

Introduction

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Contents

1	Background	3
2	Basic Ideas of TGD	4
2.1	TGD as a Poincare invariant theory of gravitation	4
2.2	TGD as a generalization of the hadronic string model	4
2.3	Fusion of the two approaches via a generalization of the space-time concept	5
3	The five threads in the development of quantum TGD	5
3.1	Quantum TGD as configuration space spinor geometry	5
3.2	p-Adic TGD	6
3.3	TGD as a generalization of physics to a theory consciousness	7
3.3.1	Quantum jump as a moment of consciousness	8
3.3.2	The notion of self	9
3.3.3	Relationship to quantum measurement theory	9
3.3.4	Selves self-organize	11
3.3.5	Classical non-determinism of Kähler action	11
3.3.6	p-Adic physics as physics of cognition and intentionality	12
3.4	TGD as a generalized number theory	13
3.5	Dynamical quantized Planck constant and dark matter hierarchy	14
3.5.1	Dark matter as large \hbar phase	14
3.5.2	Dark matter as a source of long ranged weak and color fields	15
3.5.3	p-Adic and dark matter hierarchies and hierarchy of moments of consciousness	16

4	Bird's eye of view about the topics of the book	19
5	The contents of the book	21
5.1	PART I: Number theoretical vision	21
5.1.1	TGD as a Generalized Number Theory I: p-Adicization Program	21
5.1.2	TGD as a Generalized Number Theory II: Quater- nions, Octonions and their Hyper Counterparts	23
5.1.3	TGD as a Generalized Number Theory III: Infinite Primes	30
5.2	PART II: TGD and p-Adic Numbers	36
5.2.1	p-Adic Numbers and Generalization of Number Concept	36
5.2.2	Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory	37
5.3	PART III: Related topics	40
5.3.1	Category theory, quantum TGD and TGD inspired theory of consciousness	40
5.3.2	Riemann hypothesis and physics	41
5.3.3	Topological Quantum Computation in TGD Universe	43
5.3.4	Intentionality, Cognition, and Physics as Number theory	45
5.3.5	Langlands Program and TGD	48

1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [16]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and

eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended to the interested reader.

2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M_+^4 \times CP_2$, where M_+^4 denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [17, 18, 19, 20]. The identification of the space-time as a submanifold [21, 22] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M_+^4 \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H -metric to four-surface define classical electro-weak, color gauge fields and metric in X^4 .

2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free

particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as

the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H . "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [23, 24, 25]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzlers and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics

with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics grew steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum

jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [10, 8, 9, 13, 11, 12, 14, 15].

3.3.1 Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U -matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

3.3.2 The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U . Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij} . Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

3.3.3 Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then

the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m - M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

3.3.4 Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

3.3.5 Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one

can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

3.3.6 p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, \dots$. p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive binary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [E1]. The application of this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an ex-

planation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic binary digits a p -valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ binary digits represent a Boolean logic B^k with k elementary statements (the points of the k -element set in the set theoretic realization) with n taboos which are constrained to be identically true.

3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few years ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactification stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [E2].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-

time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

3.5.1 Dark matter as large \hbar phase

D. Da Rocha and Laurent Nottale [35] have proposed that Schrödinger equation with Planck constant \hbar replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests

astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [D6].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [E9] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of \hbar occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n -branched coverings so that tension becomes n^2 -fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$ -sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [D6].

3.5.2 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry

and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

3.5.3 p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as \hbar). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [M3].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, \dots$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [C6]. Also the value of \hbar_0 has spectrum characterized by Beraha numbers $B_n = 4\cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [C6]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of \hbar at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like \hbar . Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give

rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4 CP_2$ times so that $2^{127} - 1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k = 127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p . By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p = 2$ -adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by \hbar/\hbar_0 . One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [L2].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k = 4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

4 Bird's eye of view about the topics of the book

The focus of this chapter is the number theoretical vision about physics. This vision involves three loosely related parts.

1. Combination of real physic and various p-adic physics to a single larger whole by generalizing the number concept by fusing real numbers and various p-adic number fields along common rationals. Extensions of p-adic number fields can be introduce by gluing them along common algebraic numbers to reals. Algebraic continuation of the physics from rationals and their their extensions to various number fields is the key idea and the challenge is to understand whether how one could achieve this dream. A very profound implication is that purely local p-adic physics codes for the p-adic fractality of long length length scale real physics and vice versa. As a consequence one can understand the origins of p-adic length scale hypothesis and ends up with a very concrete view about space-time correlates of cognition and intentionality.
2. Second part of the vision involves what I call hyper counterparts of the classical number fields defined as subspaces of their complexifications with Minkowskian signature of the metric. The hypothesis is

that allowed space-time surfaces correspond to what might be called hyper-quaternionic sub-manifolds of a hyper-octonionic space. Second hypothesis is that space-time surfaces can be also regarded as what I call Kähler calibrations (Kähler action density reduces to a closed 4-form at the allowed extremals). Hence the basic variational principle of TGD would have deep number theoretic content. Reduction to a closed form would also mean that classical TGD would define a generalized topological field theory with Noether charges defining topological invariants.

3. The third part of the vision involves infinite primes, which can be identified in terms of an infinite hierarchy of second quantized arithmetic quantum fields theories on one hand, and as having representations as space-time surfaces analogous to zero surfaces of polynomials on the other hand. In this framework space-time surface would represent an infinite number. This vision leads also the conclusion that single point of space-time has an infinitely complex structure since real unity can be represented as a ratio of infinite numbers in infinitely many manners each having its own number theoretic anatomy. Thus single space-time point is in principle able to represent in its structure the quantum state of the entire universe. This number theoretic variant of Brahman=Atman identity also means that Universe is an algebraic hologram.

Besides this holy trinity I will discuss also loosely related topics. The possible applications of the category theory in TGD; various TGD inspired considerations related to Riemann hypothesis, in particular a strategy for proving it; topological field computation in TGD Universe; and intentionality and cognition in TGD Universe.

The seven online books about TGD [1, 2, 4, 5, 3, 6, 7] and eight online books about TGD inspired theory of consciousness and quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended for the reader willing to get overall view about what is involved.

5 The contents of the book

5.1 PART I: Number theoretical vision

5.1.1 TGD as a Generalized Number Theory I: p-Adicization Program

The vision about a number theoretic formulation of quantum TGD is based on the gradual accumulation of wisdom coming from different sources. The attempts to find a formulation allowing to understand real and p-adic physics as aspects of some more general scenario have been an important stimulus and generated a lot of, not necessarily mutually consistent ideas, some of which might serve as building blocks of the final formulation.

The first part of the 3-part chapter is devoted to the p-adicization program attempting to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals. Highly non-trivial number theoretic conjectures are an outcome of the program.

1. Real and p-adic regions of the space-time as geometric correlates of matter and mind

The solutions of the equations determining space-time surfaces are restricted by the requirement that the imbedding space coordinates are real. When this is not the case, one might apply instead of a real completion with some rational-adic or p-adic completion: this is how rational-adic p-adic physics could emerge from the basic equations of the theory. One could interpret the resulting rational-adic or p-adic regions as geometrical correlates for 'mind stuff'.

p-Adic non-determinism implies extreme flexibility and therefore makes the identification of the p-adic regions as seats of cognitive representations very natural. Unlike real completion, p-adic completions preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with 'mind like' regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of 'self' and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves!

2. The generalization of the notion of number

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

3. p-Adicization by algebraic continuation

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

For instance, residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of

matter and p-adic physics would describe correlates of cognition and intentionality. The basic stumbling block of this program is integration and algebraic continuation should allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. Two types of cutoffs are predicted: p-adic length scale cutoff and a cutoff due to phase resolution. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and Beraha numbers $B_n = 4\cos^2(\pi/n)$, $n \geq 3$ related closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra. This cutoff hierarchy seems to relate closely to the hierarchy of cutoffs defined by the hierarchy of subalgebras of the super-canonical algebra defined by the hierarchy of sets (z_1, \dots, z_n) , where z_i are the first n non-trivial zeros of Riemann Zeta. Hence there are good hopes that the p-adicization program might unify apparently unrelated branches of mathematics.

4. Number theoretic democracy

The interpretation allows all finite-dimensional extensions of p-adic number fields and even infinite-P p-adics. The notion arithmetic quantum theory generalizes to include Gaussian and Eisenstein variants of infinite primes and corresponding arithmetic quantum field theories. Also the notion of p-adicity generalizes: it seems that one can indeed assign to Gaussian and Eisenstein primes what might be called G-adic and E-adic numbers. These number fields seem to be tailor made for modelling logarithmic spirals which represent the basic fractal like structures in a living matter and excitable media.

p-Adicization by algebraic continuation gives hopes of continuing quantum TGD from reals to various p-adic number fields. The existence of this continuation poses extremely strong constraints on theory and has already now inspired several number theoretic conjectures.

5.1.2 TGD as a Generalized Number Theory II: Quaternions, Octonions and their Hyper Counterparts

This chapter is the second part of the multi-chapter devoted to the vision about TGD as a generalized number theory.

1. *Hyper-quaternions and octonions*

The original idea was that space-time surfaces could be regarded as four-surfaces in 8-D imbedding space with the property that the tangent spaces of these spaces can be locally regarded as 4- *resp.* 8-dimensional number fields of quaternions and octonions.

The difficulties caused by the Euclidian metric signature of the number theoretical norm have however forced to give up the original idea as such, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart of the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity.

The problem is that $H = M^4 \times CP_2$ cannot be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces are assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO . Since the hyper-quaternionic sub-spaces of HO with fixed complex structure are labelled by CP_2 , each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

2. *Space-time-surface as a HQ or CHQ sub-manifold of hyper-octonionic imbedding space?*

Space-time identified as a hyper-quaternionic (HQ) or co-hyper-quaternionic (coHQ) sub-manifold of the hyper-octonionic space in the sense that the tangent space or normal space of the space-time surface defines a hyper-quaternionic sub-algebra of the hyper-octonionic tangent space of H at each space-time point, looks an attractive idea. Second possibility is that the tangent space-algebra of the space-time surface is either associative or co-associative at each point.

One can also consider possibility that the dynamics of the space-time surface is determined from the requirement that space-time surface is algebraically closed in the sense that tangent space at each point has this property. Also the possibility that the property in question is associated with the normal space at each point of X^4 can be considered. Some delicacies are caused by the question whether the induced algebra at X^4 is just

the hyper-octonionic product or whether the algebra product is projected to the space-time surface. If normal part of the product is projected out the space-time algebra closes automatically.

The first guess would be that space-time surfaces are hyper-quaternionic (HQ) or co-hyper-quaternionic (coHQ) sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that complex structure is fixed and same at all points of space-time surface. This corresponds to a global selection of a preferred octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2 . This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

For a theoretical physicists of my generation it is easy to guess that the next step is to realize that it is possible to fix the preferred octonionic imaginary at each point of HO separately so that local $S^6 = G_2/SU(3)$, or equivalently the local group G_2 subject to $SU(3)$ gauge invariance, characterizes the possible choices of hyper-quaternionic structure with a preferred imaginary unit. $G_2 \subset SO(7)$ is the automorphism group of octonions, and appears also in M-theory. This local choice has interpretation as a fixing of the plane of non-physical polarizations and rise to degeneracy which is a good candidate for the ground state degeneracy caused by the vacuum extremals.

$OH - M^4 \times CP_2$ duality allows to construct a foliation of HO by hyper-quaternionic space-time surfaces in terms of maps $HO \rightarrow SU(3)$ satisfying certain integrability conditions guaranteeing that the distribution of hyper-quaternionic planes integrates to a foliation by 4-surfaces. In fact, the freedom to fix the preferred imaginary unit locally extends the maps to $HO \rightarrow G_2$ reducing to maps $HO \rightarrow SU(3) \times S^6$ in the local trivialization of G_2 . This foliation defines a four-parameter family of 4-surfaces in $M^4 \times CP_2$ for each local choice of the preferred imaginary unit. The dual of this foliation defines a 4-parameter family co-hyper-quaternionic space-time surfaces and it turns out that also these surfaces are needed.

Hyper-octonion analytic functions $HO \rightarrow HO$ with real Taylor coefficients provide a physically motivated ansatz satisfying the integrability conditions. The basic reason is that hyper-octonion analyticity is not plagued by the complications due to non-commutativity and non-associativity. Indeed, this notion results also if the product is Abelianized by assuming that

different octonionic imaginary units multiply to zero. A good candidate for the HO dynamics is free massless Dirac action with Weyl condition for an octonion valued spinor field using octonionic representation of gamma matrices and coupled to the G_2 gauge potential defined by the tensor 7×7 tensor product of the imaginary parts of spinor fields.

The basic conjecture is that HQ and $coHQ$ surfaces correspond to preferred extremals of Kähler action. This conjecture has several variants. It could be that only asymptotic behavior corresponds to HQ analytic function but that HQ and $coHQ$ is a generic property. It could also be that maxima of Kähler function correspond to this kind of 4-surfaces. The encouraging hint is the fact that Hamilton-Jacobi coordinates appear naturally also in the construction of general solutions of field equations.

3. The notion of Kähler calibration

Calibration is a closed p-form, whose value for a given p-plane is not larger than its volume in the induced metric. What is important that if it is maximum for tangent planes of p-sub-manifold, minimal surface with smallest volume in its homology equivalence class results.

The idea of Kähler calibration is based on a simple observation. A hyper-octonionic spinor field defines a map $M^8 \rightarrow H = M^4 \times CP_2$ allowing to induce metric and Kähler form of H to M^8 . Also Kähler action is well defined for the local hyper-quaternion plane.

The idea is that the non-closed 4-form associated the wedge product of unit tangent vectors of HQ plane in M^8 and saturating to volume for it becomes closed by multiplication with Kähler action density L_K . If L_K is minimal for hyper-quaternion plane, hyper-quaternionic manifolds define extremals of Kähler action for which the magnitudes of positive and negative contributions to the action are separately minimized.

In $coHQ$ case dual of the Kähler calibration results. In this case L_K would be most naturally maximal for HQ normal plane. There is also an alternative option but it is not favored by physical considerations.

This variational principle is not equivalent with the absolute minimization of Kähler action. Rather, in HQ case Universe would do its best to save energy, being as near as possible to vacuum. Also vacuum extremals would become physically relevant (they carry non-vanishing density gravitational energy). The non-determinism of the vacuum extremals would have an interpretation in terms of the ability of Universe to engineer itself. The attractiveness of the number theoretical variational principle from the point of calculability of TGD would be that the initial values for the time derivatives of the imbedding space coordinates at X^3 at light-like 7-D causal

determinant could be computed by requiring that the energy of the solution is minimized. This could mean a computerizable construction of Kähler function.

In $coHQ$ phase Universe would obviously maximize fluctuations and contrasts in accordance with quantum criticality. One might say that these two phases give Universe kind of hawk-dove polarity.

One can assign to a given 3-surface both HQ and cHQ 4-surface in the generic case and the equivalence of descriptions requires that corresponding Kähler functions differ by the real part of a holomorphic function of CH coordinates.

4. Generalizing the notion of $HO - H$ duality to quantum level

The obvious question is how the $HO - H$ duality could generalize to quantum level. Number theoretical considerations combined with the general vision about generalized Feynman diagrams as a generalization of braid diagrams lead to general formulas for vertices in HO picture.

Simple arguments lead to the conclusion that strict duality can make sense only if the hyper-octonionic spinor field is second quantized in some sense. One can imagine two, not necessarily mutually exclusive, manners to quantize.

1. The construction of the spinor structure for the configuration space of 3-surfaces in HO forces to conclude that HO spinor fields induced to $X^4 \subset HO$ are second quantized as usual and define configuration space gamma matrices as super generators. The classical real-analytic HO spinor fields would represent analogs of zero modes of H spinor fields. The second quantized part of hyper-octonionic spinor fields induced to $X^4 \subset HO$ would have $1 + 1 + 3 + \bar{3}$ decomposition having interpretation in terms of quarks and leptons and second quantized oscillator operators would commute with hyper-octonionic units. The detailed realization of $HO - H$ duality suggests that the induced spinor fields at $X^4 \subset H$ resp. $X^4 \subset HO$ are restrictions of H resp. HO spinor fields. This would hold for zero modes and could hold for second quantized part too.
2. The original idea was that the real Laurent coefficients correspond to a complete set of mutually commuting Hermitian operators having interpretation as observables. This is not enough for configuration space geometry but is favored by quantum classical correspondence. Space-time concept would be well defined only for the eigen states of

these operators and physical states are mapped to space-time surfaces. The Hermitian operators would naturally correspond to the state space spanned by super Kac-Moody and super-canonical algebras, and quantum states would have precise space-time counterparts in accordance with quantum-classical correspondence.

The regions inside which the power series representing HO analytic function and matrix elements of G_2 rotation converge are identified as counterparts of maximal deterministic regions of the space-time surface. The Hermitian operators replacing Laurent coefficients are assumed to commute inside these regions identifiable also as coherence regions for the generalized Schrödinger amplitude represented by the HO spinor field.

By quantum classical correspondence these regions would be correlated for the final states of quantum jumps. The space-like 3-D causal determinants X^3 bounding adjacent regions of this kind represent quantum jumps. The hyper-octonionic part of the inner of the hyper-octonionic spinor fields at the two sides of the discontinuity defined as an integral over X^3 would give a number identifiable as complex number when imaginary unit is identified appropriately. The inner product would be identified as a representation of S-matrix element for an internal transition of particle represented by 3-surface, that is 2-vertex.

For the generalized Feynman diagrams n -vertex corresponds to a fusion of n 4-surfaces along their ends at X^3 . 3-vertex can be represented number theoretically as a triality of three hyper-octonion spinors integrated over the 3-surface in question. Higher vertices can be defined as composite functions of triality with a map $(h_1, h_2) \rightarrow \bar{h}_3$ defined by octonionic triality and by duality given by the inner product. More concretely, $m + n$ vertex corresponds in HO picture to the inner product for the local hyper-octonionic products of m outgoing and n incoming hyper-octonionic spinor fields integrated over the 3-surface defining the vertex. Both 2-vertices representing internal transitions and $n > 2$ vertices are completely fixed. This should give some idea about the power of the number theoretical vision.

One can raise objections against the need for non-conventional quantization. The number theoretic prescription does not apply to the second quantized parts of HO spinor fields and S-matrix elements can be constructed using them so that two equivalent prescriptions of S-matrix would emerge. On the other hand, TGD inspired quantum measurement theory suggests dual codings S-matrix elements based on either quantum states or classical observables (zero modes) in 1-1 correspondence with them.

5. *Does TGD reduce to 8-D WZW string model in HO picture?*

Conservation laws suggests that in the case of non-vacuum extremals the dynamics of the local G_2 automorphism is dictated by field equations of some kind. The experience with WZW model suggests that in the case of non-vacuum extremals G_2 element could be written as a product $g = g_L(h)g_R^{-1}(\bar{h})$ of hyper-octonion analytic and anti-analytic complexified G_2 elements. g would be determined by the data at hyper-complex 2-surface for which the tangent space at a given point is spanned by real unit and preferred hyper-octonionic unit. Also Dirac action would be naturally restricted to this surface. The theory would reduce in HO picture to 8-D WZW string model both classically and quantally since vertices would reduce to integrals over 1-D curves. The interpretation of generalized Feynman diagrams in terms of generalized braid/ribbon diagrams and the unique properties of G_2 provide further support for this picture. In particular, G_2 is the lowest-dimensional Lie group allowing to realize full-powered topological quantum computation based on generalized braid diagrams and using the lowest level $k=1$ Kac Moody representation. Even if this reduction would occur only in special cases, such as asymptotic solutions for which Lorentz Kähler force vanishes or maxima of Kähler function, it would mean enormous simplification of the theory.

6. Why hyper-quaternionicity corresponds to the minimization of Kähler action?

The resulting over all picture leads also to a considerable understanding concerning the basic questions why (co)-hyper-quaternionic 4-surfaces define extrema of Kähler action and why WZW strings would provide a dual for the description using Kähler action. The answer boils down to the realization that the extrema of Kähler action minimize complexity, also algebraic complexity, in particular non-commutativity. A measure for non-commutativity with a fixed preferred hyper-octonionic imaginary unit is provided by the commutator of $\mathfrak{3}$ and $\bar{\mathfrak{3}}$ parts of the hyper-octonion spinor field defining an antisymmetric tensor in color octet representation: very much like color gauge field.

Color action is a natural measure for the non-commutativity minimized when the tangent space algebra closes to complexified quaternionic, instead of complexified octonionic, algebra. On the other hand, Kähler action is nothing but color action for classical color gauge field defined by projections of color Killing vector fields. That WZW + Dirac action for hyper-octonionic strings would correspond to Kähler action would in turn be the TGD counterpart for the proposed string-YM dualities.

7. Various dualities and their physical counterparts

$HO - H$ duality is only one representative in a family of dualities characterizing TGD. It is not equivalent with $HQ - coHQ$ duality, which seems however to be equivalent with the electric-magnetic duality known for long. This duality relates descriptions based on space-like partonic 2-surfaces and time-like string orbits. $HO - H$ and $HQ - coHQ$ dualities seem to be closely correlated in the sense that HO picture is natural in HQ phase and H picture in $coHQ$ phase.

At configuration space level $HO - H$ duality means roughly following. In H picture spin and ew spin are spin-like quantum numbers whereas color is orbital quantum number and cannot be seen at space-time level directly. In HO picture the roles of these quantum numbers are changed. One could say that $HO - H$ duality acts as a super-symmetry permuting spin and orbital degrees of freedom of configuration space spinor fields. This duality allows a surprisingly detailed understanding of almost paradoxical dualities of hadron physics, and also explains proton spin crisis from first principles.

It seems possible to interpret $HO - H$ and $HQ - coHQ$ dualities as analogs of wave-particle duality in the infinite-dimensional context. For $HO - H$ duality the cotangent bundle of configuration space CH would be the unifying notion. Position q in CH would be represented by 3-surface whereas canonical momentum p would correspond to the same 3-surface but as a surface in CHO with induced metric and Kähler structure inherited from HO defining the tangent space of H . The notion of stringy configuration space might allow to understand also M-theory dualities in this manner.

5.1.3 TGD as a Generalized Number Theory III: Infinite Primes

Infinite primes are besides p-adicization and the representation of space-time surface as a hyper-quaternionic sub-manifold of hyper-octonionic space, basic pillars of the vision about TGD as a generalized number theory and will be discussed in the third part of the multi-chapter devoted to the attempt to articulate this vision as clearly as possible.

1. Why infinite primes are unavoidable

Suppose that 3-surfaces could be characterized by p-adic primes characterizing their effective p-adic topology. p-Adic unitarity implies that each quantum jump involves unitarity evolution U followed by a quantum jump. Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense. This leads to a peculiar paradox: if the number of quantum jumps already occurred

is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct what might be called generating infinite primes by repeating a procedure analogous to a quantization of a super symmetric quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions with the corresponding decomposition of the infinite prime to primes at a lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

2. Two views about the role of infinite primes and physics in TGD Universe

Two different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. The first view is based on the idea that infinite primes characterize quantum states of the entire Universe. 8-D hyper-octonions make this correspondence very concrete since 8-D hyper-octonions have interpretation as 8-momenta. By quantum-classical correspondence also the decomposition of space-time surfaces to p-adic space-time sheets should be coded by infinite hyper-octonionic primes. Infinite primes could even have a representation as hyper-quaternionic 4-surfaces of 8-D hyper-octonionic imbedding space.
2. The second view is based on the idea that infinitely structured space-time points define space-time correlates of mathematical cognition. The mathematical analog of Brahman=Atman identity would however suggest that both views deserve to be taken seriously.

3. Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes

so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means an enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes. This hierarchy can be identified with the corresponding hierarchy of space-time sheets of the many-sheeted space-time.

One can even try to understand the quantum numbers of physical particles in terms of infinite primes. In particular, the hyper-quaternionic primes correspond four-momenta and mass squared is prime valued for them. The properties of 8-D hyper-octonionic primes motivate the attempt to identify the quantum numbers associated with CP_2 degrees of freedom in terms of these primes. The representations of color group $SU(3)$ are indeed labelled by two integers and the states inside given representation by color hyper-charge and iso-spin.

It turns out that associativity constraint allows only rational infinite primes. One can however decompose rational infinite primes to hyper-octonionic infinite primes at lower level of the hierarchy. Physically this would mean that the number theoretic 8-momenta have only time-component. This decomposition is completely analogous to the decomposition of hadrons to its colored constituents and might be even interpreted in terms of color confinement. The interpretation of the decomposition of rational primes to primes in the algebraic extensions of rationals, hyper-quaternions, and hyper-octonions would have an interpretation as an increase of number theoretic resolution and the principle of number theoretic confinement could be seen as a fundamental physical principle implied by associativity condition.

4. Infinite primes as a bridge between quantum and classical

An important stimulus came from the observation stimulated by algebraic number theory. Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes.

This in turn led to the idea that it might be possible represent infinite primes (integers) geometrically as surfaces defined by the polynomials associated with infinite primes (integers).

Obviously, infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite num-

bers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

5. Conjecture about various equivalent characterizations of space-times as surfaces

One can imagine several number-theoretic characterizations of the space-time surface.

1. The approach based on octonions and quaternions suggests that space-time surfaces correspond to associative, or equivalently, hyper-quaternionic surfaces of hyper-octonionic imbedding space HO . Also co-associative, or equivalently, co-hyper-quaternionic surfaces are possible. These foliations can be mapped in a natural manner to the foliations of $H = M^4 \times CP_2$ by space-time surfaces which are identified as preferred extremals of the Kähler action (absolute minima or maxima for regions of space-time surface in which action density has definite sign). These views are consistent if hyper-quaternionic space-time surfaces correspond to so called Kähler calibrations [E2].
2. Hyper-octonion real-analytic surfaces define foliations of the imbedding space to hyper-quaternionic 4-surfaces and their duals to co-hyper-quaternionic 4-surfaces representing space-time surfaces.
3. Rational infinite primes can be mapped to rational functions of n arguments. For hyper-octonionic arguments non-associativity makes these functions poorly defined unless one assumes that arguments are related by hyper-octonion real-analytic maps so that only single independent variable remains. These hyper-octonion real-analytic functions define foliations of HO to space-time surfaces if b) holds true.

The challenge of optimist is to prove that these characterizations are equivalent.

6. The representation of infinite hyper-octonionic primes as 4-surfaces

The difficulties caused by the Euclidian metric signature of the number theoretical norm forced to give up the idea that space-time surfaces could be regarded as quaternionic sub-manifolds of octonionic space, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the

algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart for the transition from Riemannian to pseudo-Riemannian geometry performed already in Special Relativity.

The commutative $\sqrt{-1}$ relates naturally to the algebraic extension of rationals generalized to an algebraic extension of rational quaternions and octonions and conforms with the vision about how quantum TGD could emerge from infinite dimensional Clifford algebra identifiable as a hyper-finite factor of type II_1 [C6, A9].

The notions of hyper-quaternionic and octonionic manifold make sense but it is implausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces can be assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO . Since the hyper-quaternionic sub-spaces of HO with a locally fixed complex structure (preferred imaginary unit contained by tangent space at each point of HO) are labelled by CP_2 , each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

Any hyper-octonion analytic function $HO \rightarrow HO$ defines a function $g : OH \rightarrow SU(3)$ acting as the group of octonion automorphisms leaving a preferred imaginary unit invariant, and g in turn defines a foliation of HO and $H = M^4 \times CP_2$ by space-time surfaces. The selection can be local which means that G_2 appears as a local gauge group.

Since the notion of prime makes sense for the complexified octonions, it makes sense also for the hyper-octonions. It is possible to assign to infinite prime of this kind a hyper-octonion analytic polynomial $P : HO \rightarrow HO$ and hence also a foliation of HO and $H = M^4 \times CP_2$ by 4-surfaces. Therefore space-time surface can be seen as a geometric counterpart of a Fock state. The assignment is not unique but determined only up to an element of the local octonionic automorphism group G_2 acting in HO and fixing the local choices of the preferred imaginary unit of the hyper-octonionic tangent plane. In fact, a map $HO \rightarrow S^6$ characterizes the choice since $SO(6)$ acts effectively as a local gauge group.

The construction generalizes to all levels of the hierarchy of infinite primes if one poses the associativity requirement implying that hyper-octonionic variables are related by hyper-octonion real-analytic maps, and produces also representations for integers and rationals associated with hyper-octonionic numbers as space-time surfaces. A close relationship with algebraic geometry results and the polynomials define a natural hierarchical structure in

the space of 3-surfaces. By the effective 2-dimensionality naturally associated with infinite primes represented by real polynomials 4-surfaces are determined by data given at partonic 2-surfaces defined by the intersections of 3-D and 7-D light-like causal determinants. In particular, the notions of genus and degree serve as classifiers of the algebraic geometry of the 4-surfaces. The great dream is to prove that this construction yields the solutions to the absolute minimization of Kähler action.

7. Generalization of ordinary number fields: infinite primes and cognition

Both fermions and p-adic space-time sheets are identified as correlates of cognition in TGD Universe. The attempt to relate these two identifications leads to a rather concrete model for how bosonic generators of super-algebras correspond to either real or p-adic space-time sheets (actions and intentions) and fermionic generators to pairs of real space-time sheets and their p-adic variants obtained by algebraic continuation (note the analogy with fermion hole pairs).

The introduction of infinite primes, integers, and rationals leads also to a generalization of real numbers since an infinite algebra of real units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems to which the reduction to ordinary rational is simplest cure which would however allow interpretation as decomposition of infinite prime to hyper-octonionic lower level constituents. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real units becomes infinitely degenerate.

Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point is even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units.

One might argue that for the real physics this structure is completely invisible and is relevant only for the physics of cognition. On the other

hand, one can consider the possibility of mapping the configuration space and configuration space spinor fields to the number theoretical anatomies of a single point of imbedding space so that the structure of this point would code for the world of classical worlds and for the quantum states of the Universe. Quantum jumps would induce changes of configuration space spinor fields interpreted as wave functions in the set of number theoretical anatomies of single point of imbedding space in the ordinary sense of the word, and evolution would reduce to the evolution of the structure of a typical space-time point in the system. Physics would reduce to space-time level but in a generalized sense. Universe would be an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

5.2 PART II: TGD and p-Adic Numbers

5.2.1 p-Adic Numbers and Generalization of Number Concept

In this chapter the general TGD inspired mathematical ideas related to p-adic numbers are discussed. The extensions of the p-adic numbers including extensions containing transcendentals, the correspondences between p-adic and real numbers, p-adic differential and integral calculus, and p-adic symmetries and Fourier analysis belong the topics of the chapter.

The basic hypothesis is that p-adic space-time regions correspond to cognitive representations for the real physics appearing already at the elementary particle level. The interpretation of the p-adic physics as a physics of cognition is justified by the inherent p-adic non-determinism of the p-adic differential equations making possible the extreme flexibility of imagination.

p-Adic canonical identification and the identification of reals and p-adics by common rationals are the two basic identification maps between p-adics and reals and can be interpreted as two basic types of cognitive maps. The concept of p-adic fractality is defined and p-adic fractality is the basic property of the cognitive maps mapping real world to the p-adic internal world. Canonical identification is not general coordinate invariant and at the fundamental level it is applied only to map p-adic probabilities and predictions of p-adic thermodynamics to real numbers. The correspondence via common rationals is general coordinate invariant correspondence when general coordinate transformations are restricted to rational or extended rational maps: this has interpretation in terms of fundamental length scale unit provided by CP_2 length.

A natural outcome is the generalization of the notion of number. Differ-

ent number fields form a book like structure with number fields and their extensions representing the pages of the book glued together along common rationals representing the rim of the book. This generalization forces also the generalization of the manifold concept: both imbedding space and configuration space are obtained as union of copies corresponding to various number fields glued together along common points, in particular rational ones. Space-time surfaces decompose naturally to real and p-adic space-time sheets. In this framework the fusion of real and various p-adic physics reduces more or less to to an algebraic continuation of rational number based physics to various number fields and their extensions.

p-Adic differential calculus obeys the same rules as real one and an interesting outcome are p-adic fractals involving canonical identification. Perhaps the most crucial ingredient concerning the practical formulation of the p-adic physics is the concept of the p-adic valued definite integral. Quite generally, all general coordinate invariant definitions are based on algebraic continuation by common rationals. Integral functions can be defined using just the rules of ordinary calculus and the ordering of the integration limits is provided by the correspondence via common rationals. Residy calculus generalizes to p-adic context and also free Gaussian functional integral generalizes to p-adic context and is expected to play key role in quantum TGD at configuration space level.

The special features of p-adic Lie-groups are briefly discussed: the most important of them being an infinite fractal hierarchy of nested groups. Various versions of the p-adic Fourier analysis are proposed: ordinary Fourier analysis generalizes naturally only if finite-dimensional extensions of p-adic numbers are allowed and this has interpretation in terms of p-adic length scale cutoff. Also p-adic Fourier analysis provides a possible definition of the definite integral in the p-adic context by using algebraic continuation.

5.2.2 Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory

The mathematical aspects of p-adicization of quantum TGD are discussed. In a well-defined sense Nature itself performs the p-adicization and p-adic physics can be regarded as physics of cognitive regions of space-time which in turn provide representations of real space-time regions. Cognitive representations presumably involve the p-adicization of the geometry at the level of the space-time and imbedding space by a mapping of a real space time region to a p-adic one. One can differentiate between two kinds of maps: the identification induced by the common rationals of real and p-adic space

time region and the representations of the external real world to internal p-adic world induced by a canonical identification type maps.

Only the identification by common rationals respects general coordinate invariance, and it leads to a generalization of the number concept. Different number fields form a book like structure with number fields and their extensions representing the pages of the book glued together along common rationals representing the rim of the book. This generalization forces also the generalization of the manifold concept: both imbedding space and configuration space are obtained as union of copies corresponding to various number fields glued together along common points, in particular rational ones. Space-time surfaces decompose naturally to real and p-adic space-time sheets. In this framework the fusion of real and various p-adic physics reduces more or less to to an algebraic continuation of rational number based physics to various number fields and their extensions.

The program makes sense only if also extensions containing transcendentals are allowed: the p-dimensional extension containing powers of e is perhaps the most important transcendental extension involved. Entire cognitive hierarchy of extension emerges and the dimension of extension can be regarded as a measure for the cognitive resolution and the higher the dimension the shorter the length scale of resolution. Cognitive resolution provides also number theoretical counterpart for the notion of length scale cutoff unavoidable in quantum field theories: now the length scale cutoffs are part of the physics of cognition rather than reflecting the practical limitations of theory building.

There is a lot of p-adicizing to do.

1. The p-adic variant of classical TGD must be constructed. Field equations make indeed sense also in the p-adic context. The strongest assumption is that real space time sheets have the same functional form as real space-time sheet so that there is non-uniqueness only due to the hierarchy of dimensions of extensions.
2. Probability theory must be generalized. Canonical identification playing central role in p-adic mass calculations using p-adic thermodynamics maps genuinely p-adic probabilities to their real counterparts. p-Adic entropy can be defined and one can distinguish between three kinds of entropies: real entropy, p-adic entropy mapped to its real counterpart by canonical identification, and number theoretical entropies applying when probabilities are in finite-dimensional extension of rationals. Number theoretic entropies can be negative and provide

genuine information measures, and it turns that bound states should correspond in TGD framework to entanglement coefficients which belong to a finite-dimensional extension of rationals and have negative number theoretic entanglement entropy. These information measures generalize by quantum-classical correspondence to space-time level.

3. p-Adic quantum mechanics must be constructed. p-Adic unitarity differs in some respects from its real counterpart: in particular, p-adic cohomology allows unitary S-matrices $S = 1 + T$ such that T is hermitian and nilpotent matrix. p-Adic quantum measurement theory based on Negentropy Maximization Principle (NMP) leads to the notion of monitoring, which might have relevance for the physics of cognition.
4. Generalized quantum mechanics results as fusion of quantum mechanics in various number fields using algebraic continuation from the field of rational as a basic guiding principle. It seems possible to generalize the notion of unitary process in such a manner that unitary matrix leads from rational Hilbert space H_Q to a formal superposition of states in all Hilbert spaces H_F , where F runs over number fields. If this is accepted, state function reduction is a pure number theoretic necessity and involves a reduction to a particular number field followed by state function reduction and state preparation leading ultimately to a state containing only entanglement which is rational or finitely-extended rational and because of its negative number theoretic entanglement entropy identifiable as bound state entanglement stable against NMP.
5. Generalization of the configuration space and related concepts is also necessary and again gluing along common rationals and algebraic continuation is the basic guide line also now. Configuration space is a union of symmetric spaces and this allows an algebraic construction of the configuration space Kähler metric and spinor structure, whose definition reduces to the super canonical algebra defined by the function basis at the light cone boundary. Hence the algebraic continuation is relatively straightforward. Even configuration space functional integral could allow algebraic continuation. The reason is that symmetric space structure together with Duistermaat Hecke theorem suggests strongly that configuration space integration with the constraints posed by infinite-dimensional symmetries on physical states is effectively equivalent to Gaussian functional integration in free field theory around the unique maximum of Kähler function using contravariant

configuration space metric as a propagator. Algebraic continuation is possible for a subset of rational valued zero modes if Kähler action and Kähler function are rational functions of configuration space coordinates for rational values of zero modes.

5.3 PART III: Related topics

5.3.1 Category theory, quantum TGD and TGD inspired theory of consciousness

Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. Category theory might provide the desired systematic approach to fuse together the bundles of general ideas related to the construction of quantum TGD proper. Category theory might also have natural applications in the general theory of consciousness and the theory of cognitive representations.

1. The ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences could be expressed elegantly using the language of the category theory. Quantum classical correspondence might allow a mathematical formulation in terms of structure respecting functors mapping the categories associated with the three kinds of existences to each other.
2. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience.
3. Categories possess inherent generalized logic based on set theoretic inclusion which in TGD framework is naturally replaced with topological condensation: the outcome is quantum variants for the notions of sieve, topos, and logic. This suggests the possibility of geometrizing the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through three-valued logic. Also the right-wrong logic of moral rules and beautiful-ugly logic of aesthetics seem to be too naive and might be replaced with a more general quantum logic.

5.3.2 Riemann hypothesis and physics

Riemann hypothesis states that the nontrivial zeros of Riemann Zeta function lie on the axis $x = 1/2$. Since Riemann zeta function allows interpretation as a thermodynamical partition function for a quantum field theoretical system consisting of bosons labelled by primes, it is interesting to look Riemann hypothesis from the perspective of physics. Quantum TGD and also TGD inspired theory of consciousness provide additional view points to the hypothesis and suggests sharpening of Riemann hypothesis, detailed strategies of proof of the sharpened hypothesis, and heuristic arguments for why the hypothesis is true.

The idea that the evolution of cognition involves the increase of the dimensions of finite-dimensional extensions of p-adic numbers associated with p-adic space-time sheets emerges naturally in TGD inspired theory of consciousness. A further input that led to a connection with Riemann Zeta was the work of Hardmuth Mueller [34] suggesting strongly that e and its $p - 1$ powers at least should belong to the extensions of p-adics. The basic objects in Mueller's approach are so called logarithmic waves $exp(iklog(u))$ which should exist for $u = n$ for a suitable choice of the scaling momenta k .

Logarithmic waves appear also as the basic building blocks (the terms $n^s = exp(log(n)(Re[s] + iIm[s]))$) in Riemann Zeta. This inspires naturally the hypothesis that also Riemann Zeta function is universal in the sense that it is defined at its zeros $s = 1/2 + iy$ not only for complex numbers but also for all p-adic number fields provided that an appropriate finite-dimensional extensions involving also transcendentals are allowed. This allows in turn to algebraically continue Zeta to any number field. The zeros of Riemann zeta are determined by number theoretical quantization and are thus universal and should appear in the physics of critical systems. The hypothesis $log(p) = \frac{q_1(p)exp[q_2(p)]}{\pi}$ explains the length scale hierarchies based on powers of e , primes p and Golden Mean.

Mueller's logarithmic waves lead also to an elegant concretization of the Hilbert Polya conjecture and to a sharpened form of Riemann hypothesis: the phases q^{-iy} for the zeros of Riemann Zeta belong to a finite-dimensional extension of R_p for any value of primes q and p and any zero $1/2 + iy$ of ζ . The question whether the imaginary parts of the Riemann Zeta are linearly independent (as assumed in the previous work) or not is of crucial physical significance. Linear independence implies that the spectrum of the super-canonical weights is essentially an infinite-dimensional lattice. Otherwise a more complex structure results. The numerical evidence supporting the translational invariance of the correlations for the spectrum of zeros to-

gether with p-adic considerations leads to the working hypothesis that for any prime p one can express the spectrum of zeros as the product of a subset of Pythagorean prime phases and of a fixed subset U of roots of unity. The spectrum of zeros could be expressed as a union over the translates of the same basic spectrum defined by the roots of unity translated by the phase angles associated with a subset of Pythagorean phases: this is consistent with what the spectral correlations strongly suggest. That decompositions defined by different primes p yield the same spectrum would mean a powerful number theoretical symmetry realizing p-adicities at the level of the spectrum of Zeta.

A second strategy is based on, what I call, Universality Principle. The function, that I refer to as $\hat{\zeta}$, is defined by the product formula for ζ and exists in the infinite-dimensional algebraic extension Q_∞ of rationals containing all roots of primes. $\hat{\zeta}$ is defined for all values of s for which the partition functions $1/(1-p^{-z})$ appearing in the product formula have value in Q_∞ . Universality Principle states that $|\hat{\zeta}|^2$, defined as the product of the p-adic norms of $|\zeta|^2$ by reversing the order of producting in the adelic formula, equals to $|\zeta|^2$ and, being an infinite dimensional vector in Q_∞ , vanishes only if it contains a rational factor which vanishes. This factor is present only provided an infinite number of partition functions appearing in the product formula of $\hat{\zeta}$ have rational valued norm squared: this locates the plausible candidates for the zeros on the lines $Re[s] = n/2$.

Universality Principle implies the following stronger variant about sharpened form of the Riemann hypothesis: the real part of the phase p^{-iy} is rational for an infinite number of primes for zeros of ζ . Universality Principle, even if proven, does not however yield a proof of the Riemann hypothesis. The failure of the Riemann hypothesis becomes however extremely implausible. An important outcome of this approach is the realization that super-conformal invariance is a natural symmetry associated with ζ (not surprisingly, since the symmetry group of complex analysis is in question!).

Super-conformal invariance inspires a strategy for proving the Riemann hypothesis. The vanishing of the Riemann Zeta reduces to an orthogonality condition for the eigenfunctions of a non-Hermitian operator D^+ having the zeros of Riemann Zeta as its eigenvalues. The construction of D^+ is inspired by the conviction that Riemann Zeta is associated with a physical system allowing super-conformal transformations as its symmetries and second quantization in terms of the representations of the super-conformal algebra. The eigenfunctions of D^+ are analogous to coherent states of a harmonic oscillator and in general they are not orthogonal to each other. The states orthogonal to a vacuum state (having a negative norm squared)

correspond to the zeros of Riemann Zeta. The physical states having a positive norm squared correspond to the zeros of Riemann Zeta at the critical line. Riemann hypothesis follows both from the hermiticity and positive definiteness of the metric in the space of states corresponding to the zeros of ζ . Also conformal symmetry in appropriate sense implies Riemann hypothesis and after one year from the discovery of the basic idea it became clear that one can actually construct a rigorous twenty line long analytic proof for the Riemann hypothesis using a standard argument from Lie group theory.

These approaches concretize the vision about TGD based physics as a generalized number theory. Two new realizations of the super-conformal algebra result and the second realization has direct application to the modelling of $1/f$ noise. The zeros of ζ code for the states of an arithmetic quantum field theory coded also by infinite primes: also the hierarchical structure of the many-sheeted space-time is coded. Even some basic quantum numbers of particles of TGD Universe might have number theoretical representation.

5.3.3 Topological Quantum Computation in TGD Universe

Topological quantum computation (TQC) is one of the most promising approaches to quantum computation. The coding of logical qubits to the entanglement of topological quantum numbers promises to solve the de-coherence problem whereas the S-matrices of topological field theories (modular functors) providing unitary representations for braids provide a realization of quantum computer programs with gates represented as simple braiding operations. Because of their effective 2-dimensionality anyon systems are the best candidates for realizing the representations of braid groups.

TGD allows several new insights related to quantum computation. TGD predicts new information measures as number theoretical negative valued entanglement entropies defined for systems having extended rational entanglement and characterizes bound state entanglement as bound state entanglement. Negentropy Maximization Principle and p-adic length scale hierarchy of space-time sheets encourage to believe that Universe itself might do its best to resolve the de-coherence problem. The new view about quantum jump suggests strongly the notion of quantum parallel dissipation so that thermalization in shorter length scales would guarantee coherence in longer length scales. The possibility of negative energies and communications to geometric future in turn might trivialize the problems caused by long computation times: computation could be iterated again and again by turning the computer on in the geometric past and TGD inspired theory

of consciousness predicts that something like this occurs routinely in living matter.

The absolute minimization of Kähler action is the basic variational principle of classical TGD and predicts extremely complex but non-chaotic magnetic flux tube structures, which can get knotted and linked. The dimension of CP_2 projection for these structures is $D = 3$. These structures are the corner stone of TGD inspired theory of living matter and provide the braid structures needed by TQC.

Anyons are the key actors of TQC and TGD leads to detailed model of anyons as systems consisting of track of a periodically moving charged particle realized as a flux tube containing the particle inside it. This track would be a space-time correlate for the outcome of dissipative processes producing the asymptotic self-organization pattern. These tracks in general carry vacuum Kähler charge which is topologized when the CP_2 projection of space-time sheet is $D = 3$. This explains charge fractionization predicted to occur also for other charged particles. When a system approaches chaos periodic orbits become slightly aperiodic and the correlate is flux tube which rotates N times before closing. This gives rise to Z_N valued topological quantum number crucial for TQC using anyons ($N = 4$ holds true in this case). Non-Abelian anyons are needed by TQC, and the existence of long range classical electro-weak fields predicted by TGD is an essential prerequisite of non-Abelianity.

Negative energies and zero energy states are of crucial importance of TQC in TGD. The possibility of phase conjugation for fermions would resolve the puzzle of matter-antimatter asymmetry in an elegant manner. Anti-fermions would be present but have negative energies. Quite generally, it is possible to interpret scattering as a creation of pair of positive and negative energy states, the latter representing the final state. One can characterize precisely the deviations of this Eastern world view with respect to the Western world view assuming an objective reality with a positive definite energy and understand why the Western illusion apparently works. In the case of TQC the initial *resp.* final state of braided anyon system would correspond to positive *resp.* negative energy state.

The light-like boundaries of magnetic flux tubes are ideal for TQC. The point is that 3-dimensional light-like quantum states can be interpreted as representations for the time evolution of a two-dimensional system and thus represented self-reflective states being "about something". The light-likeness (no geometric time flow) is a space-time correlate for the ceasing of subjective time flow during macro-temporal quantum coherence. The S-matrices of TQC can be coded to these light-like states such that each elementary

braid operation corresponds to positive energy anyons near the boundary of the magnetic flux tube A and negative energy anyons with opposite topological charges residing near the boundary of flux tube B and connected by braided threads representing the quantum gate. Light-like boundaries also force Chern-Simons action as the only possible general coordinate invariant action since the vanishing of the metric determinant does not allow any other candidate. Chern-Simons action indeed defines the modular functor for braid coding for a TQC program.

5.3.4 Intentionality, Cognition, and Physics as Number theory

In this chapter a braid of ideas inspired by the work with topological quantum computation and ideas about mathematical cognition is discussed.

1. The first bundle of ideas relates to quantum TGD and emerged when I learned about braid groups and type II_1 factors of von Neumann algebras. The connection with infinite-dimensional Clifford algebra of configuration space led to the idea about the realization of quantum geometry at the level of configuration space without non-commutative coordinates. Classical quantum correspondence led to very concrete view about how join along boundaries bonds defining braids serve as correlates for quantum bound state formation and bound state entanglement. Even a physicist's "proof" of four-color theorem emerges as an outcome.

The non-integer quantization of dimensions for effective II_1 tensor factors implies quantization of Planck constant: the values of h are given $h(n) = [2\log(2)/\log(r)] \times h$, where r is the dimension of the effective II_1 tensor factor. The spectrum of r contains continuum $r \geq 4$ and discrete spectrum $r = B_n = 4\cos^2(\pi/n)$ ($n \geq 3$) below it. B_n is so called Beraha number. The interpretation in terms of discrete bound states and continuum of unbound states is a suggestive, and in fact a sensible, interpretation. For $n = 3$ with $r = 1$ Planck constant becomes infinite and this corresponds to extremely quantal regime. Planck constant would approach zero at the limit $r \rightarrow \infty$. TGD approach mildly suggests that $r \geq 4$ is not possible.

2. Physics as a generalized number theory is the most ambitious dream inspired by TGD approach. The dimensions 4 and 8 for space-time and imbedding space stimulated the development, which led to notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative,

4-surfaces in M^8 regarded as the space of hyper-octonions or as surfaces in $M^4 \times CP_2$. What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit.

3. The idea about p-adic physics as physics of cognition and intentionality inspired the generalization of a number concept so that reals and various extensions of p-adic numbers are glued together along common rationals and form a book like structure with the rim of book being represented by rationals. This in turn inspired the vision that p-adicization of physics should correspond to an algebraic continuation of rational physics to various number fields.
4. p-Adic continuation leads typically to the need of finite-dimensional extensions of p-adic numbers and also transcendental extensions are needed (e^p is ordinary p-adic number and defines finite-dimensional transcendental extension). This inspired the idea that the evolution of mathematical consciousness corresponds to the gradual increase of both p and dimension of extension of p-adics and inspired various number theoretical conjectures relating different transcendentals to each other. The first form of these conjectures turned out to be wrong but in this chapter the conjectures are already much more realistic being minimal conjectures guaranteeing the universality of physics.

The p-adication problem led to quite unexpected developments in the understanding of space-time correlates of mathematical cognition. The challenge is to locate the Platonia of mathematical ideas at space-time level, that is to identify space-time correlates of algebraic structures, manipulations and equations. The arguments evolved in the following manner.

1. The p-adicization of the vacuum functional defined as an exponent of Kähler function requires that the exponent proportional to the inverse of Kähler coupling constant converges for most primes p . The observation that this is highly improbable led the question whether infinite primes might be of help in the problem. The notion of infinite primes, integers and rationals generalized to complex, (hyper-)quaternionic and (hyper-)octonionic case was one of the first deep ideas inspired by TGD inspired theory of consciousness. The basic observation was that infinite primes correspond to quantum states of an arithmetic quantum field theory second quantized repeatedly.

The key observation was that the multiplication of inverse Kähler coupling strength with a power of quantity $Y = X/(1 + X)$, where X is defined as product of all finite primes, and by powers of more general quantities $Y(n/m) = (n/m)X/Q(n/m)$ (n is integer and m square free integer), where $\Pi(n/m)$ is infinite rational solves the algebraic continuation problem. In real sense the numbers Y are units but p-adically their p-adic norm is $1/p$ for all primes except those dividing n and m .

2. The unit-in-real-sense property means that these numbers define an infinite-dimensional extension of rational numbers differing from ordinary rationals in no manner in the real context. The conclusion is that in TGD Universe space-time and imbedding space points are like the monads of Leibniz having infinitely complex structure. Since infinite primes, and their complex, (hyper-)quaternionic, and (hyper-)octonionic counterparts can represent quantum states of entire Universe, Universe is an algebraic hologram in the strongest sense that one can imagine.
3. Since this structure is not visible at the level of real number based physics, the interpretation is as space-time correlate for mathematical cognition. The free algebra generated by products and sums of infinite primes can be seen as the Mother of All Algebraic Structures allowing representation of any smaller structure. In particular, quantum states and quantum entanglement is representable and all kinds of algebraic rules can be represented using entanglement of the algebra elements representing algebra elements.
4. The paths of points in space-time define paths in this algebra and p-adic continuity for all primes implies the conservation of topological energy encountered in arithmetic quantum field theories and implying that the ordinary rational defined by the algebra element is a constant of motion. These paths are correlates for algebraic manipulations forming themselves an algebra with respect to local multiplication (analogous to gauge group multiplication).

A hierarchy of paths (paths of $d = 1$ objects, $d = 2$ and $d = 3$ objects) defining d -dimensional surfaces giving rise to analogs of local gauge algebras result. In TGD Universe the maximal dimension of this kind of structure is $d = 4$. This probably has some deep algebraic meaning. At classical level these structures have interpretation as abstractions of the rules obeyed by algebraic manipulations using

a collection of examples defined by a pile of d -dimensional manipulation sequences. The algebraic manipulations correspond to Feynman diagram like structures with algebra and co-algebra operations having particle fusion and creation as their physical analogs.

5.3.5 Langlands Program and TGD

Number theoretic Langlands program can be seen as an attempt to unify number theory on one hand and theory of representations of reductive Lie groups on the other hand. So called automorphic functions to which various zeta functions are closely related define the common denominator. Geometric Langlands program tries to achieve a similar conceptual unification in the case of function fields. This program has caught the interest of physicists during last years.

TGD can be seen as an attempt to reduce physics to infinite-dimensional Kähler geometry and spinor structure of the "world of classical worlds" (WCW). Since TGD ce be regarded also as a generalized number theory, it is difficult to escape the idea that the interaction of Langlands program with TGD could be fruitful.

More concretely, TGD leads to a generalization of number concept based on the fusion of reals and various p -adic number fields and their extensions implying also generalization of manifold concept, which inspires the notion of number theoretic braid crucial for the formulation of quantum TGD. TGD leads also naturally to the notion of infinite primes and rationals. The identification of Clifford algebra of WCW as a hyper-finite factors of type II_1 in turn inspires further generalization of the notion of imbedding space and the idea that quantum TGD as a whole emerges from number theory. The ensuing generalization of the notion of imbedding space predicts a hierarchy of macroscopic quantum phases characterized by finite subgroups of $SU(2)$ and by quantized Planck constant. All these new elements serve as potential sources of fresh insights.

1. The Galois group for the algebraic closure of rationals as infinite symmetric group?

The naive identification of the Galois groups for the algebraic closure of rationals would be as infinite symmetric group S_∞ consisting of finite permutations of the roots of a polynomial of infinite degree having infinite number of roots. What puts bells ringing is that the corresponding group algebra is nothing but the hyper-finite factor of type II_1 (HFF). One of the many avatars of this algebra is infinite-dimensional Clifford algebra playing

key role in Quantum TGD. The projective representations of this algebra can be interpreted as representations of braid algebra B_∞ meaning a connection with the notion of number theoretical braid.

2. Representations of finite subgroups of S_∞ as outer automorphisms of HFFs

Finite-dimensional representations of $Gal(\overline{Q}/Q)$ are crucial for Langlands program. Apart from one-dimensional representations complex finite-dimensional representations are not possible if S_∞ identification is accepted (there might exist finite-dimensional l-adic representations). This suggests that the finite-dimensional representations correspond to those for finite Galois groups and result through some kind of spontaneous breaking of S_∞ symmetry.

1. Sub-factors determined by finite groups G can be interpreted as representations of Galois groups or, rather infinite diagonal imbeddings of Galois groups to an infinite Cartesian power of S_n acting as outer automorphisms in HFF. These transformations are counterparts of global gauge transformations and determine the measured quantum numbers of gauge multiplets and thus measurement resolution. All the finite approximations of the representations are inner automorphisms but the limit does not belong to S_∞ and is therefore outer. An analogous picture applies in the case of infinite-dimensional Clifford algebra.
2. The physical interpretation is as a spontaneous breaking of S_∞ to a finite Galois group. One decomposes infinite braid to a series of n-braids such that finite Galois group acts in each n-braid in identical manner. Finite value of n corresponds to IR cutoff in physics in the sense that longer wave length quantum fluctuations are cut off. Finite measurement resolution is crucial. Now it applies to braid and corresponds in the language of new quantum measurement theory to a sub-factor $\mathcal{N} \subset \mathcal{M}$ determined by the finite Galois group G implying non-commutative physics with complex rays replaced by \mathcal{N} rays. Braids give a connection to topological quantum field theories, conformal field theories (TGD is almost topological quantum field theory at parton level), knots, etc..
3. TGD based space-time correlate for the action of finite Galois groups on braids and for the cutoff is in terms of the number theoretic braids obtained as the intersection of real partonic 2-surface and its p-adic counterpart. The value of the p-adic prime p associated with the

parton is fixed by the scaling of the eigenvalue spectrum of the modified Dirac operator (note that renormalization group evolution of coupling constants is characterized at the level free theory since p-adic prime characterizes the p-adic length scale). The roots of the polynomial would determine the positions of braid strands so that Galois group emerges naturally. As a matter fact, partonic 2-surface decomposes into regions, one for each braid transforming independently under its own Galois group. Entire quantum state is modular invariant, which brings in additional constraints.

Braiding brings in homotopy group aspect crucial for geometric Langlands program. Another global aspect is related to the modular degrees of freedom of the partonic 2-surface, or more precisely to the regions of partonic 2-surface associated with braids. $Sp(2g, R)$ (g is handle number) can act as transformations in modular degrees of freedom whereas its Langlands dual would act in spinorial degrees of freedom. The outcome would be a coupling between purely local and and global aspects which is necessary since otherwise all information about partonic 2-surfaces as basic objects would be lost. Interesting ramifications of the basic picture about why only three lowest genera correspond to the observed fermion families emerge.

3. Correspondence between finite groups and Lie groups

The correspondence between finite and Lie group is a basic aspect of Langlands.

1. Any amenable group gives rise to a unique sub-factor (in particular, compact Lie groups are amenable). These groups act as genuine outer automorphisms of the group algebra of S_∞ rather than being induced from S_∞ outer automorphism. If one gives up uniqueness, it seems that practically any group G can define a sub-factor: G would define measurement resolution by fixing the quantum numbers which are measured. Finite Galois group G and Lie group containing it and related to it by Langlands correspondence would act in the same representation space: the group algebra of S_∞ , or equivalently configuration space spinors. The concrete realization for the correspondence might transform a large number of speculations to theorems.
2. There is a natural connection with McKay correspondence which also relates finite and Lie groups. The simplest variant of McKay correspondence relates discrete groups $G \subset SU(2)$ to ADE type groups.

Similar correspondence is found for Jones inclusions with index $\mathcal{M} : \mathcal{N} \leq 4$. The challenge is to understand this correspondence.

i) The basic observation is that ADE type compact Lie algebras with n -dimensional Cartan algebra can be seen as deformations for a direct sum of n $SU(2)$ Lie algebras since $SU(2)$ Lie algebras appear as a minimal set of generators for general ADE type Lie algebra. The algebra results by a modification of Cartan matrix. It is also natural to extend the representations of finite groups $G \subset SU(2)$ to those of $SU(2)$.

ii) The idea would be that is that n -fold Connes tensor power transforms the direct sum of n $SU(2)$ Lie algebras by a kind of deformation to a ADE type Lie algebra with n -dimensional Cartan Lie algebra. The deformation would be induced by non-commutativity. Same would occur also for the Kac-Moody variants of these algebras for which the set of generators contains only scaling operator L_0 as an additional generator. Quantum deformation would result from the replacement of complex rays with \mathcal{N} rays, where \mathcal{N} is the sub-factor.

iii) The concrete interpretation for the Connes tensor power would be in terms of the fiber bundle structure $H = M_{\pm}^4 \times CP_2 \rightarrow H/G_a \times G_b$, $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$, which provides the proper formulation for the hierarchy of macroscopic quantum phases with a quantized value of Planck constant. Each sheet of the singular covering would represent single factor in Connes tensor power and single direct $SU(2)$ summand. This picture has an analogy with brane constructions of M-theory.

4. Could there exist a universal rational function giving rise to the algebraic closure of rationals?

One could wonder whether there exists a universal generalized rational function having all units of the algebraic closure of rationals as roots so that S_{∞} would permute these roots. Most naturally it would be a ratio of infinite-degree polynomials.

With motivations coming from physics I have proposed that zeros of zeta and also the factors of zeta in product expansion of zeta are algebraic numbers. Complete story might be that non-trivial zeros of Zeta define the closure of rationals. A good candidate for this function is given by $(\xi(s)/\xi(1-s)) \times (s-1)/s$, where $\xi(s) = \xi(1-s)$ is the symmetrized variant of ζ function having same zeros. It has zeros of zeta as its zeros and poles

and product expansion in terms of ratios $(s - s_n)/(1 - s + s_n)$ converges everywhere. Of course, this might be too simplistic and might give only the algebraic extension involving the roots of unity given by $\exp(i\pi/n)$. Also products of these functions with shifts in real argument might be considered and one could consider some limiting procedure containing very many factors in the product of shifted ζ functions yielding the universal rational function giving the closure.

5. *What does one mean with S_∞ ?*

There is also the question about the meaning of S_∞ . The hierarchy of infinite primes suggests that there is entire infinity of infinities in number theoretical sense. Any group can be formally regarded as a permutation group. A possible interpretation would be in terms of algebraic closure of rationals and algebraic closures for an infinite hierarchy of polynomials to which infinite primes can be mapped. The question concerns the interpretation of these higher Galois groups and HFFs. Could one regard these as local variants of S_∞ and does this hierarchy give all algebraic groups, in particular algebraic subgroups of Lie groups, as Galois groups so that almost all of group theory would reduce to number theory even at this level?

Be it as it may, the expressive power of HFF:s seem to be absolutely marvellous. Together with the notion of infinite rational and generalization of number concept they might unify both mathematics and physics!

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