Basic Extremals of the Kähler Action

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Abstract

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable.

1. General considerations

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all extremals or at least the absolute minima of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Kähler current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional

to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

2. Absolute minimization of Kähler action and second law of thermodynamics
By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that absolute minima of Kähler action correspond to space-time sheets which asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Absolute minimization abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. Also the equivalence of absolute minimization with the second law strongly suggests itself. Of course, one must keep mind open for the possibility that it is the second law of thermodynamics which replaces absolute minimization as the fundamental principle.

3. The dimension of CP_2 projection as classifier for the fundamental phases of matter
The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. This phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase is the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

4. Specific extremals of Kähler action
The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

a) The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

b) The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

c) There are also various non-vacuum extremals.

i) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

ii) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

iii) In the so called Maxwell's phase, ordinary Maxwell equations for the induced Kähler field are satisfied in an excellent approximation. A special case is provided by a radially symmetric extremal having an interpretation as the space-time exterior to a topologically condensed particle. The sign of the gravitational mass correlates with that of the Kähler charge and one can understand the generation of the matter antimatter asymmetry from the basic properties of this extremal. The possibility to understand the generation of the matter antimatter asymmetry directly from the basic equations of the theory gives strong support in favor of TGD in comparison to the ordinary EYM theories, where the generation of the matter antimatter asymmetry is still poorly understood.

1 Introduction

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes in the third part of the book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the configuration space geometry and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

a) The 4-surface associated with given 3-surface defined by Kähler function K as an absolute minimum or some more general preferred extremal of the Kähler action is identifiable as a classical space-time. In [E2] it was proposed that absolute value of Kähler action inside regions where the action density has definite sign is minimized or maximized. Number theoretically these extremals would correspond to Kähler calibrations and their duals having representation has hyper-quaternionic or co-hyper-quaternionic surfaces of hyper-octonionic imbedding space. . . Hence the notion of space-time would not not completely objective: similar situation is encountered in M-theory (mirror symmetry [**?**]).

Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

It must be emphasized that absolute minimization could be replaced by any other principle allowing to select a unique extremal going through X^3 belonging to 7-D light-like causal determinant determined as the boundary of the union of future or past directed light cones $M^4_\pm \times CP_2$. Indeed, the number theoretical considerations of [E2] favor the separate minimization of magnitudes of positive and contributions to the Kähler action. This option seems to be also more physical since it reduces to the minimization of the energy and thus to fixing of time derivatives of imbedding space coordinates at X^3. Since the considerations of this chapter do to depend on the detailed form of the variational principle, I leave it for the reader to replace "absolute minimization" by some more general phrase everywhere.

b) The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation.

c) Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy.

d) In spin degrees of freedom the massless Dirac equation for the induced spinor fields with modified Dirac action defines classical theory: this is in complete accordance with the proposed definition of the configuration space spinor structure.

In the case that absolute minimization is taken as the fundamental principle, the assumption about the form of the vacuum functional leads to principle, which we shall call "Yin-Yang" principle in what follows.

a) Kähler function is essentially a Maxwell action and as such not positive definite: the generation of the Kähler electric fields gives negative contribution to the Kähler action. Therefore the absolute minima of the Kähler action are expected to have in general non-positive Kähler action and to correspond to space-times carrying Kähler- electric fields. This tendency of the Kähler function to become negative corresponds to the "Yin"-aspect of our principle.

b) Vacuum functional favors 3-surfaces with the property that the value of the Kähler function is positive. This tendency is the "Yang" aspect of the principle. Together these tendencies stabilize the theory since they imply that for very large systems the average Kähler action per volume is essentially zero to guarantee that vacuum amplitude is non-vanishing: in particular vacuum functional doesn't diverge for any configurations.

If preferred extrema correspond to Kähler calibrations or their duals [E2], Yin-Yang principle is modified to a local principle. For Kähler calibrations (their duals) the absolute value of action in given region is minimized (maximized). A given region with positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically but positive values of Kähler function are of course favored.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a longstanding problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

a) The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

b) The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Kähler current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

c) By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that absolute minima of Kähler action correspond to space-time sheets which asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Absolute minimization abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context. The equivalence of the absolute minimization with the second law of thermodynamics strongly suggests itself. Of course, one must keep mind open for the possibility that it is the second law of thermodynamics which replaces the proposed candidates as the fundamental principle.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions asymptotically. It is perhaps good to emphasize that only extremals rather than preferred extremals are in question. These extremals can however form building blocks of genuine absolute minima or space-time surface satisfying an action principle reducing to extremization of absolute value of Kähler action in regions where action density has definite sign, and are therefore interesting. Also the small deformations of them, say vacuum extremals, are expected to be important physically. Extremals decompose in a natural manner to vacuum and non-vacuum extremals and both kinds of extremals are studied.

2 General considerations

In this section field equations and their physical interpretation are discussed. Quantum classical correspondence suggests that the non-deterministic dynamics of Kähler action makes possible self-referential dynamics in the sense that larger space-time sheet perform smoothed out mimicry of the dynamics at smaller space-time sheets. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the absolute minimization of Kähler action so that this principle would be essentially equivalent with the second law of thermodynamics.

2.1 Long range classical weak and color gauge fields as correlates for dark massless weak bosons

Long ranged electro-weak gauge fields are unavoidably present when the dimension D of the CP_2 projection of the space-time sheet is larger than 2. Classical color gauge fields are non-vanishing for all non-vacuum extremals. This poses deep interpretational problems. If ordinary quarks and leptons are assumed to carry weak charges feeded to larger space-time sheets within electro-weak length scale, large hadronic, nuclear, and atomic parity breaking effects, large contributions of the classical Z^0 force to Rutherford scattering, and strong isotopic effects, are expected. If weak charges are screened within electro-weak length scale, the question about the interpretation of long ranged classical weak fields remains.

2.1.1 Various interpretations for the long ranged classical electro-weak fields

During years I have discussed several solutions to the problems listed above.

Option I: The trivial solution of the constraints is that Z^0 charges are neutralized at electro-weak length scale. The problem is that this option leaves open the interpretation of classical long ranged electro-weak gauge fields unavoidably present in all length scales when the dimension for the CP_2 projection of the space-time surface satisfies $D > 2$.

Option II: Second option involves several variants but the basic assumption is that nuclei or even quarks feed their Z^0 charges to a space-time sheet with size of order neutrino Compton length. The large parity breaking effects in hadronic, atomic, and nuclear length scales is not the only difficulty. The scattering of electrons, neutrons and protons in the classical long range Z^0 force contributes to the Rutherford cross section and it is very difficult to see how neutrino screening could make these effects small enough. Strong isotopic effects in condensed matter due to the classical Z^0 interaction energy are expected. It is far from clear whether all these constraints can be satisfied by any assumptions about the structure of topological condensate.

Option III: During 2005 (27 years after the birth of TGD!) third option solving the problems emerged based on the progress in the understanding of the basic mathematics behind TGD.

In ordinary phase the Z^0 charges of elementary particles are indeed neutralized in intermediate boson length scale so that the problems related to the parity breaking, the large contributions of classical Z^0 force to Rutherford scattering, and large isotopic effects in condensed matter, trivialize.

Classical electro-weak gauge fields in macroscopic length scales are identified as space-time correlates for the gauge fields created by dark matter, which corresponds to a macroscopically quantum coherent phase for which elementary particles possess complex conformal weights such that the net conformal weight of the system is real.

In this phase $U(2)_{ew}$ symmetry is not broken below the scaled up weak scale except for fermions so that gauge bosons are massless below this length scale whereas fermion masses are essentially the same as for ordinary matter. Of course, also scale down copies of also fermionic spectra are possible and infinite hierarchy of scaled down copies of standard model physics is expected. By charge screening gauge bosons look massive in length scales much longer than the relevant p-adic length scale. The large parity breaking effects in living matter (chiral selection for bio-molecules) support the view that dark matter is what makes living matter living.

2.1.2 Classical color gauge fields

Classical long ranged color gauge fields always present for non-vacuum extremals are interpreted as space-time correlates of gluon fields associated with dark copies of hadron physics. It seems that this picture is indeed what TGD predicts. A very special feature of classical color fields is that the holonomy group is Abelian. This follows directly from the expression $g^A_{\alpha\beta} \propto H^A J_{\alpha\beta}$ of induced gluon fields in terms of Hamiltonians H^A of color isometries and induced Kähler form $J_{\alpha\beta}$. This means that classical color magnetic and electric fluxes reduce to the analogs of ordinary magnetic fluxes appearing in the construction of configuration space geometry [B2, B3].

By a local color rotation the color field can be rotated to a fixed direction so that genuinely Abelian field would be in question apart from the possible presence of gauge singularities making impossible a global selection of color direction. These singularities could be present since Kähler form defines a magnetic monopole field. An interesting question inspired by quantum classical correspondence is what the Abelian holonomy tells about the sources of color gauge fields and color confinement.

For instance, could Abelian holonomy mean that colored gluons (and presumably also other colored particles) do not propagate in the p-adic length scale considered? Color neutral gluons would propagate but since also their sources must be color neutral, they should have vanishing net color electric fluxes. This form of confinement would allow those states of color multiplets which have vanishing color charges and obviously symmetry breaking down to $U(1) \times U(1)$ would be in question. This would serve as a signal for monopole confinement which would not exclude higher multipoles for the Abelian color fields. This kind of fields appear in the the TGD based model for nuclei as nuclear strings bound together by color flux tubes [F8].

2.2 Is absolute minimization the correct variational principle?

One can criticize the assumption that extremals correspond to absolute minima, and the number theoretical vision discussed in [E2] indeed favors the separate minimization of magnitudes of positive and negative contributions to the Kähler action.

For this option Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant: note that they would be only inertial vacua and carry non-vanishing density gravitational energy. The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself.

The 3-surfaces for which CP_2 projection is at least 2-dimensional and not a Lagrange manifold would correspond to non-vacua since conservation laws do not leave any other option. The variational principle would favor equally magnetic and electric configurations whereas absolute minimization of action based on S_K would favor electric configurations. The positive and negative contributions would be minimized for 4-surfaces in relative homology class since the boundary of X^4 defined by the intersections with 7-D light-like causal determinants would be fixed. Without this constraint only vacuum bubbles would result.

The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable solution to the construction of Kähler function.

The number theoretic approach based on the properties of quaternions and octonions discussed in the chapter [E2] leads to a proposal for the general solution of field equations based on the generalization of the notion of calibration [16] providing absolute minima of volume to that of Kähler calibration. This approach will not be discussed in this chapter.

2.3 Field equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k_{\ l}\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu^{\ \beta} - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha^{\ \beta}h^k_{\ \beta} + J^k_{\ l}\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{2}$$

$H^k_{\alpha\beta}$ denotes the components of the second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 the field equations are given by the expression

$$T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha^{\ \beta}\partial_\beta h^k + J^k_{\ l})\partial_\alpha h^k) = 0 \ . \tag{3}$$

A general manner manner to solve the field equations on the boundaries is to assume that the induced Kähler field associated with the boundaries vanishes:

$$J_{\alpha\beta}(\delta) \ = \ 0 \ . \tag{4}$$

In this case the energy-momentum tensor vanishes identically on the boundary component. On the outer boundaries of the 3-surface this solution ansatz makes sense only provided the gauge fluxes and gravitational flux (defined by Newtonian potential in the non-relativistic limit) associated with the matter in the interior go somewhere. The only possibility seems to be that 3-surface is topologically condensed on a larger 3-surface and feeds its gauge fluxes to the larger 3-surface via # contacts (topological sum). This assumption forces the concept of topological condensate defined as a hierarchical structure of 3-surfaces condensed on each other and thus giving rise to the many-sheeted space-time.

An important thing to notice is that the boundary conditions do not force the normal components of the gauge fields to zero even if the Kähler electric field vanishes near the boundaries. This makes in principle possible gauge charge renormalization classically resulting from the hierarchical structure of the topological condensation.

2.4 Topologization and light-likeness of the Kähler current as alternative manners to guarantee vanishing of Lorentz 4-force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

2.4.1 Topologization of the Kähler current for $D_{CP_2}=3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2}<4$. For $D_{CP_2}=2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

$$j^{\alpha} \equiv D_{\beta}J^{\alpha\beta} = \psi \times j_I^{\alpha} = \psi \times \epsilon^{\alpha\beta\gamma\delta}J_{\beta\gamma}A_{\delta} \ . \tag{5}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2}<4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2}<4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^{\alpha}J_{\alpha\beta} &= \psi \times j_I^{\alpha}J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta}J_{\mu\nu}A_{\delta}J_{\alpha\beta} \ . \end{aligned} \tag{6}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2}<4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^{\alpha}J^{k_l}\partial_{\alpha}s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{7}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

2.4.2 Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi \overline{B} \ , \quad \rho_I = \overline{B} \cdot \overline{A} \ . \tag{8}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla \times \overline{B} - \partial_t \overline{E} &= \overline{j} = \psi \overline{j}_I = \psi \left(\phi \overline{B} + \overline{E} \times \overline{A} \right) \ , \\ \nabla \cdot E &= \rho = \psi \rho_I \ . \end{aligned} \tag{9}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla \times \overline{B} = \alpha \overline{B} \ , \quad \alpha = \psi \phi \ . \tag{10}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [19].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I \overline{E} + \overline{j} \times \overline{B} = \psi \overline{B} \cdot \overline{AE} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \times \overline{B} = 0 \ . \tag{11}$$

The fourth component of the Lorentz force reads as

$$\bar{j} \cdot \overline{E} = \psi \overline{B} \cdot \overline{E} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \cdot \overline{E} = 0 \ . \tag{12}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi \rho_I = \psi B \cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

2.4.3 Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

2.4.4 Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

a) The $\overline{E} \times \overline{A}$ term contributing besides $\phi\overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla\Phi - \partial_t\overline{A} = \beta\overline{A} \quad (13)$$

This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinates and and $\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

b) The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta\overline{B} \quad (14)$$

The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B{\cdot}A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A},\overline{B}) \rightarrow_{\nabla\times} (\overline{B},\overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

2.4.5 Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the imbedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [20].

In hydrodynamics the role of the magnetic field is taken by the velocity field. TGD based models for nuclei [F6] and condensed matter [F9] involve in an essential manner valence quarks having large $\hbar$ and exotic quarks giving nucleons anomalous color and weak charges creating long ranged color and weak forces. Weak forces have a range of order atomic radius and could be responsible for the repulsive core in van der Waals potential.

This raises the idea that the incompressible flow could occur along the field lines of the Z^0 magnetic field so that the velocity field would be proportional to the Z^0 magnetic field and the Beltrami condition for the velocity field would reduce to that for Z^0 magnetic field. Thus the flow lines of hydrodynamic flow would directly correspond to those of Z^0 magnetic field. The generalized Beltrami flow based on the topologization of the Z^0 current would allow to model also the non-stationary incompressible non-viscous hydrodynamical flows.

It would seem that one cannot describe viscous flows using flows satisfying generalized Beltrami conditions since the vanishing of the Lorentz 4-force says that there is no local dissipation of the classical field energy. One might claim that this is not a problem since in TGD framework viscous flow could be seen as a practical description of a quantum jump sequence by replacing the corresponding sequence of space-time surfaces with a single space-time surface.

One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Kähler fields, which are dissipative, and thus correspond to a non-vanishing Lorentz 4-force, represent one candidate for correlates of this kind. If this is the case, then the fields satisfying the generalized Beltrami condition provide space-time correlates only for the asymptotic self organization patterns for which the viscous effects are negligible, and also the solutions of field equations describing effects of viscosity should be possible.

One must however take this argument with a grain of salt. Dissipation, that is the transfer of conserved quantities to degrees of freedom corresponding to shorter scales, could correspond to a transfer of these quantities between different space-time sheets of the many-sheeted space-time. Here the opponent could however argue that larger space-time sheets mimic the dissipative dynamics in shorter scales and that classical currents represent "symbolically" averaged currents in shorter length scales, and that the local non-conservation of energy momentum tensor consistent with local conservation of isometry currents provides a unique manner to mimic the dissipative dynamics. This view will be developed in more detail below.

2.4.6 The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds [21]. Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta} A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta} J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

a) The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.

b) This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α

and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [21], and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [21]!

Beltrami flows associated with Euler equations are known to be unstable [21]. Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

2.5 How to satisfy field equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \qquad (15)$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection. In the following somewhat different approach is however considered utilizing the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

2.5.1 String model as a starting point

String model serves as a starting point.

a) In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u,v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

b) For Euclidian minimal surfaces (u,v) is replaced by complex coordinates $(w,\overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

2.5.2 The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{16}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E} \times \overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E} \times \overline{B}$ such that the coordinates $u = t - z$ and $v = t + z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

2.5.3 Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

a) Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t + z, S^- \equiv v = t - z, w = x + iy, \overline{w} = x - iy)$.

b) In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_{\pm})^2 = 0 \quad .$$

The gradients of $S_{\pm}$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_{\pm} = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_{\pm}$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t - z, t + z, x, y) \rightarrow S^-(t - z, t + z, x, y) = S^+(t + z, t - z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

c) The coordinates $(S_{\pm}, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \quad . \end{aligned} \tag{17}$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

d) The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w \partial_{\overline{w}} K \quad , \quad g_{+-} = \partial_{S^+} \partial_{S^-} K \quad , \\ g_{w\pm} &= \partial_w \partial_{S^{\pm}} K \quad , \quad g_{\overline{w}\pm} = \partial_{\overline{w}} \partial_{S^{\pm}} K \quad . \end{aligned} \tag{18}$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard lightcone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{19}$$

The Christoffel symbols satisfy the conditions

$$\{ {}^{k}_{w\ \overline{w}} \} = 0 \ , \quad \{ {}^{k}_{+-} \} = 0 \ . \tag{20}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

e) The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

2.5.4 Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2

projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

b) Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \ \ , \ \ s_{ws} = \partial_w \partial_s K \ \ , \ \ s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \ \ . \tag{21}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{{}_{\xi}{}^{k}{}_{\overline{\xi}}\}$.

$$\{{}_{\xi}{}^{k}{}_{\overline{\xi}}\} \ = \ 0 \ . \tag{22}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta (J^{\alpha\beta}\sqrt{g}) &= 0 \text{ for } \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} \neq 0 \ . \end{aligned} \tag{23}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

2.5.5 A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

a) The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{24}$$

Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

b) Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

c) By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi} s} = 0 \ . \tag{25}$$

Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

d) The condition guaranteing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi,s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi,s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{26}$$

Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

e) The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \tag{27}$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \ = \ 0 \tag{28}$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

a) The light-like character of the Kähler current brings in mind CP_2 extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\bar{\xi}}, J^{\xi -}, J^{\bar{\xi} -})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

b) Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If absolute minima correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

c) The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \bar{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

d) Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

2.5.6 Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

The following ansatz gives good hopes for obtaining solutions with space-like and time-like Kähler currents.

a) Assign to light-like coordinates coordinates (T, Z) by the formula

$T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

b) The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z,s) \ , \quad w = w(\xi, s) \ . \tag{29}$$

If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

c) The components of the induced metric are

$$\begin{aligned}
g_{ZZ} &= m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} &= m_{TT}\partial_Z T \partial_s T \ , \\
g_{ss} &= s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} &= s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\
g_{s\xi} &= s_{s\xi} \ , & g_{s\overline{\xi}} &= s_{s\overline{\xi}} \ .
\end{aligned} \tag{30}$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \ = \ 0 \tag{31}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned}
\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\
\partial_\xi(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) &= 0 \ .
\end{aligned} \tag{32}$$

In the special case that the induce metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} \{^{\ k}_{T\ w}\} &= 0 \ , \quad \{^{\ k}_{T\ \overline{w}}\} = 0 \ , \\ \{^{\ k}_{Z\ w}\} &= 0 \ , \quad \{^{\ k}_{Z\ \overline{w}}\} = 0 \end{aligned} \tag{33}$$

hold true. The conditions are equivalent with the conditions

$$\{^{\ k}_{\pm\ w}\} = 0 \ , \quad \{^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \tag{34}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s,Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

2.5.7 $D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

a) Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.

b) One might hope that for $D = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations are indeed satisfied.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1,\xi^2) \ , \quad w = w(\xi^1,\xi^2) \ , \quad S^- = constant \ . \tag{35}$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{36}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1,ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{^{\ k}_{w+}\} = 0 \ , \quad \{^{\ k}_{\overline{w}+}\} = 0 \ , \quad \{^{\ k}_{++}\} = 0 \ . \tag{37}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \quad = \quad 0 \ . \tag{38}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \quad = \quad a_k\xi^k + \overline{a}_k\overline{\xi}^k \ . \tag{39}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u,w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j(J^{j\bar{i}}\sqrt{g}) = 0 \ , \quad \partial_{\bar{j}}(J^{\bar{j}i}\sqrt{g}) = 0 \ , \tag{40}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i} S^+ \partial_{\xi^j} S^- + m_{+-}\partial_{\xi^i} S^- \partial_{\xi^j} S^+)$ are non-vanishing.

a) The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

b) The solution ansatz has the same general form as in $D = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \qquad (41)$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of

M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

2.5.8 $D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

a) String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the imbedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfwen waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfwen waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

b) Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u, v), r(u, v)) \quad , \quad \xi = constant \quad , \tag{42}$$

and corresponds to a vanishing Kähler current.

c) Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B}\cdot\overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s,r) = (s,r)\,(u,v,w,\overline{w}) \ , \quad \xi = constant \ . \tag{43}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

a) Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w,\overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

b) The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfwen waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \ , \ \ w = w(\xi) \ , \ \ S^- = s^- \ , \ \ S^+ = s^+ + f(\xi,\overline{\xi}) \ .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi\partial_+h^k$ and $\partial_{\overline{\xi}}\partial_+h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

3. Do scalar wave pulses represent a solution type with non-vanishing but not light-like Kähler current?

Since longitudinal polarizations are possible only for off mass shell virtual photons, physical intuition suggests that scalar wave pulse solutions describing the propagation of longitudinal electric field with light velocity cannot appear as asymptotic field patterns. This is also consistent with the claim that scalar wave pulses are associated with the transients involved with sudden switching of electric voltage on or off. Let $M^4 = M^2 \oplus E^2$ be the standard decomposition of M^4 to flat longitudinal and transversal spaces, and S^2 a homologically non-trivial geodesic sphere of CP_2. The simplest solution ansatz corresponds to a surface $X^2 \times Y^2$, $X^2 \subset E^2$, such that Y^2 is a surface defined by a map $S^2 \rightarrow M^2$ (or vice versa).

Energy momentum tensor is in both longitudinal and transversal degrees of freedom proportional to the corresponding part of the induced metric. Field equations are trivially true in the transversal degrees of freedom. The calculation of the divergence of energy momentum tensor demonstrates that Kähler current can be regarded as a vector field

$$j^\alpha = \frac{1}{4} J^{\alpha\beta} \partial_\beta L$$

defined by the Kähler action density acting as Hamiltonian. Poisson bracket is defined by the pseudo-symplectic form associated with the induced Kähler form with respect to the induced metric rather that that of S^2 (using S^2-coordinates as coordinates for Y^2, the square of this pseudo-symplectic form is equal to metric multiplied by the ratio $det(g(Y^2))/det(g(S^2))$).

In longitudinal degrees of freedom field equations are minimal surface equations with a source term proportional to the Kähler current divided by the Kähler action density. The vanishing of the Kähler current is possible only if Kähler action density is constant. This condition is true in the approximation that the induced metric for Y^2 is flat, that is at the limit when M^4 projection has size larger than size of CP_2 projection and that induced metric has Minkowskian signature). It is not clear whether the minimal surface property of Y^2 in $M^2 \times S^2$ is consistent with the constancy of the Kähler action density. This would suggest that classical gravitational interactions eliminate scalar wave pulses as asymptotic field patterns and cause the deviation from the minimal surface property and the non-vanishing of the Kähler current. The fact that solution becomes "instanton" like Euclidian solution when S^+ and S^- become constant suggests that the M^4 projection of the solution quite generally has a finite extension in time direction.

2.5.9 Could $D_{CP_2} = 2 \to 3$ transition occur in rotating magnetic systems?

I have studied the imbeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [G2].

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta), \Phi)$ and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{aligned} U = U(\rho) \ , &\quad \Phi = n\phi + kz \ , \\ J_{\rho\phi} = n\partial_\rho U \ , &\quad J_{\rho z} = k\partial_\rho U \ . \end{aligned} \tag{44}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \to 3$. This could help to understand various strange effects related to the rotating magnetic systems [G2]. For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

2.6 $D = 3$ phase allows infinite number of topological charges characterizing the linking of magnetic field lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = fdQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [22]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of configuration space of 3-surfaces defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to configuration space Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to configuration space Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

2.7 Is absolute minimization of Kähler action equivalent with the topologization/light-likeness of Kähler current and with second law?

The basic question is whether the Kähler current is either topologized or light-like for all extremals or only for the absolute minima of Kähler action in some sense, presumably asymptotically as suggested by the fact that generalized Beltrami fields correspond to asymptotic self-organization patterns, when dissipation has become insignificant.

a) The generalized Beltrami conditions or light-likeness can hold true only asymptotically. First of all, generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$.

b) The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields could represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles.

The obvious objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all 3-surfaces approaching same asymptotic state have the same value of Kähler function. This is actually not a problem since it means an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature. The absolute minimization of Kähler action would guarantee that the predicted both signs of Kähler coupling strength and depending on sign Kähler magnetic or electric field configurations are stable. The two phases would relate by time reversal to each other. The role of time reversed dissipation manifesting itself as processes like self assembly would involve in essential manner the time reversed negative energy world. The fact that Kähler coupling strength is of opposite sign for the time reversed dynamics is essential for internal consistency. For instance, creation of matter from vacuum in big bang can be seen as time reflection of stable negative energy cosmic strings as positive energy cosmic strings unstable against decay to magnetic flux tubes.

2.7.1 Is absolute minimization equivalent with generalized Beltrami conditions?

Previous findings inspire the hypothesis that generalized Beltrami conditions express algebraically the absolute minimization conditions so that they make sense also in the p-adic case.

a) Generalized Beltrami conditions are satisfied by the asymptotic field configurations representing self-organization patterns. For non-asymptotic fields vacuum Lorentz force is non-vanishing and does work in Maxwellian sense so that $\overline{j} \cdot \overline{E}$ is non-vanishing. This would mean that the dynamics defined by Kähler action could in principle predict even the values of the parameters related to dissipation such as conductivities and viscosities. The space-time sheets of the many-sheeted space-time would be busily modelling its own physics in shorter length scales.

b) Absolute minimization of Kähler action implies that single space-time surface goes through given 3-surface apart from the non-uniqueness caused by the non-determinism of Kähler action. This gives *four* additional local conditions to the initial values of field equations fixing the time derivatives of the four dynamical imbedding space coordinates (conditions are analogous to Bohr conditions).

The topologization of the Kähler current current gives also *four* local conditions:

i) For $D_{CP_2} < 4$ the vanishing of instanton density gives one condition, and the proportionality of the Kähler current to instanton current gives 3 conditions since the proportionality factor is an arbitrary function of CP_2 coordinates. Altogether this makes four conditions.

ii) For $D_{CP_2=4}$ the vanishing of the Kähler current gives four conditions. This encourages to think that the absolute minimization of Kähler action forces the asymptotic behavior (final values instead of initial values) to correspond to dissipation-less state characterized by the generalized Beltrami conditions.

c) Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number except perhaps by algebraic continuation procedure described in the first part of the book. The generalized Beltrami conditions are however purely algebraic conditions and make sense also in the p-adic context. Therefore it possible to give a precise content to the notion of absolute minimization also in p-adic context.

2.7.2 Is absolute minimization equivalent with the second law?

The fact that Beltrami conditions are associated with the asymptotic dynamics suggests that absolute minimization is equivalent with the second law at space-time level. Or putting it more cautiously: second law at space-time level could be equivalent with absolute minimization. If not, one must give up absolute minimization and replace it with the second law.

For space-time sheets with negative time orientation and negative energy, say "massless extremals" representing phase conjugate laser waves, field configurations would approach non-dissipating ones in the geometric past, and the arrow of geometric time would be opposite to the standard one in this case. This situation is possible for space-time sheets of finite duration, in particular virtual particle like space-time sheets or the negative energy space-time sheets extending down to the boundary of imbedding space (moment of "big bang"). This would explain at the space-time level the change of arrow of time and breaking of the second law observed for the phase conjugate laser waves (used to generate healing and error correction for instance). In TGD framework second law is not a producer of a thermal chaos but Darwinian selector since state function reduction and state preparation by self measurements lead from a state with positive entanglement entropy to that with a negative entanglement entropy (defined number theoretically), and possessing only finitely extended rational entanglement identifiable as a bound state entanglement.

Absolute minimization of Kähler action indeed induces long range correlations since the positive Kähler action of space-time sheets carrying magnetic fields must be compensated by the negative Kähler action of space-time sheets dominated by Kähler electric fields. The resulting non-local long range correlations could serve as correlates for bound state entanglement. More concretely, the stable join along boundaries bonds would be the correlates for bound state entanglement whereas topological light rays analogous to the exchange of virtual photons could serve as classical correlates for unbound entanglement. The closedness (periodicity) of the field lines of Beltrami fields for space-like Kähler current and periodicity of the field pattern for the time like Kähler current could be space-time correlates for the rational entanglement. The pinary expansions of rational numbers which are periodic after finite number of pinary digits indeed represent closed orbits in the set of integers modulo p. Amusingly, the first non-periodic pits of the expansion would in fact be analogous to the dissipative period.

Macro-temporal quantum coherence integrates sequences of quantum jumps to single effective quantum jump so that effectively a fractal hierarchy of quantum jumps emerges having the fractal hierarchy of time scales of dissipation resulting from many-sheetedness as a correlate. Even the anatomy of quantum jump could have space-time correlate. The final state of the quantum jump would correspond to highly negentropic and non-dissipating topologically quantized generalized Beltrami fields. State function reduction and preparation would correspond to the non-deterministic dissipative approach to the non-dissipative Beltrami field configuration. The points of space-time sheets with vanishing Kähler 4-currents would be unstable against quantum jumps generating an instability of the Beltrami field leading to a field configuration with a non-vanishing Lorentz 4-force and emission of topological light rays representing unstable entanglement. Quantum jump would have this kind of instability as a natural space-time correlate.

To sum up, the main lessons would be following.

a) The ability of basically non-dissipative dynamics to mimic dissipative dynamics in terms of energy momentum tensor would be the basic reason for why space-times must be 4-surfaces.

b) If absolute minimization of the Kähler action is correct principle, it must correspond to the second law, which is the Darwinian selector of the most information rich patterns rather than a thermal killer.

2.8 Generalized Beltrami fields and biological systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos.

2.8.1 Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

a) Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

b) Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [20].

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

2.8.2 $D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary dead matter. If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D = 2$ and the vanishing of em field requires that that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A} \cdot \overline{B} \neq 0$ and Kähler current $\overline{E} \times \overline{A} + \phi\overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [L4]. For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D = 3$ phase to $D = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. Since the field lines of the vector potential define chaotic orbits in this phase, the loss of coherence of the order parameter implying the loss of superconductivity by random collisions of particles is what one expects to happen.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

2.9 About small perturbations of field equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M^4_+ \rightarrow CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k}))$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

2.9.1 Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

a) Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where

k_T is transverse momentum, make algebraization possible.

For time-like longitudinal momenta one can choose the local M^4 coordinates in such a manner that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^{\pm})$, where ω has continuum of values.

c) It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force for the absolute minima has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

2.9.2 High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta}k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

2.9.3 Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

a) The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the 4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is of order $d=8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w=\omega_0^2$ results.

b) Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail catastrophe [23] with three control parameters applies to the situation.

c) The general form of the vanishing determinant is

$$D(w,a,b,c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w=\hat{w}+e/4$ the determinant reduces to the canonical form

$$D(\hat{w},a,b,c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w=\omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe.

d) The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a=0$ the swallowtail reduces to $\hat{w}=0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp in turn reduces for $b=0$ to $\hat{w}=0$ and fold catastrophe

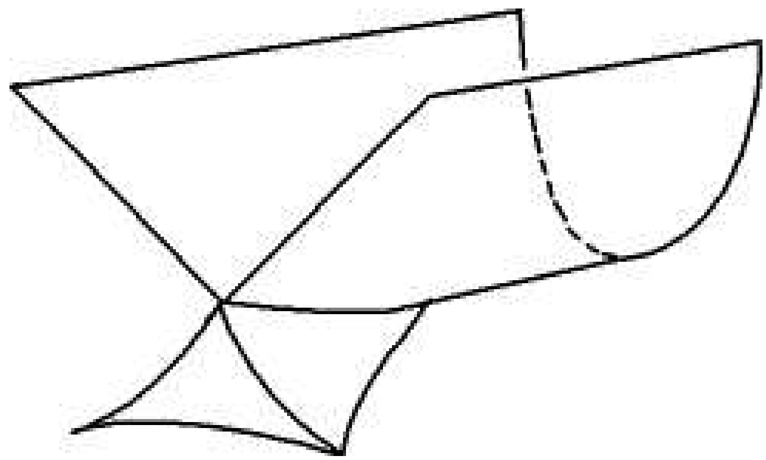

Figure 1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

$\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta}k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.

e) It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense. Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.

f) Additional complications are encountered at the points, where the induced Kähler field vanishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

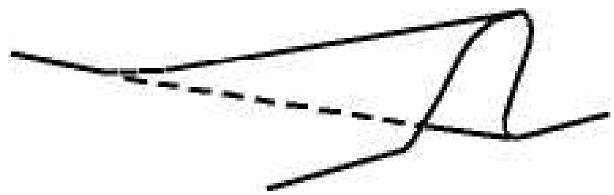

Figure 2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

3 Gerbes and TGD

The notion of gerbes has gained much attention during last years in theoretical physics and there is an abundant gerbe-related literature in hep-th archives. Personally I learned about gerbes from the excellent article of Jouko Mickelson [24] (Jouko was my opponent in PhD dissertation for more than two decades ago: so the time flows!).

I have already applied the notion of bundle gerbe in TGD framework in the construction of the Dirac determinant which I have proposed to define the Kähler function for the configuration space of 3-surfaces (see the chapter "Configuration Space Spinor Structure"). The insights provided by the general results about bundle gerbes discussed in [24] led, not only to a justification for the hypothesis that Dirac determinant exists for the modified Dirac action, but also to an elegant solution of the conceptual problems related to the construction of Dirac determinant in the presence of chiral symmetry. Furthermore, on basis of the special properties of the modified Dirac operator there are good reasons to hope that the determinant exists even without zeta function regularization. The construction also leads to the conclusion that the space-time sheets serving as causal determinants must be geodesic sub-manifolds (presumably light like boundary components or "elementary particle horizons"). Quantum gravitational holography is realized since the exponent of Kähler function is expressible as a Dirac determinant determined by the local data at causal determinants and there would be no need to find absolute minima of Kähler action explicitly.

In the sequel the emergence of 2-gerbes at the space-time level in TGD framework is discussed and shown to lead to a geometric interpretation of the somewhat mysterious cocycle conditions for a wide class of gerbes generated via the $\wedge d$ products of connections associated with 0-gerbes. The resulting conjecture is that gerbes form a graded-commutative Grassmman algebra like structure generated by -1- and 0-gerbes. 2-gerbes provide also a beautiful topological characterization of space-time sheets as structures carrying Chern-Simons charges at boundary components and the 2-gerbe variant of Bohm-Aharonov effect occurs for perhaps the most interesting asymptotic solutions of field equations especially relevant for anyonics systems, quantum Hall effect, and living matter [E9].

3.1 What gerbes roughly are?

Very roughly and differential geometrically, gerbes can be regarded as a generalization of connection. Instead of connection 1-form (0-gerbe) one considers a connection $n+1$-form defining n-gerbe. The curvature of n-gerbe is closed $n+2$-form and its integral defines an analog of magnetic charge. The notion of holonomy generalizes: instead of integrating n-gerbe connection over curve one integrates its connection form over n+1-dimensional closed surface and can transform it to the analog of magnetic flux.

There are some puzzling features associated with gerbes. Ordinary $U(1)$-bundles are defined in terms of open sets U_α with gauge transformations $g_{\alpha\beta} = g^{-1}_{\beta\alpha}$ defined in $U_\alpha \cap U_\beta$ relating the connection forms in the patch U_β to that in patch U_α. The 3-cocycle condition

$$g_{\alpha\beta} g_{\beta\gamma} g_{\gamma\alpha} = 1 \tag{45}$$

makes it possible to glue the patches to a bundle structure.

In the case of 1-gerbes the transition functions are replaced with the transition functions $g_{\alpha\beta\gamma} = g^{-1}_{\gamma\beta\alpha}$ defined in triple intersections $U_\alpha \cap U_\beta \cap U_\gamma$ and 3-cocycle must be replaced with 4-cocycle:

$$g_{\alpha\beta\gamma} g_{\beta\gamma\delta} g_{\gamma\delta\alpha} g_{\delta\alpha\beta} = 1 \ . \tag{46}$$

The generalizations of these conditions to n-gerbes is obvious.

In the case of 2-intersections one can build a bundle structure naturally but in the case of 3-intersections this is not possible. Hence the geometric interpretation of the higher gerbes is far from obvious. One possible interpretation of non-trivial 1-gerbe is as an obstruction for lifting projective bundles with fiber space CP_n to vector bundles with fiber space C^{n+1} [24]. This involves the lifting of the holomorphic transition functions g_α defined in the projective linear group $PGL(n+1, C)$ to $GL(n+1, C)$. When the 3-cocycle condition for the lifted transition functions $\overline{g}_{\alpha\beta}$ fails it can be replaced with 4-cocycle and one obtains 1-gerbe.

3.2 How do 2-gerbes emerge in TGD?

Gerbes seem to be interesting also from the point of view of TGD, and TGD approach allows a geometric interpretation of the cocycle conditions for a rather wide class of gerbes.

Recall that the Kähler form J of CP_2 defines a non-trivial magnetically charged and self-dual $U(1)$-connection A. The Chern-Simons form $\omega = A\wedge J = A\wedge dA$ having CP_2 Abelian instanton density $J\wedge J$ as its curvature form and can thus be regarded as a 3-connection form of a 2-gerbe. This 2-gerbe is induced by 0-gerbe.

The coordinate patches U_α are same as for $U(1)$ connection. In the transition between patches A and ω transform as

$$\begin{aligned} A &\rightarrow A + d\phi \ , \\ \omega &\rightarrow \omega + dA_2 \ , \\ A_2 &= \phi\wedge J \ . \end{aligned} \tag{47}$$

The transformation formula is induced by the transformation formula for $U(1)$ bundle. Somewhat mysteriously, there is no need to define anything in the intersections of U_α in the recent case.

The connection form of the 2-gerbe can be regarded as a second $\wedge d$ power of Kähler connection:

$$A_3 \equiv A\wedge dA \ . \tag{48}$$

The generalization of this observation allows to develop a different view about n-gerbes generated as $\wedge d$ products of 0-gerbes.

3.2.1 The hierarchy of gerbes generated by 0-gerbes

Consider a collection of $U(1)$ connections $A^{i)}$. They generate entire hierarchy of gerbe-connections via the $\wedge d$ product

$$A_3 = A^{1)}\wedge dA^{2)} \tag{49}$$

defining 2-gerbe having a closed curvature 4-form

$$F_4 \;=\; dA^{1)}\wedge dA^{2)} \;\; . \tag{50}$$

$\wedge d$ product is commutative apart from a gauge transformation and the curvature forms of $A^{1)}\wedge dA^{2)}$ and $A^{2)}\wedge dA^{1)}$ are the same.

Quite generally, the connections A_m of $m-1$ gerbe and A_n of $n-1$-gerbe define $m+n+1$ connection form and the closed curvature form of $m+n$-gerbe as

$$\begin{aligned} A_{m+n+1} &= A^{1)}_m\wedge dA^{2)}_n \;\; , \\ F_{m+n+2} &= dA^{1)}_m\wedge dA^{2)}_n \;\; . \end{aligned} \tag{51}$$

The sequence of gerbes extends up to $n=D-2$, where D is the dimension of the underlying manifold. These gerbes are not the most general ones since one starts from 0-gerbes. One can of course start from $n>0$-gerbes too.

The generalization of the $\wedge d$ product to the non-Abelian situation is not obvious. The problems stem from the that the Lie-algebra valued connection forms $A^{1)}$ and $A^{2)}$ appearing in the covariant version $D=d+A$ do not commute.

3.3 How to understand the replacement of 3-cycles with n-cycles?

If n-gerbes are generated from 0-gerbes it is possible to understand how the intersections of the open sets emerge. Consider the product of 0-gerbes as the simplest possible case. The crucial observation is that the coverings U_α for $A^{1)}$ and V_β for $A^{2)}$ need not be same (for CP_2 this was the case). One can form a new covering consisting of sets $U_\alpha\cap V_{\alpha_1}$. Just by increasing the index range one can replace V with U and one has covering by $U_\alpha\cap U_{\alpha_1}\equiv U_{\alpha\alpha_1}$.

The transition functions are defined in the intersections $U_{\alpha\alpha_1}\cap U_{\beta\beta_1}\equiv U_{\alpha\alpha_1\beta\beta_1}$ and cocycle conditions must be formulated using instead of intersections $U_{\alpha\beta\gamma}$ the intersections $U_{\alpha\alpha_1\beta\beta_1\gamma\gamma_1}$. Hence the transition functions can be written as $g_{\alpha\alpha_1\beta\beta_1}$ and the 3-cocycle are replaced with 5-cocycle conditions since the minimal co-cycle corresponds to a sequence of 6 steps instead of 4:

$$U_{\alpha\alpha_1\beta\beta_1}\rightarrow U_{\alpha_1\beta\beta_1\gamma}\rightarrow U_{\beta\beta_1\gamma\gamma_1}\rightarrow U_{\beta_1\gamma\gamma_1\alpha}\rightarrow U_{\gamma\gamma_1\alpha\alpha_1} \;\; .$$

The emergence of higher co-cycles is thus forced by the modification of the bundle covering necessary when gerbe is formed as a product of lower gerbes. The conjecture is that any even gerbe is expressible as a product of 0-gerbes.

An interesting application of the product structure is at the level of configuration space of 3-surfaces ("world of classical worlds"). The Kähler form of the configuration space defines a connection 1-form and this generates infinite hierarchy of connection $2n+1$-forms associated with $2n$-gerbes.

3.4 Gerbes as graded-commutative algebra: can one express all gerbes as products of -1 and 0-gerbes?

If one starts from, say 1-gerbes, the previous argument providing a geometric understanding of gerbes is not applicable as such. One might however hope that it is possible to represent the connection 2-form of any 1-gerbe as a $\wedge d$ product of a connection 0-form ϕ of "-1"-gerbe and connection 1-form A of 0-gerbe:

$$A_2 = \phi dA \equiv A \wedge d\phi \ ,$$

with different coverings for ϕ and A. The interpretation as an obstruction for the modification of the underlying bundle structure is consistent with this interpretation.

The notion of -1-gerbe is not well-defined unless one can define the notion of -1 form precisely. The simplest possibility that 0-form transforms trivially in the change of patch is not consistent. One could identify contravariant n-tensors as $-n$-forms and d for them as divergence and d^2 as the antisymmetrized double divergence giving zero. ϕ would change in a gauge transformation by a divergence of a vector field. The integral of a divergence over closed M vanishes identically so that if the integral of ϕ over M is non-vanishing it corresponds to a non-trivial 0-connection. This interpretation of course requires the introduction of metric.

The requirement that the minimal intersections of the patches for 1-gerbes are of form $U_{\alpha\beta\gamma}$ would be achieved if the intersections patches can be restricted to the intersections $U_{\alpha\beta\gamma}$ defined by $U_\alpha \cap V_\gamma$ and $U_\beta \cap V_\gamma$ (instead of $U_\beta \cap V_\delta$), where the patches V_γ would be most naturally associated with -1-gerbe. It is not clear why one could make this restriction. The general conjecture is that any gerbe decomposes into a multiple $\wedge d$ product of -1 and 0-gerbes just like integers decompose into primes. The $\wedge d$ product of two odd gerbes is anti-commutative so that there is also an analogy with the decomposition of the physical state into fermions and bosons, and gerbes for a graded-commutative super-algebra generalizing the Grassmann algebra of manifold to a Grassmann algebra of gerbe structures for manifold.

3.5 The physical interpretation of 2-gerbes in TGD framework

2-gerbes could provide some insight to how to characterize the topological structure of the many-sheeted space-time.

a) The cohomology group H^4 is obviously crucial in characterizing 2-gerbe. In TGD framework many-sheetedness means that different space-time sheets with induced metric having Minkowski signature are separated by elementary particle horizons which are light like 3-surfaces at which the induced metric becomes degenerate. Also the time orientation of the space-time sheet can change at these surfaces since the determinant of the induced metric vanishes.

This justifies the term elementary particle horizon and also the idea that one should treat different space-time sheets as generating independent direct summands in the homology group of the space-time surface: as if the space-time sheets not connected by join along boundaries bonds were disjoint. Thus the homology group H^4 and 2-gerbes defining instanton numbers would become important topological characteristics of the many-sheeted space-time.

b) The asymptotic behavior of the general solutions of field equations can be classified by the dimension D of the CP_2 projection of the space-time sheet. For $D = 4$ the instanton density defining the curvature form of 2-gerbe is non-vanishing and instanton number defines a topological charge. Also the values of the Chern-Simons invariants associated with the boundary components of the space-time sheet define topological quantum numbers characterizing the space-time sheet and their sum equals to the instanton charge. CP_2 type extremals represent a basic example of this kind of situation. From the physical view point $D = 4$ asymptotic solutions correspond to what might be regarded chaotic phase for the flow lines of the Kähler magnetic field. Kähler current vanishes so that empty space Maxwell's equations are satisfied.

c) For $D = 3$ situation is more subtle when boundaries are present so that the higher-dimensional analog of Aharonov-Bohm effect becomes possible. In this case instanton density vanishes but the Chern-Simons invariants associated with the boundary components can be non-vanishing. Their sum obviously vanishes. The space-time sheet can be said to be a neutral C-S multipole. Separate space-time sheets can become connected by join along boundaries bonds in a quantum jump replacing a space-time surface with a new one. This means that the cohomology group H^4 as well as instanton charges and C-S charges of the system change.

Concerning the asymptotic dynamics of the Kähler magnetic field, $D = 3$ phase corresponds to an extremely complex but highly organized phase serving as an excellent candidate for the modelling of living matter. Both the TGD based description of anyons and quantum Hall effect and the model for topological quantum computation based on the braiding of magnetic flux tubes rely heavily on the properties $D = 3$ phase [E9].

The non-vanishing of the C-S form implies that the flow lines of the Kähler magnetic are highly entangled and have as an analog mixing hydrodynamical flow. In particular, one cannot define non-trivial order parameters, say phase factors, which would be constant along the lines. The interpretation in terms of broken super-conductivity suggests itself. Kähler current can be non-vanishing so that there is no counterpart for this phase at the level of Maxwell's equations.

4 Vacuum extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

4.1 CP_2 type extremals

4.1.1 CP_2 type vacuum extremals

These extremals correspond to various isometric imbeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the imbedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{52}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-submanifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of the configuration space geometry and quantum TGD and appears both at the level of imbedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the imbedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} . \quad (53)$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action. The absolute minimization of Kähler action suggests that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. Indeed, the nice generalization of the area law for the entropy of black hole [E5] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [25]. A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but nonvanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

4.1.2 Are CP_2 type non-vacuum extremals possible?

The isometric imbeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however nonvacuum extremals as deformations of these solutions. There are several types of deformations leading to nonvacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r,Θ,Ψ,Φ) [26] are given by

$$\begin{aligned}
\frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2} + \frac{r}{2(1+r^2))^2}(d\Psi + cos(\Theta)d\Phi)^2 \\
&+ \frac{r^2}{(4(1+r^2)}(d\Theta^2 + sin^2\Theta d\Phi^2) \ ,
\end{aligned}$$

$$
\begin{aligned}
J &= \frac{r}{(1+r^2)} dr \wedge (d\Psi + cos(\Theta) d\Phi) \\
&- \frac{r^2}{(2(1+r^2)} sin(\Theta) d\Theta \wedge d\Phi \ .
\end{aligned} \tag{54}
$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as "cyclic" coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4 = M^2 \times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of nonvacuum extremals.

a) "Virtual particle" extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

b) Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$
m^k = a^k \Psi + b^k \Phi \ . \tag{55}
$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$
\begin{aligned}
m^0 &= m^3 = a\Psi + b\Phi \ , \\
(m^1, m^2) &= (m^1(r,\Theta), m^2(r,\Theta)) \ .
\end{aligned} \tag{56}
$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2} - (dm^1)^2 - (dm^2)^2 \ . \tag{57}$$

Field equations reduce to conservation condition for the componenents of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$\begin{aligned} (J^i_a)_{,i} &= 0 \ , \\ J^i_a &= T^{ij} f^a_{,j} \sqrt{g} \ . \end{aligned} \tag{58}$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik}g^{jl} + g^{ij}g^{kl}/2)s_{kl} \ . \tag{59}$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J^i_a = \varepsilon^{ij} H^a_{,j} \ , \tag{60}$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \tag{61}$$

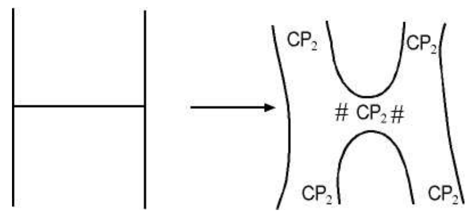

Figure 3: Topological sum of CP_2:s as Feynmann graph with lines thickened to four-manifolds

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all nonvacuum solutions have boundary components in accordance with basic ideas of TGD.

4.1.3 $CP_2\#CP_2\#...\#CP_2$:s as generalized Feynmann graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynmann diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynmann graphs are essentially connected sums of CP_2:s (see Fig. 4.1.3): $X^4 = CP_2\#CP_2....\#CP_2$).

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naive geometric argument suggests that the cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the

CP_2 type extremals whereas virtual particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

4.2 Vacuum extremals with vanishing Kähler field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{62}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{63}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the configuration space in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D = 3$ for CP_2 type extremals, $D = 2$ for string like objects, $D = 1$ for membranes and $D = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynmann diagram is not supressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

The reason would "Yin-Yang principle" discussed in [**?**].

a) Consider first the option for which Kähler function corresponds to an absolute minimum of Kähler action. Vacuum functional as an exponent of Kähler function is expected to concentrate on those 3-surfaces for which the Kähler action is non-negative. On the other hand, the requirement that Kähler action is absolute minimum for the space-time associated with a given 3-surface, tends to make the action negative. Therefore the vacuum functional is expected to differ considerably from zero only for 3-surfaces with a vanishing Kähler action per volume. It could also occur that the degeneracy of 3-surfaces with same large negative action compensates the exponent of Kähler function.

b) If preferred extrema correspond to Kähler calibrations or their duals [E2], Yin-Yang principle is modified to a more local principle. For Kähler calibrations (their duals) the absolute value of action in given region is minimized (maximized). A given region with positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically and implies that average Kähler action per volume vanishes. Positive and finite values of Kähler function are of course favored.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

5 Non-vacuum extremals

5.1 Cosmic strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{64}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

5.2 Massless extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{aligned} & s^k = f^k(u,v) \ , \\ & u = p \cdot m \ , \qquad v = \varepsilon \cdot m \ , \\ & p \cdot \varepsilon = 0 \ , \qquad p^2 = 0 \ . \end{aligned} \tag{65}$$

CP_2 coordinates are arbitrary functions of $p \cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{aligned} & H(p \cdot m, \varepsilon \cdot m, \varepsilon_1 \cdot m) = 0 \ , \\ & \varepsilon \cdot p = 0 \ , \qquad \varepsilon_1 \cdot p = 0 \ , \quad \varepsilon \cdot \varepsilon_1 = 0 \ . \end{aligned} \tag{66}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many manners to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta}J^k_{\ l}\partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a manner to satisfy all boundary conditions but it is not clear whether there are any other manners to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are are bounded: this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

5.3 Generalization of the solution ansatz defining massless extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

5.3.1 Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

a) Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

b) With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{67}$$

If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

c) This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

5.3.2 A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ (\nabla_3 f)^2 = 1 \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many manners.

5.3.3 Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

a) Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

b) The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k \nabla S_+ \ ,$$

where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

c) The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$

is satisfied: in this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

d) The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$

$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

5.3.4 General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

a) Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

b) Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

c) The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then 'free' solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

d) For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

e) One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the imbedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \quad (68)$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{69}$$

The diagonalization of the metric seems to be a general aspect of absolute minima. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

5.3.5 Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

a) The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

b) Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-canonical and super-conformal symmetries just like the boundary of the imbedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

c) These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of imbedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

5.4 Maxwell phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^{\alpha}J^{k}_{\ l}s^{l}_{,\alpha} \;=\; 0 \ . \tag{70}$$

These equations are satisfied if Maxwell's equations

$$j^{\alpha} \;=\; 0 \tag{71}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an imbedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of the point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulombic term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This implies the counter part of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M^4_+)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field: $\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g^A_{\alpha\beta} = kH^A J_{\alpha\beta} \ . \qquad (72)$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

5.5 Stationary, spherically symmetric extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow imbedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields.

contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter. This model is developed in more detail in [F6] by applying general number theoretic ideas and p-adic length scale hypothesis.

5.5.1 General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t \ , \\ m^0 &= \lambda t \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{73}$$

The induced metric is given by the expression

$$ds^2 = \left[\lambda^2 - \frac{R^2}{4}\omega^2(1-u^2)\right]dt^2 - (1+\frac{R^2}{4}\theta_{,r}^2)dr^2 - r^2 d\Omega^2 \ . \tag{74}$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$\lambda^2 - \frac{R^2}{4}\omega^2(1-u(\infty)^2) = 1 \ . \tag{75}$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} = \frac{\omega}{4}u_{,r} \ . \tag{76}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$\begin{aligned} \gamma &= (3-p/2)J \ , \\ Z^0 &= J \ . \end{aligned} \tag{77}$$

From this formula one obtains the expressions

$$\begin{aligned} Q_{em} &= \frac{(3-p/2)}{4\pi\alpha_{em}}Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z}Q \ , \\ Q &\equiv \frac{J_{tr}4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ . \end{aligned} \tag{78}$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr}z_{,r})_{,r} + (T^{\theta\theta}z_{,\theta})_{,\theta} = 0 \ . \tag{79}$$

Using the explicit expressions for the components of the energy momentum tensor

$$\begin{aligned} T^{rr} &= g^{rr}L/2 \ , \\ T^{\theta\theta} &= -g^{\theta\theta}L/2 \ , \\ L &= g^{tt}g^{rr}(J_{tr})^2\sqrt{g}/2 \ , \end{aligned} \tag{80}$$

and the following notations

$$\begin{aligned} A &= g^{tt}g^{rr}r^2\sqrt{-g_{tt}g_{rr}} \ , \\ X &\equiv (J_{tr})^2 \ , \end{aligned} \tag{81}$$

the field equations reduce to the following form

$$(g^{rr}AX)_{,r} - \frac{2AX}{r} = 0 \ . \tag{82}$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (82), one obtains integral equation for X

$$J_{tr} = \frac{q}{r_c}(|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r})\frac{1}{r} \ , \tag{83}$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity and spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a topologically condensed CP_2 extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) = u_0 + \frac{4q}{\omega}\int_{r_c}^{r}(-g_{rr}^3 g_{tt})^{1/4}\frac{1}{r}exp(\int_{r_c}^{r}dr\frac{g_{rr}}{r}) \ . \qquad (84)$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) = T_{n-1} \ , \qquad (85)$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n$: $A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 = u_\infty \ , \qquad (86)$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u = u_\infty - \frac{4q}{\omega r} \ , \qquad (87)$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the imbedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \ , \quad q < 0 \ , \quad \omega > 0 \qquad (88)$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1) The condition $g_{tt} > 0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r = r_c$ given by the the condition $u = 1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c &\geq \frac{4q}{(|u_\infty| + 1)\omega} \\ &= \frac{4\alpha Q_{em}}{(3 - p/2)} \frac{1}{(|u_\infty| + 1)\omega} \ . \end{aligned} \tag{89}$$

The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r \theta$ grows near the origin without bound and $u = 1$ is reached at some finite value of r. Boundary conditions require that the quantity $X = T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2) g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r = r_c$. Boundary conditions can be satisfied only provided $r_c = 0$. Thus it seems that for the values of ω satisfying the condition $\omega^2 = \frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u ($V = 1/r$)

$$\begin{aligned} u &= \sum_n u_n A^n V^n \ , \\ u_0 &= u_\infty(< 0) \ , \quad u_1 = 1 \ , \\ u_2 &= K|u_\infty| \ , \quad u_3 = K(1 + 4K|u_\infty|) \ , \\ A &\equiv \frac{4q}{\omega} \ , \quad K \equiv \omega^2 \frac{R^2}{4} \ . \end{aligned}$$

(90)

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q = 4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

5.5.2 Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

a) When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".

i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.

ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases at short distances.

iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.

iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.

b) For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

5.5.3 Exterior solution cannot be identified as a counter part of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

TGD predicts the possibility of negative classical energy for space-time sheets with negative time orientation, and the only manner to second quantize induced spinor fields without diverging vacuum energy is by assuming that fermions have positive energies and anti-fermions negative energies (vice versa for phase conjugate fermions). This modifies the original form of Equivalence Principle: gravitational mass can be interpreted as absolute value of inertial mass so that the density of gravitational mass becomes the difference of densities of inertial mass for matter and antimatter (or vice versa). This interpretation leads to an elegant solution of the basic interpretational difficulties created by the conservation of inertial four-momentum and non-conservation of gravitational four-momentum.

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$M_{gr} = \frac{R^2}{G}\omega q u_\infty \ , \qquad (91)$$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$\begin{aligned} M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\ &\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\ \lambda &= \sqrt{1+\frac{R^2}{4}\omega^2(1-u_\infty^2)}\ (>1) \ . \end{aligned} \qquad (92)$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ . \tag{93}$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$\begin{aligned} \frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\ x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ . \end{aligned} \tag{94}$$

In the approximation used the the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ . \tag{95}$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z = \frac{\alpha_K}{\alpha_Z}\frac{1}{\pi(1+|u_\infty|)\Omega R}\sqrt{\frac{G}{R}} \ . \tag{96}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} = -\frac{1}{(1-\frac{2GM}{r})} , \qquad (97)$$

and

$$g_{rr} \simeq 1 - \frac{\frac{Rq}{\omega^2}}{\left[1-(u_\infty - \frac{4q}{\omega r})^2\right] r^4} , \qquad (98)$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about $2/3$ times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the imbedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned} cos(\Theta) &= u(r) , \\ \Phi &= \omega t + f(r) , \\ m^0 &= \lambda t + h(r) , \\ r_M &= r , \quad \theta_M = \theta , \quad \phi_M = \phi . \end{aligned} \qquad (99)$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda \partial_r h - \frac{R^2}{4} sin^2(\Theta)\omega\partial_r f = 0 . \qquad (100)$$

The expression for the radial component of the metric transforms to

$$g_{rr} \simeq \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r\Theta)^2 - \frac{R^2}{4} sin^2(\Theta)\partial_r f^2 \ , \tag{101}$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\rightarrow\infty} \rightarrow \omega r \ , \ \Lambda^2 - 1 = \frac{R^2\omega^2}{4} sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r\rangle} \ . \tag{102}$$

In the second equation $\langle r\rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \rightarrow \Phi + \epsilon$ and gives equation for f.

$$[T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g}]_{,r} = 0 \ . \tag{103}$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S^2_I should be satisfied identically. This equation can be readily integrated to give

$$T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g_{tt}g_{rr}} = \frac{C}{r^2} \ . \tag{104}$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \rightarrow \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \quad .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4} sin^2(\Theta) k^2 \ .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the imbeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [D3] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

5.6 The scalar waves of Tesla in TGD framework

The scalar waves or so called non-Hertzian waves of Nikola Tesla belong to the fringe region of science. Many proponents of free energy believe that scalar waves might provide a basis for a new energy and communication technologies. Tesla himself was isolated from the official science and found no place in text books because his hypothesis about scalar waves did not fit within the framework of the Maxwell's electrodynamics. Personally I justified my personal prejudices against scalar waves by the observation that the formulations for the notion of scalar waves that I had seen seemed to be in a conflict with the cherished gauge invariance of gauge theories. The discussions with a Finnish free energy enthusiast Juha Hartikka however led me to reconsider the status of the scalar waves.

The surprise was that one can understand the non-Hertzian waves of Tesla in TGD framework and that they are basically predicted by the electric-magnetic duality of TGD. As a matter fact, TGD predicts huge number of solutions of field equations representing constant energy density configurations of electric field assignable to bioelectrets which are in a well defined sense dual to the magnetic flux tube structures with analogous properties [I4, I5]. Thus the deep symmetry principle of electric-magnetic duality allows to understand the basic structures of living matter. Also classical gravitational fields generated by classical field energy are predicted to be important in the living matter. In the following scalar waves with constant electric action density are discussed with the understanding that the solution ansatz generalizes also to the case of magnetic field with a constant action density.

5.6.1 The properties of the scalar waves

Perhaps the most important properties of the scalar waves are following.

a) Scalar waves involve some kind of oscillatory process in the direction of the propagation of the wave. The analogy with sound waves suggests that the oscillation could relate to charge density, or more generally to 4-current in the direction of the wave. Even massless extremals (MEs), which are essentially topological light rays, involve vacuum current and vacuum charge density which oscillates in the direction of propagation.

b) Scalar waves are believed to carry electric field in the direction of the wave motion so that the identification of MEs as scalar waves is not possible. The presence of only electric field means that scalar wave is characterized solely by the scalar potential. This kind of solution is excluded by the gauge invariance and linearity of Maxwell's electrodynamics in vacuum.

5.6.2 Could nonlinearity of TGD allow scalar waves?

One is led to ask whether the nonlinearity of TGD might allow existence for scalar waves.

a) In TGD based electrodynamics CP_2 coordinates are the primary dynamical degrees of freedom gauge fields being secondary dynamical variables induced from the spinor curvature of CP_2. Field equations are extremely nonlinear allowing among other things vacuum 4-currents (even Faraday's unipolar generator involves vacuum charge density changing its sign when the direction of rotation of magnet changes its sign). This gives hopes about finding solutions of field equations with the properties assigned to the hypothetical scalar waves.

b) Interestingly, in TGD framework the canonical symmetries of CP_2 are dynamical symmetries and act as isometries of the configuration space of 3-surfaces. Canonical transformations act formally as $U(1)$ gauge transformations but, rather than being gauge symmetries, they are dynamical generating new physical configurations and are partially responsible for the quantum spin glass degeneracy of the TGD universe. As a matter fact, also diffeormorphisms of M^4 act as dynamical symmetries in the lowest order.

c) Magnetic flux tubes represent fundamental solutions of field equations and the simplest magnetic flux tubes can be characterized as maps from a region of a 2-dimensional Euclidian hyperplane E^2 of Minkowski space to a geodesic sphere S^2 of CP_2.

d) Electric-magnetic duality is a fundamental symmetry of the configuration space geometry. Therefore there should exist solutions dual to the magnetic flux tubes carrying only electric fields and perhaps allowing interpretation as waves. These solutions would be characterized by a map from a region of the Minkowskian hyperplane M^2 of Minkowski space to S^2. This kind solution ansatz makes sense since it formally provides the solutions of a field theory from M^2 to S^2.

5.6.3 Lowest order solution ansatz

One can write the field equations explicitely. They are however extremely nonlinear and without physical intuition one cannot say much about the solution spectrum of these equations. One can however make simplifying assumptions to get grasp to the problem.

i) The effect of classical gravitation can be assumed to be extremely weak except possibly at some singular regions associated with the solutions. Interestingly, the electrogravitational effects associated with scalar waves are standard free energy folklore. Since TGD predicts that classical field energy couples to gravitation with a coupling strength which is 10^8 times stronger than the ordinary gravitational coupling [G1], gravitational effects could indeed become important.

ii) In Maxwellian theory without sources gauge current vanishes identically. This would suggest that it is good to start from a zeroth order solution ansatz with this property so that the non-vanishing of the vacuum current would be solely due to gravitational effects. It deserves to be noticed that Tesla proposed also that non-Hertzian radiation fields involve a kind of radiation charge.

In principle, one can imbed a portion of any solution of Maxwell's equations in empty space as a space-time sheet (note the occurrence of the topological quantization) using M^4 coordinates as preferred coordinates. Field equations are satisfied in the lowest order in R^2. The canonical symmetries of CP_2 act as dynamical symmetries for these solution ansätze and one obtains infinite degeneracy of the space-time surfaces representing the same Kähler field.

iii) Constant electric field represents the simplest field configuration one can imagine. Therefore it is reasonable to start with this kind of solution ansatz and to look whether gravitational corrections affect the solution and bring in the wave aspect.

iv) Since wave motion is hoped to result, it is useful to choose the space-time coordinates in an appropriate manner. Light like coordinates (x^+, x^-, x, y) of M^4 are thus very natural. They are defined by the conditions

$$t = (x^+ + x^-)/2 \ , \quad z = (x^+ - x^-)/2 \ ,$$

with (t, x, y, z) referring to the linear Minkowski coordinates such that t is time coordinate. In these coordinates the line element of M^2 has the form $ds^2 = -2dx^+dx^-$ so that one has $g_{+-} = -1$.

v) Using the spherical coordinates $(u = cos(\Theta), \Phi)$ for the geodesic sphere S^2 of CP_2, the zeroth order solution ansatz has the following form:

$$u \equiv u_0 = \omega_1 x^+ \ , \quad \Phi \equiv \Phi_0 = \omega_2 x^- \ . \tag{105}$$

Since electromagnetic, Z^0 and color fields are proportional to Kähler form for the solution type considered, one can restrict the consideration to the induced Kähler form. Denoting the Kähler form of CP_2 by J_{kl}, by noticing that S^2 Kähler form is given by $J_{u\Phi} = 1$ (forgetting the precise normalization factor), and using the expressions $[s_{uu} = R^2/(1-u^2), s_{\Phi\Phi} = R^2(1-u^2)]$ for the metric of S^2, one can write the induced line element and the non-vanishing component of the induced Kähler form as

$$\begin{aligned}
ds^2 &= -2dx^+dx^- + \frac{R^2\omega_1^2}{1-u^2}(dx^+)^2 + R^2\omega_2^2(1-u^2)(dx^-)^2 - dx^2 - dy^2 \ , \\
J_{+-} &= \partial_+ u \partial_- \Phi = \omega_1\omega_2 \ , \\
J^{+-} &= \frac{\omega_1\omega_2}{det(g)} \ .
\end{aligned} \tag{106}$$

Since the determinant of the induced metric is constant, J^{+-} describes constant electric field and that Kähler current j^α is vanishes. This means that Maxwell's equations hold true in the zeroth order approximation as required.

Apart from the normalization factors the energy momentum tensor in the longitudinal degrees of freedom is given by

$$T^{\alpha\beta}(long) = g^{\alpha\beta} L/4 \ ,$$

In the transversal degrees of freedom similar expression but with opposite sign holds true. Here L is Kähler action which is essentially electric energy density and constant.

In M^4 degrees of freedom the field equations express conservation of the energy momentum currents and are satisfied to order R^2 since the action is constant. These equations imply that action density is constant. This forces to ask whether all perturbatively constructible solutions represent a constant Kähler electric field locally.

In CP_2 degrees of freedom field equations involve a sum of two terms: the first term involves the contraction of the energy momentum tensor with the second fundmental form whereas the second term involves Kähler current. Since Kähler current vanishes, the latter term vanishes and one can say that field equations are satisfied in zeroth order approximation (the term involving energy momentum tensor is proportional to CP_2 length squared and thus small). For exactly vanishing vacuum current the field equations would reduce to the equations for a minimal surface:

$$g^{\alpha\beta}D_\beta\partial_\alpha h^k = 0 \ , \tag{107}$$

where the imbedding space coordinates h^k corresponds to u and Φ now. The same equations result also in M^4 degrees of freedom by requiring that the terms of order R^2 in the equation for the energy momentum conservation vanish.

This equation is not satisfied exactly as is easy to see. The non-vanishing components of the trace of the second fundamental form are given by

$$\begin{aligned} g^{\alpha\beta}D_\beta\partial_\alpha u &= -\{{}^{\ u}_{\Phi\ \Phi}\}\omega_2^2 \times \left[1 - g^{++}\omega_1^2 R^2/(1-u^2)\right] \ , \\ g^{\alpha\beta}D_\beta\partial_\alpha \Phi &= -\{{}^{\ \Phi}_{u\ \Phi}\}\omega_1\omega_2 \times \left[1 - g^{--}\omega_2^2 R^2(1-u^2)\right] \ . \end{aligned} \tag{108}$$

Here $\{{}^{\ \alpha}_{\beta\ \gamma}\}$ denote the components of the Riemann connection for sphere. It is seen that the connection term gives contributions which vanish only at $u = 0$ which corresponds to the equator of the geodesic sphere S^2. At poles the minimal surface condition fails to be satisfied.

5.6.4 First order corrections to the solution ansatz

To take into account gravitational corrections one must modify the solution ansatz in such a manner that x^- does not appear in the field equations at all: this guarantees that field equations reduce to ordinary differential equations. The modification is following:

$$u = u_0 + u_1(x^+) \ , \quad \Phi = \Phi_0 + \Phi_1(x^+) \ . \tag{109}$$

The modification affects the electric field and vacuum current and allows the compensation of the terms resulting form the contractions of the energy momentum tensor and vacuum current. The modification means that wave equations are still satisfied for u and Φ. Note that second fundamental form does not contain second derivative terms in the lowest order approximation.

The derivation of the differential equations for u_1 and Φ_1 is completely straightforward but requires some patience with numerical factors (reader should check sign factors and numerical factors).

a) Calculate the the current contraction term

$$j^{\alpha}\left[J_r^k\,\partial_\alpha h^r - J^{\mu}_{\ \alpha}\partial_\mu h^k\right]$$

and energy momentum tensor contraction term

$$T^{\alpha\beta}D_\beta\partial_\alpha h^k$$

and equate these terms. Effective two-dimensionality makes the explicit calculations relatively simple.

b) The equations for u and Φ in terms of $j^{\pm}$ read as

$$j^-(1-u_0^2)+j^+\epsilon_1\epsilon_2 = \left\{{}^{\ u}_{\Phi\ \Phi}\right\}\frac{K\epsilon_2^2}{2} \equiv X_1 \ ,$$

$$j^+\frac{1}{(1-u_0^2)}j^-\epsilon_2^2 = -2\left\{{}^{\ \Phi}_{u\ \Phi}\right\}K\epsilon_1\epsilon_2 \equiv X_2 \ ,$$

Here the notations $\epsilon_i = \omega_i R$ and $K = \omega_1\omega_2^2$ are used. Linear second order differential equations are in question with the right side serving as an inhomogenuity term.

c) One can solve j^+ and j^- from these equations to get

$$\begin{pmatrix} j^+ \\ j^- \end{pmatrix} = \frac{1}{\epsilon_1\epsilon_2^3-1}\times\begin{pmatrix} \epsilon_2^2 & -(1-u_0^2) \\ -1/(1-u_0^2) & \epsilon_1\epsilon_2 \end{pmatrix}\times\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \equiv \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

From this form one can see that j^- becomes singular at $u_0 = \pm 1$ as $1/(1-u_0^2)$ which means that light like vacuum current is generated. The physical interpretation is that vacuum charge density at these points which correspond to the boundaries of the solution acting as the source of the vacuum electric field is in question.

d) One can calculate $j^{\pm}$ by calculating the covariant divergence of the induce Kähler field in the lowest non-trivial order. The calculation gives the following expression

$$\begin{pmatrix} j^+ \\ \\ j^- \end{pmatrix} = \omega_1 \begin{pmatrix} u_0\partial_+^2 u_1 + \epsilon_1\epsilon_2(1-u_0^2)\partial_+^2\Phi_1 \\ \\ \omega_1\epsilon_2\partial_+^2 u_1 - \epsilon_1(1-u_0^2)\partial_+^2\Phi_1 \end{pmatrix}$$

e) For u_1 one finds the equation

$$\begin{aligned} \partial_+^2 u_1 \quad + \quad & \epsilon_1\epsilon_2^2\omega_1 u_0\partial_+ u_1 = \frac{1}{\omega_1}\times (Y_1+\epsilon_1 Y_2) \\ = \quad & \frac{\omega_2^2}{2}\frac{\epsilon_1}{\epsilon_1\epsilon_2^3-1}\times u_0\times\left[-\epsilon_2^4(1-u_0^2)+\epsilon_1\epsilon_2(-2+\epsilon_1)-\epsilon_1^3\epsilon_2\frac{1}{1-u_0^2}\right] \end{aligned} \tag{110}$$

This equation reduces to a first order differential equation for u_1 and one can solve it by variation of integration constants. The singularity at $u=\pm 1$ implies a logarithmic singularity of the derivative

$$\partial_+ u_1 \sim log(1-u_0^2)$$

but u remains finite as it should.

f) One can integrate Φ_1 from the second order inhomogenous and linear equation

$$\begin{aligned} \partial_2^+\Phi_1 &= \frac{1}{\epsilon_1\epsilon_2(1-u_0^2)}\left[j^- - \omega_2\partial_+^2 u_1\right] \ , \\ j^- &= \frac{\omega_1\omega_2^2\epsilon_1\epsilon_2}{2(\epsilon_1\epsilon_2^3-1)}\times u_0\times\left[1-\frac{2\epsilon_1^2}{1-u_0^2}\right] \ , \end{aligned} \tag{111}$$

once the solution for u_1 is known. Note that the most singular part corresponds to $u_0/(1-u_0^2)^2$ type term and one obtains logarithmic singularity also now.

5.6.5 Properties of the solution ansatz

The form of the differential equations for the first order corrections allows to conclude that the North and South poles of the geodesic sphere S^2 (the points $u_0=\pm 1$) correspond to singularities of the solution. Both the components of the induced metric and the induced Kähler form become singular at these points. This means that classical gravitation becomes important near these points. These points correspond in the lowest order approximation to the lines $x^+ = \pm 1/\omega_1 \equiv T$ plus possibly the lines obtained by continuing the solution by assuming that $x^- = constant$ lines define a motion identifiable constant rotation along the big circle from $\theta = 0$ ($x_+ = T$) to $\theta = \pi$ ($x_+ = -T$) continuing in the same manner to $\theta = 0$ at ($x = 2T$) and so on. Therefore gravitational effects induce a periodical behavior of the solution such that gravitational effects become strong at $x^+ = (2n+1)T$.

In the next order electric field is not constant anymore and vacuum current is generated. The contravariant component of electric field, being proportional to $1/\partial_+ u$ near singularity, vanishes at the singularity whereas the tangential component j^- of the vacuum current diverges. The vacuum current should generate coherent photons.

By a straightforward calculation one finds that the curvature scalar behaves as $R \propto 1/(1-u_0^2)$ at the singularities so that the energy density of vacuum becomes singular and could generate a coherent state of gravitons. Since Einstein tensor vanishes identically in two-dimensional case, the longitudinal components G^{++}, G^{--} and G^{+-} of Einstein tensor vanish. The components of Einstein tensor in transverse degrees of freedom are given by $G^{alpha\beta} = -g^{\alpha\beta}R/2$. Therefore the energy momentum tensor defined by Einstein's equations would involve only space-like momentum currents. The singularity is amplified by the fact that field energy couples to the classical gravitation with coupling which is 10^8 times stronger than the ordinary gravitational coupling. The singularity might relate to the claimed gravitational anomalies associated with the scalar waves.

As already found, Einstein tensor and gauge current have no components in the direction of x^+. Energy-momentum tensor behaves as $1/det(g)^{3/2}$ at the end points of the interval $[-T, T]$ and thus vanishes. Therefore conservation laws allow to restrict the solution into the x^+ interval $(-T, T)$. This restricted solution defines geometrically a particle like structure moving in x^- direction but with fields moving in x^+ direction so that one would have rather exotic kind of particle-wave dualism. In accordance with the quantum-classical correspondence, one could interpret this as classical space-time representation of the particle wave duality and the solution would be a particular example of topological field quantization.

5.6.6 More general solutions representing electric field of constant action density are possible

The solution ansatz just discussed represents a constant electric field in a region of space-time moving with light velocity in the direction of x^- coordinate. Also ordinary constant electric fied is a possible solution and is constructed iteratively in an essentially identical manner by starting from the solution ansatz

$$u = kz \ , \quad \Phi = \omega t \ . \tag{112}$$

Also now Kähler current vanishes in the lowest order and action density is constant so that lowest order field equations are satisfied. Higher order corrections are obtained using the ansatz $u_1 = u_1(z)$, $\Phi = \Phi_1(z)$. Minimal surface condition gives now essentially same kind of expressions for u_1 and Φ_1. Also now the singularities where gravitational interaction becomes strong are at $u = \pm 1$ and one can select the solution to represent a membrane like structure with thickness $L = 2/k$.

Cell membrane space-time sheets are good candidates for the realization of this kind of solutions. If so, one might expect that classical gravitational effects become important at the boundaries of the cell membrane. More generally, bio-systems are electrets and the proposed solution type might provide a fundamental model for bio-electrets. In particular, electrogravitic effects due to the energy of the classical electric field might be of importance.

This observation relates interestingly to the sol-gel phase transitions occurring inside cell. In these transitions large scale bound states of water molecules are formed and could make possible macrotemporally quantum coherent systems able to perform quantum computations in time scales of order say .1 seconds. These bound states would be characterized by spin glass degeneracy broken only by the classical gravitation and spin glass degeneracy would make these bound states longlived. In the case of the proposed solution ansätze spin glass degeneracy corresponds to the canonical symmetries of CP_2 generating new solutions representing constant electric field.

Also M^4 diffeomorphisms are symmetries of the field equations broken only by the classical gravitation. Approximate diffeomorphism invariance means that one obtains solutions for which the lines of electric flux are curved and only the action density stays constant. In the case of magnetic flux tubes this symmetry makes possible curved magnetic flux tubes. Both electric fields and the magnetic flux tubes are fundamental for the TGD based model of living matter and relate deeply to the electric-magnetic duality symmetry and to the quantum criticality predicting that magnetic and electric space-time regions having opposite signs of Kähler action play a role similar to the ice and water regions at critical point of water, are important physically.

6 Can one determine experimentally the shape of the space-time surface?

If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the imbedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

6.1 Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

a) All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly overdetermined system is in question.

b) The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.

c) The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the imbedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

6.2 Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency. Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

a) The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

Do the p-adic length scales correspond to the length units of the induced metric or of M^4_+ metric? If the topological condensation a meter stick space-time sheet at a larger space-time sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M^4_+ metric.

b) If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

c) At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of the configuration space geometry uses as configuration space coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

a) Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of the configuration space and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

b) Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

c) Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

d) The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

References

Online books about TGD

[1] M. Pitkänen (2006), *Topological Geometrodynamics: Overview.* http://www.helsinki.fi/∼matpitka/tgdview/tgdview.html.

[2] M. Pitkänen (2006), *Quantum Physics as Infinite-Dimensional Geometry*.
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen (2006), *Physics in Many-Sheeted Space-Time*.
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen (2006), *Quantum TGD*.
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen (2006), *TGD as a Generalized Number Theory*.
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen (2006), *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*.
http://www.helsinki.fi/∼matpitka/paddark/paddark.html.

[7] M. Pitkänen (2006), *TGD and Fringe Physics*.
http://www.helsinki.fi/∼matpitka/freenergy/freenergy.html.

Online books about TGD inspired theory of consciousness and quantum biology

[8] M. Pitkänen (2006), *Bio-Systems as Self-Organizing Quantum Systems*.
http://www.helsinki.fi/∼matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen (2006), *Quantum Hardware of Living Matter*.
http://www.helsinki.fi/∼matpitka/bioware/bioware.html.

[10] M. Pitkänen (2006), *TGD Inspired Theory of Consciousness*.
http://www.helsinki.fi/∼matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory*.
http://www.helsinki.fi/∼matpitka/genememe/genememe.html.

[12] M. Pitkänen (2006), *TGD and EEG*.
http://www.helsinki.fi/∼matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen (2006), *Bio-Systems as Conscious Holograms*.
http://www.helsinki.fi/∼matpitka/hologram/hologram.html.

[14] M. Pitkänen (2006), *Magnetospheric Consciousness*.
http://www.helsinki.fi/∼matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen (2006), *Mathematical Aspects of Consciousness Theory.* http://www.helsinki.fi/∼matpitka/magnconsc/mathconsc.html.

References to the chapters of books

[B2] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#compl1.

[B3] The chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [2].
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html#compl2.

[D3] The chapter *The Relationship Between TGD and GRT* of [3].
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html#tgdgrt.

[E2] The chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#visionb.

[E5] The chapter *p-Adic Physics: Physical Ideas* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#phblocks.

[E9] The chapter *Topological Quantum Computation in TGD Universe* of [5].
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html#tqc.

[F6] The chapter *Topological Condensation and Evaporation* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padaelem.

[F8] The chapter *TGD and Nuclear Physics* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#padnucl.

[F9] The chapter *Dark Nuclear Physics and Living Matter* of [6].
http://www.helsinki.fi/∼matpitka/paddark/paddark.html#exonuclear.

[G1] The chapter *Anomalies Related to the Classical Z^0 Force and Gravitation* of [7].
http://www.helsinki.fi/∼matpitka/freenergy/freenergy.html#Zanom.

[G2] The chapter *The Notion of Free Energy and Many-Sheeted Space-Time Concept* of [7].
http://www.helsinki.fi/∼matpitka/freenergy/freenergy.html#freenergy.

[I4] The chapter *Quantum Control and Coordination in Bio-systems: Part I* of [8].
http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html#qcococI.

[I5] The chapter *Quantum Control and Coordination in Bio-Systems: Part II* of [8].
http://www.helsinki.fi/~matpitka/bioselforg/bioselforg.html#qcococII.

[L4] The chapter *Pre-Biotic Evolution in Many-Sheeted Space-Time* of [11].
http://www.helsinki.fi/~matpitka/genememe/genememe.html#prebio.

Mathematics related references

[16] R. Harvey (1990), *Spinors and Calibrations*, Academic Press, New York.

[17] A. Lakthakia (1994), *Beltrami Fields in Chiral Media*, Series in Contemporary Chemical Physics - Vol. 2, World Scientific, Singapore.

[18] D. Reed (1995), in *Advanced Electromagnetism: Theories, Foundations, Applications*, edited by T. Barrett (Chap. 7), World Scientific, Singapore.

[19] O. I Bogoyavlenskij (2003), *Exact unsteady solutions to the Navier-Stokes equations and viscous MHD equations*. Phys. Lett. A, 281-286.

[20] J. Etnyre and R. Ghrist (2001), *An index for closed orbits in Beltrami field*. ArXiv:math.DS/0101095.

[21] J. Etnyre and G. Ghrist (2003)*Generic hydrodynamic instability of curl eigen fields*, arXiv:hat.DS/0306310.

[22] G. E. Marsh (1995), *Helicity and Electromagnetic Field Topology* in *Advanced Electromagnetism*, Eds. T. W. Barrett and D. M. Grimes, Word Scientific.

[23] E. C. Zeeman (ed.)(1977), *Catastrophe Theory*, Addison-Wessley Publishing Company.

[24] J. Mickelson (2002), *Gerbes, (Twisted) K-Theory, and the Supersymmetric WZW Model*, hep-th/0206139.

[25] Gibbons, G., W., Pope, C., N. (1977): *CP_2 as gravitational instanton.* Commun. Math. Phys. 55, 53.

[26] Eguchi, T., Gilkey, B., Hanson, J. (1980): Phys. Rep. 66, 6.