matrices of configuration space with fermion number ± 2 are non-vanishing since corresponding gamma matrices anti-commute. Second strange feature is that fermionic generators correspond to 3+3 super-coordinates of the flag-manifold $SU(3)/U(1)\times U(1)$ plus 2 fermions and their conjugates. Perhaps the coset realization in CP_2 degrees of freedom is not appropriate in TGD framework and that one should work directly with the realization based on second quantized induced spinor fields.

5.5 Are both quark and lepton like chiralities needed/possible?

Before the formulation of quantum TGD based on the identification of light-like 3-surfaces as a representation of parton orbits emerged, one had to consider two different physical realizations of $N = 4$ super-conformal symmetry. The original option for which leptons and quarks correspond to different H-chiralities of the induced spinor field is consistent with the partonic picture and definitely favored so that this subsection can be regarded as as interesting side track.

On the other hand, only lepton like chiralities are needed if one can accepts a possible instability of proton. This option is mathematically the minimal but it is not at all clear whether the SU(3) associated with A_2 characterizing Jones inclusion can correspond to color SU(3). One can go further and ask whether it is even possible to have both chiralities.

5.5.1 Option I: $N = 4$ SCA and fractionally charged quarks

Quarks generate super affinization of quaternions, which involves in no manner the Kähler charge of quarks but for fractional quark charges only SCA in the leptonic sector is possible since covariant constancy fails. At the fundamental level one the spectrum generating algebra for quarks would thus emerge and they could appear as primary fields of $N = 4$ conformal field theory. Configuration space gamma matrices could be uniquely constructed in terms of the leptonic oscillator operators since they could correspond to super-generators of super-Kac Moody algebra. Furthermore, if the solutions of the modified Dirac equation generate super-conformal symmetries, it might be possible to have super-conformal symmetry acting also in the quark sector.

A possible manner to understand quarks is as a phase with $N = 2$ super-conformal symmetry with U(1) Kac-Moody algebra. Using just the requirement that the charges in the $k = 1, c = 1$ phase for $N = 2$ super-conformal symmetry are proportional to factor 1/3, one can conclude that this phase can contain ordinary quarks and fractionally charged leptons whose charge results from the phase factors depending on the sheet of the 3-fold covering of CP_2. Also phases with $n > 3$ are possible and require fractionization of both quark and lepton charges. For quarks the condition $n \ mod \ 3 = 0$ must be satisfied in this case.

5.5.2 Option II: $N = 4$ super-conformal algebra and quarks as fractionally charged leptons

For the simplest option realizing $N = 4$ SCA only leptons are fundamental particles and quarks would be leptons in the anyonic $k = 1, c = 1, n = 3$ phase of the theory. This option would resolve elegantly the problem whether one should construct configuration space gamma matrices using leptonic or quark like gamma matrices. Fermion number fractionization might in principle allow the decay of proton to positron plus pion as in GUTs. This decay might be however excluded for purely mathematical reasons. Indeed, the worlds corresponding to different value of $q = exp(i\pi/n)$ could communicate only via exchanges of bosons having a vanishing fermion number.

In the interactions between leptons and quarks the gauge bosons would penetrate to the space-time sheets corresponding to the hadrons. In $k = 1$ phase weak interactions would become strong since arbitrarily high parton vertices would become possible and strong interactions could be simply electro-weak interactions which have become strong in the anyonic phases as HO-H duality strongly suggests [E2]. By the same duality strong interactions wold have dual descriptions as non-perturbative electro-weak interactions and as color interactions.

There are objections against this picture.

1. p-Adic mass calculations rely strongly on the fact that free quarks have fractional charges and move in CP_2 partial waves and it would be pity to lose the nice results of these calculations.

2. This option requires that the SU(3) associated with A_2 characterizing $n = 3$ Jones inclusion produces states equivalent with triality 1 partial waves for quarks in order to reproduce the results of p-adic mass calculations. This does not seem to be the case although one can understand how effective triality 1 states results by considering 3-fold coverings of CP_2 points by M^4 points defined by the space-time surfaces in question. The essential point is that 2π rotation in CP_2 phase angle leads to a different M^4 point than original and 6π rotation brings back to the original point. This might not be however enough.

5.5.3 Option III: Integer charged leptons and quarks

For the third option $N = 4$ superconformal symmetry can be realized in both lepton and quark sector but by the previous arguments $N = 8$ SCA is not possible. Both imbedding space chiralities would possess leptonic quantum numbers and would be allowed as fundamental fermions. At the level of configuration space the choice of either chirality to realize the configuration space gamma matrices would correspond to the selection of quark or lepton like chirality. This presumably leads to problems with continuity unless the two chiralities correspond to completely disjoint parts of the configuration space.

Finding an explanation for the experimental absence of the free integer charged quarks is the basic challenge met by the advocate of integer charged free quarks. A possible explanation could rely on the fact that also gauge bosons would be doubled. There are two options.

1. The two kinds of gauge bosons couple to only single H-chirality. One can indeed argue that if one allows at given space-time sheet only quark or lepton like chirality then it is not possible to have quantum superpositions of fermion-antifermion pairs of opposite chiralities at a given space-time sheet so that bosons would couple to either quark or lepton like chirality. This would mean that leptons and free quarks would have no electro-weak interactions. Even gravitational interaction would be absent. This would however imply that ordinary hadrons should consist of fractionally charged leptons so that second chirality would not appear at all in known or experimentally testable physics.

2. An option allowing ordinary hadrons to consist of genuine quarks is that the couplings of these two bosons are vectorial and axial with respect to H-chirality (the simplest option) and left-right permutation occurs for electro-weak couplings. This would induce a breaking of the chiral symmetry at the level of H just as the ordinary weak interactions do at the level of M^4 and the masses of integer charged quarks could differ from those of genuine leptons.

If H-vectorial and H-axial gauge bosons have same coupling strengths and masses, the diagrams representing exchanges of vectorial and axial gauge bosons would interfere to zero so that free leptons and quarks would not see each other at all. This should be true in $(c = 6, n = \infty)$ phase. This could be the case for even gravitons. On the other hand, the interactions between free quarks and hadronic quarks would be possible and would make free quarks visible so that this option seems to produce more problems than to solve them.

In $(c = 1, k = 1, n = 3)$ phase leptons and quarks should interact and this is achieved if the masses and couplings of H-vectorial or H-axial electro-weak bosons are different in this phase. It is far from clear whether this picture can be consistent with what is known about lepton-hadron interactions.

5.5.4 Common features of the options I and II

Consider now the common features of options I and II which on basis of the previous arguments look the only realistic ones.

1. For both options only $c = 6$ would correspond to the integer charged world and hadrons would be represented by primary fields in this phase. Hadrons would correspond to $k = 1, c = 1$ representation for the reduced $N = 2$ conformal symmetry. Elementary fermions inside hadrons would correspond to the lowest $n = 3$ Jones inclusion having $k = 1$ which indeed corresponds to A_2 Dynkin diagram and thus SU(3). Ordinary leptons

and quarks (whether fractionally charged leptons or not) would thus live in different CP_2:s (recall that the generalized imbedding space has fan like structure with different $M^4 \times CP_2$:s meeting along M^4). This would explain the impossibility to observe free fractionally charged quarks.

Anyonic color triplet leptons and fractionally charged quarks would live at the three branches of the covering of CP_2. The observation that leptonic spinors possess anomalous color hyper-charge identifiable as lepton number and that this charge corresponds to weak hyper-charge explains why the electromagnetic charge of lepton can be fractionized but not its weak isospin.

2. An infinite hierarchy of states with fractionally charged fermions would be predicted with charges of form m/n appearing as dark matter so that the counterparts of quarks would represent only the simplest Jones inclusion. For quarks one would have $n = k + 2 \ mod \ 3 = 0$. The invisibility of free fractionally charged fermions would be equivalent with the invisibility of dark matter with scaled up value of CP_2 Planck constant in both options. For option I the phase transition transforming leptons to quarks and vice versa would require three leptons per quark in order to achieve conservation of fermion number.

3. I have already proposed the idea that antimatter is dark matter [D7] and the obvious possibility is that matter-antimatter asymmetry corresponds to the transformation of n anti-leptons to baryon like entities consisting of n fractionally charged leptons inside which they behave like dark matter. For option II anti-leptons would correspond to baryons and antimatter would be directly observable. The notion of N-atom in TGD based model for quantum biology is based on the same idea: in this case $n = 2^{11}$ electrons would form a similar structure [M3].

5.5.5 Lepton-hadron interactions for various options

The interactions between leptons and quarks and their fractionally charged counterparts can be also understood. The following arguments favor option I and II over option III.

1. Quite generally, the CP_2 type extremal representing virtual electroweak boson must tunnel between two CP_2:s in the fan formed by $M^4 \times CP_2$:s glued together along M^4 and in this process transform to hadronic weak boson. This means that also strong interactions between leptons and hadrons are generated but these interactions could be seen as secondary strong interactions occurring inside hadron in any case via the decay of photon to quark pair in turn interacting strongly with other partons.

   The coupling constant characterizing the tunnelling must be such that correct results for electro-weak interactions between quarks and leptons are obtained in the lowest order. The notion of vector meson dominance meaning that weak bosons transform to strongly interacting mesons with same electro-weak quantum numbers conforms with this picture.

2. For option II the lowest order contributions to electro-weak interactions inside hadrons could be identified as direct lepton-quark interaction and there are no obvious problems involved.

3. For option I gauge bosons must couple to both chiralities in order to make possible the interaction between leptons and quarks. This is possible and the prediction is that gauge bosons should appear as H-vectorial and H-axial variants or their mixtures. A doubling of ordinary vector bosons is predicted. This however does not have any dramatic effects if ordinary gauge bosons correspond to H-vectorial gauge bosons and axial ones are heavy enough. Nothing new is predicted for situation in which leptons do not penetrate inside hadrons. A lepton penetrating into hadron must suffer an anyonization and becomes fractionally charged and decomposes into a triplet of leptons with fractional fermion number. This implies that lepton has strong interactions with quarks.

4. For option III the understanding of the interactions between leptons and hadrons consisting of genuine quarks becomes a highly non-trivial problem for several reasons.

   i) The hypothesis that only fermions of fixed chirality are possible at a given space-time sheet would exclude the possibility of non-trivial interactions between leptons and hadrons. If one gives up this assumption the doubling of electro-weak interactions gives however hopes for describing the interactions. The non-observability of free quarks in $c = 6$ phase is guaranteed if the masses and couplings of H-vectorial and -axial bosons are identical in this phase. To have interactions in $k = 1$ phase, these couplings and masses must be different. This would look nice at first since one could hope of explaining strong interactions in terms of this symmetry breaking.

   ii) However, if H-vectorial and -axial couplings are different inside hadrons, the expectation is that the resulting low energy lepton-hadron electro-weak interactions are quite different from what they are known to be experimentally. The most natural guess suggested by the masslessness of gluons is that all (say) H-axial weak bosons are massless inside hadrons. However, if both H-vectorial and -axial photons are massless there would be no electromagnetic coupling between quarks and leptons and hadrons would look like em neutral particles at low energies.

   iii) The coupling constant characterizing this tunnelling should have a value making possible to reproduce the standard model picture about lepton-quark scattering. If only (say) H-vectorial ew bosons can tunnel to hadron and the amplitude A for the tunnelling equals to $A = 2$ it gives amplitude equal to $V - V + A - A = 2V - V$ between leptons then quark-lepton scattering can be reproduced correctly. This kind of transformation is however not described by a unitary S-matrix.

5.5.6 New view about strong interactions

The proposed picture suggests the identification of strong interactions as electro-weak interactions which have become strong in $k = 1$ anyonic phase. HO-H duality leads to the same proposal [E2].

1. Strong interactions as electro-weak interactions in a non-perturbative phase?

Consider the situation in $k = 1, c = 1$ hadronic sector at the sheets of 3-fold covering of M^4 at which fractionally charged fermions reside. It is an experimental fact that their electro-weak interactions allow a perturbative description. One would however obtain all higher order stringy diagrams allowed by rational conformal field theories. This looks like a paradox but one can consider the possibility that electro-weak interactions give rise also to strong interactions.

For all options the non-vanishing of higher n-point functions in $k = 1, c = 1$ phase would give rise to and additional non-perturbative contribution to electro-weak interactions having a natural interpretation as strong interactions. Weak isospin and hypercharge could be interpreted also as strong isospin and hyper-charge as is indeed found to be the case experimentally. Conserved vector current hypothesis and partially conserved axial current hypothesis of the old-fashioned hadron physics indeed support this kind of duality.

For option I one can consider the possibility that H-axial bosons define the dual counterparts of gluons and are massless. H-axial electro-weak interactions would give rise also to strong interactions between quarks and anyonic leptons inside hadrons. The idea that color interactions have dual description as H-axial electro-weak interactions is admittedly rather seductive.

For option III different masses and couplings of H-vectorial and H-axial bosons inside hadrons would allow to interpret strong interactions as (say) axial weak interactions. The simplest option would be that H-axial weak bosons are massless so that strong isospin and hyper-charge would correspond to their H-axial variants. The problems relating to the interaction between leptons and hadrons have been already mentioned: for instance, em interactions between leptons and quarks would vanish if they vanish in $c = 6$ phase.

2. HO-H duality and equivalence with QCD type description

One can ask how QCD type description emerges if strong interactions are non-perturbative electro-weak interactions (option II) or H-axial counterparts of them (option I). In [E2] I have discussed a possible duality suggested by the fact that space-time surfaces can be regarded as 4-surfaces in hyper-octonionic $H = M^8$ or in $H = M^4 \times CP_2$. In the first picture spinors would be octonionic spinors and correspond to two leptonic singlets and color triplet and its conjugate: there would be no trace about spin and electro-weak quantum numbers besides electro-weak hyper charge.

The absence of spin in HO description could provide a resolution of the spin puzzle of proton (quarks do not seem to contribute to the spin of proton). In H picture spinors would carry only electro-weak quantum numbers and spin besides anomalous color hypercharge. The question is whether quark like spinors in HO are equivalent with leptonic spinors in H and whether the descriptions based on (possibly) doubled electro-weak and color interactions are equivalent for many-sheeted coverings.

6 Generalization of the notion of imbedding space and the notion of number theoretic braid

This section summarizes the the attempt to meet following challenges.

1. Try to understand how the hierarchy of Planck constants is realized at the level of imbedding space and what quantum criticality for phase transitions changing Planck constant means.

2. Try to understand the notion of number theoretic braid in terms of quantum criticality. Identification as subset of real and p-adic variant of partonic 2-surface is not enough since some kind of inherent cutoff is needed to make the braid finite. Here the generalization of imbedding space comes in rescue and leads to an identification of two kinds of number theoretic braids assignable to phase transitions in M^4 and CP_2 degrees changing the value of Planck constant and corresponding symmetries. One should also be also able to identify the braiding operation.

6.1 Generalization of the notion of imbedding space

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

6.1.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. H_4 represents a straight cosmic string. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labelled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{M}^4 \times \hat{CP}_2$ implying that surfaces in $M^4 \times S^2$ and $M^2 \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

3. The covering spaces in question would correspond to the Cartesian products $\hat{M}^4{}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{M}^4$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing M^2 and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{M}^4 \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

4. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5. Also the orbifolds $\hat{M}^4/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{M}^4/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{M}^4 \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

6.1.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{M}^2 \hat{\times} G_a$ and $\hat{CP}_2 \hat{\times} G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labelled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

   By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

5. How do the Planck constants associated with factors and coverings relate? One might argue that Planck constant defines a homomorphism respecting the multiplication and division (when possible) by G_i. If so, then Planck constant in units of $\hbar_0$ would be equal to n_a/n_b for $\hat{H}/G_a \times G_b$ option and n_b/n_a for $\hat{H}\hat{\times}(G_a \times G_b)$ with obvious formulas for hybrid cases. This option would put M^4 and CP_2 in a very symmetric role and allow much more flexibility in the identification of symmetries associated with large Planck constant phases.

6.2 Phase transitions changing the value of Planck constant

There are two basic kinds of phase transitions changing the value of Planck constant inducing a leakage between sectors of imbedding space. There are three cases to consider corresponding to

1. leakage in M^4 degrees of freedom changing G_a: the critical manifold is $R_+ \times CP_2$;
2. leakage in CP_2 degrees of freedom changing G_b: the critical manifold is $\delta M^4_+ \times S^2_{II}$;
3. leakage in both degrees of freedom changing both G_a and G_b: the critical manifold is $R_+ \times S^2_{II}$. This is the non-generic case

For transitions of type 2) and 3) X^2 must go through vacuum extremal in the classical picture about transition.

Covering space can also change to a factor space in both degrees of freedom or vice versa and in this case G can remain unchanged as a group although its interpretation changes.

The phase transitions satisfy also strong group theoretical constraints. For the transition $G_1 \to G_2$ either $G_1 \subset G_2$ or $G_2 \subset G_1$ must hold true. For maximal cyclic subgroups Z_n associated with quantization axes this means that n_1 must divide n_2 or vice versa. Hence a nice number theoretic view about transitions emerges.

One can classify the points of critical manifold according to the degree of criticality. Obviously the maximally critical points corresponds to fixed points of G_i that its points $z = 0, \infty$ of the spheres S^2_r and S^2_{II}. In the case of δM^4_+ the points $z = 0$ and ∞ correspond to the light-like rays R_+ in opposite directions. This ray would define the quantization direction of angular momentum. Quantum phase transitions changing the value of M^4 Planck constant could occur anywhere along this ray (partonic 2-surface would have 1-D projection along this ray). At the level of cosmology this would bring in a preferred direction.

Light-cone dip, the counterpart of big bang, is the maximally quantum critical point since it remains invariant under entire group $SO(3,1)$.

Interesting questions relate to the groups generated by finite discrete subgroups of $SO(3)$. As noticed the groups generated as products of groups leaving R_+ invariant and three genuinely 3-D groups are infinite discrete subgroups of SO(3) and could also define Jones inclusions. In this case orbifold is replaced with orbifold containing infinite number of rotated versions of R_+. These phases could be important in elementary particle length scales or in early cosmology.

6.3 The identification of number theoretic braids

Number theoretic braids should be known once partonic surface and corresponding p-adic prime is known. Braid should belong to the intersection of real and p-adic variant of partonic 2-surface and the definition of should automatically give rise to a finite braid in case of non-vacuum extremals. Quantum criticality suggests that there are two kinds of braids. First kind of braid would relate to phase transitions changing G_a and would correspond to intersection of X^2 with R_+ and for given point in intersection would consist of points of CP_2 with same R_+ projection. Second kind of braid would relate to phase transitions changing G_b and correspond to intersection of X^2 with S^2_{II} and would consist for given point of points of S^2_r with same S^2_{II} coordinates.

6.3.1 Why a discrete set of points of partonic 2-surface must be selected?

As already noticed, p-adicization might provide a deeper motivation for the selection of discrete subset of points of partonic 2-surface in the construction of S-matrix elements in the case of non-diagonal transitions between different number fields.

1. The fusion of p-adic variants of TGD with real TGD, could be possible by algebraic continuation. This however requires the restriction of n-point functions to a finite set of algebraic points of X^2 with the usual stringy formula formula for S-matrix elements involving an integral over a circle of X^2 replaced with a sum over these points.

2. The same universal formula would give not only ordinary S-matrix elements but also those for p-adic-to-real transitions describing transformation intentions to actions. Quite generally, the formula would express S-matrix elements for transitions between two arbitrary number fields as algebraic numbers so that p-adicization of the theory would become trivial.

3. The interpretation of this finite set of points as a braid suggests a connection with the representation of Jones inclusions in terms of a hierarchy of braids [A8, E9] with the increasing number of strands meaning a continually improved finite-dimensional approximation of the hyper-finite

factor of type II_1 identifiable as the Clifford algebra for the configuration space. The hierarchy of approximations for the hyper-finite factor would correspond to a genuine physical hierarchy of S-matrices corresponding to increasing dimension of algebraic extension of various p-adic numbers. This hierarchy would also define a cognitive hierarchy.

What could then be this discrete set of points having interpretation as a braid?

1. Number theoretical vision suggests that quantum TGD involves the sequence hyper-octonions $\rightarrow$ hyper-quaternions$\rightarrow$ complex numbers$\rightarrow$ reals $\rightarrow$ finite field $G(p,1)$ or of its algebraic extension. These reductions would define number theoretical counterparts of dimensional reductions. The points in the finite field $G(p,1)$ could be defined by p-adic integers modulo p so that a connection with p-adic numbers would emerge. Also more general algebraic extensions of p-adic numbers are allowed.

2. Number theoretical braids must belong to the intersection of real partonic 2-surface and its p-adic counterpart and thus the points must be algebraic points. Besides this a natural cutoff determined by X^2 itself is needed in order to have only finite number of points.

3. The generalization of the imbedding space inspired by the hierarchy of Planck constants suggest a very concrete identification of number theoretic braids in terms of intersections of partonic 2-surface and critical manifolds R_+ and S^2_{II} involving no ad hoc assumptions and giving braids having finite number of points.

6.3.2 About the precise definition of number theoretical braid

The precise definition of number theoretic braids has been a challenge for long time. The generalization of the notion of imbedding space however leads to good guess for the identification of number theoretical braids.

What is clear that the points of number theoretic braid belong to the intersection of the real and p-adic variant of partonic 2-surfaces consisting of rationals and algebraic points in the extension used for p-adic numbers. The points of braid have same projection on an algebraic point of a geodesic sphere of $S^2 \subset CP_2$ belonging to the algebraic extension of rationals considered.

There are two different geodesic spheres in CP_2 and the homologically trivial geodesic sphere S^2_{II} is the most natural choice from the point of view of the generalized imbedding space since $M^2 \times S^2_{II}$, which defines the intersection of all sectors of H, is a vacuum extremal so that the ill-definedness of Planck constant does not matter. Note that also the M^4 part of the metric is discontinuous at $M^2 \times S^2_{II}$.

One can argue that algebraicity condition is not strong enough and gives too many points unless one introduces a cutoff in some manner. Since TQFT like theory can naturally assigned with the partonic 2-surfaces in $M^2 \times S^2_{II}$, the natural identification of the intersection points of number theoretical braids with $\delta M^4_\pm \times CP_2$ would be as the intersection of the 2-D CP_2 projection of the partonic 2-surface in $\delta M^4_\pm \times CP_2$ with S^2_{II}. In the generic case the intersection would consist of discrete points and for non-vacuum extremals this would certainly be the case. The intersection should consist of algebraic points allowing also p-adic interpretation: the condition that CP_2 projection is an algebraic surface is a necessary condition for this.

The points of braid are obtained as solutions of polynomial equation and thus one can assign to them a Galois group permuting the points of the braid. In this case finite Galois group could be realized as left or right translation or conjugation in S_∞ or in braid group.

To make the notion of number theoretic braid more concrete, suppose that the complex coordinate w of $\delta M^4_\pm$ is expressible as a polynomial of the complex coordinate z of CP_2 geodesic sphere and the radial light-like coordinate r of $\delta M^4_\pm$ is obtained as a solution of polynomial equation $P(r, z, w) = 0$. By substituting w as a polynomial $w = Q(z, r)$ of z and r this gives polynomial equation $P(r, z, Q(z, r)) = 0$ for r for a given value of z. Only real roots can be accepted. Local Galois group (in a sense different as it is used normally in literature) associated with the algebraic point of S^2 defining the number theoretical braid is thus well defined.

If the partonic 2-surface involves all roots of an irreducible polynomial, one indeed obtains a braid for each point of the geodesic sphere $S^2 \subset CP_2$. In this case the action of Galois group is naturally a braid group action realized as the action on induced spinor fields and configuration space spinors.

The choice of the points of braid as points common to the real and p-adic partonic 2-surfaces would be unique so that the obstacle created by the fact that the finite Galois group as function of point of S^2 fluctuates wildly (when some roots become rational Galois group changes dramatically: the simplest example is provided by $y - x^2 = 0$ for which Galois group is Z_2 when y is not a square of rational and trivial group if y is rational).

This picture looks nice but a closer inspection show that it is not quite correct. The identification of Higgs field as a purely geometrical object leads to the identification of intersection points as unstable extrema of negative valued Higgs potential for which Higgs vanishes. The stable minima correspond to the extrema in the vicinity of these maxima and correspond to non-vanishing Higgs field (the nearest valley for a peak of 2-D landscape defined on sphere). The minima (bottoms of valleys) define both physically and mathematically natural candidate for the number theoretical braids. At quantum criticality these braids approach to zero braids. Thus one can say that the original identification is correct apart from Higgs mechanism.

6.3.3 What is the fundamental braiding operation?

The basic quantum dynamics of TGD could define the braiding operation for the braid defined by a discrete set of points of X^2 satisfying the algebraicity conditions. I have considered several candidates for braiding operation and the situation is still partially unsettled.

One promising candidate for the braiding operation is found by observing that both Kähler gauge potential and Kähler magnetic field define flows at light-like partonic 3-surface. The dual of the induced Kähler form defines a conserved topological current, whose flow lines are field lines of the Kähler magnetic field in the light like direction. This flow is incompressible. Vector potential defines also a flow in the interior of space-time surface, and Chern-Simons action at partonic 3-surface defines a topological invariant of this flow known as helicity in hydrodynamics. The non-gauge invariance of helicity is not a problem since symplectic transformations of CP_2 do not define gauge degeneracy but spin glass degeneracy. The flow defined by the vector potential is perhaps the most attractive option but one cannot exclude the possibility that the braids defined by both flows play a role in the definition of S-matrix. Number theoretical braid (tangle if flow line fuse or split) would correspond to the unique orbit for the points of the number theoretic braid at the initial partonic 2-surface. The points of the braid would be algebraic only in suitably chosen discrete time slices but this would not lead to a loss of uniqueness. Hence cobordism would become discrete. This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]). This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]).

The second candidate for the braiding operation emerges naturally when one identifies the points defining the number theoretic braid in terms of minima of Higgs field defined on X^2 (the details of this identification are discussed later). In this case time evolution takes minima to minima and can induce braiding for the projections of the points of braid to S^2_{II} *resp.* S^2_r. One might hope that the braiding associated with S^2_r *resp.* S^2_{II} is topologically equivalent with the braiding defined by Kähler gauge potential *resp.* Kähler magnetic field.

7 Could a symplectic analog of conformal field theory be relevant for quantum TGD?

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have.

7.1 Symplectic QFT at sphere

Actually the notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years [D8]. In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where there is homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctution $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon can be constructed from 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k \Phi_l = c_{kl}^m \Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = \int c_{kl}^m f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s \ . \quad (84)$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1,s_2,s))Idd\mu_s$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = \int c_{kl} f(A(s_1,s_2,s))d\mu_s \ . \quad (85)$$

Hence 2-point function is average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that $n=1$- an are constant, most naturally vanishing, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

7.2 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1,s_2,s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1,s_2,s_3) \equiv A(s_1,s_2,s_3,N) - A(s_1,s_2,s_3,S) \quad (86)$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\begin{aligned} \langle (\Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle &= c_{kl}^r \int f(\Delta A(s_1,s_2,s))\langle \Phi_r(s)\Phi_m(s_3)\rangle d\mu_s \\ &= \end{aligned} \quad (87)$$

$$c_{kl}^r c_{rm} \int f(\Delta A(s_1,s_2,s)) f(\Delta A(s,s_3,t)) d\mu_s d\mu_t \ . \quad (88)$$

Associativity requires that this expression equals to $\langle \Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1,s_2,s))d\mu_s \quad (89)$$

4. There is a clear difference between $n>3$ and $n=3$ cases: for $n>3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n=4$ theory is certainly well-defined, but one can argue that so are also $n>4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0)=0$ n-point correlation functions vanish if any two arguments coincide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

7.3 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments co-incide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components

of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. This framework gives dimensionless areas (unit sphere is considered). p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

The recent view about M-matrix described in [C3] is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Iteration* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *recursion* replaces iteration in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD [E2].

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with recursive instead of iterative approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time [E10].

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2- surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$

calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_\pm$ associated with initial, final and, and intermediate states so that symplectic n-point functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately.

8 Could local zeta functions take the role of Riemann Zeta in TGD framework?

The recent view about TGD leads to some conjectures about Riemann Zeta.

1. Non-trivial zeros should be algebraic numbers.
2. The building blocks in the product decomposition of ζ should be algebraic numbers for non-trivial zeros of zeta.
3. The values of zeta for their combinations with positive imaginary part with positive integer coefficients should be algebraic numbers.

These conjectures are motivated by the findings that Riemann Zeta seems to be associated with critical systems and by the fact that non-trivial zeros of zeta are analogous to complex conformal weights or perhaps more naturally, to complex square roots of real conformal weights [A9]. The necessity to make such a strong conjectures, in particular conjecture c), is an unsatisfactory feature of the theory and one could ask how to modify this picture. Also a clear physical interpretation of Riemann zeta is lacking.

It was also found that there are good reasons for expecting that the zetas in question should have only a finite number zeros. In the same section the self-referentiality hypothesis for ζ was proposed on basis of physical arguments. In this section (written before the emergence of self-referentiality hypothesis) the situation will be discussed from different view point.

8.1 Local zeta functions and Weil conjectures

Riemann Zeta is not the only zeta [49, 48]. There is entire zoo of zeta functions and the natural question is whether some other zeta sharing the basic properties of Riemann zeta having zeros at critical line could be more appropriate in TGD framework.

The so called local zeta functions analogous to the factors $\zeta_p(s) = 1/(1-p^{-s})$ of Riemann Zeta can be used to code algebraic data about say numbers about solutions of algebraic equations reduced to finite fields. The local zeta functions appearing in Weil's conjectures [50] associated with finite fields $G(p,k)$ and thus to single prime. The extensions $G(p,nk)$ of this finite field are considered. These local zeta functions code the number for the points of algebraic variety for given value of n. Weil's conjectures also state that if X is a mod p reduction of non-singular complex projective variety then the degree for the polynomial multiplying the product $\zeta(s) \times \zeta(s-1)$ equals to Betti number. Betti number is 2 times genus in 2-D case.

It has been proven that the zetas of Weil are associated with single prime p, they satisfy functional equation, their zeros are at critical lines, and rather remarkably, they are rational functions of p^{-s}. For instance, for elliptic curves zeros are at critical line [50].

The general form for the local zeta is $\zeta(s) = exp(G(s))$, where $G = \sum g_n p^{-ns}$, $g_n = N_n/n$, codes for the numbers N_n of points of algebraic variety for n^{th} extension of finite field F with nk elements assuming that F has $k = p^r$ elements. This transformation resembles the relationship $Z = exp(F)$ between partition function and free energy $Z = exp(F)$ in thermodynamics.

The exponential form is motivated by the possibility to factorize the zeta function into a product of zeta functions. Note also that in the situation when N_n approaches constant N_∞, the division of N_n by n gives essentially $1/(1 - N_\infty p^{-s})$ and one obtains the factor of Riemann Zeta at a shifted argument $s - log_p(N_\infty)$. The local zeta associated with Riemann Zeta corresponds to $N_n = 1$.

8.2 Local zeta functions and TGD

The local zetas are associated with single prime p, they satisfy functional equation, their zeros lie at the critical lines, and they are rational functions of p^{-s}. These features are highly desirable from the TGD point of view.

8.2.1 Why local zeta functions are natural in TGD framework?

In TGD framework modified Dirac equation assigns to a partonic 2-surface a p-adic prime p and inverse of the zeta defines local conformal weight. The intersection of the real and corresponding p-adic parton 2-surface is the set containing the points that one is interested in. Hence local zeta sharing the basic properties of Riemann zeta is highly desirable and natural. In particular, if the local zeta is a rational function then the inverse images of rational points of the geodesic sphere are algebraic numbers. Of course, one might consider a stronger constraint that the inverse image is rational. Note that one must still require that p^{-s} as well as s are algebraic numbers for the zeros of the local zeta (conditions a) and b) listed in the beginning) if one wants the number theoretical universality.

Since the modified Dirac operator assigns to a given partonic 2-surface a p-adic prime p, one can ask whether the inverse $\zeta_p^{-1}(z)$ of some kind of local zeta directly coding data about partonic 2-surface could define the generalized eigenvalues of the modified Dirac operator and radial super-canonical conformal weights so that the conjectures about Riemann Zeta would not be needed at all.

The eigenvalues of the modified Dirac operator would in a holographic manner code for information about partonic 2-surface. This kind of algebraic geometric data are absolutely relevant for TGD since U-matrix and probably also S-matrix must be formulated in terms of the data related to the intersection of real and partonic 2-surfaces (number theoretic braids) obeying same algebraic equations and consisting of algebraic points in the appropriate algebraic extension of p-adic numbers. Note that the hierarchy of algebraic extensions of p-adic number fields would give rise to a hierarchy of zetas so that the algebraic extension used would directly reflect itself in the eigenvalue spectrum of the modified Dirac operator and super-canonical conformal weights. This is highly desirable but not achieved if one uses Riemann Zeta.

One must of course leave open the possibility that for real-real transitions the inverse of the zeta defined as a product of the local zetas (very much analogous to Riemann Zeta) defines the conformal weights. This kind of picture would conform with the idea about real physics as a kind of adele formed from p-adic physics.

8.2.2 Finite field hierarchy is not natural in TGD context

That local zeta functions are assigned with a hierarchy of finite field extensions do not look natural in TGD context. The reason is that these extensions are regarded as abstract extensions of $G(p,k)$ as opposed to a large number of algebraic extensions isomorphic with finite fields as abstract number fields and induced from the extensions of p-adic number fields. Sub-field property is clearly highly relevant in TGD framework just as the sub-manifold property is crucial for geometrizing also other interactions than gravitation in TGD framework.

The $O(p^n)$ hierarchy for the p-adic cutoffs would naturally replace the hierarchy of finite fields. This hierarchy is quite different from the hierarchy of finite fields since one expects that the number of solutions becomes constant at the limit of large n and also at the limit of large p so that powers in the function G coding for the numbers of solutions of algebraic equations as function of n should not increase but approach constant N_∞. The possibility to factorize $exp(G)$ to a product $exp(G_0)exp(G_\infty)$ would mean a reduction to a product of a rational function and factor(s) $\zeta_p(s) = 1/(1-p^{-s_1})$ associated with Riemann Zeta with argument s shifted to $s_1 = s - log_p(N_\infty)$.

8.2.3 What data local zetas could code?

The next question is what data the local zeta functions could code.

1. It is not at clear whether it is useful to code global data such as the numbers of points of partonic 2-surface modulo p^n. The notion of number theoretic braid occurring in the proposed approach to S-matrix suggests that the zeta at an algebraic point z of the geodesic sphere S^2 of CP_2 or of light-cone boundary should code purely local data such as the numbers N_n of points which project to z as function of p-adic cutoff p^n. In the generic case this number would be finite for non-vacuum extremals with 2-D S^2 projection. The n^{th} coefficient $g_n = N_n/n$ of the function G_p would code the number N_n of these points in the approximation $O(p^{n+1}) = 0$ for the algebraic equations defining the p-adic counterpart of the partonic 2-surface.

2. In a region of partonic 2-surface where the numbers N_n of these points remain constant, $\zeta(s)$ would have constant functional form and therefore the information in this discrete set of algebraic points would allow to deduce deduce information about the numbers N_n. Both the algebraic points and generalized eigenvalues would carry the algebraic information.

3. A rather fascinating self referentiality would result: the generalized eigen values of the modified Dirac operator expressible in terms of inverse of zeta would code data for a sequence of approximations for the p-adic variant of the partonic 2-surface. This would be natural since second quantized induced spinor fields are correlates for logical thought in TGD inspired theory of consciousness. Even more, the data would be given at points $\zeta(s)$, s a rational value of a super-canonical conformal weight or a value of generalized eigenvalue of modified Dirac operator (which is essentially function $s = \zeta_p^{-1}(z)$ at geodesic sphere of CP_2 or of light-cone boundary).

8.3 Galois groups, Jones inclusions, and infinite primes

Langlands program [52, 53] is an attempt to unify mathematics using the idea that all zeta functions and corresponding theta functions could emerge as automorphic functions giving rise to finite-dimensional representations for Galois groups (Galois group is defined as a group of automorphisms of the extension of field F leaving invariant the elements of F). The basic example corresponds to rationals and their extensions. Finite fields $G(p,k)$ and their extensions $G(p,nk)$ represents another example. The largest extension of rationals corresponds to algebraic numbers (algebraically closed set). Although this non-Abelian group is huge and does not exist in the usual sense of the word its finite-dimensional representations in groups $GL(n,Z)$ make sense.

For instance, Edward Witten is working with the idea that geometric variant of Langlands duality could correspond to the dualities discovered in string model framework and be understood in terms of topological version of four-dimensional $N = 4$ super-symmetric YM theory [33]. In particular, Witten assigns surface operators to the 2-D surfaces of 4-D space-time. This brings unavoidably in mind partonic 2-surfaces and TGD as $N = 4$ super-conformal almost topological QFT.

This observation stimulates some ideas about the role of zeta functions in TGD if one takes the vision about physics as a generalized number theory seriously.

8.3.1 Galois groups, Jones inclusions, and quantum measurement theory

The Galois representations appearing in Langlands program could have a concrete physical/cognitive meaning.

1. The Galois groups associated with the extensions of rationals have a natural action on partonic 2-surfaces represented by algebraic equations. Their action would reduce to permutations of roots of the polynomial equations defining the points with a fixed projection to the above mentioned geodesic sphere S^2 of CP_2 or δM^4_+. This makes possible to define modes of induced spinor fields transforming under representations of Galois groups. Galois groups would also have a natural action on configuration space-spinor fields. One can also speak about configuration space spinors invariant under Galois group.

2. Galois groups could be assigned to Jones inclusions having an interpretation in terms of a finite measurement resolution in the sense that the discrete group defining the inclusion leaves invariant the operators generating excitations which are not detectable.

3. The physical interpretation of the finite resolution represented by Galois group would be based on the analogy with particle physics. The field extension K/F implies that the primes (more precisely, prime ideals) of F decompose into products of primes (prime ideals) of K. Physically this corresponds to the decomposition of particle into more elementary constituents, say hadrons into quarks in the improved resolution implied by the extension $F \rightarrow K$. The interpretation in terms of cognitive resolution would be that the primes associated with the higher extensions of rationals are not cognizable: in other words, the observed states are singlets under corresponding Galois groups: one has algebraic/cognitive counterpart of color confinement.

4. For instance, the system labelled by an ordinary p-adic prime could decompose to a system which is a composite of Gaussian primes. Interestingly, the biologically highly interesting p-adic length scale range 10 nm-5 μm contains as many as four Gaussian Mersennes ($M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$), which suggests that the emergence of living matter means an improved cognitive resolution.

8.3.2 Galois groups and infinite primes

In particular, the notion of infinite prime suggests a manner to realize the modular functions as representations of Galois groups. Infinite primes might also provide a new perspective to the concrete realization of Langlands program.

1. The discrete Galois groups associated with various extensions of rationals and involved with modular functions which are in one-one correspondence with zeta functions via Mellin transform defined as $\sum x_n n^{-s} \rightarrow \sum x_n z^n$ [51]. Various Galois groups would have a natural action in the space of infinite primes having interpretation as Fock states and more general bound states of an arithmetic quantum field theory.

2. The number theoretic anatomy of space-time points due to the possibility to define infinite number of number theoretically non-equivalent real units using infinite rationals [17] allows the imbedding space points themselves to code holographically various things. Galois groups would have a natural action in the space of real units and thus on the number theoretical anatomy of a point of imbedding space.

3. Since the repeated second quantization of the super-symmetric arithmetic quantum field theory defined by infinite primes gives rise to a huge space of quantum states, the conjecture that the number theoretic anatomy of imbedding space point allows to represent configuration space (the world of classical worlds associated with the light-cone of a given point of H) and configuration space spinor fields emerges naturally [17].

4. Since Galois groups G are associated with inclusions of number fields to their extensions, this inclusion could correspond at quantum level to a generalized Jones inclusion $\mathcal{N} \subset \mathcal{M}$ such that G acts as automorphisms of $\mathcal{M}$ and leaves invariant the elements of $\mathcal{N}$. This might be possible if one allows the replacement of complex numbers as coefficient fields of hyper-finite factors of type II_1 with various algebraic extensions of rationals. Quantum measurement theory with a finite measurement resolution defined by Jones inclusion $\mathcal{N} \subset \mathcal{M}$ [16] could thus have also a purely number theoretic meaning provided it is possible to define a non-trivial action of various Galois groups on configuration space spinor fields via the imbedding of the configuration space spinors to the space of infinite integers and rationals (analogous to the imbedding of space-time surface to imbedding space).

This picture allows to develop rather fascinating ideas about mathematical structures and their relationship to physical world. For instance, the functional form of a map between two sets the points of the domain and target rather than only its value could be coded in a holographic manner by using the number theoretic anatomy of the points. Modular functions giving rise to generalized zeta functions would emerge in especially natural manner in this framework. Configuration space spinor fields would allow a physical realization of the holographic representations of various maps as quantum states.

8.4 Connection between Hurwitz zetas, quantum groups, and hierarchy of Planck constants?

The action of modular group SL(2,Z) on Riemann zeta [55] is induced by its action on theta function [56]. The action of the generator $\tau \rightarrow -1/\tau$ on theta function is essential in providing the functional equation for Riemann Zeta. Usually the action of the generator $\tau \rightarrow \tau+1$ on Zeta is not considered explicitly. The surprise was that the action of the generator $\tau \rightarrow \tau + 1$ on Riemann Zeta does not give back Riemann zeta but a more general function known as Hurwitz zeta $\zeta(s,z)$ for $z = 1/2$. One finds that Hurwitz zetas for certain rational values of argument define in a well defined sense representations of fractional modular group to which quantum group can be assigned naturally. This could allow to code the value of the quantum phase $q = exp(i2\pi/n)$ to the solution spectrum of the modified Dirac operator D. It has later turned out that there is very natural Zeta function associated with the generalized eigenvalue spectrum of the modified Dirac operator and since the number of various kinds of zeta functions is so immense, the hopes that this conjecture would hold true, are meager. Despite this it is worth to discuss Hurwitz zetas here: one of the reasons is that one end up with a very nice argument for why the number of observed fermion families is three.

8.4.1 Hurwitz zetas

Hurwitz zeta is obtained by replacing integers m with $m+z$ in the defining sum formula for Riemann Zeta:

$$\zeta(s,z) = \sum_m (m+z)^{-s} \quad . \tag{90}$$

Riemann zeta results for $z = n$.
Hurwitz zeta obeys the following functional equation for rational $z = m/n$ of the second argument [57]:

$$\zeta(1-s,\frac{m}{n}) = \frac{2\Gamma(s)}{2\pi n}^s \sum_{k=1}^{n} cos(\frac{\pi s}{2} - \frac{2\pi k m}{n})\zeta(s,\frac{k}{n}) \quad . \tag{91}$$

The representation of Hurwitz zeta in terms of θ [57] is given by the equation

$$\int_0^{\infty} \left[\theta(z,it) - 1\right] t^{s/2}\frac{dt}{t} = \pi^{(1-s)/2}\Gamma(\frac{1-s}{2})\left[\zeta(1-s,z) + \zeta(1-s,1-z)\right] \quad . \tag{92}$$

By the periodicity of theta function this gives for $z = n$ Riemann zeta.

8.4.2 The action of $\tau \to \tau + 1$ transforms $\zeta(s,0)$ to $\zeta(s,1/2)$

The action of the transformations $\tau \to \tau + 1$ on the integral representation of Riemann Zeta [55] in terms of θ function [56]

$$\theta(z;\tau) - 1 = 2\sum_{n=1}^{\infty}[exp(i\pi\tau)]^{n^2}cos(2\pi nz) \tag{93}$$

is given by

$$\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s) = \int_0^{\infty}[\theta(0;it) - 1]t^{s/2}\frac{dt}{t} \quad . \tag{94}$$

Using the first formula one finds that the shift $\tau = it \to \tau + 1$ in the argument θ induces the shift $\theta(0;\tau) \to \theta(1/2;\tau)$. Hence the result is Hurwitz zeta $\zeta(s,1/2)$. For $\tau \to \tau + 2$ one obtains Riemann Zeta.

Thus $\zeta(s,0)$ and $\zeta(s,1/2)$ behave like a doublet under modular transformations. Under the subgroup of modular group obtained by replacing $\tau \to \tau + 1$ with $\tau \to \tau + 2$ Riemann Zeta forms a singlet. The functional equation for Hurwitz zeta relates $\zeta(1-s,1/2)$ to $\zeta(s,1/2)$ and $\zeta(s,1) = \zeta(s,0)$ so that also now one obtains a doublet, which is not surprising since the functional equations directly reflects the modular transformation properties of theta functions. This doublet might be the proper object to study instead of singlet if one considers full modular invariance.

8.4.3 Hurwitz zetas form n-plets closed under the action of fractional modular group

The inspection of the functional equation for Hurwitz zeta given above demonstrates that $\zeta(s,m/n)$, $m = 0,1,...,n$, form in a well-defined sense an n-plet under fractional modular transformations obtained by using generators $\tau \to -1/\tau$ and $\tau \to \tau + 2/n$. The latter corresponds to the unimodular matrix $(a,b;c,d) = (1,2/n;0,1)$. These matrices obviously form a group. Note that Riemann zeta is always one member of the multiplet containing n Hurwitz zetas.

These observations bring in mind fractionization of quantum numbers, quantum groups corresponding to the quantum phase $q = exp(i2\pi/n)$, and the inclusions for hyper-finite factors of type II_1 partially characterized by these quantum phases. Fractional modular group obtained using generator $\tau \to \tau + 2/n$ and Hurwitz zetas $\zeta(s,k/n)$ could very naturally relate to these and related structures.

8.4.4 Hurwitz zetas and TGD

These observations suggest a direct application to quantum TGD.

1. In TGD framework inclusions of HFFs of type II_1 are directly related to the hierarchy of Planck constants involving a generalization of the notion of imbedding space obtained by gluing together copies of 8-D $H = M^4 \times CP_2$ with a discrete bundle structure $H \rightarrow H/Z_{n_a} \times Z_{n_b}$ together along the 4-D intersections of the associated base spaces [A9]. A book like structure results and various levels of dark matter correspond to the pages of this book. One can say that elementary particles proper are maximally quantum critical and live in the 4-D intersection of these imbedding spaces whereas their "field bodies" reside at the pages of the Big Book. Note that analogous book like structures results when real and various p-adic variants of the imbedding space are glued together along common algebraic points.

2. The integers n_a and n_b give Planck constant as $\hbar/\hbar_0 = n_a/n_b$, whose most general value is a rational number. In Platonic spirit one can argue that number theoretically simple integers involving only powers of 2 and Fermat primes are favored physically. Phase transitions between different matters occur at the intersection.

3. The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs relate also to quantum measurement theory with finite measurement resolution with $\mathcal{N}$ defining the measurement resolution so that N-rays replace complex rays in the projection postulate and quantum space $\mathcal{M}/\mathcal{N}$ having fractional dimension effectively replaces $\mathcal{M}$.

4. The basic hypothesis is that the inverses of zeta function or of more general variants of zeta coding information about the algebraic structure of the partonic 2-surface appear in the admittedly speculative fundamental formula for the generalized eigenvalues of modified Dirac operator D. This formula is consistent with the generalized eigenvalue equation for D but is not the only one that one can imagine.

5. The generalized eigen spectrum of D should code information both about the p-adic prime p characterizing particle and about quantum phases $q = exp(i2\pi/n)$ assignable to the particle in M^4 and CP_2 degrees of freedom. I understand how p-adic primes appear in the spectrum of D and therefore how coupling constant evolution emerges at the level of free field theory so that radiative corrections can vanish without the loss of coupling constant evolution [C6]. The problem has been to understand how the quantum phase characterizing the sector of the generalized imbedding space could make itself visible in these formulas and therefore in quantum dynamics at the level of free spinor fields. The replacement of Riemann zeta with an n-plet of Hurwitz zetas would resolve this problem.

6. Geometrically the fractional modular invariance would naturally relate to the fact that Riemann surface (partonic 2-surface) can be seen as an $n_a \times n_b$-fold covering of its projection to the base space of H: fractional modular transformations corresponding to n_a and n_b would relate points

at different sheets of the covering of M^4 and CP_2. This suggests that the fractionization could be a completely general phenomenon happening also for more general zeta functions.

8.4.5 What about exceptional cases $n = 1$ and $n = 2$?

Also $n = 1$ and $n = 2$ are present in the hierarchy of Hurwitz zetas (singlet and doublet). They do not correspond to allowed Jones inclusion since one has $n > 2$ for them. What could this mean?

1. It would seem that the fractionization of modular group relates to Jones inclusions ($n > 2$) giving rise to fractional statistics. $n = 2$ corresponding to the full modular group Sl(2,Z) could relate to the very special role of 2-valued logic, to the degeneracy of $n = 2$ polygon in plane, to the very special role played by 2-component spinors playing exceptional role in Riemann geometry with spinor structure, and to the canonical representation of HFFs of type II_1 as fermionic Fock space (spinors in the world of classical worlds). Note also that SU(2) defines the building block of compact non-commutative Lie groups and one can obtain Lie-algebra generators of Lie groups from n copies of SU(2) triplets and posing relations which distinguish the resulting algebra from a direct sum of SU(2) algebras.

2. Also $n = 2$-fold coverings $M^4 \rightarrow M^4/Z_2$ and $CP_2 \rightarrow CP_2/Z_2$ seem to make sense. One can argue that by quantum classical correspondence the spin half property of imbedding space spinors should have space-time correlate. Could $n = 2$ coverings allow to define the space-time correlates for particles having half odd integer spin or weak isospin? If so, bosons would correspond to $n = 1$ and fermions to $n = 2$. One could of course counter argue that induced spinor fields already represent fermions at space-time level and there is no need for the doubling of the representation.

   The trivial group Z_1 and Z_2 are exceptional since Z_1 does not define any quantization axis and Z_2 allows any quantization axis orthogonal to the line connecting two points. For $n \geq 3$ Z_n fixes the direction of quantization axis uniquely. This obviously correlates with $n \geq 3$ for Jones inclusions.

8.4.6 Dark elementary particle functionals

One might wonder what might be the dark counterparts of elementary particle vacuum functionals [F1]. Theta functions $\theta_{[a,b]}(z, \Omega)$ with characteristic $[a, b]$ for Riemann surface of genus g as functions of z and Teichmueller parameters Ω are the basic building blocks of modular invariant vacuum functionals defined in the finite-dimensional moduli space whose points characterize the conformal equivalence class of the induced metric of the partonic 2-surface. Obviously, kind of spinorial variants of theta functions are in question with $g + g$ spinor indices for genus g.

The recent case corresponds to $g = 1$ Riemann surface (torus) so that a and b are $g = 1$-component vectors having values 0 or 1/2 and Hurwitz zeta corresponds to $\theta_{[0,1/2]}$. The four Jacobi theta functions listed in Wikipedia [56] correspond to these thetas for torus. The values for a and b are 0 and 1 for them but this is a mere convention.

The extensions of modular group to fractional modular groups obtained by replacing integers with integers shifted by multiples of $1/n$ suggest the existence of new kind of q-theta functions with characteristics $[a, b]$ with a and b being g-component vectors having fractional values k/n, $k = 0, 1...n-1$. There exists also a definition of q-theta functions working for $0 \leq |q| < 1$ but not for roots of unity [58]. The q-theta functions assigned to roots of unity would be associated with Riemann surfaces with additional Z_n conformal symmetry but not with generic Riemann surfaces and obtained by simply replacing the value range of characteristics $[a, b]$ with the new value range in the defining formula

$$\Theta[a,b](z|\Omega) = \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a) + i2\pi(n+a)\cdot(z+b)\right] \quad . \tag{95}$$

for theta functions. If Z_n conformal symmetry is relevant for the definition of fractional thetas it is probably so because it would make the generalized theta functions sections in a bundle with a finite fiber having Z_n action.

This hierarchy would correspond to the hierarchy of quantum groups for roots of unity and Jones inclusions and one could probably define also corresponding zeta function multiplets. These theta functions would be building blocks of the elementary particle vacuum functionals for dark variants of elementary particles invariant under fractional modular group. They would also define a hierarchy of fractal variants of number theoretic functions: it would be interesting to see what this means from the point of view of Langlands program [52] discussed also in TGD framework [E11] involving ordinary modular invariance in an essential manner.

This hierarchy would correspond to the hierarchy of quantum groups for roots of unity and Jones inclusions and one could probably define also corresponding zeta function multiplets. These theta functions would be building blocks of the elementary particle vacuum functionals for dark variants of elementary particles invariant under fractional modular group.

8.4.7 Dark matter hierarchy and hierarchy of quantum critical systems in modular degrees of freedom

Dark matter hierarchy corresponds to a hierarchy of conformal symmetries Z_n of partonic 2-surfaces with genus $g \geq 1$ such that factors of n define subgroups of conformal symmetries of Z_n. By the decomposition $Z_n = \prod_{p|n} Z_p$, where $p|n$ tells that p divides n, this hierarchy corresponds to an hierarchy of increasingly quantum critical systems in modular degrees of freedom. For a given prime p one has a sub-hierarchy Z_p, $Z_{p^2} = Z_p \times Z_p$, etc... such that the moduli at n+1:th level are contained by n:th level. In the similar manner the moduli of Z_n are sub-moduli for each prime factor of n. This mapping of integers to quantum critical systems conforms nicely with the general vision that biological evolution corresponds to the increase of quantum criticality as Planck constant increases.

The group of conformal symmetries could be also non-commutative discrete group having Z_n as a subgroup. This inspires a very short-lived conjecture that only the discrete subgroups of SU(2) allowed by Jones inclusions are possible as conformal symmetries of Riemann surfaces having $g \geq 1$. Besides Z_n one could have tedrahedral and icosahedral groups plus cyclic group Z_{2n} with reflection added but not Z_{2n+1} nor the symmetry group of cube. The conjecture is wrong. Consider the orbit of the subgroup of rotational group on standard sphere of E^3, put a handle at one of the orbits such that it is invariant under rotations around the axis going through the point, and apply the elements of subgroup. You obtain a Riemann surface having the subgroup as its isometries. Hence all discrete subgroups of SU(2) can act even as isometries for some value of g.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself.

Spherical topology is exceptional since in this case the space of conformal moduli is trivial and conformal symmetries correspond to the entire $SL(2,C)$. This would suggest that only the fermions of lowest generation corresponding to the spherical topology are maximally quantum critical. This brings in mind Jones inclusions for which the defining subgroup equals to $SU(2)$ and Jones index equals to $\mathcal{M}/\mathcal{N} = 4$. In this case all discrete subgroups of $SU(2)$ label the inclusions. These inclusions would correspond to fiber space $CP_2 \rightarrow CP_2/U(2)$ consisting of geodesic spheres of CP_2. In this case the discrete subgroup might correspond to a selection of a subgroup of $SU(2) \subset SU(3)$ acting non-trivially on the geodesic sphere. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ having geodesic spheres of CP_2 as their ends could correspond to this phase dominating the very early cosmology.

8.4.8 Fermions in TGD Universe allow only three families

What is nice that if fermions correspond to $n = 2$ dark matter with Z_2 conformal symmetry as strong quantum classical correspondence suggests, the number of ordinary fermion families is three without any further assumptions. To see this suppose that also the sectors corresponding to $M^4 \rightarrow M^4/Z_2$ and $CP_2 \rightarrow CP_2/Z_2$ coverings are possible. Z_2 conformal symmetry implies that partonic Riemann surfaces are hyper-elliptic. For genera $g > 2$ this means that some theta functions of $\theta_{[a,b]}$ appearing in the product of theta functions defining the vacuum functional vanish. Hence fermionic elementary particle vacuum functionals would vanish for $g > 2$ and only 3 fermion families would be possible for $n = 2$ dark matter.

This results can be strengthened. The existence of space-time correlate for the fermionic 2-valuedness suggests that fermions quite generally to even values of n, so that this result would hold for all fermions. Elementary bosons (actually exotic particles belonging to Kac-Moody type representations) would correspond to odd values of n, and could possess also higher families. There is a nice argument supporting this hypothesis. n-fold discretization provided by covering associated with H corresponds to discretization for angular momentum eigen states. Minimal discretization for $2j+1$ states corresponds to $n = 2j+1$. $j = 1/2$ requires $n = 2$ at least, $j = 1$ requires $n = 3$ at least, and so on. $n = 2j+1$ allows spins $j \leq n - 1/2$. This spin-quantum phase connection at the level of space-time correlates has counterpart for the representations of quantum SU(2).

These rules would hold only for genuinely elementary particles corresponding to single partonic component and all bosonic particles of this kind are exotics (excitations in only "vibrational" degrees of freedom of partonic 2-surface with modular invariance eliminating quite a number of them): ordinary gauge bosons correspond to fermion pairs at throats of a wormhole contact and decompose to SU(3) singlet and octet, whose states are labelled by handle-number pairs (g_1, g_2): they define new kind of heavy bosons giving rise to neutral flavor changing currents (could they be visible in LHC?). Note that gravitons necessarily correspond to pairs of fermions or gauge bosons connected by flux tubes so that they are stringy objects in this sense.

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