Configuration Space Spinor Structure

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Abstract

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of configuration space spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the configuration space spinor structure in the sense that the anti-commutation relations for configuration space gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

a) One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin $3/2$ fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the configuration space are analogous to spin $3/2$ fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

As a consequence, configuration space spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of configuration space spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field.

b) The classical theory for the bosonic fields is an essential part of the configuration space geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

c) Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the

space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

d) It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

e) The only possible option is that second quantized induced spinor fields are defined at 3-D light-like causal determinants associated with 4-D space-time sheet. The unique partonic dynamics is almost topological QFT defined by Chern-Simons action for the induced Kähler gauge potential and by the modified Dirac action constructed from it by requiring super-conformal symmetry. The resulting theory has all the desired super-conformal symmetries and is exactly solvable at parton level. It is 3-dimensional lightlike 3-surfaces rather than generic 3-surfaces which are the fundamental dynamical objects in this approach.

The classical dynamics of the interior of space-time surface defines a classical correlate for the partonic quantum dynamics and provides a realization of quantum measurement theory. It is determined by the vacuum functional identified as the Dirac determinant. There are good arguments suggesting that it reduces to an exponent of absolute extremum of Kähler action in each region of the space-time sheet where the Kähler action density has a definite sign.

2. Modified Dirac equation for induced classical spinor fields

The identification of the light-like partonic 3-surfaces as carriers of elementary particle quantum numbers inspired by the TGD based quantum measurement theory forces the identification of the modified Dirac action as that associated with the Chern-Simons action for the induced Kähler gauge potential. At the fundamental level TGD would be almost-topological super-conformal QFT in the sense that only the light-likeness condition for the partonic 3-surfaces would involve the induced metric. Chern-Simons dynamics would thus involve the induced metric only via the generalized eigenvalue equation for the modified Dirac operator involving the light-like normal of $X_l^3 \subset X^4$. $N = 4$ super-conformal symmetry emerges as a maximal Super-Kac Moody symmetry for this option. The application of D to any generalized eigen-mode gives a zero mode and zero modes and generalized eigen-modes define a cohomology.

The interpretation of the solutions of the modified Dirac equation differs from the conventional one and is motivated by the construction of the Kähler function in terms of Dirac determinants associated with the modified Dirac action. This gives hopes of evaluating the exponent of Kähler function as a product of Dirac determinants of the partonic 3-surfaces associated with the space-time sheet without solving the field equations and using only the data provided by the light-likeness.

The solutions of the modified Dirac equation are interpreted as generators of exact $N = 4$ super-conformal symmetries in both quark and lepton sectors. These super-symmetries correspond to pure super gauge transformations and no spartners of ordinary particles are predicted: in particular $N = 2$ space-time super-symmetry is generated by the righthanded neutrino is absent contrary to the earlier beliefs. There is no need to emphasize the experimental implications of this finding. The original conjecture was that the eigenvalues correspond to Riemann Zeta. Zeta is however naturally replaced with that defined by the eigenvalues of the modified Dirac operator.

An essential difference with respect to standard super-conformal symmetries is that Majorana condition is not satisfied and the usual superspace formalism does not apply. Chern-Simons action however leads to an elegant mechanism allowing to obtain modified Dirac action from the super-symmetrized Chern-Simons action by integrating over Grassmann parameters using a modification of the usual Grassman integration measure.

Configuration space gamma matrices identified as super generators of super-canonical or super Kac-Moody algebras (depending on CH coordinates used) are expressible in terms of the oscillator operators associated with the eigen modes of the modified Dirac operator. Super-canonical and super Kac-Moody charges are expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X_l^3 as indeed required by quantum measurement theory. In fact, for certain subalgebra of super-canonical transformations the charge densities correspond to closed 2-forms expressible as 1-dimensional integrals so that stringy picture results and leads to anti-commutation relations for the induced spinor fiels consistent with those of conformal field theory. The resulting situation is highly reminiscent of WZW model and the results imply that at technical level the methods of 2-D conformal field theories should allow to construct quantum TGD.

3. The exponent of Kähler function as Dirac determinant for the modified Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

a) The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X_l^3 associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. One can of course worry about the finiteness of the Dirac determinant. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

b) The generalization of the imbedding space implied by the hierarchy of Planck constants turns out to be essential for the explicit construction. Generalized imbedding space is obtained by gluing together infinite number of covering spaces and factor spaces of $M^4\backslash M^2 \times CP_2\backslash S^2_{II}$, where S^2_{II} is homologically trivial geodesic sphere of CP_2. The gluing takes place along the quantum critical manifolds $M^2 \times CP_2$, $M^4 \times S^2_{II}$, and $M^2 \times S^2_{II}$.

c) Simple consistency conditions imply that D can have only one generalized eigenvalue whose over-all scale is expected to depend on p-adic prime. λ can be deduced from the requirement that it represents geometric data about X^2 and depends on the transversal coordinates of the partonic 2-surface holomorphically. Higgs vacuum expectation is naturally proportional to $\lambda(w)$, w the point of X^2. This leads to an identification of the points of the number theoretic braid as minima of the purely geometric Higgs potential associated $\lambda(w_k)$.

d) Dirac determinant can be defined as the product of $\lambda(w_k)$ at the points of the braid divided with the product of $M^4_+\pm$ distances to the quantum critical manifold $R_+ \times S^2_{II}$. Dirac should give rise to the exponents of Kähler function and Chern-Simons action. The consistency with the vacuum degeneracy leads to an essentially unique geometric construction of $\lambda(w)$. Both Kähler function and Chern-Simons action and also super-canonical conformal weights - identified as zeros of zeta associated with $\lambda(w_k)$ at points of braid - are invariant under overall scalings of λ in accordance with renormalization group invariance. One can understand p-adic coupling constant evolution in terms of dependence of the dependence of the scaling factor of λ on p-adic prime p.

e) What is remarkable, the construction of λ automatically gives rise to a construction of 4-D space-time sheet assigned to the 3-D light-like surface and it remains to be shown that preferred extremum of Kähhler action is in question.

4. Super-conformal symmetries

The almost topological QFT property of partonic formulation based on Chern-Simons action and corresponding modified Dirac action allows a rich structure of $N = 4$ super-conformal symmetries. In particular, the generalized Kac-Moody symmetries leave corresponding X^3-local isometries respecting the light-likeness condition. A rather detailed view about various aspects of super-conformal symmetries emerge leading to identification of fermionic anti-commutation relations and explicit expressions for configuration space gamma matrices and Kähler metric. This picture is consistent with the conditions posed by p-adic mass calculations.

Number theoretical considerations play a key role and lead to the picture in which effective discretization occurs so that partonic two-surface is effectively replaced by a discrete set of algebraic points belonging to the intersection of the real partonic 2-surface and its p-adic counterpart obeying the same algebraic equations. The restriction to the minima of the purely geometric correlate of the vacuum expectation value of Higgs field defined by the eigenvalue of the modified Dirac operator selects a subset of points defining number theoretic braids. This implies effective discretization of super-conformal field theory giving N-point functions defining vertices via discrete versions of stringy formulas.

1 Introduction

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the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. Contrary to the long held belief it seems that covariantly constant right handed neutrino does not generate . The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \quad ,$$

where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

5. What second quantization for spinor fields at space-time surface really means is a highly non-trivial problem. The simplest option would be that free spinor fields of the imbedding space are second quantized and simply restricted to the space-time surface. This however leads to several problems. The relationship between color and electro-weak quantum numbers

comes out incorrectly. The number of degrees of freedom for second quantized imbedding space spinor fields seems to be too small. The explicit construction of the configuration space gamma matrices as super-charges fails. Thus the only possible option is that induced spinor fields are second quantized at the space-time surface.

TGD as a generalized number theory vision leads to the understanding of how the second quantization of the induced spinor fields should be carried out and space-time conformal symmetries allow to explicitly solve the Dirac equation associated with the modified Dirac action in the interior and at the 3-D light like causal determinants (spinorial shock waves). These solutions represent super gauge degrees of freedom. For 3-surfaces at 7-D CDs the anti-commutation relations for the induced spinor fields are fixed by the anti-commutation relations of the canonical super-charges whereas at light like 3-D CDs they are fixed by the anti-commutation relations of Kac Moody super charges. These two manners to fix the anti-commutation relations must be equivalent.

1.2 Dualities and representations of configuration space gamma matrices as super-canonical and super Kac-Moody super-generators

There are several approaches to the construction of the configuration space geometry and dualities provide a powerful framework for unifying these seemingly disparate approaches. One must however be extremely cautious in order to avoid exaggerations here and must honestly admit that the proposed dualities are just interesting hypothesis rather than proven mathematical facts.

1. The construction of the configuration space geometry led to the notion of for more than decade ago. One can imagine a variant of this duality by allowing both signs of Kähler coupling strength. This would correspond to the dominance of electric/magnetic fields and to the two different arrows of the geometric time and 7-D causal determinants which are future/past directed. The first guess was that the possibility to identify space-time surfaces as surfaces for which tangent/normal space defines a quaternionic sub-algebra of octonionic tangent space of H could be the geometric counterpart of this duality.

   A more promising identification is based on the conjecture stating that space-time surfaces can be regarded either as or co-hyper-quaternionic 4-surfaces of imbedding space M^8 or as 4-surfaces in $H = M^4 \times CP_2$ [E2]. Here the attribute "hyper" means that the imaginary units of quaternion/octonion algebra are multiplied by $\sqrt{-1}$ to obtain a sub-space of complexified quaternions/octonions and Minkowskian signature of number theoretic norm.

2. A duality between 7-D and 3-D light like CDs would simplify a lot the construction of the theory. The basic idea behind is that space-time surface

is fixed completely once either 3-D light like CDs or space-like 3-surfaces at 7-D CDs are given: one cannot fix both arbitrarily. Also other variants can be considered such as fixing 3-D and 7-D light like CDs. The two choices would roughly correspond to the possibility of fixing either initial values or boundary values to solve field equations.

This picture re-emerged in a modified form quite recently in the construction of S-matrix [C1, C2] accompanied by a more detailed formulation of the TGD counterpart of the quantum measurement theory. One can say that the classical dynamics in the interior dictated by the Kähler action (or number theoretically) is in a precise correspondence with the quantum dynamics at light-like partonic 3-surfaces X^3_l in the sense that conserved classical charges correspond to a maximum commuting set of quantal charges. Furthermore, the Dirac determinant associated with the modified Dirac action at X^3_l gives rise to the exponent of Kähler function of CH. The modified Dirac action would be simply Chern-Simons action for the induced Kähler gauge potential so that TGD would reduce to almost-topological QFT.

3. The strongest form of 7–3 duality would be quantum gravitational holography in a strong form: light like 3-D CDs would provide a representation for the theory and a very intimate connection with closed super-string models would result. This alternative leads to a remarkable simplification of the basic formulas for configuration space Hamiltonians, Kähler metric, and gamma matrices since one can restrict the integrals in the defining formulas to 2-D intersections X^2 of 3-D CDs with 7-D CDs identifiable as sub-manifolds of space-like 3-surfaces.

   Duality would means that configuration space gamma matrices identified as should also anticommute to configuration space metric just like the super-canonical charges do. This is quite possible: the representations would correspond to two different coordinates for the tangent space of CH determined by the Hamiltonians of $\delta M^4_\pm \times CP_2$ and by Kac Moody Lie-algebra and if the coordinatizations are faithful 7–3 duality corresponds to a change of CH coordinates.

The objection is that super-canonical representations associated with 7-D CDs and Super-Kac Moody algebras associated with 3-D CDs do not seem to be in dual relation. Super-canonical and Super Kac-Moody representations can be realized at the above mentioned 2-D intersections X^2_i, and the action of Kac Moody algebra on super-canonical algebra is well defined and does not lead out of super-canonical algebra. Hence one can hope that same representation space defines representations of both algebras, at least when one allows the representation to consist of several irreducible representations of both algebras.

There are good reasons to consider the possibility that the corresponding algebras are in same relation and Virasoro algebras of group G and its subgroup H in giving rise to super Virasoro representation with a vanishing central charge using differences of the super Virasoro generators in question. Hence the super

Virasoro algebras would be dual in the sense that their actions cancel each other. Together with the so called zero energy ontology [C1, C2] according to which physical states in TGD Universe have always vanishing net conserved charges, this leads to a very elegant general picture about super-conformal symmetries and about the construction of S-matrix in TGD framework.

The realization that coset construction is possible for super-canonical and Super Kac-Moody algebras provides a more convincing justification and more precise formulation for 7-3 duality. Coset Super-Virasoro conditions provide TGD counterparts of Einstein's equations and realize Equivalence Principle in TGD framework. Coset construction justifies also p-adic thermodynamics. The construction will be discussed in detail in the last section of the book devoted to conformal symmetries.

1.3 Modified Dirac equation for induced classical spinor fields

The earlier approach to the definition of the configuration space spinor structure relied on the second quantized ordinary massless Dirac action for the induced spinors. This action had some anomalous looking features. The first anomaly was the appearance of the effective tachyonic mass term proportional to the trace of the second fundamental form vanishing only for minimal surfaces. The breaking of $N = 2$ super symmetry generated by right-handed neutrinos for other than minimal surfaces was the second anomalous feature. It became also clear that the divergences of the fermionic isometry currents can have a non-vanishing c-number anomaly unless one varies Dirac action also with respect to the configuration space coordinates. This anomaly obviously might destroy the definition of the configuration space spinor structure.

The vision about quantum TGD as a generalized number theory [E1, E2, E3] comes in rescue here. One of its outcomes was the realization that, in order to achieve exact super-symmetry, one must modify Dirac action so that its variation with respect to the imbedding space coordinates gives the field equations derivable from the action principle in question. By taking the modified Dirac action as the fundamental action, one can identify vacuum functional as the Dirac determinant. If this determinant equals to exponent of absolute extremum of Kähler action containing partonic 3-surfaces, one can predict the value of the Kähler coupling constant.

The identification of the light-like partonic 3-surfaces as carriers of elementary particle quantum numbers inspired by the TGD based quantum measurement theory forces the identification of the modified Dirac action as that associated with the Chern-Simons action for the induced Kähler gauge potential. At the fundamental level TGD would be almost-topological super-conformal QFT in the sense that only the light-likeness condition for the partonic 3-surfaces would involve the induced metric. Chern-Simons dynamics would thus involve the induced metric only via the generalized eigenvalue equation for the modified Dirac operator involving the light-like normal of $X^3_l \subset X^4$. $N = 4$ super-conformal symmetry emerges as a maximal Super-Kac Moody symmetry for this option. The application of D to any generalized eigen-mode gives a zero mode and zero modes and generalized eigen-modes define a cohomology.

The interpretation of the solutions of the modified Dirac equation differs from the conventional one and is motivated by the construction of the Kähler function in terms of Dirac determinants associated with the modified Dirac action. This gives hopes of evaluating the exponent of Kähler function as a product of Dirac determinants of the partonic 3-surfaces associated with the space-time sheet without solving the field equations and using only the data provided by the light-likeness.

The solutions of the modified Dirac equation are interpreted as generators of exact $N = 4$ super-conformal symmetries in both quark and lepton sectors. These super-symmetries correspond to pure super gauge transformations and no spartners of ordinary particles are predicted: in particular $N = 2$ space-time super-symmetry is generated by the righthanded neutrino is absent contrary to the earlier beliefs. There is no need to emphasize the experimental implications of this finding.

An essential difference with respect to standard super-conformal symmetries is that Majorana condition is not satisfied and the usual super-space formalism does not apply. Chern-Simons action however leads to an elegant mechanism allowing to obtain modified Dirac action from the super-symmetrized Chern-Simons action by integrating over Grassmann parameters using a modification of the usual Grassman integration measure.

Configuration space gamma matrices identified as super generators of super-canonical or super Kac-Moody algebras (depending on CH coordinates used) are expressible in terms of the oscillator operators associated with the eigen modes of the modified Dirac operator. Super-canonical and super Kac-Moody charges are expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X^3_l as indeed required by quantum measurement theory. The resulting situation is highly reminiscent of WZW model and the results imply that at technical level the methods of 2-D conformal field theories should allow to construct quantum TGD.

1.4 The exponent of Kähler function as Dirac determinant for the modified Dirac action?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

1. The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X^3_l associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. One can of course worry

about the finiteness of the Dirac determinant. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

2. The generalization of the imbedding space implied by the hierarchy of Planck constants turns out to be essential for the explicit construction. Generalized imbedding space is obtained by gluing together infinite number of covering spaces and factor spaces of $M^4\backslash M^2 \times CP_2\backslash S^2_{II}$, where S^2_{II} is homologically trivial geodesic sphere of CP_2. The gluing takes place along the quantum critical manifolds $M^2 \times CP_2$, $M^4 \times S^2_{II}$, and $M^2 \times S^2_{II}$.

3. Simple consistency conditions imply that D can have only one generalized eigenvalue whose over-all scale is expected to depend on p-adic prime. λ can be deduced from the requirement that it represents geometric data about X^2 and depends on the transversal coordinates of the partonic 2-surface holomorphically. Higgs vacuum expectation is naturally proportional to $\lambda(w)$, w the point of X^2. This leads to an identification of the points of the number theoretic braid as minima of the purely geometric Higgs potential associated $\lambda(w_k)$.

4. Dirac determinant can be defined as the product of $\lambda(w_k)$ at the points of the braid divided with the product of $M^4_+\pm$ distances to the quantum critical manifold $R_+ \times S^2_{II}$. Dirac should give rise to the exponents of Kähler function and Chern-Simons action. The consistency with the vacuum degeneracy leads to an essentially unique geometric construction of $\lambda(w)$. Both Kähler function and Chern-Simons action and also super-canonical conformal weights - identified as zeros of zeta associated with $\lambda(w_k)$ at points of braid - are invariant under overall scalings of λ in accordance with renormalization group invariance.

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The relationship between super-canonical (SC) and Super Kac-Moody (SKM) symmetries has been one of the central themes in the development of TGD. The progress in the understanding of the number theoretical aspects of TGD gives good hopes of lifting $SKMV$ (V denotes Virasoro) to a subalgebra of SCV so that coset construction works meaning that the differences of SCV and $SKMV$ generators annihilate physical states. This condition has interpretation in terms of Equivalence Principle with coset Super Virasoro conditions defining a generalization of Einstein's equations in TGD framework. Also p-adic thermodynamics finds a justification since the expectation values of SKM conformal weights can be non-vanishing in physical states.

Number theoretical considerations play a key role and lead to the picture in which effective discretization occurs so that partonic two-surface is effectively replaced by a discrete set of algebraic points belonging to the intersection of the real partonic 2-surface and its p-adic counterpart obeying the same algebraic equations. This discretization is unique [E2]. The restriction to the minima of the purely geometric correlate of the vacuum expectation value of Higgs field defined by the eigenvalue of the modified Dirac operator selects a subset of points defining number theoretic braids. This implies effective discretization of super-conformal field theory giving N-point functions defining vertices via discrete versions of stringy formulas.

2 Configuration space spinor structure: general definition

The basic problem in constructing configuration space spinor structure is clearly the construction of the explicit representation for the gamma matrices of the configuration space. One should be able to identify the space, where these gamma matrices act as well as the counterparts of the "free" gamma matrices, in terms of which the gamma matrices would be representable using generalized vielbein coefficients.

2.1 Defining relations for gamma matrices

The ordinary definition of the gamma matrix algebra is in terms of the anti-commutators

$$\{\gamma_A, \gamma_B\} = 2g_{AB} \ .$$

This definition served implicitly also as a basic definition of the gamma matrix algebra in TGD context until the difficulties related to the understanding of the configuration space d'Alembertian defined in terms of the square of the Dirac operator forced to reconsider the definition. If configuration space allows

Kähler structure, the most general definition allows to replace the metric any covariantly constant Hermitian form. In particular, g_{AB} can be replaced with

$$\{\Gamma_A^\dagger, \Gamma_B\} = iJ_{AB} \quad , \tag{1}$$

where J_{AB} denotes the matrix element of the Kähler form of the configuration space. The reason is that gamma matrices carry fermion number and are non-hermitian in all coordinate systems. This definition is numerically equivalent with the standard one in the complex coordinates but in arbitrary coordinates situation is different since in general coordinates iJ_{kl} is a nontrivial positive square root of g_{kl}. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly. Obviously, what one must do is the equivalent of replacing $D^2 = (\Gamma^k D_k)^2$ with $D\hat{D}$ with $\hat{D}$ defined as

$$\hat{D} = iJ^{kl}\Gamma_l^\dagger D_k \quad .$$

2.2 General vielbein representations

There are two ideas, which make the solution of the problem obvious.

1. Since the classical time development in bosonic degrees of freedom (induced gauge fields) is coded into the geometry of the configuration space it seems natural to expect that same applies in the case of the spinor structure. The time development of the induced spinor fields dictated by TGD counterpart of the massless Dirac action should be coded into the definition of the configuration space spinor structure. This leads to the challenge of defining what classical spinor field means.

2. Since classical scalar field in the configuration space corresponds to second quantized boson fields of the imbedding space same correspondence should apply in the case of the fermions, too. The spinor fields of configuration space should correspond to second quantized fermion field of the imbedding space and the space of the configuration space spinors should be more or less identical with the Fock space of the second quantized fermion field of imbedding space or $X^4(X^3)$. Since classical spinor fields at space-time surface are obtained by restricting the spinor structure to the space-time surface, one might consider the possibility that life is really simple: the second quantized spinor field corresponds to the free spinor field of the imbedding space satisfying the counterpart of the massless Dirac equation and more or less standard anti-commutation relations. Unfortunately life is not *so* simple as the construction of configuration space spinor structure demonstrates: second quantization must be performed for induced spinor fields.

It is relatively simple to fill in the details once these basic ideas are accepted.

1. The only natural candidate for the second quantized spinor field is just the on X^4. Since this field is free field, one can indeed perform second quantization and construct fermionic oscillator operator algebra with unique anti-commutation relations. The space of the configuration space spinors can be identified as the associated with these oscillator operators. This space depends on 3-surface and strictly speaking one should speak of the Fock bundle having configuration space as its base space.

2. The gamma matrices of the configuration space (or rather fermionic Kac Moody generators) are representable as super positions of the fermionic oscillator algebra generators:

$$\begin{aligned} \Gamma_A^+ &= E_A^n a_n^\dagger \\ \Gamma_A^- &= \bar{E}_A^n a_n \\ iJ_{A\bar{B}} &= \sum_n E_A^n \bar{E}_B^n \end{aligned} \tag{2}$$

   where E_A^n are the vielbein coefficients. Induced spinor fields can possess zero modes and there is no oscillator operators associated with these modes. Since oscillator operators are spin 1/2 objects, configuration space gamma matrices are analogous to spin 3/2 spinor fields (in a very general sense). Therefore the generalized vielbein and configuration space metric is analogous to the pair of spin 3/2 and spin 2 fields encountered in super gravitation! Notice that the contractions $j^{Ak}\Gamma_k$ of the complexified gamma matrices with the isometry generators are genuine spin 1/2 objects labelled by the quantum numbers labelling isometry generators. In particular, in CP_2 degrees of freedom these fermions are color octets.

3. A further great idea inspired by the symplectic and Kähler structures of the configuration space is that configuration gamma matrices are actually generators of super-canonical symmetries. This simplifies enormously the construction allows to deduce explicit formulas for the gamma matrices.

2.3 Inner product for configuration space spinor fields

The conjugation operation for configuration space spinors corresponds to the standard $ket \rightarrow bra$ operation for the states of the Fock space:

$$\begin{aligned} \Psi &\leftrightarrow |\Psi\rangle \\ \bar{\Psi} &\leftrightarrow \langle\Psi| \end{aligned} \tag{3}$$

The inner product for configuration space spinors at a given point of the configuration space is just the standard Fock space inner product, which is unitary.

$$\bar{\Psi}_1(X^3)\Psi_2(X^3) = \langle\Psi_1|\Psi_2\rangle_{|X^3} \quad (4)$$

Configuration space inner product for two configuration space spinor fields is obtained as the integral of the Fock space inner product over the whole configuration space using the vacuum functional $exp(K)$ as a weight factor

$$\langle\Psi_1|\Psi_2\rangle = \int \langle\Psi_1|\Psi_2\rangle_{|X^3} exp(K)\sqrt{G}dX^3 \quad (5)$$

This inner product is obviously unitary. A modified form of the inner product is obtained by including the factor $exp(K/2)$ in the definition of the spinor field. In fact, the construction of the central extension for the isometry algebra leads automatically to the appearance of this factor in vacuum spinor field.

The inner product differs from the standard inner product for, say, Minkowski space spinors in that integration is over the entire configuration space rather than over a time= constant slice of the configuration space. Also the presence of the vacuum functional makes it different from the finite dimensional inner product. These are not un-physical features. The point is that (apart from classical non-determinism forcing to generalized the concept of 3-surface) $Diff^4$ invariance dictates the behavior of the configuration space spinor field completely: it is determined form its values at the moment of the big bang. Therefore there is no need to postulate any Dirac equation to determine the behavior and therefore no need to use the inner product derived from dynamics.

2.4 Holonomy group of the vielbein connection

Generalized vielbein allows huge gauge symmetry. An important constraint on physical observables is that they do not depend at all on the gauge chosen to represent the gamma matrices. This is indeed achieved using vielbein connection, which is now quadratic in fermionic oscillator operators. The holonomy group of the vielbein connection is the configuration space counterpart of the electroweak gauge group and its algebra is expected to have same general structure as the algebra of the configuration space isometries. In particular, the generators of this algebra should be labelled by conformal weights like the elements of Kac Moody algebras. In present case however conformal weights are complex as the construction of the configuration space geometry demonstrates.

2.5 Realization of configuration space gamma matrices in terms of super symmetry generators

In string models super symmetry generators behave effectively as gamma matrices and it is very tempting to assume that configuration space gamma matrices can be regarded as generators of the canonical algebra extended to super-canonical Kac Moody type algebra. The experience with string models suggests also that radial Virasoro algebra extends to Super Virasoro algebra. There are good reasons to expect that configuration space Dirac operator and its square give automatically a realization of this algebra. It this is indeed the case, then configuration space spinor structure as well as Dirac equation reduces to mere group theory.

One can actually guess the general form of the super-canonical algebra. The form is a direct generalization of the ordinary super Kac Moody algebra. The complexified super generators S_A are identifiable as configuration space gamma matrices:

$$\Gamma_A = S_A \ . \tag{6}$$

The anti-commutators $\{\Gamma_A^\dagger, \Gamma_B\}_+ = i2J_{A,B}$ define a Hermitian matrix, which is proportional to the Kähler form of the configuration space rather than metric as usually. Only in complex coordinates the anti-commutators equal to the metric numerically. This is, apart from the multiplicative constant n, is expressible as the Poisson bracket of the configuration space Hamiltonians H_A and H_B. Therefore one should be able to identify super generators $S_A(r_M)$ for each values of r_M as the counterparts of fluxes. The anti-commutators between the super generators S_A and their Hermitian conjugates should read as

$$\{S_A, S_B^\dagger\}_+ = iQ_m(H_{[A,B]}) \ . \tag{7}$$

and should be induced directly from the anti-commutation relations of free second quantized spinor fields of the imbedding space restricted to the light cone boundary.

The commutation relations between s and super generators follow solely from the transformation properties of the super generators under canonical transformations, which are same as for the Hamiltonians themselves

$$\{H_{Am}, S_{Bn}\}_- = S_{[Am,Bn]} \ , \tag{8}$$

and are of the same form as in the case of Super-Kac-Moody algebra.

The task is to derive an explicit representation for the super generators S_A in both cases. For obvious reason the spinor fields restricted to the 3-surfaces on the light cone boundary $\delta M^4_+ \times CP_2$ can be used. Leptonic/quark like oscillator operators are used to construct Ramond/NS type algebra.

What is then the strategy that one should follow?

1. Configuration space Hamiltonians correspond to either magnetic or electric flux Hamiltonians and the conjecture is that these representations are equivalent. It turns out that this electric-magnetic duality generalizes to the level of super charges. It also turns out that quark representation is the only possible option whereas leptonic super charges super-symmetrize the ordinary function algebra of the light cone boundary.

2. The simplest option would be that second quantized imbedding space spinors could be used in the definition of super charges. This turns out to not work and one must second quantize the induced spinor fields.

3. The task is to identify a super-symmetric variational principle for the induced spinors: ordinary Dirac action does not work. It turns out that in the most plausible scenario the modified Dirac action varied with respect to *both* imbedding space coordinates and spinor fields is the fundamental action principle. The c-number parts of the conserved canonical charges associated with this action give rise to bosonic conserved charges defining configuration space Hamiltonians. The second quantization of the spinor fields reduces to the requirement that super charges and Hamiltonians generate super-canonical algebra determining the anti-commutation relations for the induced spinor fields.

2.6 Central extension as symplectic extension at configuration space level

The earlier attempts to understand the emergence of central extension of super-canonical algebra were based on the notion of symplectic extension. This general view is not given up although it seems that this abstract approach is not very practical. Symplectic extension emerged originally in the attempts to construct formal expression for the configuration space Dirac equation. The rather obvious idea was that the Dirac equation reduces to super Virasoro conditions with Super Virasoro generators involving the Dirac operator of the imbedding space. The basic difficulty was the necessity to assign to the gamma matrices of the imbedding space fermion number. In the recent formulation the Dirac operator of H does not appear in in the Super Virasoro conditions so that this problem disappears.

The proposal that Super Virasoro conditions should replaced with conditions stating that the commutator of super-canonical and super Kac-Moody algebras annihilates physical states, looks rather feasible. One could call these conditions as configuration space Dirac equation but at this moment I feel that this would be just play with words and mask the group theoretical content of these conditions. In any case, the formulas for the symplectic extension and action of isometry generators on configuration space spinor deserve to be summarized.

2.6.1 Symplectic extension

The Abelian extension of the super-canonical algebra is obtained by an extremely simple trick. Replace the ordinary derivatives appearing in the definition of, say spinorial isometry generator, by the covariant derivatives defined by a coupling to a multiple of the Kähler potential.

$$\begin{aligned} j^{Ak}\partial_k &\rightarrow j^{Ak}D_k \ , \\ D_k &= \partial_k + ikA_k/2 \ . \end{aligned} \tag{9}$$

where A_k denotes Kähler potential. The reality of the parameter k is dictated by the Hermiticity requirement and also by the requirement that Abelian extension reduces to the standard form in Cartan algebra. k is expected to be integer also by the requirement that covariant derivative corresponds to connection (quantization of magnetic charge).

The commutation relations for the centrally extended generators J^A read:

$$[J^A, J^B] = J^{[A,B]} + ikj^{Ak} J_{kl} j^{Bl} \equiv J^{[A,B]} + ikJ_{AB} \ . \tag{10}$$

Since Kähler form defines symplectic structure in configuration space one can express Abelian extension term as a Poisson bracket of two Hamiltonians

$$J_{AB} \equiv j^{Ak} J_{kl} j^{Bl} = \{H^A, H^B\} \ . \tag{11}$$

Notice that Poisson bracket is well defined also when Kähler form is degenerate.

The extension indeed has acceptable properties:

1. Jacobi-identities reduce to the form

$$\sum_{cyclic} H^{[A,[B,C]]} = 0 \ , \tag{12}$$

   and therefore to the Jacobi identities of the original Lie- algebra in Hamiltonian representation.

2. In the Cartan algebra Abelian extension reduces to a constant term since the Poisson bracket for two commuting generators must be a multiple of a unit matrix. This feature is clearly crucial for the non-triviality of the Abelian extension and is encountered already at the level of ordinary (q, p) Poisson algebra: although the differential operators ∂_p and ∂_q commute the Poisson bracket of the corresponding Hamiltonians p and q is nontrivial: $\{p, q\} = 1$. Therefore the extension term commutes with the generators of the Cartan subalgebra. Extension is also local $U(1)$ extension since Poisson algebra differs from the Lie-algebra of the vector fields in that it contains constant Hamiltonian ("1" in the commutator), which commutes with all other Hamiltonians and corresponds to a vanishing vector field.

3. For the generators not belonging to Cartan sub-algebra of CH isometries Abelian extension term is not annihilated by the generators of the original algebra and in this respect the extension differs from the standard central extension for the loop algebras. It must be however emphasized that for the super-canonical algebra generators correspond to products of δM^4_+ and CP_2 Hamiltonians and this means that generators of say δM^4_+-local $SU(3)$ Cartan algebra are non-commuting and the commutator is completely analogous to central extension term since it is symmetric with respect to $SU(3)$ generators.

4. The proposed method yields a trivial extension in the case of $Diff^4$. The reason is the (four-dimensional!) Diff degeneracy of the Kähler form. Abelian extension term is given by the contraction of the $Diff^4$ generators with the Kähler potential

$$j^{Ak}J_{kl}j^{Bl} = 0 \ , \tag{13}$$

which vanishes identically by the Diff degeneracy of the Kähler form. Therefore neither 3- or 4-dimensional Diff invariance is not expected to cause any difficulties. Recall that 4-dimensional Diff degeneracy is what is needed to eliminate time like vibrational excitations from the spectrum of the theory. By the way, the fact that the loop space metric is not Diff degenerate makes understandable the emergence of Diff anomalies in string models [37, 38].

5. The extension is trivial also for the other zero norm generators of the tangent space algebra, in particular for the $k_2 = Im(k) = 0$ canonical generators possible present so that these generators indeed act as genuine $U(1)$ transformations.

6. Concerning the solution of configuration space Dirac equation the maximum of Kähler function is expected to be special, much like origin of Minkowski space and symmetric space property suggests that the construction of solutions reduces to this point. At this point the generators and Hamiltonians of the algebra h in the defining Cartan decomposition $g = h + t$ should vanish. h corresponds to integer values of $k_1 = Re(k)$ for Cartan algebra of super-canonical algebra and integer valued conformal weights n for Super Kac-Moody algebra. The algebra reduces at the maximum to an exceptionally simple form since only central extension contributes to the metric and Kähler form. In the ideal case the elements of the metric and Kähler form could be even diagonal. The degeneracy of the metric might of course pose additional complications.

2.6.2 Super canonical action on configuration space spinors

The generators of canonical transformations are obtained in the spinor representation of the isometry group of the configuration space by the following formal construction. Take isometry generator in the spinor representation and add to the covariant derivative D_k defined by vielbein connection the coupling to the multiple of the Kähler potential: $D_k \rightarrow D_k + ikAk/2$.

$$\begin{aligned} J^A &= j^{Ak}D_k + D_l j_k \Sigma^{kl}/2 \ , \\ &\rightarrow \hat{J}^A = j^{Ak}(D_k + ikA_k/2) + D_l j^A_k \Sigma^{kl}/2 \ , \end{aligned} \tag{14}$$

This induces the required central term to the commutation relations. Introduce complex coordinates and define bosonic creation and annihilation operators as $(1,0)$ and $(0,1)$ parts of the modified isometry generators

$$\begin{aligned} B_A^\dagger &= J_+^A = j^{Ak}(D_k + ... \ , \\ B_A &= J_-^A = j^{A\bar{k}}(D_{\bar{k}} + ... \ . \end{aligned} \tag{15}$$

where "k" refers now to complex coordinates and "$\bar{k}$" to their conjugates.

Fermionic generators are obtained as the contractions of the complexified gamma matrices with the isometry generators

$$\begin{aligned} \Gamma_A^\dagger &= j^{Ak}\Gamma_k \ , \\ \Gamma_A &= j^{A\bar{k}}\Gamma_{\bar{k}} \ . \end{aligned} \tag{16}$$

Notice that the bosonic Cartan algebra generators obey standard oscillator algebra commutation relations and annihilate fermionic Cartan algebra generators. Hermiticity condition holds in the sense that creation type generators are hermitian conjugates of the annihilation operator type generators. There are two kinds of representations depending on whether one uses leptonic or quark like oscillator operators to construct the gammas. These will be assumed to correspond to Ramond and NS type generators with the radial plane waves being labelled by integer and half odd integer indices respectively.

The non-vanishing commutators between the Cartan algebra bosonic generators are given by the matrix elements of the Kähler form in the basis of formed by the isometry generators

$$[B_A^\dagger, B_B] = J(j^{A\dagger}, j^B) \equiv J_{\bar{A}B} \ . \tag{17}$$

and are isometry invariant quantities. The commutators between local $SU(3)$ generators not belonging to Cartan algebra are just those of the local gauge algebra with Abelian extension term added.

The anti-commutators between the fermionic generators are given by the elements of the metric (as opposed to Kähler form in the case of bosonic generators) in the basis formed by the isometry generators

$$\{\Gamma_A\dagger, \Gamma_B\} = 2g(j^{A\dagger}, j^B) \equiv 2g_{\bar{A}B} \ . \tag{18}$$

and are invariant under isometries. Numerically the commutators and anti-commutators differ only the presence of the imaginary unit and the scale factor R relating the metric and Kähler form to each other (the factor R is same for CP_2 metric and Kähler form).

The commutators between bosonic and fermionic generators are given by

$$[B_A, \Gamma_B] = \Gamma_{[A,B]} \ . \tag{19}$$

The presence of vielbein and rotation terms in the representation of the isometry generators is essential for obtaining these nice commutations relations. The commutators vanish identically for Cartan algebra generators. From the commutation relations it is clear that Super Kac Moody algebra structure is directly related to the Kähler structure of the configuration space: the anti-commutator of fermionic generators is proportional to the metric and the commutator of the bosonic generators is proportional to the Kähler form. It is this algebra, which should generate the solutions of the field equations of the theory.

The vielbein and rotational parts of the bosonic isometry generators are quadratic in the fermionic oscillator operators and this suggests the interpretation as the fermionic contribution to the isometry currents. This means that the action of the bosonic generators is essentially non-perturbative since it creates fermion antifermion pairs besides exciting bosonic degrees of freedom.

2.7 Configuration space Clifford algebra as a hyper-finite factor of type II_1

The naive expectation is that the trace of the unit matrix associated with the Clifford algebra spanned by configuration space sigma matrices is infinite and thus defines an excellent candidate for a source of divergences in perturbation theory. This potential source of infinities remained un-noticed until it became clear that there is a connection with von Neumann algebras [34]. In fact, for a separable Hilbert space defines a standard representation for so called [18]. This guarantees that the trace of the unit matrix equals to unity and there is no danger about divergences.

2.7.1 Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [18].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless.

2.7.2 von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac based on the notion of delta function, plus the emergence of Feynman graphs, the possibility to formulate the notion of delta function rigorously in terms of distributions, and the emergence of path integral approach meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [24, 25] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [20, 21] relate closely to type II_1 factors. In topological quantum computation [19] based on braid groups [23] modular S-matrices they play an especially important role.

2.7.3 Clifford algebra of configuration space as von Neumann algebra

The Clifford algebra of the configuration space provides a school example of a hyper-finite factor of type II_1, which means that fermionic sector does not produce divergence problems. Super-symmetry means that also "orbital" degrees of freedom corresponding to the deformations of 3-surface define similar factor. The general theory of hyper-finite factors of type II_1 is very rich and leads to rather detailed understanding of the general structure of S-matrix in TGD framework. For instance, there is a unitary evolution operator intrinsic to the von Neumann algebra defining in a natural manner single particle time evolution. Also a connection with 3-dimensional topological quantum field theories and knot theory, conformal field theories, braid groups, quantum groups, and quantum counterparts of quaternionic and octonionic division algebras emerges naturally. These aspects are discussed in detail in [C7].

3 Dualities and conformal symmetries in TGD framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the modified Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of configuration space geometry. Also the duality between 7-D and 3-D CDs relating closely to quantum gravitational holography and YM-gravity duality and representing basically field-particle duality suggests itself. In this case strict duality seems however too strong an assumption.

3.1 Electric-magnetic duality

Electric-magnetic duality for the induced Kähler induced field is present also in TGD (CP_2 Kähler form is self-dual). My original belief was that it corresponds to a self duality leaving Kähler coupling constant invariant as an analog of critical temperature: $\alpha_K \rightarrow \alpha_K$ in this transformation [B2, B3]. This duality would allow to construct configuration space Kähler metric in terms of Kähler electric or magnetic fluxes.

This duality relates in an interesting manner to the conjecture about number theoretic spontaneous compactification [E2] in the sense that space-time surfaces in $M^4 \times CP_2$ could be equivalently regarded as hyper-quaternionic 4-surfaces in M^8 possessing hyper-octonionic structure. The point is that one can consider also the dual definition for which the 4-D normal space defines 4-D subalgebra of 8-D algebra at each point of the space-time surface. Future-past duality could basically reduce to this purely geometric duality and would basically reflect bra-ket duality.

It is also possible to imagine a second variant of electric-magnetic duality, not in fact a genuine duality.

1. This duality like transformation is defined formally via the replacement $\alpha_K \rightarrow -\alpha_K$ [B2, B3, E3] in Kähler action, and could be formally regarded as a logarithmic version of $g \rightarrow 1/g$ duality, and makes sense in TGD framework since g_K does not appear as a coupling constant. The requirement that the Kähler metric of CH defined by Kähler function is positive definite, poses very strong conditions and might exclude this kind duality. Certainly, the space-time sheet of one phase cannot correspond to that for another phase. For instance, electric *resp.* magnetic flux tubes would be favored in the two phases if absolute minimization of Kähler action defines the variational principle. TGD inspired cosmology encourages to consider this picture seriously [D5].

2. A possible interpretation is that this duality allows to distinguish between positive energy particle propagating to the geometric future and negative energy particle propagating to the geometric past (phase conjugate photons would represent an example of negative energy particles propagating in the direction of geometric past). Quantum TGD indeed allows to make this distinction at the level of quantum states: the super-canonical conformal weights are complex and the sign of the imaginary part distinguishes between partons and their phase conjugates. The two directions of geometric time and two possible signs of inertial energy and Kähler action could correspond to the two signs g_K^2. At parton level this difference would correspond to two different sign of Chern-Simons action. Both the sign of the imaginary part of the conformal weight (quantum level) and of g_K^2 (classical space-time correlates) would characterize the direction of inherent time arrow of particle. The imaginary part of the conformal weight would also correspond to irreversibility and p-adic mass calculations suggest interpretation in terms of a decay rate.

3. On the other hand, the geometric construction recipe for S-matrix [C2] suggests that $g_K^2 \rightarrow -g_K^2$ duality could relate the incoming and outgoing lines of particle reaction represented by space-time sheets. Just as one assigns to the incoming and outgoing lines positive and negative energies one could assign to them an opposite sign of Kähler action. As a matter fact, the definition of action in mechanical systems is as $S = \int_0^t dtLdt$ so that the sign of S depends on time orientation. In the recent case action is general coordinate invariant and its sign does not depend on time orientation. The different choice of sign of α_K or equivalently different inherent time orientation, for incoming and outgoing particles would have same net effect. This would however imply a profound difference between incoming and and outgoing particles of particle reaction. This interpre-

tation is consistent with the previous one if outgoing particles are always phase conjugate particles.

4. One could argue that the change of sign of α_K breaks unitarity (this is not related to the naive expectation that g_K becomes imaginary since the absolute minima would be different and tend to be dominated by Kähler magnetic fields). The defining property of hyper-finite factors of type II_1 is that infinite-dimensional unit matrix has unit and this makes possible to identify S-matrix as unitary entanglement coefficients between positive and negative energy components of a zero energy state. Unitary S-matrix is thus a property of zero energy state now and the counter argument does not bite.

3.2 Duality of 3-D and 7-D causal determinants as particle-field duality

As already described, TGD predicts two kinds of super-conformal symmetries corresponding to 7-D and 3-D causal determinants and that their duality would generalize the age-old field-particle particle duality so that quantum gravitational holography and YM-gravitational duality could be seen as particular aspects of field particle duality. The two dual super symmetry algebras defined by super-canonical and Super Kac-Moody algebras at configuration space level define spectrum generating algebras whereas at space-time level they define pure super gauge symmetry algebras eliminating half of the helicities of the induced Dirac spinor fields at each point.

1. The conformal symmetries associated with 7-D CDs and space-time interior

The super-canonical conformal invariance is associated with 7-dimensional light like CDs $\delta M^4_\pm(a) \times CP_2$ and their unions at the level of the imbedding space. The GRT counterpart is the moment of local "big bang". The string model counterpart is a Kaluza-Klein type representation of quantum numbers used in super string models relying on closed strings. Obviously this corresponds to the field aspect of the duality.

At space-time level these dynamical super-conformal symmetries have gauge super-symmetries as their counterpart. The number theoretic spontaneous compactification conjecture [E2] suggests that a possibly equivalent identification is in terms of deformations of the boundaries of the space-time surface induced by deformations of $X^4 \subset M^8$ caused by hyper-quaternion analytic maps of $X^4 \rightarrow X^4$ inducing no deformation in the interior but moving the boundaries of X^4 [E2]. These deformations would correspond to deformations of the corresponding surface $X^4 \subset M^4 \times CP_2$.

This symmetry could be equivalent with $N = 4$ local gauge super-symmetry ($N = 4$ super-symmetric YM theory has been proposed to be closely linked with string models). There would be no global super symmetry. Since the hyper-quaternion conformal symmetry can make sense only in interior, and since only the induced spinor field in the interior of space-time surface contributes to the configuration space super charges, hyper-quaternion conformal symmetry would indeed correspond to the field aspect of the field-particle duality.

2. Conformal symmetries associated with 3-D light like CDs and quantum gravitational holography

The Super-Kac Moody symmetry at the 3-D light like causal determinants and super-canonical symmetry at 7-D causal determinants define configuration space super-symmetries. These super-symmetries are dynamical and contrary to the original beliefs do not imply the existence of sparticles.

The conformal symmetry associated with the 3-dimensional light like CDs reduces to a generalization of the ordinary super-conformal symmetry. The derivative of the normal coordinate disappears from the modified Dirac operator and solutions are 3-dimensional spinorial shock waves having a very natural interpretation as representations of elementary particles. The GRT analog is black hole horizon. $N = 4$ superconformal symmetry in an almost ordinary sense is in question.

The induced spinor fields carry electro-weak quantum numbers as YM type quantum numbers, Poincare and color isometry charges, but not color as spin like degrees of freedom. Hence color degrees of freedom are analogous to rigid body rotational degrees of freedom of the 3-D causal determinant and genuine configuration space degrees of freedom having no counterpart at space-time level although conserved classical color charges make sense: obviously type quantum numbers are in question.

7–3 duality however suggests that it is possible to code the information about configuration space color partial wave to the induced spinor field at X^3_l (for 7-D CDs this is not necessary) as a functional of X^3_l. The guess is that the shock wave solutions of the modified Dirac equation at 3-D CDs can be constructed by taking imbedding space spinor harmonics, operating on them by appropriate color Kac Moody generators to get a correct correlation between electro-weak and color quantum numbers, and applying the modified Dirac operator D to get a spinor basis $D\Psi_m$. If the spinor basis obtained in this manner satisfies $D\Psi_m = c_{mn}o\Psi_n$, where o is the contraction of the light like normal vector of CD with the induced gamma matrices appearing in the eigenvalue equation $D\Psi = \lambda o\Psi$ and defining boundary states for the induced spinor fields and Dirac determinant, the construction works. The fact that quark color does not have a direct space-time counterpart (imbedding space spinors allow color partial waves but induced spinors do not) might correlate with color confinement and with the impossibility to detect free quarks.

3. 7–3 duality and effective 2-dimensionality of 3-surfaces

Whether super-canonical and Kac-Moody algebras are dual is not at all obvious. The assumption that the situation reduces to the intersections X^2_i of the 3-D CDs X^3_l with 7-D CDs defining 2-sub-manifolds of X^3 concretizes the idea about duality. Duality would imply effective 2-dimensionality of 3-surfaces and the task is to understand what this could mean.

1. By duality both X_i^2-local H-isometries and the Hamiltonians of $\delta M_+^4 \times CP_2$ restricted to X_i^2 span the tangent space of CH. A highly non-trivial implication would be a dramatic simplification of the construction of the configuration space Hamiltonians, Kähler metric, and gamma matrices since one could just sum only over the flux integrals over the sub-manifolds X_i^2. The best that one might hope is that it is possible to fix both 3-D light like CDs and their sub-manifolds X_i^2 and 7-D CDs freely. There would be good hopes about achieving the p-adicization of the basic definitions.

   One could assign unique modular degrees of freedom to X_i^2: this would be crucial for the unique definition of the elementary particle vacuum functionals [F1]. This would give rather good hopes of achieving a better understanding of why particle families corresponding to genera $g > 2$ are effectively absent from the spectrum. Elementary particle vacuum functionals vanish when $g > 2$ is hyper-elliptic, that is allows Z_2 conformal symmetry. The requirement that the tangent space 2-surface defines at each point a commutative sub-space of the octonionic tangent space might force $g > 2$ surfaces to be hyper-elliptic.

2. The reduction to dimension 2 could be understood in terms of the impossibility to choose X^3 freely once light like 3-D CDs are fixed but this does not remove the air of paradox. The resolution of the paradox comes from the following observation. The light likeness condition for 3-D CD can be written in the coordinates for which the induced metric is diagonal as a vanishing of one of the diagonal components of the induced metric, say g_{11}:

$$g_{1i}h_{kl}\partial_1 h^k \partial_i h^l = 0 \ , \ i = 1,2,3 \ . \tag{20}$$

   The condition $g_{11} = 0$ is exactly like the light likeness condition for the otherwise random M^4 projection of CP_2 type extremals [D1]. When written in terms of the Fourier expansion this conditions gives nothing but classical Virasoro conditions. This analog of the conformal invariance is different from the conformal invariance associated with transversal degrees of freedom and and from hyper-quaternion conformal invariance. This symmetry conforms nicely with the duality idea since also the boundary of the light cone allows conformal invariance in both light like direction and transversal degrees of freedom.

   One can consider two interpretations of this symmetry.

   i) The degrees of freedom generating different light like 3-D CDs X_l^3 with a given intersection X^2 with 7-D CD correspond to zero modes. Physically this would mean that in each quantum jump a complete localization occurs in these degrees of freedom so that particles behave effectively classically. With this interpretation these degrees of freedom could perhaps be seen as dual for the zero mode degrees of freedom associated with the space-like

3-surfaces X^3 at 7-D CDs: deformation of X^3_l would induce deformation of X^3.

ii) Gauge degrees of freedom could be in question so that one can make a gauge choice fixing the orbits within certain limits. If ones assumes a suitable correlation between the longitudinal and transversal conformal degrees of freedom, hyper-quaternion conformal invariance could be the outcome. The two symmetries could correspond to two different choices of gauge reflected as a choice of different space-time sheets. This would mean additional flexibility in the interpretation of this symmetry at the level of solutions of the modified Dirac equation.

At the level of configuration space geometry the result would mean that one can indeed code all data using only two-dimensional surfaces X^2_i of X^3. This brings in mind a number theoretic realization for the quantum measurement theory. That only mutually commuting observables can be measured simultaneously would correspond to the assumption that all data about configuration space geometry and quantum physics must be given at 2-dimensional surfaces of H for which the tangent space at each point corresponds to an Abelian sub-algebra of octonions. Quantum TGD would reduce to something having very high resemblance with WZW model. One cannot deny the resemblance with M-theories with M interpreted as a membrane.

4. 7-3 duality and quantum measurement theory

The action of Super Kac-Moody generators on configuration space Hamiltonians is well defined and one might hope that as a functional of 2-surface it could give rise to a unique superposition of super-canonical Hamiltonians. Same should apply to the action of super-canonical algebra on Kac Moody algebra. At the level of gamma matrices the question is whether the configuration space metric can be defined equivalently in terms of anti-commutators of super-canonical and Super Kac-Moody generators. If the answer is affirmative, then 7–3 duality would be nothing but a transformation between two preferred coordinates of the configuration space.

TGD inspired quantum measurement theory suggests however that the two super-conformal algebras correspond to each other like classical and quantal degrees of freedom. Super Kac-Moody algebra and super conformal algebra would act as transformations preserving the conformal equivalence class of the partonic 2-surfaces X^2 associated with the maxima of the Kähler function whereas super-canonical algebra in general changes conformal moduli and induces a conformal anomaly in this manner. Hence Kac-Moody algebra seems to act in the zero modes of the configuration space metric. In TGD inspired quantum measurement zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement.

Duality would be also realized in a well-defined sense at the level of configuration space conformal symmetries. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-canonical algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-canonical local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-canonical generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and thus mass.

Number theoretic vision provides strong support for the belief that the coset construction is possible for super-canonical and Super Kac-Moody algebras provides a more convincing justification and more precise formulation for 7-3 duality. Coset Super-Virasoro conditions provide TGD counterparts of Einstein's equations and realize Equivalence Principle in TGD framework. Coset construction justifies also p-adic thermodynamics. The construction will be discussed in detail in the section devoted to conformal symmetries.

3.3 Quantum gravitational holography

The so called AdS/CFT of [39] correspondence relates to quantum-gravitational holography states roughly that the gravitational theory in 10-dimensional $AdS_{10-n}\times S^n$ manifold is equivalent with the conformal field theory at the boundary of AdS_D factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD quantum gravitational holography is realized in terms of the modified Dirac action at light like 3-D CDs, which by their metric 2-dimensionality allow superconformal invariance and are very much like world sheets of closed super string or the ends of an open string.

It is possible to deduce the values of Kähler action at maximally deterministic regions of space-time sheets from the Dirac determinants at the CDs [E3] so that the enormously difficult solution of the problem selecting preferred extremals of Kähler action as generalized Bohr orbits would be reduced to local data stating that CDs are light like 3-surfaces which are also minimal surfaces in the case that Kähler action density is non-vanishing at them. This reduction has enormous importance for the calculability of the theory. Also the values of Kähler coupling strength and gravitational constant are predicted [E3].

Here a word of warning is in order: I do not know how to prove that the minimal surface property of the CDs implies the absolute minimization of Kähler action or the more general variational principle discussed in [E2].

Perhaps the most practical form of the quantum gravitational holography would be that the correlation functions of $N = 4$ super-conformal field theory at the light like 3-D CDs allow to construct the vertices needed to construct S-matrix of quantum TGD. Computationally TGD would reduce to almost string model since light like CD:s are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand. There is also an analogy with the Wess-Zumino-Witten model: light like CDs would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

Quantum gravitational holography could also mean that light like CDs define what might be called fundamental central nervous systems able to represent and process conscious information about the interior of the space-time surface in terms of its own quantum states which have interpretation either as a time evolution or state (duality again!). Topological quantum computation might be one of the activities associated with the light like CDs as proposed in [E9].

3.4 Super-symmetry at the space-time level

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X^3_l. Gauge symmetries are in question when the intersections of X^3_l with 7-D CDs X^7 are not changed. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced configuration space consisting of the light like 3-D CDs. The conformal symmetries in question imply a further degeneracy of the configuration space metric and effective metric 2-dimensionality of 3-surfaces as a consequence. These conformal symmetries are accompanied by $N = 4$ local super conformal symmetries defined by the solutions of the induced spinor fields.

Contrary to the original beliefs these conformal symmetries do not seem to be continuable to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate. The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional. If the number theoretic spontaneous compactification conjecture [E2] holds true, hyper-quaternionic symmetries make however sense for the representation of the space-time surface as a 4-surface in M^8 possessing hyper-octonion structure.

The solutions of the modified Dirac equation $D\Psi = 0$, define the modes which do not contribute to the Dirac determinant of the modified Dirac operator in terms of which the vacuum functional assumed to correspond to the exponent of the Kähler action is defined. Thus they define gauge super-symmetries. Usually D selects the physical helicities by the requirement that it annihilates physical states: now the situation is just the opposite. D^2 annihilates the generalized eigen states both at space-like and light like 3-surfaces. Hence the roles of the physical and non-physical helicities are switched. It is the generalized eigen modes of D with non-vanishing eigenvalues λ, which code for the physics whereas the solutions of the modified Dirac equation define super gauge symmetries.

At the space-like 3-surfaces associated with 7-D CDs the spinor harmonics of the configuration space satisfy the $M^4 \times CP_2$ counterpart of the massless Dirac equation so that non-physical helicities are eliminated in the standard sense at the imbedding space level. The righthanded neutrino does not generate an $N = 1$ space-time super-symmetry contrary to the long held belief.

3.5 Super-symmetry at the level of configuration space

The gamma matrices of the configuration space are defined as matrix elements of properly chosen operators between right-handed neutrino and second quantized induced spinor field at space-like boundaries X^3. These generators define the fermionic generators of what I call super-canonical algebra. The right handed neutrino can be replaced with any spinor harmonic of the imbedding space to obtain an extended super-algebra, which can be used to construct the physical states.

The requirement that super-generators vanish for the vacuum extremals requires that the modified Dirac operator D_+ or the inverse of D_- appearing in the matrix element of the "Hermitian conjugate" $S^- = (S^+)^\dagger$ of the super charge S^+. Here $\pm$ refers to the negative and positive energy space-time sheets meeting at X^3 or to the two maximally deterministic space-time regions separated by the causal determinant. The operators D_+ and D_-^{-1} are restricted to the spinor modes not annihilated by $D_\pm$. The super-generator generated by the covariantly constant right handed neutrino vanishes identically: a more rigorous argument showing that $N = 1$ global super symmetry is indeed absent.

If the configuration space decomposes into a union of sectors labelled by unions of light cones having dips at arbitrary points of M^4, the spinor harmonics can be assumed to define plane waves in M^4 and even possess well-defined four-momenta and mass squared values. Same applies to the super-canonical generators defined by their commutators. This means that the generators of the super-canonical algebra generated in this manner would possess well defined four-momenta and thus their action would change the mass of the state. Space-time super-symmetries would be absent. Similar argument applies to the Kac Moody algebras associated with the light like 3-D CDs if super-canonical Super Kac-Moody algebras provide dual representations of quantum states.

If the gist of these admittedly heuristic arguments is correct, they force to modify drastically the existing view about space-time super-symmetries. The problem how to break super-symmetry disappears since there is no space-time symmetry be broken down. Super-symmetries are realized as a spectrum generating algebra rather than symmetries in the standard sense.

I hasten to admit that I have myself believed that right handed neutrino defines a global super-symmetry and proposed that the topological condensation of sparticles and particles at space-time sheets with different p-adic primes would provide an elegant model for super-symmetry breaking using same general mass formulas but only a different mass scale. Giving up this assumption causes however only a sigh of relief. The predicted spectrum of massless states is reduced dramatically [F3]. p-Adic mass calculations based on p-adic thermodynamics and representations of super-conformal algebra are not affected since the global $N = 1$ super-symmetry implies only an additional vacuum degeneracy. Most predictions of TGD remain intact. The speculation that sneutrinos might be light and play a role in TGD based condensed matter physics is the only possible exception [F9]. One can however consider the possibility of light colored sneutrinos obtained by applying to a neutrino state a colored and thus non-vanishing super-canonical generator defined by right handed antineutrino.

It deserves to be noticed that the notion super-symmetry in configuration space sense was discovered with the advent of super string models and generalized to a space-time super-symmetry when gauge theories made their breakthrough. The notion of spontaneous compactification (we meet our friend again and again!) inspired then the hypothesis that this super-symmetry has a space-time counterpart and everyone believed. There is now an entire industry making similar purely formal out of context applications and generalizations of quantum groups, which originally emerged naturally in knot and braid theory and in the theory of von Neumann algebras [E10, E9].

4 Generalization of the notion of imbedding space and the notion of number theoretic braid

This section is not directly related to the configuration space spinor structure but is necessary in order to represent the recent view about configuration space spinor structure. What follows summarizes the attempt to meet the following challenges.

1. Try to understand how the hierarchy of Planck constants is realized at the level of imbedding space and what quantum criticality for phase transitions changing Planck constant means.
2. Try to understand the notion of number theoretic braid in terms of quantum criticality. Identification as subset of real and p-adic variant of partonic 2-surface is not enough since some kind of inherent cutoff is needed to make the braid finite. Here the generalization of imbedding space comes in rescue and leads to an identification of two kinds of number theoretic braids assignable to phase transitions in M^4 and CP_2 degrees changing the value of Planck constant and corresponding symmetries. One should also be also able to identify the braiding operation.

4.1 Generalization of the notion of imbedding space

The original idea was that the proposed modification of the imbedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the imbedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of imbedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

4.1.1 Both covering spaces and factor spaces are possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. H_4 represents a straight cosmic string. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labelled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{M}^4 \times \hat{CP}_2$ implying that surfaces in $M^4 \times S^2$ and $M^2 \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

3. The covering spaces in question would correspond to the Cartesian products $\hat{M}^4{}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{M}^4$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing M^2 and S^2 with its orbit under G_a (say tedrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{M}^4 \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

4. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a

set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tedrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

5. Also the orbifolds $\hat{M}^4/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{M}^4/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{M}^4 \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the imbedding space to another one.

1. How the gluing of copies of imbedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are

very relevant for the understanding of phase transitions changing Planck constant.

4.1.2 Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{M}^2\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labelled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tedrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tedrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in the configuration space labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

5. How do the Planck constants associated with factors and coverings relate? One might argue that Planck constant defines a homomorphism respecting the multiplication and division (when possible) by G_i. If so, then Planck constant in units of $\hbar_0$ would be equal to n_a/n_b for $\hat{H}/G_a \times G_b$ option and n_b/n_a for $\hat{H}\hat{\times}(G_a \times G_b)$ with obvious formulas for hybrid cases. This option would put M^4 and CP_2 in a very symmetric role and allow much more flexibility in the identification of symmetries associated with large Planck constant phases.

4.2 Phase transitions changing the value of Planck constant

There are two basic kinds of phase transitions changing the value of Planck constant inducing a leakage between sectors of imbedding space. There are three cases to consider corresponding to

1. leakage in M^4 degrees of freedom changing G_a: the critical manifold is $R_+ \times CP_2$;

2. leakage in CP_2 degrees of freedom changing G_b: the critical manifold is $\delta M^4_+ \times S^2_{II}$;

3. leakage in both degrees of freedom changing both G_a and G_b: the critical manifold is $R_+ \times S^2_{II}$. This is the non-generic case

For transitions of type b) and c) X^2 must go through vacuum extremal in the classical picture about transition.

Covering space can also change to a factor space in both degrees of freedom or vice versa and in this case G can remain unchanged as a group although its interpretation changes.

The phase transitions satisfy also strong group theoretical constraints. For the transition $G_1 \rightarrow G_2$ either $G_1 \subset G_2$ or $G_2 \subset G_1$ must hold true. For maximal cyclic subgroups Z_n associated with quantization axes this means that n_1 must divide n_2 or vice versa. Hence nice number theoretic view about transitions emerges [A9].

One can classify the points of critical manifold according to the degree of criticality. Obviously the maximally critical points corresponds to fixed points of G_i that its points $z = 0, \infty$ of the spheres S^2_r and S^2_{II}. In the case of δM^4_+ the points $z = 0$ and ∞ correspond to the light-like rays R_+ in opposite directions. This ray would define the quantization direction of angular momentum. Quantum phase transitions changing the value of M^4 Planck constant could occur anywhere along this ray (partonic 2-surface would have 1-D projection along this ray). At the level of cosmology this would bring in a preferred direction. Light-cone dip, the counterpart of big bang, is the maximally quantum critical point since it remains invariant under entire group $SO(3,1)$.

Interesting questions relate to the groups generated by finite discrete subgroups of $SO(3)$. As noticed the groups generated as products of groups leaving R_+ invariant and three genuinely 3-D groups are infinite discrete subgroups of SO(3) and could also define Jones inclusions. In this case orbifold is replaced with orbifold containing infinite number of rotated versions of R_+. These phases could be important in elementary particle length scales or in early cosmology.

4.3 The identification of number theoretic braids

Number theoretic braids should be known once partonic surface and corresponding p-adic prime is known. Braid should belong to the intersection of real and p-adic variant of partonic 2-surface and the definition of should automatically give rise to a finite braid in case of non-vacuum extremals. Quantum criticality suggests that there are two kinds of braids. First kind of braid would relate to phase transitions changing G_a and would correspond to intersection of X^2 with R_+ and for given point in intersection would consist of points of CP_2 with same R_+ projection. Second kind of braid would relate to phase transitions changing G_b and correspond to intersection of X^2 with S^2_{II} and would consist for given point of points of S^2_r with same S^2_{II} coordinates.

4.3.1 Why a discrete set of points of partonic 2-surface must be selected?

As already noticed, p-adicization might provide a deeper motivation for the selection of discrete subset of points of partonic 2-surface in the construction of S-matrix elements in the case of non-diagonal transitions between different number fields.

1. The fusion of p-adic variants of TGD with real TGD, could be possible by algebraic continuation. This however requires the restriction of n-point functions to a finite set of algebraic points of X^2 with the usual stringy formula formula for S-matrix elements involving an integral over a circle of X^2 replaced with a sum over these points.

2. The same universal formula would give not only ordinary S-matrix elements but also those for p-adic-to-real transitions describing transformation intentions to actions. Quite generally, the formula would express S-matrix elements for transitions between two arbitrary number fields as algebraic numbers so that p-adicization of the theory would become trivial.

3. The interpretation of this finite set of points as a braid suggests a connection with the representation of Jones inclusions in terms of a hierarchy of braids [C7, E9] with the increasing number of strands meaning a continually improved finite-dimensional approximation of the hyper-finite factor of type II_1 identifiable as the Clifford algebra for the configuration space. The hierarchy of approximations for the hyper-finite factor would correspond to a genuine physical hierarchy of S-matrices corresponding to increasing dimension of algebraic extension of various p-adic numbers. This hierarchy would also define a cognitive hierarchy.

What could then be this discrete set of points having interpretation as a braid?

1. Number theoretical vision suggests that quantum TGD involves the sequence hyper-octonions $\rightarrow$ hyper-quaternions$\rightarrow$ complex numbers$\rightarrow$ reals $\rightarrow$ finite field $G(p,1)$ or of its algebraic extension. These reductions would define number theoretical counterparts of dimensional reductions. The points in the finite field $G(p,1)$ could be defined by p-adic integers modulo p so that a connection with p-adic numbers would emerge. Also more general algebraic extensions of p-adic numbers are allowed.

2. Number theoretical braids must belong to the intersection of real partonic 2-surface and its p-adic counterpart and thus the points must be algebraic points. Besides this a natural cutoff determined by X^2 itself is needed in order to have only finite number of points.

3. The generalization of the imbedding space inspired by the hierarchy of Planck constants suggest a very concrete identification of number theoretic braids in terms of intersections of partonic 2-surface and critical manifolds R_+ and S^2_{II} involving no ad hoc assumptions and giving braids having finite number of points.

4.3.2 Precise definition of number theoretical braid

The precise identification of number theoretic braids is quite demanding challenge. In the following three guesses are discussed.

1. Number theoretic approach

The progress in the understanding of physics as generalized number theory vision [E2] leads to a unique identification of number theoretic braids. The point is that associativity condition for N-point functions in hyper-octonionic imbedding space $HO = M^8$ is satisfied only if the arguments belong to a hyper-quaternionic plane $HQ = M^4 \subset M^8$. The intersection $M^4 \cap X^4$ is discrete in the generic case.

A further physically well-motivated assumption is that the pre-images of the partonic 2-surfaces X^2 at the boundaries of the causal diamond belong to M^4. This implies that the intersection of the commutative hyper-plane M^2 with X^2 consists of discrete set of points. If also the pre-images of X^3 belong to M^4, the intersections $X^3 \cap M^2$ are 1-dimensional curves and can be identified as strands of number theoretical braids. The M^4 projections of braids are in M^2 and thus planar containing intersections of strands: actual braiding takes place in CP_2 degrees of freedom so that genuine intersections are not present. There good hopes that rationality/algebraicity conditions for intersection points at partonic 2-surfaces can be satisfied with rather natural restrictions on X^2.

2. Number theoretic braids and extrema of Higgs

The identification of Higgs field as a purely geometrical object leads to the identification of intersection points as unstable extrema of negative valued Higgs potential for which Higgs vanishes. The stable minima correspond to the extrema in the vicinity of these maxima and correspond to non-vanishing Higgs field (the nearest valley for a peak of 2-D landscape defined on sphere). The minima (bottoms of valleys) define both physically and mathematically natural candidate for the number theoretical braids. At quantum criticality these braids approach to zero braids. An open question is whether this identification is consistent with the first one.

3. The first guess

The generalization of the notion of imbedding space leads to good guess for the identification of number theoretical braids.

What is clear that the points of number theoretic braid belong to the intersection of the real and p-adic variant of partonic 2-surfaces consisting of rationals and algebraic points in the extension used for p-adic numbers. The points of braid have same projection on an algebraic point of a geodesic sphere of $S^2 \subset CP_2$ belonging to the algebraic extension of rationals considered (the reader willing to understand the details can consult [C1]).

There are two different geodesic spheres in CP_2 and the homologically trivial geodesic sphere S^2_{II} is the most natural choice from the point of view of the generalized imbedding space since $M^2 \times S^2_{II}$, which defines the intersection of all sectors of H, is a vacuum extremal so that the ill-definedness of Planck constant does not matter. Note that also the M^4 part of the metric is discontinuous at $M^2 \times S^2_{II}$.

One can argue that algebraicity condition is not strong enough and gives too many points unless one introduces a cutoff in some manner. Since TQFT like theory can naturally assigned with the partonic 2-surfaces in $M^2 \times S^2_{II}$, the natural identification of the intersection points of number theoretical braids with $\delta M^4_\pm \times CP_2$ would be as the intersection of the 2-D CP_2 projection of the partonic 2-surface in $\delta M^4_\pm \times CP_2$ with S^2_{II}. In the generic case the intersection would consist of discrete points and for non-vacuum extremals this would certainly be the case. The intersection should consist of algebraic points allowing also p-adic interpretation: the condition that CP_2 projection is an algebraic surface is a necessary condition for this.

The points of braid are obtained as solutions of polynomial equation and thus one can assign to them a Galois group permuting the points of the braid. In this case finite Galois group could be realized as left or right translation or conjugation in S_∞ or in braid group.

To make the notion of number theoretic braid more concrete, suppose that the complex coordinate w of $\delta M^4_\pm$ is expressible as a polynomial of the complex coordinate z of CP_2 geodesic sphere and the radial light-like coordinate r of $\delta M^4_\pm$ is obtained as a solution of polynomial equation $P(r,z,w) = 0$. By substituting w as a polynomial $w = Q(z,r)$ of z and r this gives polynomial equation $P(r,z,Q(z,r)) = 0$ for r for a given value of z. Only real roots can be accepted. Local Galois group (in a sense different as it is used normally in literature) associated with the algebraic point of S^2 defining the number theoretical braid is thus well defined.

If the partonic 2-surface involves all roots of an irreducible polynomial, one indeed obtains a braid for each point of the geodesic sphere $S^2 \subset CP_2$. In this case the action of Galois group is naturally a braid group action realized as the action on induced spinor fields and configuration space spinors.

The choice of the points of braid as points common to the real and p-adic partonic 2-surfaces would be unique so that the obstacle created by the fact that the finite Galois group as function of point of S^2 fluctuates wildly (when some roots become rational Galois group changes dramatically: the simplest example is provided by $y - x^2 = 0$ for which Galois group is Z_2 when y is not a square of rational and trivial group if y is rational).

4.3.3 What is the fundamental braiding operation?

The basic quantum dynamics of TGD could define the braiding operation for the braid defined by a discrete set of points of X^2 satisfying the algebraicity conditions. I have considered several candidates for braiding operation and the situation is still partially unsettled.

One promising candidate for the braiding operation is found by observing that both Kähler gauge potential and Kähler magnetic field define flows at light-like partonic 3-surface. The dual of the induced Kähler form defines a conserved topological current, whose flow lines are field lines of the Kähler magnetic field in the light like direction. This flow is incompressible. Vector potential defines also a flow in the interior of space-time surface, and Chern-Simons action at partonic 3-surface defines a topological invariant of this flow known as helicity in hydrodynamics. The non-gauge invariance of helicity is not a problem since symplectic transformations of CP_2 do not define gauge degeneracy but spin glass degeneracy. The flow defined by the vector potential is perhaps the most attractive option but one cannot exclude the possibility that the braids defined by both flows play a role in the definition of S-matrix. Number theoretical braid (tangle if flow line fuse or split) would correspond to the unique orbit for the points of the number theoretic braid at the initial partonic 2-surface. The points of the braid would be algebraic only in suitably chosen discrete time slices but this would not lead to a loss of uniqueness. Hence cobordism would become discrete. This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]). This picture makes sense also for macroscopic 2-surfaces defining outer boundaries of physical systems (quantum Hall effect and topological quantum computation [E9]).

The second candidate for the braiding operation emerges naturally when one identifies the points defining the number theoretic braid in terms of minima of Higgs field defined on X^2 (the details of this identification are discussed later). In this case time evolution takes minima to minima and can induce braiding for the projections of the points of braid to S^2_{II} *resp.* S^2_r. One might hope that the braiding associated with S^2_r *resp.* S^2_{II} is topologically equivalent with the braiding defined by Kähler gauge potential *resp.* Kähler magnetic field.

5 Does the modified Dirac action define the fundamental action principle?

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography. The Dirac determinant associated with the modified Dirac action is an excellent candidate in this respect.

5.1 Modified Dirac equation

In the following the problems of the ordinary Dirac action are discussed and the notion of modified Dirac action is introduced. In particular, the following problems are discussed.

1. Try to guess general formula for the spectrum of the modified Dirac operator and for super-canonical conformal weights by assuming that the eigenvalues are expressible in terms of the data assignable to the two kinds of of number theoretical braids and that the product of vacuum functional expressible as exponent of Kahler function and of the exponent of Chern-Simons action is identifiable as Dirac determinant expressible as product of M^4 and CP_2 parts. Since Kähler function is isometry invariant only the Dirac determinant defined by M^4 braid can contribute to it.

Chern-Simons action is not isometry invariant and can be identified as the Dirac determinant associated with CP_2 braid.

2. Try to understand whether the zeta functions involved can be identified as Riemann Zeta or some zeta coding geometric data about partonic 2-surface. Try to understand whether the assignment of a fixed prime p to a partonic 2-surface implies that the zeta function is actually an analog for basic building block of Riemann Zeta.

5.1.1 Problems associated with the ordinary Dirac action

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or a more general principle selecting preferred extremals as Bohr orbits [E2].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the configuration space geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of the configuration space geometry so that there is internal inconsistency.

5.1.2 Super-symmetry forces modified Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$D_\alpha T^\alpha_k = 0 \ ,$$

$$T_k^{\alpha} = \frac{\partial}{\partial h_{\alpha}^k} L_K \quad . \tag{21}$$

If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R}\Gamma^k T_l^{\alpha}\Gamma^l \Psi \quad , \\ D_{\alpha} J^{\alpha k} &= 0 \quad . \end{aligned} \tag{22}$$

having a vanishing covariant divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^{\alpha} = \overline{\nu_R} T_l^{\alpha}\Gamma^l \Psi \tag{23}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_{\alpha} J^{\alpha k} = \overline{\nu_R}\Gamma^k T_l^{\alpha}\Gamma^l D_{\alpha}\Psi \quad . \tag{24}$$

The requirement that this current vanishes is guaranteed if one assumes that modified Dirac equation

$$\begin{aligned} \hat{\Gamma}^{\alpha} D_{\alpha}\Psi &= 0 \quad , \\ \hat{\Gamma}^{\alpha} &= T_l^{\alpha}\Gamma^l \quad . \end{aligned} \tag{25}$$

This equation must be derivable from a modified Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^{\alpha} D_{\alpha}\Psi \quad . \tag{26}$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with effective induced gamma matrices and the requirement

$$D_{\mu}\hat{\Gamma}^{\mu} = 0 \tag{27}$$

guaranteing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

5.1.3 How can one avoid minimal surface property?

These observations suggest how to avoid the emergence of the minimal surface property as a consequence of field equations. It is not induced metric which appears in field equations. Rather, the effective n metric appearing in the field equations is defined by the anti-commutators of $\hat{\gamma}_\mu$

$$\hat{g}_{\mu\nu} = \{\hat{\Gamma}_\mu, \hat{\Gamma}_\nu\} = 2T^k_\mu T_{\nu k} \ . \tag{28}$$

Here the index raising and lowering is however performed by using the induced metric so that the problems resulting from the non-invertibility of the effective metric are avoided. It is this dynamically generated effective metric which must appear in the number theoretic formulation of the theory.

Field equations state that space-time surface is minimal surface with respect to the effective metric. Note that a priori the choice of the bosonic action principle is arbitrary. The requirement that effective metric defined by energy momentum tensor has only non-diagonal components except in the case of non-light-like coordinates, is satisfied for the known solutions of field equations.

5.1.4 Does the modified Dirac action define the fundamental action principle?

There is quite fundamental and elegant interpretation of the modified Dirac action as a fundamental action principle discussed also in [E2]. In this approach vacuum functional can be defined as the Grassmannian functional integral associated with the exponent of the modified Dirac action. This definition is invariant with respect to the scalings of the Dirac action so that theory contains no free parameters.

An alternative definition is as a Dirac determinant which might be calculated in TGD framework without applying the poorly defined functional integral. There are good reasons to expect that the Dirac determinant exponent of Kähler function for a preferred Bohr orbit like extremal of the Kähler action with the value of Kähler coupling strength coming out as a prediction. Hence the dynamics of the modified Dirac action at light-like partonic 3-surfaces X^3_l, even when restricted to almost-topological dynamics induced by Chern-Simons action, would dictate the dynamics at the interior of the space-time sheet.

The knowledge of the canonical currents and super-currents, together with the anti-commutation relations stating that the fermionic super-currents S_A and S_B associated with Hamiltonians H_A and H_B anti-commute to a bosonic current $H_{[A,B]}$, allows in principle to deduce the anti-commutation relations satisfied by the induced spinor field. In fact, these conditions replace the usual anti-commutation relations used to quantize free spinor field. Since the normal ordering of the Dirac action would give Kähler action,

Kähler coupling strength would be determined completely by the anti-commutation relations of the super-canonical algebra. Kähler coupling strength would be dynamical and the selection of preferred extremals of Kähler action would be more or less equivalent with quantum criticality because criticality corresponds to conformal invariance and the hyper-quaternionic version of the super-conformal invariance results only for the extrema of Kähler action. p-Adic (or possibly more general) coupling constant evolution and quantum criticality would come out as a prediction whereas in the case that Kähler action is introduced as primary object, the value of Kähler coupling strength must be fixed by quantum criticality hypothesis.

The mixing of the M^4 chiralities of the imbedding space spinors serves as a signal for particle massivation and breaking of super-conformal symmetry. The induced gamma matrices for the space-time surfaces which are deformations of M^4 indeed contain a small contribution from CP_2 gamma matrices: this implies a mixing of M^4 chiralities even for the modified Dirac action so that there is no need to introduce this mixing by hand.

5.2 The association of the modified Dirac action to Chern-Simons action and explicit realization of super-conformal symmetries

Super Kac-Moody symmetries should correspond to solutions of modified Dirac equation which are in some sense holomorphic. The discussion below is based on the same general ideas but differs radically from the previous picture at the level of details. The additional assumption inspired by the considerations of this section is that the action associated with the partonic 3-surfaces is non-singular and therefore Chern-Simons action for the induced Kähler gauge potential.

This means that TGD is at the fundamental level almost-topological QFT: only the light-likeness of the partonic 3-surfaces brings in the induced metric and gravitational and gauge interactions and induces the breaking of scale and super-conformal invariance. The resulting theory possesses the expected super Kac-Moody and super-canonical symmetries albeit in a more general form than suggested by the considerations of this section. A connection of the spectrum of the modified Dirac operator with the zeros or Riemann Zeta is suggestive and provides support for the earlier number theoretic speculations concerning the spectrum of super-canonical conformal weights. One can safely say, that if this formulation is correct, TGD could not differ less from a physically trivial theory.

5.2.1 Zero modes and generalized eigen modes of the modified Dirac action

Consider net the zero modes and generalized eigen modes for the modified Dirac operator.

1. The modified gamma matrices appearing in the modified Dirac equation are expressible in terms of the Lagrangian density L assignable to the light-like partonic 3-surface X^43_l as

$$\hat{\Gamma}^{\alpha} = \frac{\partial L}{\partial_{\alpha} h^k} \Gamma_k \ , \tag{29}$$

where Γ_k denotes gamma matrices of imbedding space. The modified Dirac operator is defined as

$$D = \hat{\Gamma}^{\alpha} D_{\alpha} \ , \tag{30}$$

where D_{α} is the covariant derivative defined by the induced spinor connection. Modified gamma matrices satisfy the condition

$$D_{\alpha} \hat{\Gamma}^{\alpha} = 0 \tag{31}$$

if the field equations associated with L are satisfied. This guarantees that one indeed obtains the analog of the massless Dirac equation. Zero modes of the modified Dirac equation should define the conformal supersymmetries.

2. The generalized eigenvalues and eigen solutions of the modified Dirac operator are defined as

$$\begin{aligned} D\Psi &= \lambda N \Psi \ , \\ N &= n^k \Gamma_k \ . \end{aligned} \tag{32}$$

Here n^k denotes a light-like vector which must satisfy the integrability condition

$$\left[D, n^k \Gamma_k \right] = 0 \ . \tag{33}$$

if the analog $D^2\Psi = 0$ for the square of massless Dirac equation is to hold true. n^k should be determined by the field equations associated with L somehow and commutativity condition could fix n more or less uniquely.

If the commutativity condition holds true then any generalized eigen mode Ψ_{λ} gives rise to a zero mode as $\Psi = N\Psi_{\lambda}$. One can add to a given non-zero mode any superposition of zero modes without affecting the generalized eigen mode property.

The commutativity condition can be satisfied if the tangent space at each point of X^4 contains preferred plane M^2 guaranteing $HO-H$ duality and having interpretation as a preferred plane of non-physical polarizations. In this case n can be chosen to be constant light-like vector in M^2.

3. The hypothesis is that Kähler function is expressible in terms of the Dirac determinant of the modified Dirac operator defined as the product of the generalized eigenvalues. The Dirac determinant must carry information about the interior of the space-time surface determined as preferred extremal of Kähler action or (as the hypothesis goes) as hyper-quaternionic or co-hyper-quaternionic 4-surface of M^8 defining unique 4-surface of $M^4 \times CP_2$. The assumption that X^3_L is light-like brings in an implicit dependence on the induced metric. The simplest but non-necessary assumption is that n^k is a light-like vector field tangential to X^3_l so that the knowledge of X^3_l fixes completely the dynamics.

4. If the action associated with the partonic light-like 3-surfaces contains induced metric, the field equations become singular and ill-defined unless one defines the field equations at X^3_l via a limiting procedure and poses additional conditions on the behavior of Ψ at X^3_l. Situation changes if the action does not contain the induced metric. The classical field equations are indeed well-defined at light-like partonic 3-surfaces for Chern-Simons action for the induced Kähler gauge potential

$$L = L_{C-S} = k\epsilon^{\alpha\beta\gamma}J_{\alpha\beta}A_\gamma \ . \tag{34}$$

One obtains the analog of WZW model with gauge field replaced with the induced Kähler form. This action does not depend on the induced metric explicitly so that in this sense a topological field theory results. There is no dependence on M^4 gamma matrices so that local Lorentz transformations act as super-conformal symmetries of both classical field equations and modified Dirac equation and $SL(2,C)$ defines the analog of the $SU(2)$ Kac-Moody algebra for $N = 4$ SCA.

The facts that the induced metric is light-like for X^3_l, that the modified Dirac equation contains information about this and therefore about induced metric, and that Dirac determinant is the product of the non-vanishing eigen values of the modified Dirac operator, imply the failure of topological field theory property at the level of Kähler function identified as the logarithm of the Dirac determinant.

A more complicated option would be that the modified Dirac action contains also interior term corresponding to the Kähler action. This alternative would break super-conformal symmetries explicitly and almost-topological QFT property would be lost. This option is not consistent with the idea that quantum-classical correspondence relates the partonic dynamics at X^3_l with the classical dynamics in the interior of space-time providing first principle justification for the basic assumptions of the quantum measurement theory.

The classical field equations defined by L_{C-S} read as

$$D_\mu \frac{\partial L_{C-S}}{\partial_\mu h^k} = 0 \ ,$$

$$\frac{\partial L_{C-S}}{\partial_\mu h^k} = \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k)\right] \quad . \tag{35}$$

From the explicit form of equations it is obvious that the most general solution corresponds to a X^3_l with at most 2-dimensional CP_2 projection.

Although C-S action vanishes, the color isometry currents are in general non-vanishing. One can assign currents also to super-Kac Moody and super-canonical transformations using standard formulas and the possibility that the corresponding charges define configuration space Hamiltonians and their super-counterparts must be considered seriously.

Suppose that the CP_2 projection is 2-dimensional and not a Lagrange manifold. One can introduce coordinates for which the coordinates for X^2 are same as those for CP_2 projection. For instance, complex coordinates $(z, \overline{z})$ of a geodesic sphere could be used as local coordinates for X^2. One can also assign one M^4 coordinate, call it r, with M^4 projection X^1 of X^3_l. Locally this coordinate can be taken to be one of the standard M^4 coordinates. The remaining five H-coordinates can be expressed in terms of $(r, z, \overline{z})$ and light-likeness condition boils down to the vanishing of the metric determinant:

$$det(g_3) = 0 \ . \tag{36}$$

All diffeomorphisms of H respecting the light-likeness condition are symmetries of the solution ansatz.

Consider some special cases serve as examples.

1. The simplest situation results when X^4_l is of form $X^1 \times X^2$, where X^1 is light-like random curve in M^4 as for CP_2 type vacuum extremals. In this case light-likeness boils down to Virasoro conditions with real parameter r playing the role analogous to that of a complex coordinate: this conformal symmetry is dynamical and must be distinguished from conformal symmetries assignable to X^2. A plausible guess is that light-likeness condition quite generally reduces to the classical Virasoro conditions.

2. A solution in which CP_2 projection is dynamical is obtained by assuming that for a given value of M^4 time coordinate CP_2- and M^4- projections are one-dimensional curves. For instance, CP_2 projection could be the circle $\Theta = \Theta(m^0 \equiv t)$ whereas M^4 projection could be the circle $\rho = \sqrt{x^2 + y^2} = \rho(m^0)$. Light-likeness condition reduces to the condition $g_{tt} = 1 - R^2\partial_t\Theta^2 - \partial_t\rho^2 = 0$.

5.2.2 Classical field equations for the modified Dirac equation defined by Chern-Simons action

The modified Dirac operator is given by

$$D = \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k D_\mu$$

$$\begin{aligned} &= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ \hat{\epsilon}^{\alpha\beta\gamma} &= \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \tag{37}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric. The operator is non-trivial only for 3-surfaces for which CP_2 projection is 2-dimensional non-Lagrangian sub-manifold. The modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{38}$$

The solutions of the modified Dirac equation are obtained as spinors which are covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{39}$$

Non-vanishing spinors $\Psi_1 = \partial_r\Psi$ satisfying $\Gamma_r\Psi_1 = 0$ are not possible. Ψ defines super-symmetry for the generalized eigen modes if the additional condition

$$\Psi = N\Psi_0 \tag{40}$$

is satisfied. The interpretation as super-conformal symmetries makes sense if the Fourier coefficients of zero modes and their conjugates are anticommuting Grassmann numbers. The zero modes which are not of this form do not generate super-conformal symmetries and might correspond to massless particles. TGD based vision about Higgs mechanism suggest the interpretation of n^k as a non-conserved gravitational four-momentum whose time average defines inertial four-momentum of parton. The sum of the partonic four-momenta would be identified as the classical four-momentum associated with the interior of the space-time sheet.

The covariant derivatives D_α involve only CP_2 spinor connection and the metric induced from CP_2. D_r involves CP_2 spinor connection unless X_l^3 is of form $X^1 \times X^2 \subset M^4 \times CP_2$. The eigen modes of D correspond to the solutions of

$$D\Psi = \lambda N\Psi \tag{41}$$

The first guess is that $N = n^k\gamma_k$ corresponds to the tangential light-like vector $n^k = \Phi\partial_r h^k$ where Φ is a normalization factor which can depend on position.

The obvious objection is that with this assumption it is difficult to understand how Dirac determinant can correspond to an absolute extremum of Kähler action for 4-D space-time sheet containing partonic 3-surfaces as causal determinants ($\sqrt{g_4} = 0$). However, if one can select a unique M^4 time coordinate, say as that associated with the rest system for the average four-momentum defined as Chern-Simons Noether charge, then one can assign to n^k a unique dual obtained by changing the sign of its spatial components. The condition that this vector is tangential to the 4-D space-time sheet would provide information about the space-time sheet and bring in 4-dimensionality. At this stage one must however leave the question about the choice of n^k open.

One should be able to fix Φ apart from overall normalization. First of all, the requirement that zero modes defines super symmetries implies the condition $[D, n^k\Gamma_k]\Psi = 0$ for zero modes. This condition boils down to the requirement

$$D_r(\Phi\partial_r h^k\Gamma_k)\Psi = 0 \ . \tag{42}$$

This in turn boils down to a condition

$$D_r\partial_r h^k + \frac{\partial_r\Phi}{\Phi}\partial_r h^k = 0 \ . \tag{43}$$

These conditions in turn guarantee that the condition

$$D_r(h_{kl}\partial_r h^k\partial_r h^l) = 0 \tag{44}$$

implied by the light-likeness condition are satisfied. Since Φ is determined apart from a multiplicative constant from the light-likeness condition the system is internally consistent. The conditions above are not general coordinate invariant so that the coordinate r must correspond to a physically preferred coordinate perhaps defined by the conditions above.

One can express the eigenvalue equation in the form

$$\begin{aligned} \partial_r\Psi &= \lambda O\Psi \ , \\ O &= (\hat{\Gamma}^r)^{-1}N \ , \\ (\hat{\Gamma}^r)^{-1} &= \frac{\hat{\Gamma}^r}{a^k a^l h_{kl}} \ , \ \hat{\Gamma}^r \equiv a^k\Gamma_k \ . \end{aligned} \tag{45}$$

This equation defines a flow with r in the role of a time parameter. The solutions of this equation can be formally expressed as

$$\Psi(r,z,\overline{z}) = Pe^{\lambda\int O(r,z,\overline{z})dr}\Psi_0(z,\overline{z}) \ . \tag{46}$$

Here P denotes the ordered exponential needed because the operators $O(r, z\overline{z})$ need not commute for different values of r.

5.2.3 Can one allow light-like causal determinants with 3-D CP_2 projection?

The standard quantum field theory wisdom would suggest that light-like partonic 3-surfaces which are extremals of the Chern-Simons action correspond only to what stationary phase approximation gives when vacuum functional is the product of exponent of Kähler function resulting from Dirac determinant and an imaginary exponent of Chern-Simons action whose coefficient is proportional to the central charge of Kac-Moody algebras associated with CP_2 degrees of freedom.

One cannot exclude the possibility that 3-D light-like causal determinants might be required by the general consistency of the theory. The identification of the exponent of Kähler function as Dirac determinant remains a viable hypothesis for this option. "Off mass shell" breaking of super-conformal symmetries is implied since modified Dirac equation implies the conservation of super conformal currents only when CP_2 projection is at most 2-dimensional.

5.2.4 Some problems of TGD as almost-topological QFT and their resolution

There are some problems involved with the precise definition of the quantum TGD as an almost-topological QFT at the partonic level and the resolution of these problems leads to an unexpected connection between cosmology and parton level physics.

1. Three problems

The proposed view about partonic dynamics is plagued by three problems.

1. The definition of supercanonical and super-Kac-Moody charges in M^4 degrees of freedom poses a problem. These charges are simply vanishing since M^4 coordinates do not appear in field equations.

2. Classical field equations for the C-S action imply that this action vanishes identically which would suggest that the dynamics does not depend at all on the value of k. The central extension parameter k determines the over-all scaling of the eigenvalues of the modified Dirac operator. $1/k$-scaling occurs for the eigenvalues so that Dirac determinant scales by a finite power k^N if the number N of the allowed eigenvalues is finite for the algebraic extension considered. A constant $Nlog(k)$ is added to the Kähler function and its effect seems to disappear completely in the normalization of states.

3. The general picture about Jones inclusions and the possibility of separate Planck constants in M^4 and CP_2 degrees of freedom suggests a close symmetry between M^4 and CP_2 degrees of freedom at the partonic level. Also in the construction of the geometry for the world of classical worlds the symplectic and Kähler structures of both light-cone boundary and

CP_2 are in a key role. This symmetry should be somehow coded by the Chern-Simons action.

2. A possible resolution of the problems

A possible cure to the above described problems is based on the modification of Kähler gauge potential by adding to it a gradient of a scalar function Φ with respect to M^4 coordinates.

1. This implies that super-canonical and super Kac-Moody charges in M^4 degrees of freedom are non-vanishing.

2. Chern-Simons action is non-vanishing if the induced CP_2 Kähler form is non-vanishing. If the imaginary exponent of C-S action multiplies the vacuum functional, the presence of the central extension parameter k is reflected in the properties of the physical states.

3. The function Φ could code for the value of $k(M^4)$ via a proportionality constant

$$\Phi = \frac{k(M^4)}{k(CP_2)}\Phi_0 \ , \tag{47}$$

   Here $k(CP_2)$ is the central extension parameter multiplying the Chern-Simons action for CP_2 Kähler gauge potential. This tricks does just what is needed since it multiplies the Noether currents and super currents associated with M^4 degrees of freedom with $k(M^4)$ instead of $k(CP_2)$.

The obvious breaking of $U(1)$ gauge invariance looks strange at first but it conforms with the fact that in TGD framework the canonical transformations of CP_2 acting as $U(1)$ gauge symmetries do not give to gauge degeneracy but to spin glass degeneracy since they act as symmetries of only vacuum extremals of Kähler action.

3. How to achieve Lorentz invariance?

Lorentz invariance fixes the form of function Φ uniquely as the following argument demonstrates.

1. Poincare invariance would be broken in any case for a given light-cone in the decomposition $CH = \cup_m CH_m$ of the configuration space to sub-configuration spaces associated with light-cones at various locations of M^4 but since the functions Φ associated with various light cones would be related by a translation, translation invariance would not be lost.

2. The selection of Φ should not break Lorentz invariance. If Φ depends on the Lorentz proper time a only, this is partially achieved. Momentum

currents would be proportional to m^k and become light like at the boundary of the light-cone. This fits very nicely with the interpretation that the matter emanates from the tip of the light cone in Robertson-Walker cosmology.

Lorentz invariance poses even stronger conditions on Φ.

1. Partonic four-momentum defined as Chern-Simons Noether charge is definitely not conserved and must be identified as gravitational four-momentum whose time average corresponds to the conserved inertial four-momentum assignable to the Kähler action [D3, D5]. This identification is very elegant since also gravitational four-momentum is well-defined although not conserved.

2. Lorentz invariance implies that mass squared is constant of motion. Hence it is interesting to look what expression for Φ results if the gravitational mass defined as Noether charge for C-S action is conserved. The components of the four-momentum for Chern-Simons action are given by

$$P^k = \frac{\partial L_{C-S}}{\partial(\partial_\alpha a)} m^{kl}\partial_{m^l} a \ .$$

   Chern-Simons action is proportional to $A_\alpha = A_a \partial_\alpha a$ so that one has

$$P^k \propto \partial_a \Phi \partial_{m^k} a = \partial_a \Phi \frac{m^k}{a} \ .$$

   The conservation of gravitational mass gives $\Phi \propto a$. Since CP_2 projection must be 2-dimensional, M^4 projection is 1-dimensional so that mass squared is indeed conserved.

   Thus one could write

$$\Phi = \frac{k(M^4)}{k(CP_2)} x\theta(a)\frac{a}{R} \ , \tag{48}$$

   where R the radius of geodesic sphere of CP_2 and x a numerical constant which could be fixed by quantum criticality of the theory. Chern-Simons action density does not depend on a for this choice and this independence guarantees that the earlier ansatz satisfies field equations. The presence of the step function $\theta(a)$ tells that Φ is non-vanishing only inside light-cone and gives to the gauge potential delta function term which is non-vanishing only at the light-cone boundary and makes possible massless particles.

3. If M^4 projection is 1-dimensional, only homologically charged partonic 3-surfaces can carry gravitational four-momentum. This is not a problem since M^4 projection can be 2-dimensional in the general case. For CP_2

type extremals, ends of cosmic strings, and wormhole contacts the non-vanishing of homological charge looks natural. For wormhole contacts 3-D CP_2 projection suggests itself and is possible only if one allows also quantum fluctuations around light-like extremals of Chern-Simons action. The interpretation could be that for a vanishing homological charge boundary conditions force X^4 to approach vacuum extremal at partonic 3-surfaces.

This picture does not fit completely with the picture about particle massivation provided by CP_2 type extremals. Massless partons must correspond to 3-surfaces at light-cone boundary in this picture and light-likeness allows only linear motion so that inertial mass defined as average must vanish.

5. Comment about quantum classical correspondence

The proposed general picture allows to define the notion of quantum classical correspondence more precisely. The identification of the time average of the gravitational four-momentum for C-S action as a conserved inertial four-momentum associated with the Kähler action at a given space-time sheet of a finite temporal duration (recall that we work in the zero energy ontology) is the most natural definition of the quantum classical correspondence and generalizes to all charges.

In this framework the identification of gravitational four-momentum currents as those associated with 4-D curvature scalar for the induced metric of X^4 could be seen as a phenomenological manner to approximate partonic gravitational four-momentum currents using macroscopic currents, and the challenge is to demonstrate rigorously that this description emerges from quantum TGD.

For instance, one could require that at a given moment of time the net gravitational four-momentum of $Int(X^4)$ defined by the combination of the Einstein tensor and metric tensor equals to that associated with the partonic 3-surfaces. This identification, if possible at all, would certainly fix the values of the gravitational and cosmological constants and it would not be surprising if cosmological constant would turn out to be non-vanishing.

5.2.5 The eigenvalues of D as complex square roots of conformal weight and connection with Higgs mechanism?

An alternative interpretation for the eigenvalues of D emerges from the TGD based description of particle massivation. The eigenvalues could be interpreted as complex square roots of conformal weights in the sense that $|\lambda|^2$ would have interpretation as a conformal weight. There is of course the possibility of numerical constant of proportionality.

The physical motivation for the interpretation is that λ is in the same role as the mass term in the ordinary Dirac equation and thus indeed square root of mass squared proportional to the conformal weight. The vacuum expectation of Higgs would correspond to that for λ and Higgs contribution to the mass squared would correspond to the p-adic thermodynamical expectation value $\langle|\lambda|^2\rangle$ [A9]. Additional contributions to mass squared would come from super conformal and modular degrees of freedom. The interpretation of the generalized eigenvalue as a Higgs field is also natural because the generalized eigen values of the modified Dirac operator can depend on position.

5.2.6 Super-conformal symmetries

The topological character of the solutions spectrum makes possible the expected and actually even larger conformal symmetries in X^2 degrees of freedom. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the canonical transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the canonical transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

Diffeomorphisms of M^4 respecting the light-likeness condition define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-analytic deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2,C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only. Also electro-weak $U(2)$ acts as X^2-local Kac-Moody algebra of symmetries. Hence all the desired Kac-Moody symmetries are realized.

The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D to a given eigen mode. This defines a symmetry if zero modes satisfy the additional condition $N\Psi = 0$ implied by $\Psi = N\Psi_0$ in turn guaranteed by the already described conditions. These symmetries are super-conformal symmetries with respect to z and $\overline{z}$.

The radial conformal symmetries generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-canonical conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_\pm$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X^3_l carrying information about H metric makes possible these symmetries.

$N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2) \times SU(2) \times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

5.2.7 How the super-conformal symmetries of TGD relate to the conventional ones?

The representation of super-symmetries as an addition of anticommuting zero modes to the second quantized spinor field defined by the superposition of non-zero modes of the modified Dirac equation differs radically from the standard realization based on the replacement of the world sheet or target space coordinates with super-coordinates. Also the fundamental role of the generalized eigen modes of the modified Dirac operator is something new and absolutely essential for the understanding of how super-conformal invariance is broken: the breaking of super-symmetries is indeed the basic problem of the super-string theories.

Since the spinor fields in question are not Majorana spinors the standard super-field formalism cannot work in TGD context. It is however interesting to look to what extent this formalism generalizes and whether it allows some natural modification allowing to formally integrate the notions of the bosonic action and corresponding modified Dirac action.

1. One can consider the formal introduction of super fields by replacing of X^3_l coordinates by super-coordinates requiring the introduction of anticommuting parameters θ and $\overline{\theta}$ transforming as H-spinors of definite chirality, which is not consistent with Majorana condition. Using real coordinates x^α for X^3_l, one would have

   $$x^\alpha \rightarrow X^\alpha = x^\alpha + \overline{\theta}\hat{\Gamma}^\alpha\Psi + \overline{\Psi}\hat{\Gamma}^\alpha\theta \ ,$$

   Super-conformal symmetries would add to θ a zero mode with Grassmann number valued coefficient. The replacement $z^\alpha \rightarrow X^\alpha$ for the arguments of CP_2 and M^4 coordinates would super-symmetrize the field C-S action density. As a matter fact, the super-symmetrization is non-trivial only in radial degree of freedom since only $\hat{\Gamma}^r$ is non-vanishing.

2. Also imbedding space coordinates could be formally replaced with super-fields using a similar recipe and super-symmetries would act on them. The topological character of Chern-Simons action would allow the super-symmetries induced by the translation of θ by an anticommuting zero mode as formal symmetries at the level of the imbedding space. In both cases it is however far from clear whether the formal super-symmetrization has any real physical meaning.

3. The notion of super-surface suggests itself and would mean that imbedding space Θ parameters are functions of single θ parameter assignable with X^3_l. A possible representation of super-part of the imbedding is a generalization of ordinary imbedding in terms of constraints $H_{i)}(h^k) = 0$, $i = 1,2....$ Symmetries allow only linear functions so that one would have

   $$c^\alpha_{i)}(r, z, \overline{z})\Theta_\alpha = 0 \ .$$

A hyper-plane in the space of theta parameters is obtained. Since only single theta parameter is possible in integral the number of constraints is seven and one obtains the modified Dirac action from the super-space imbedding.

Consider next the basic difficulty and its resolution.

1. The super-conformal symmetries do not generalize to the level of action principle in the standard sense of the word and the reason is the failure of the Majorana property forced by the separate conservation of quark and lepton numbers so that the standard super-space formalism remains empty of physical content.

2. One can however consider the modification of the integration measure $\prod_i d\theta_i d\overline{\theta}_i$ over Grassmann parameters by replacing the product of bilinears with

$$d\overline{\theta}\gamma_1 d\theta d\overline{\theta}\gamma_2 d\theta ...$$

   analogous to the product $dx^1 \wedge dx^2 ...$ (where γ^k would be gamma matrices of the imbedding space) transforming like a pseudoscalar. It seems that the replacement of product with wedge product leads to a trivial theory. This formalism could work for super fields obeying Weyl condition instead of Majorana condition and it would be interesting to find what kind of super-symmetric field theories it would give rise to.

   The requirement that the number of Grassmann parameters given by $2D$ is the number of spinor components of definite chirality (counting also conjugates) given by $2 \times 2^{D/2-1}$ gives critical dimension $D = 8$, which suggest that this kind of quantum field theory might exist. As found, the zero modes which are not of form $\Psi = N\Psi_0$ do not generate super-conformal symmetries in the strict sense of the word and might correspond to light particles. One could ask whether chiral SUSY in $M^4 \times CP_2$ might describe the low energy dynamics of corresponding light parton states. General arguments do not however support space-time super-symmetry.

3. Because of the light-likeness the super-symmetric variant of C-S action should involve the modified gamma matrices $\hat{\Gamma}^\alpha$ instead of the ordinary ones. Since only $\hat{\Gamma}^r$ is non-vanishing for the extremals of C-S action and since super-symmetrization takes place for the light-like coordinate r only, the integration measure must be defined as $d\overline{\theta}\hat{\Gamma}_r d\theta$, with θ perhaps assignable to a fixed covariantly constant right-handed neutrino spinor and $\hat{\Gamma}_r$ the inverse of $\hat{\Gamma}^r$. This action gives rise to the modified Dirac action with the modified gamma matrices emerging naturally from the Taylor expansion of the C-S action in powers of super-coordinate.

5.3 Why the cutoff in the number superconformal weights and modes of D is needed?

Two kinds of cutoffs are necessary in the number theoretic approach involving a hierarchy of algebraic extensions of rationals with increasing algebraic dimension.

5.3.1 Spatial cutoff realized in terms of number theoretical braids

The first cutoff corresponds to a spatial discretization selecting a subset of algebraic points of the partonic 2-surface X^2 as a subset of the points common to the real and p-adic variants of X^2 obeying the same algebraic equations. Almost topological field theory property allows to assume algebraic equations and also quantum criticality and generalization of the imbedding space concept are crucial for achieving the cutoff as a completely inherent property of X^2.

5.3.2 Cutoff in the number of super-canonical conformal weights

It is not quite clear whether the number of radial conformal weights should be finite or not. The assumption HFF property is realized also in configuration space degrees of freedom would motivate finiteness for the number of conformal weights and would effectively replace the world of the classical worlds with a finite-D space. Also super-symmetry suggests the same. Finiteness would be guaranteed if the ζ function involved characterizes partonic 2-surface and is labelled by p-adic prime: this would also guarantee that zeros of ζ are algebraic numbers. If the zeta function in question characterizes the spectrum of modified Dirac operator and the number of eigenvalues is finite then this goal is achieved. In the case of Riemann Zeta one would be forced to use cutoff due related to the algebraic extension of p-adic numbers used and to assume that zeros and even more general arguments are algebraic numbers.

5.3.3 Cutoff in the number of generalized eigenvalues of the modified Dirac operator

Second cutoff corresponds to a cutoff in the number of generalized eigenvalues of the modified Dirac operator and also now almost TQFT provides the needed flexibility.

1. If the generalized eigenvalues are interpreted as Higgs field then the number of eigenvalues is just one and also orthogonality condition for the modes is achieved without posing ad hoc correlations between longitudinal and transversal degrees of freedom.

2. A priori the dependence of the eigenmodes on transversal degrees of freedom of X^2 is arbitrary. This looks strange on basis of experience with quantum field theory and would imply non-stringy anti-commutation relations. Holomorphic dependence however leads to stringy anti-commutations.

3. Anti-commutativity at braid points only would be highly satisfactory since it would allow to avoid delta functions but would require that the transverse degrees of freedom reduce to a finite number of modes. The reduction of this cutoff to inherent properties of X^2 remains to be understood. What is clear is that the number of conformal modes in transversal degrees of freedom corresponds essentially to the number of points in the braid and the precise realization of this cutoff remains to be understood. Since this cutoff relates to finite measurement resolution, the idea that non-commutative S^2_{II} coordinates provides an elegant manner to realize the anti-commutativity at finite number of points.

It is natural to choose the modes to be S^2_{II} partial waves with a well defined color isospin quantum numbers I, I_3. The Abelianity of the color holonomy group of induced spinor connection suggests also color confinement in weak sense meaning vanishing of I_3 and Y for the physical states.

Since cutoff hierarchy must relate closely to the hierarchy of quantum phases, it seems natural to assume that for given value of $q = exp(i2\pi/n_b)$ only the angular momentum values $I \leq n_b$ are allowed. Here n_b is the order of the maximal cyclic subgroup of G_b involved with the Jones inclusion. In the similar manner one can introduce cutoff for S^2 partial waves in $\delta M^4_\pm$ as cutoff $l \leq n_b$ for angular momentum. Both cutoffs are needed in the definition of configuration space Hamiltonians and super-Hamiltonians allowing to approximate configuration space with a finite-dimensional space which is obviously in spirit with the hyper-finiteness.

Cutoffs imply that n-point functions are finite and non-trivial since the anti-commutators of second quantized induced spinor fields are non-local and delta function singularity is smoothed out. Non-locality implies that vertices are non-trivial and pair creation becomes possible. It is of course essential that the dynamics of the space-time interior induces correlations between different partonic 2-surfaces.

That this picture can give rise to the basic vertices of quantum theory seems clear. For instance, suppose that bosons can be assigned to the fermionic representation of Hamiltonians and fermions by super Hamiltonians. The idea would be that right handed neutrino represents vacuum state to which imbedding space gamma matrices act like creation operators. The vertex for the emission of boson would involve sum of vacuum expectation values for the product of the operators $\overline{\Psi} J_A \Psi(x)$, $\overline{\nu} J_B \Psi(y)$ $\overline{\Psi} J_C \nu(z)$, $J_A = j^k_A \Gamma_k$ with various choices of arguments. Anticommutation relations would give sum over the values of the quantity $\overline{\nu} J_A(x) J_B(y) J_C(z) \nu$ multiplied by "wave functions" coming the modes of Ψ. Summation would be over the discrete set of points of the number theoretical braid. A discretized version of stringy scattering amplitude would be in question.

5.3.4 Attempt to form an overall view

This approach leads to both a hierarchy of discretized theories and continuum theory. Continuum theory indeed seems to be completely well defined and would correspond to string theory with free fermions with $N = 4$ super-conformal symmetry as far vertices are considered.

The interpretation encouraged by Jones inclusion hierarchy is that the limit $n \rightarrow \infty$ for quantum phase $q = exp(i2\pi/n)$ is not equivalent with the exact real theory based on stringy amplitudes defined using 1-D integrals over the inverse image of the image of the critical line. The natural interpretation for the stringy option without discretization could be in terms of Jones inclusions with group $SU(2)$ and classified by extended ADE diagrams relating to the monodromies of the theory. This interpretation would also conform with the full Kac-Moody invariance whereas for quantum version infinite-dimensional symmetries are reduced to finite-dimensional ones. Note that quantum trace should be equivalent with the condition that the trace of the unit matrix is unity for hyper-finite factors of type II_1.

The number theoretic cutoff hierarchy for the allowed zeros of ζ relates closely to the hierarchy of finite-dimensional extensions of p-adic numbers and to the quantum criticality realized in terms of the generalized imbedding space. This hierarchy of extensions defines a hierarchy of number theoretic braids with an increasing number of strands since the number of points in the intersection between real and corresponding p-adic surface increases and does also the number of allowed zeros. Also the hierarchy of finite-dimensional approximations for the inclusions of hyperfinite factors of type II_1 can be visualized in terms of a hierarchy of braid inclusions with increasing number of braids and is described in terms of Temperley-Lieb algebras. This hierarchy of approximate representations of the inclusion means the replacement of the Beraha number $B_n = 4cos^2(\pi/n)$ by a rational number defining the ratio of dimensions of two subsequent finite-dimensional algebras in the hierarchy. Hence the number theoretic braid hierarchy could provide a concrete representation for the hierarchy of approximations for the hyper-finite factors of type II_1 and their Jones inclusions in terms of inclusions of Temperley Lieb algebras assignable to the number theoretic braids. Physics itself would define this sequence of approximations via p-adicization which basically means space-time realization of cognitive representations.

5.4 The spectrum of Dirac operator and radial conformal weights from physical and geometric arguments

The identification of the generalized eigenvalues of the modified Dirac operator as Higgs field suggests the possibility of understanding the spectrum of D purely geometrically by combining physical and geometric constraints.

The standard zeta function associated with the eigenvalues of the modified Dirac action is the best candidate concerning the interpretation of super-canonical conformal weights as zeros of ζ. This ζ should have very concrete geometric and physical interpretation related to the quantum criticality if these eigenvalues have geometric meaning based on geometrization of Higgs field.

Before continuing it its convenient to introduce some notations. Denote the complex coordinate of a point of X^2 w, its $H = M^4 \times CP_2$ coordinates by $h = (m, s)$, and the H coordinates of its $R_+ \times S^2_{II}$ projection by $h_c = (r_+, s_{II})$.

5.4.1 Generalized eigenvalues

The generalized eigenvalue equation defined by the modified Dirac equation is a differential equation involving only the derivative with respect to r. Hence the eigenvalues λ can depend on X^2 coordinate w and on the coordinates of the critical manifold $R_+ \times S^2_{II}$ via the dependence of w these. As a function of $R_+ \times S^2_{II}$ coordinates they would be many-valued functions of these coordinates since several points of X^2 can project at given point of $R_+ \times S^2_{II}$.

The replacement of the ordinary eigenvalues with continuous functions would be a space-time analog for generalized eigenvalues identified as Hermitian operators (or equivalently, their spectra) inspired by the quantum measurement theory based on inclusions of hyper-finite factors of type II$_1$ [C7]. The replacement of these functions with their values in a discrete set defined by number theoretic braid would in turn be the counterpart for the finite measurement resolution.

The interpretation of eigenvalue as a complex Higgs field gives the most concrete interpretation for the generalized eigenvalues. Of course, only single eigenvalue would be realized in this kind of situation. Also the requirement that different modes are orthogonal with respect to the inner product at the partonic 2-surface allows only single generalized eigenvalue. Hence the modes in transversal degrees of freedom would code for physics as in the usual QFT.

This interpretation does not kill the idea about eigenvalues as inverses of zeta function $\lambda = \zeta^{-1}(z)$, S^2_{II}. The point is that one can regard X^2 as a covering of S^2 and assign different branches of ζ^{-1} to the different sheets of covering. Different branches of $\zeta^{-1}(z)$, call them $\zeta_k^{-1}(z)$, would combine to single function of the coordinate w of X^2. In the case of Riemann zeta the corresponding construction would replaced complex plane with its infinite-fold covering.

5.4.2 General definition of Dirac determinant

The first guess is that Dirac determinant can defined as a product of determinants assignable to the points $z = z_k$ of the number theoretic braids:

$$det(D) = \prod_{z_k)} det(D(z_k)) \ . \tag{49}$$

The determinant $det(D(z))$ at point z of S^2 would be defined as the product of the eigenvalues $\lambda(z)$ at points associated with the number theoretic braids.

$$det(D)(z_k) = \left[\prod_i \zeta_i^{-1}(z_k)\right]^{n(z_k)} , \quad (50)$$

$n(z_k)$ is the number of strands in the number theoretical braid of associated with z_k. Higgs interpretation would imply that only single value of Higgs contributes for a given point of X^2. Dirac determinant must be an algebraic number. This is the case if the total number of points of number theoretic braids involved is finite. It turns out that this guess is quite not general enough: it turns out that actual Dirac determinant must be identified as a ratio of two determinants.

5.4.3 Interpretation of eigenvalues of D as Higgs field

The eigenvalues of the modified Dirac operator have a natural interpretation as Higgs field which vanishes for the unstable extrema of Higgs potential. These unstable extrema correspond naturally to quantum critical points resulting as intersection of M^4 *resp.* CP_2 projection of the partonic 2-surface X^2 with R_+ *resp.* S^2_{II}.

Quantum criticality suggests that the counterpart of Higgs potential could be identified as the modulus square of ζ:

$$V(H(s)) = -|H(s)|^2 . \quad (51)$$

which indeed has the points s with $V(H(s)) = 0$ as extrema which would be unstable in accordance with quantum criticality. The fact that for ordinary Higgs mechanism minima of V are the important ones raises the question whether number theoretic braids might more naturally correspond to the minima of V rather than intersection points with S^2. This turns out to be the case. It will also turn out that the detailed form of Higgs potential does not matter: the only thing that matters is that $|V|$ is monotonically decreasing function of the distance from the critical manifold.

5.4.4 Purely geometric interpretation of Higgs

Geometric interpretation of Higgs field suggests that critical points with vanishing Higgs correspond to the maximally quantum critical manifold $R_+ \times S^2_{II}$. The value of H should be determined once $h(w)$ and $R_+ \times S^2_{II}$ projection $h_c(w)$ are known. $|H|$ should increase with the distance between these points. The question is whether one can assign to a given point pair $(h(w), h_c(w))$ naturally a value of H. The first guess is that value of H is most determined by the shortest piece of the geodesic line connecting the points which is a product of geodesics of δM^4_+ and CP_2.

This guess need not be quite correct. An alternative guess is that M^4 projection is indeed geodesic but that CP_2 projection extremizes its length subject to the constraint that the absolute value of the phase defined by the one-dimensional Kähler action $\int A_\mu dx^\mu$ is minimized: this point will be discussed below.

The value should be in general complex and invariant under the isometries of δH affecting h and h_c. The minimal distance $d(h, h_c)$ between the two points constrained by extremal property of phase would define the first candidate for the modulus of H.

The phase factor should relate close to the Kähler structure of CP_2 and one possibility would be the non-integrable phase factor $U(s, s_{II})$ defined as the integral of the induced Kähler gauge potential along the geodesic line in question. Hence the first guess for the Higgs would be as

$$\begin{aligned} H(w) &= d(h, h_c) \times U(s, s_{II}) \ , \\ d(h, h_c) &= \int_h^{h_c} ds \ , \ U(s, s_{II}) = exp\left[i \int_s^{s^1} A_k ds^k \right] \ . \end{aligned} \tag{52}$$

This gives rise to a holomorphic function in X^2 the local complex coordinate of X^2 is identified as $w = d(h, h_s)U(s, s_{II})$ so that one would have $H(w) = w$ locally. This view about H would be purely geometric.

One can ask whether one should include to the phase factor also the phase obtained using the Kähler gauge potential associated with S^2_r having expression $(A_\theta, A_\phi) = (k, cos(\theta))$ with k even integer from the requirement that the non-integral phase factor at equator has the same value irrespective of whether it is calculated with respect to North or South pole. For $k = 0$ the contribution would be vanishing. The value of k might correlate directly with the value of quantum phase. The objection against inclusion of this term is that Kähler action defining Kähler function should contain also M^4 part if this term is included. If this inclusion is allowed then internal consistency requires also the extremization of $\int A_\mu dx^\mu$ so that geodesic lines are not allowed.

In each coordinate patch Higgs potential could be simply the quadratic function $V = -w\overline{w}$. Negative sign is required by quantum criticality. As noticed any monotonically increasing function of V works as well since the minima of the potential remain unaffected. Potential could indeed have minima as minimal distance of X^2 point from $R_+ \times S^2_{II}$. Earth's surface with zeros as tops of mountains and bottoms of valleys as minima would be a rather precise visualization of the situation for given value of r_+. Mountains would have a shape of inverted rotationally symmetry parabola in each local coordinate system.

5.4.5 Consistency with the vacuum degeneracy of Kähler action and explicit construction of preferred extremals

An important constraint comes from the condition that the vacuum degeneracy of Kähler action should be understood from the properties of the Dirac determinant. In the case of vacuum extremals Dirac determinant should have unit modulus.

Suppose that the space-time sheet associated with the vacuum parton X^2 is indeed vacuum extremal. This requires that also X_l^3 is a vacuum extremal: in this case Dirac determinant must be real although it need not be equal to unity. The CP_2 projection of the vacuum extremal belongs to some Lagrangian sub-manifold Y^2 of CP_2. For this kind of vacuum partons the ratio of the product of minimal H distances to corresponding $M^4_\pm$ distances must be equal to unity, in other words minima of Higgs potential must belong to the intersection $X^2 \cap S^2_{II}$ or to the intersection $X^2 \cap R_+$ so that distance reduces to M^4 or CP_2 distance and Dirac determinant to a phase factor. Also this phase factor should be trivial.

It seems however difficult to understand how to obtain non-trivial phase in the generic case for all points if the phase is evaluated along geodesic line in CP_2 degrees of freedom. There is however no deep reason to do this and the way out of difficulty could be based on the requirement that the phase defined by the Kähler gauge potential is evaluated along a curve either minimizing the absolute value of the phase modulo 2π.

One must add the condition that curve is not shorter than the geodesic line between points. For a given curve length s_0 the action must contain as a Lagrange multiplier the curve length so that the action using curve length s as a coordinate reads as

$$S = \int A_s ds + \lambda(\int ds - s_0) \ . \qquad (53)$$

This gives for the extremum the equation of motion for a charged particle with Kähler charge $Q_K = 1/\lambda$:

$$\begin{aligned} \frac{D^2 s^k}{ds^2} + \frac{1}{\lambda} \times J^k_{\ l} \frac{ds^l}{ds} &= 0 \ , \\ \frac{D^2 m^k}{ds^2} &= 0 \ . \end{aligned} \qquad (54)$$

The magnitude of the phase must be further minimized as a function of curve length s.

If the extremum curve in CP_2 consists of two parts, first belonging to X^2_{II} and second to Y^2, the condition is certainly satisfied. Hence if $X^2_{CP_2} \times Y^2$ is not empty, the phases are trivial. In the generic case 2-D sub-manifolds of CP_2 have intersection consisting of discrete points (note again the fundamental role of 4-dimensionality of CP_2). Since S^2_{II} itself is a Lagrangian sub-manifold, it has especially high probably to have intersection points with S^2_{II}. If this is not the case one can argue that X^3_l cannot be vacuum extremal anymore.

Radial conformal invariance of $\delta M^4_\pm$ raises the question whether $\delta M^4_\pm$ geodesics should be defined by allowing $r_M(s)$ to be arbitrary rather than constant. The minimization of $\delta M^4_\pm$ distance would favor geodesics for which $r_M(s)$ decreases as fast as possible so that one has a light-like geodesics going directly to the tip of $\delta M^4_\pm$. Therefore this option does not seem to work.

The construction gives also a concrete idea about how the 4-D space-time sheet $X^4(X^3_l)$ becomes assigned with X^3_l. The point is that the construction extends X^2 to 3-D surface by connecting points of X^2 to points of S^2_{II} using the proposed curves. This process can be carried out in each intersection of X^3_l and M^4_+ shifted to the direction of future. The natural conjecture is that the resulting space-time sheet defines the 4-D preferred extremum of Kähler action.

The most obvious objection is that this construction might not work for cosmic strings of form $X^2 \times S^2_I$, where S^2_I is a homologically non-trivial geodesic sphere of CP_2. In this case X^2 would correspond to string ends, copies of S^2_I at different points of $\delta M^4_\pm$. There seems to be however no real problem. If $S^2_I \cap S^2_{II}$ is not empty, the orbits representing motion in the induced Kähler gauge field could simply define a flow at S^2_I connecting the points of S^2_I to one of the intersection points. Since geodesic manifold is in question one expects that the orbits indeed belong to S^2_I and cosmic string is obtained. Also a flow with several sources and sinks is possible. Situation should be the same for complex 2-sub-manifolds of CP_2. The 3-D character of the resulting surface would be due to the fact that $\delta M^4_\pm$ projections of the orbits are not points. If the second end of the string is at R_+ string and has the same value of r_M coordinate, single string would result. Otherwise one would obtain two strings with second end point at R_+ with the same value of r_M.

5.4.6 About the definition of the Dirac determinant and number theoretic braids

The definition of Dirac determinant should be independent of the choice of complex coordinate for X^2 and local complex coordinate implied by the definition of Higgs is a unique choice for this coordinate. The physical intuition based on Higgs mechanism suggests that apart from normalization factor the Dirac determinant should be defined simply as product of the eigenvalues of D, that is those of Higgs field, associated with the number theoretic braids.

If only single kind of braid is allowed then the minima of Higgs field define the points of the braid very naturally. The points in $R_+ \times S^2_{II}$ cannot contribute to the Dirac determinant since Higgs vanishes at the critical manifold. Note that at S^2_{II} criticality Higgs values become real and the exponent of Kähler action should become equal to one. This is guaranteed if Dirac determinant is normalized by dividing it with the product of $\delta M^4_\pm$ distances of the extrema from R_+. The value of the determinant would equal to one also at the limit $R_+ \times S^2_{II}$.

One would define the Dirac determinant as the product of the values of Higgs field over all minima of local Higgs potential

$$det(D) = \frac{\prod_k H_(w_k)}{\prod_k H_0(w_k)} = \prod_k \frac{w_k}{w^0_k} \ . \tag{55}$$

Here w^0_k are M^4 distances of extrema from R_+. Equivalently: one can identify the values of Higgs field as dimensionless numbers w_k/w^0_k. The modulus of Higgs field would be the ratio of H and $M^4_\pm$ distances from the critical sub-manifold. The modulus of the Dirac determinant would be the product of the ratios of H and M^4 depths of the valleys.

This definition would be general coordinate invariant and independent of the topology of X^2. It would also introduce a unique conformal structure in X^2 which should be consistent with that defined by the induced metric. Since the construction used relies on the induced metric this looks natural. The number of eigen modes of D would be automatically finite and eigenvalues would have purely geometric interpretation as ratios of distances on one hand and as masses on the other hand. The inverse of CP_2 length defines the natural unit of mass. The determinant is invariant under the scalings of H metric as are also Kähler action and Chern-Simons action. This excludes the possibility that Dirac determinant could also give rise to the exponent of the area of X^2.

Number theoretical constraints require that the numbers w_k are algebraic numbers and this poses some conditions on the allowed partonic 2-surfaces unless one drops from consideration the points which do not belong to the algebraic extension used.

5.4.7 About the detailed definition of number theoretic braids

Consider now the detailed definition of number theoretic braids. One can define a pile X^2_t of cross sections of $X^3_l \cap (\delta M^4_{\pm,t} \times CP_2)$, where $\delta M^4_{\pm,t}$ represents $\delta M^4_\pm$ shifted by t in a preferred time direction defined by M^2. In the same manner one can decompose M^2 to a pile of light-like geodesics $R_{+,t}$ defining the quantization axis of angular momentum. For each value of t one obtains a collection of minima of the "Higgs field" λ_t in 3-dimensional space $R_{+,t} \times S^2_{II}$. The minima define orbits $\gamma(t)$: $(r_{+,i}(t), s_{II}(t))$ in $M^2 \times S^2_{II}$ space.

One can consider braidings (or more generally tangles, two minima can disappear in collision or can be created from vacuum) both in X^3_l and at the level of imbedding space.

1. Braids in X^3_l

A braid in X^3_l is obtained by considering the fate of points of $X^2 t = 0$ in X^3_l and by assigning a braiding to the minima of Higgs field in X^3_l. Also the field lines of Kähler magnetic field or of Kähler gauge potential on X^3_l going through the initial positions of Higgs minima can be considered. Since the construction of the Higgs field involves induced Kähler gauge potential in an essential manner, the braiding defined by the Kähler gauge potential could be consistent with the time evolution for the positions of the minima of Higgs.

Recall that only topological rather than point-wise equivalence of the braids is required. It is not clear how much these definition depend on the coordinates used for X^3_l. For instance, could one trivialize the braid by making a time dependent coordinate change for X^2? This requires that it is possible to define global time coordinate whose coordinate lines correspond to field lines. This is possible only if the flow satisfies additional integrability conditions [D1].

2. Braidings defined by imbedding space projections

One can define braidings also by the projections to the heavenly spheres S^2_{II} of CP_2 and S^2_r of $\delta M^4_\pm$. A linear braid like structure is also obtained by considering the projections of Higgs minima in M^2.

1. The simplest option is the identification of the braid as the projections of the orbits of the minima of Higgs field to S^2_{II} or S^2_r (for various values of t). This seems to be the most elegant choice. One could decompose the braid to sub-braids such that each initial value $r_{+,i}(0)$ would define its own braid in S^2_{II} or S^2_r. Also each point of S^2_{II} or S^2_r could define its own sub-braid.

2. Factoring quantum field theories defined in M^2 [21, 36] suggest a further definition of a braid like structure based on the projections of Higgs minima to M^2. The braid like structure would result from the motion of braid points with different velocities so that they would pass by each other. This kind of pattern with constant velocities of particles describes scattering in factoring quantum field theories defined in M^2. The M^2 velocities of particles would not be constant now. S-matrix is almost trivial inducing only a permutation of the initial state momenta and S-matrix elements are mere phases. The interpretation is that each pass-by process induces a time lag. At the limit when the velocities approach to zero or infinity such that their ratios remain constant, S-matrix reduces to a braiding S-matrix.

The Higgs minima contributing to the elements of S-matrix (or at least U-matrix) should correspond to algebraic points of braids. This suggests that the information about the braids comes from the minima of Higgs in X^3_l rather than X^2_t so that only some values of t at each strand $\gamma(t)$ give rise to physically relevant braid points. The condition that the resulting numbers are algebraic poses restrictions on X^3_l as does also the condition that X^3_l have also p-adic counterparts. This does not of course mean the loss of braids. Note that the discretization allows to assign Dirac determinant and zeta function to any 3-surface X^3_l rather than only those corresponding to the maxima of Kähler function.

5.4.8 The identification of zeta function

The proposed picture supports the identification of the eigenvalues of D in terms of a Higgs fields having purely geometric meaning. It also seems that number theoretic braids must be identified as minima of Higgs potential in X^2. Furthermore, the braiding operation could be defined for all intersections of X^3_l defined by shifts $M^4_\pm$ as orbits of minima of Higgs potential. Second option is braiding by Kähler magnetic flux lines.

The question is how to understand super-canonical conformal weights for which the identification as zeros of a zeta function of some kind is highly suggestive. The natural answer would be that the normalized eigenvalues of D defines this zeta function as

$$\zeta(s) \quad = \quad \sum_k (\frac{H(w_k)}{H_0(w_k)})^{-s} \quad . \tag{56}$$

The number of eigenvalues contributing to this function would be finite and $H(w_k)/H_0(w_k)$ should be rational or algebraic at most. ζ function would have a precise meaning consistent with the usual assignment of zeta function to Dirac determinant.

The case of Riemann Zeta inspires the question whether one should allow only the moduli of the eigenvalues in the zeta or allow only real and positive eigenvalues. The moduli of eigenvalues are not smaller than unity as is the case also for Riemann Zeta. Real eigenvalues correspond to vanishing phase and thus vanishing Chern-Simons action and unit eigenvalues to the quantum critical points of S^2_{II}.

The ζ function would directly code the basic geometric properties of X^2 since the moduli of the eigenvalues characterize the depths of the valleys of the landscape defined by X^2 and the associated non-integrable phase factors. The degeneracies of eigenvalues would in turn code for the number of points with same distance from a given zero intersection point.

The zeros of the ζ function in turn define natural candidates for the super-canonical conformal weights and their number would thus be finite in accordance with the idea about inherent cutoff present also in configuration space degrees of freedom. Super-canonical conformal weights would be functionals of X^2. The scaling of λ by a constant depending on p-adic prime factors out from the zeta so that zeros are not affected: this is in accordance with the renormalization group invariance of both super-canonical conformal weights and Dirac determinant.

The zeta function should exist also in p-adic sense. This requires that the numbers λ^s at the points s of S^2_{II} which corresponds to the number theoretic braid are algebraic numbers. The freedom to scale λ could help to achieve this.

The conformal weights defined by the zeros of zeta would be constant. One could however consider also the generalization of the super-canonical conformal weights to functions of S^2_{II} or S^2_r coordinate although this is not necessary and would spoil the simple group theoretical properties of the δH Hamiltonians. The coordinate s appearing as the argument of ζ could be formally identified as S^2_{II} or S^2_r coordinate so that generalized super-canonical conformal weights could be interpreted geometrically as inverses of $\zeta^{-1}(s)$ defined as a function in S^2_{II} or S^2_r.

In this case also the notion of number theoretic braids defined as sets of points for which $X^2_{M^4}$ projection intersects R_+ at same point could make sense for super-canonical conformal weights. This would require that the number for the branches of ζ^{-1} is same as the number of points of braid.

5.4.9 The relationship between λ and Higgs field

The generalized eigenvalue $\lambda(w)$ is only proportional to the vacuum expectation value of Higgs, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and antifermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this). Gauge bosons can have Higgs expectation proportional to λ. The proportionality must be of form $\langle H\rangle \propto \lambda/p^{n/2}$ if gauge boson mass squared is of order $1/p^n$.

5.4.10 Possible objections related to the interpretation of Dirac determinant

Suppose that that Dirac determinant is defined as a product of determinants associated with various points z_k of number theoretical braids and that these determinants are defined as products of corresponding eigenvalues.

Since Dirac determinant is not real and is not invariant under isometries of CP_2 and of $\delta M^4_{\pm}$, it cannot give only the exponent of Kähler function which is real and $SU(3)\times SO(3,1)$ invariant. The natural guess is that Dirac determinant gives also the Chern-Simons exponential and possible phase factors depending on quantum numbers of parton.

1. The first manner to circumvent this objection is to restrict the consideration to maxima of Kähler function which select preferred light-like 3-surfaces X^3_l. The basic conjecture forced by the number theoretic universality and allowed by TGD based view about coupling constant evolution indeed is that perturbation theory at the level of configuration space can be restricted to the maxima of Kähler function and even more: the radiative corrections given by this perturbative series vanish being already coded by Kähler function having interpretation as analog of effective action.

2. There is also an alternative way out of the difficulty: define the Dirac determinant and zeta function using the minima of the modulus of the generalized Higgs as a function of coordinates of X^3_l so that continuous strands of braids are replaced by a discrete set of points in the generic case.

The fact that general Poincare transformations fail to be symmetries of Dirac determinant is not in conflict with Poincare invariance of Kähler action since preferred extremals of Kähler action are in question and must contain the fixed partonic 2-surfaces at $\delta M^4_{\pm}$ so that these symmetries are broken by boundary conditions which does not require that the variational principle selecting the preferred extremals breaks these symmetries.

One can exclude the possibility that the exponent of the stringy action defined by the area of X^2 emerges also from the Dirac determinant. The point is that Dirac determinant is invariant under the scalings of H metric whereas the area action is not.

The condition that the number of eigenvalues is finite is most naturally satisfied if generalized ζ coding information about the properties of partonic 2-surface and expressible as a rational function for which the inverse has a finite number of branches is in question.

5.4.11 How unique the construction of Higgs field is?

Is the construction of space-time correlate of Higgs as λ really unique? The replacement of H with its power H^r, $r > 0$, leaves the minima of H invariant as points of X^2 so that number theoretic braid is not affected. As a matter fact, the group of monotonically increasing maps real-analytic maps applied to H leaves number theoretic braids invariant.

The map $H \rightarrow H^r$ scales Kähler function to its r-multiple, which could be interpreted in terms of $1/r$- scaling of the Kähler coupling strength. Also super-canonical conformal weights identified as zeros of ζ are scaled as $h \rightarrow h/r$ and Chern-Simons charge k is replaced with k/r so that at least $r = 1/n$ might be allowed.

One can therefore ask whether the powers of H could define a hierarchy of quantum phases labelled by differen values of k and α_K. The interpretation as separate phases would conform with the idea that D in some sense has entire spectrum of generalized eigenvalues.

5.5 Quantization of the modified Dirac action

The quantization of the modified Dirac action involves a fusion of various number theoretical ideas. Stringy picture need not be correct with string being replaced number theoretic braids.

1. The first question is how M^4 and CP_2 braids relate. Since one assumes that the data associated with both braids are independent, it seems necessary to assume anti-commutativity between all points of X^2 belonging to some number theoretic braid.

2. There is no correlation between λ and eigenvalues associated with transverse degrees of freedom as in the case of d'Alember operator. Therefore an infinite number of eigen-modes of D for a given eigenvalue λ can be considered unless one poses some additional conditions. This would mean that one could have anti-commutativity for different points of X^2 and anti-commutators of Ψ and conjugate at same point would be proportional to delta function. This would not conform with the stringy picture.

3. How could one obtain stringy anticommutations? The assumption that modes are holomorphic or antiholomorphic would guarantee this since formally only single coordinate variable would appear in Ψ. Anti-commutativity along string requires that in a given sector of configuration space isometries

commute with the selection of quantization axes for the isometry algebra of the imbedding space. This might be justified by quantum classical correspondence. The unitarity for Yang-Baxter matrices and unitarity of the inner product for the radial modes r^{Δ}, $\Delta = 1/2 + iy$, is consistent with the stringy option where y would now label those points of R_+ which do not correspond to $z = 0$. String corresponds to the ζ-image of the critical line containing non-trivial zeros of zeta at the geodesic sphere of S^2_r.

4. One could ask whether number theoretic braids might have deeper meaning in terms of anticommutativity. This would be the case if the modes in transversal degrees of freedom reduce to a finite number and are actually labelled by λ. This could be achieved if there is no other dependence on transverse degrees of freedom than that coming through $\lambda(z)$. Anticommutativity would hold true only at finite number of points and that anti-commutators would be finite in general. This outcome would be very nice.

5. An interesting question is whether the number theoretic braid could be also described by introducing a non-commutativity of the complex coordinate of X^2 provided by S^2_r or S^2_{II}. This should replace anti-commutativity in X^2 with anti-commutativity for different points of the number theoretic braid. The nice outcome would be the finiteness of anti-commutators at same point.

The following is an attempt to formulate this general vision in a more detail manner.

5.5.1 Fermionic anticommutation relations: non-stringy option

The fermionic anti-commutation relations must be consistent with the vacuum degeneracy and with the anti-commutation relations of configuration space gamma matrices defining the matrix elements of configuration space metric between complexified Hamiltonians.

1. The bosonic representation of configuration space Hamiltonians is naturally as Noether charges associated with Chern-Simons action:

$$\begin{aligned} H_A &= \int d^2x \pi^0_k j^k_A \ , \\ \pi^\alpha &= \frac{\partial L_{C-S}}{\partial_\alpha h^k} \ . \end{aligned} \tag{57}$$

π^0_k denotes bosonic canonical momentum density. Note that also fermionic dynamics allows definition of Hamiltonians as fermionic charges) and this would give rise to fermionic representation of super-canonical algebra. Same applies to the super Kac-Moody algebra generators which super

Kac-Moody generators realized as X^3-local isometries of the imbedding space.

2. Super Hamiltonians identifiable as contractions of configuration space gamma matrices with Killing vector fields of symplectic transformations in CH can be defined as matrix elements of $j_A^k\Gamma_k$ between $\overline{\nu}_R$ and Ψ:

$$J_A^K\Gamma_K \quad \equiv \quad \Gamma_A = H_{S,A} = \int d^2x\overline{\nu}_R j_A^k\Gamma_k\Psi \ . \tag{58}$$

   $H_{S,A}^\dagger$ is obtained by Hermitian conjugation.

3. The anti-commutation relations read as

$$\{\overline{\Psi}(x),\Gamma_k\Psi(y)\} \quad = \quad \pi_k^0 J^{rs}\Sigma_{rs}\delta^2(x,y) \quad . \tag{59}$$

   Here J^{rs} denotes the degenerate Kähler form of $\delta M_+^4\times CP_2$. What makes these anti-commutation relations non-stringy is that anti-commutator is proportional to 2-D delta function rather than 1-D delta function at 1-D sub-manifold of X^2 as in the case of conformal field theories. Hence one would have 3-D quantum field theory with one light-like direction.

4. The matrix elements of configuration space metric for the complexified Killing vector fields of symplectic transformations give the elements of configuration space Kähler form and metric as

$$\{\Gamma_A^\dagger,\Gamma_B\} \quad = \quad iG_{\overline{A},B} = J_{\overline{A},B} = \{\overline{H_A},H_B\} = H_{[\overline{A},B]} \ . \tag{60}$$

5.5.2 Fermionic anti-commutation relations: stringy option

As already noticed, 2-dimensional delta function in the anti-commutation relations implies that spinor field is 2-D Euclidian free field rather than conformal field. The usual stringy picture would require anti-commutativity only along circle and nonlocal commutators outside this circle.

Also the original argument based on the observation that the points of CP_2 parameterize a large class of solutions of Yang-Baxter equation suggests the stringy option. The subset of commuting Yang-Baxter matrices corresponds to a geodesic sphere S^2 of CP_2 and the subset of unitary Yang-Baxter matrices to a geodesic circle of S^2 identifiable as real line plane compactified to S^2. Physical intuition strongly favors unitarity.

Stringy choice is consistent with the identification of the configuration space Hamiltonians as bosonic Noether charges only if Noether charges correspond to closed but in general not exact 2-forms and thus reduce to integrals of a 1-form over 1-dimensional manifold representing the discontinuity of the associated vector potential. That Noether charges would reduce to cohomology would conform with almost TQFT property. This is indeed the case under conditions which will be idenfied below.

1. The canonical momentum density associated with C-S action has the expression

$$\pi_k = \epsilon_{\alpha\beta 0}(\partial_\beta\left[A_\alpha A_k) - \partial_\alpha(A_\beta A_k\right] \ , \tag{61}$$

and is thus a closed two-form. Note that the discontinuity of the monopole like vector potential implies that the form in question is not exact.

2. Also the Hamiltonian densities

$$H_A = j_A^k \pi_k = J^{kl}\partial_l H_A \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k) - \partial_\alpha(A_\beta A_k\right] \tag{62}$$

should define closed forms

$$H_A = j_A^k \pi_k = \epsilon_{\alpha\beta 0}\left[\partial_\beta(A_\alpha A_k J^{kl}\partial_l H_A) - \partial_\alpha(A_\beta A_k \partial_l J^{kl} H_A\right] . \tag{63}$$

3. This is not the case in general since the derivatives coming from j_A^k give the term

$$\epsilon_{\alpha\beta 0} A_\alpha A_k J^{kl} D_r(\partial_l H_A)\partial_\beta h^r - A_\beta A_k J^{kl} D_r(\partial_l H_A)\partial_\alpha h^r \ . \tag{64}$$

which does not vanish unless the condition

$$A_k J^{kl} D_r(\partial_l H_A) = \partial_r \Phi \tag{65}$$

holds true.

The condition is equivalent with the vanishing of the Poisson bracket between Hamiltonian and components of the Kähler potential:

$$\partial_k H_A J^{kl} \partial_l A_r = 0 \ . \tag{66}$$

This poses a restriction on the group of isometries of configuration space. The restriction of Kähler potential to A_r is given by $(A_\theta, A_\phi) = (0, cos(\theta))$ and A_ϕ generates rotations in z-direction. Hence only the Hamiltonians commuting with Kähler gauge potential of $\delta M^4_\pm \times CP_2$ at X^2 would have vanishing color isospin and presumably also vanishing color hyper charge in the case of CP_2 and vanishing net spin in case of δM^4_+.

4. The discontinuity of Φ would result from the topological magnetic monopole character of the Kähler potential A_k in $\delta M^4_{\pm} \times CP_2$.

5. Quantum classical correspondence suggests that quantum measurement theory is realized at the level of the configuration space and induces a decomposition of the configuration space to a union of sub-configuration spaces corresponding to different choices of quantization axes of angular momentum and color quantum numbers. Hence the interpretation of configuration space isometries in terms of a maximal set of commuting observables would make sense. Of course, also the canonical transformations for which Hamiltonians do not reduce to 1-D integrals act as symmetries although they do not possess super counterparts. They play same role as Lorentz boosts whereas the super-symmetrizable part of the algebra is analogous to the little group of Lorentz group leaving momentum invariant. This means that complete reduction to string model type theory does not occur even at the level of quantum states.

Consider now the basic formulas for the stringy option.

1. Hamiltonians can be expressed as

$$H_A = \int dx A A_k J^{kl} \partial_l H_A \quad . \tag{67}$$

   where A denotes the projection of Kähler gauge potential to the 1-dimensional manifold in question.

2. The fermionic super-currents defining super-Hamiltonians and configuration space gamma matrices would be given by

$$J_A^K \Gamma_K \equiv \Gamma_A = H_{S,A} = \int dx \overline{\nu}_R j_A^k \Gamma_k \Psi \quad . \tag{68}$$

   $H^{\dagger}_{S,A}$ is obtained by Hermitian conjugation.

3. The anti-commutation relations would read as

$$\{\overline{\Psi}(x), \Gamma_k \Psi(y)\} = A A_k J^{kl} \partial_l H_A J^{rs} \Sigma_{rs} \delta(x,y) \quad . \tag{69}$$

   The general formulas for the matrix elements of the configuration space metric and Kähler form are as for the non-stringy option.

5.5.3 String as the inverse image for image of critical line for zeros of zeta

Number theoretical argument suggests that 1-D dimensional delta function corresponds to the point set for which δM^4_+ projection corresponds to the line of non-trivial zeros for ζ: $z = \zeta(1/2+iy)$ that is intersection of X^2 with R_+. Thus stringy anti-commutation would be along R_+. In CP_2 the discrete set of points along which anticommutations would be given would be subset in S^2_{II}. Anti-commutativity on quantum critical set which corresponds to vacuum extremals would be indeed very natural.

In case of Riemann zeta one must consider also trivial zeros at $x = -2n$, $n = 1, 2....$ These would correspond to the integer powers of r^n for which the definition of inner product is problematic. Note however that for negative powers $-2n$ corresponding to zeros of ζ there are no problems if there is cutoff $r > r_0$.

The number theoretic counterpart of string would be most naturally a curve whose S^2_r projection belongs to the image of the critical line consisting of points $\zeta(1/2+iy)$. This image consist of the real axis of S^2 interpreted as compactified plane since ζ is real at the critical line. Note that in case of Riemann zeta also real axis is mapped to the real line so that it gives nothing new. Also this has a number theoretical justification since the basis $r^{1/2+iy}$, where r could correspond to the light-like coordinate of both $\delta M^4_\pm$ and partonic 3-surface, forms an orthogonal basis with respect to the inner product defined by the scaling invariant integration measure dx/x.

For number theoretical reasons which should be already clear, the values of y would be restricted to $y = \sum_k n_k y_k$ of imaginary parts of zeros of ζ. In the case of partonic 3-surface this would mean that eigenvalues of the modified Dirac operator would be of form $1/2 + i\sum_k n_k y_k$ and the number theoretical cutoff regularizing the Dirac determinant would emerge naturally. The important implication would be that not only q^{iy_k} but also y_k must be algebraic numbers. Note that the zeros of Riemann zeta at this line correspond to quantum criticality against phase transitions changing Planck constant meaning geometrically a leakage between different sectors of the imbedding space.

5.6 Number theoretic braids and global view about anti-commutations of induced spinor fields

The anti-commutations of induced spinor fields are reasonably well understood locally. The basic objects are 3-dimensional light-like 3-surfaces. These surfaces can be however seen as random light-like orbits of partonic 2-surfaces taking which would thus seem to take the role of fundamental dynamical objects. Conformal invariance in turn seems to make the 2-D partons 1-D objects and number theoretical braids in turn discretizes strings. And it also seems that the strands of number theoretic braid can in turn be discretized by considering the minima of Higgs potential in 3-D sense.

Somehow these apparently contradictory views should be unifiable in a more global view about the situation allowing to understand the reduction of effective dimension of the system as one goes to short scales. The notions of measurement resolution and number theoretic braid indeed provide the needed insights in this respect.

5.6.1 Anti-commutations of the induced spinor fields and number theoretical braids

The understanding of the number theoretic braids in terms of Higgs minima and maxima allows to gain a global view about anti-commutations. The coordinate patches inside which Higgs modulus is monotonically increasing function define a division of partonic 2-surfaces $X^2_t = X^3_l \cap \delta M^4_{\pm,t}$ to 2-D patches as a function of time coordinate of X^3_l as light-cone boundary is shifted in preferred time direction defined by the quantum critical sub-manifold $M^2 \times CP_2$. This induces similar division of the light-like 3-surfaces X^3_l to 3-D patches and there is a close analogy with the dynamics of ordinary 2-D landscape.

In both 2-D and 3-D case one can ask what happens at the common boundaries of the patches. Do the induced spinor fields associated with different patches anti-commute so that they would represent independent dynamical degrees of freedom? This seems to be a natural assumption both in 2-D and 3-D case and correspond to the idea that the basic objects are 2- *resp.* 3-dimensional in the resolution considered but this in a discretized sense due to finite measurement resolution, which is coded by the patch structure of X^3_l. A dimensional hierarchy results with the effective dimension of the basic objects increasing as the resolution scale increases when one proceeds from braids to the level of X^3_l.

If the induced spinor fields associated with different patches anti-commute, patches indeed define independent fermionic degrees of freedom at braid points and one has effective 2-dimensionality in discrete sense. In this picture the fundamental stringy curves for X^2_t correspond to the boundaries of 2-D patches and anti-commutation relations for the induced spinor fields can be formulated at these curves. Formally the conformal time evolution scaled down the boundaries of these patches. If anti-commutativity holds true at the boundaries of patches for spinor fields of neighboring patches, the patches would indeed represent independent degrees of freedom at stringy level.

The cutoff in transversal degrees of freedom for the induced spinor fields means cutoff $n \leq n_{max}$ for the conformal weight assignable to the holomorphic dependence of the induced spinor field on the complex coordinate. The dropping of higher conformal weights should imply the loss of the anti-commutativity of the induced spinor fields and its conjugate except at the points of the number theoretical braid. Thus the number theoretic braid should code for the value of n_{max}: the naive expectation is that for a given stringy curve the number of braid points equals to n_{max}.

5.6.2 The decomposition into 3-D patches and QFT description of particle reactions at the level of number theoretic braids

What is the physical meaning of the decomposition of 3-D light-like surface to patches? It would be very desirable to keep the picture in which number theoretic braid connects the incoming positive/negative energy state to the partonic 2-surfaces defining reaction vertices. This is not obvious if X_l^3 decomposes into causally independent patches. One can however argue that although each patch can define its own fermion state it has a vanishing net quantum numbers in zero energy ontology, and can be interpreted as an intermediate virtual state for the evolution of incoming/outgoing partonic state.

Another problem - actually only apparent problem - has been whether it is possible to have a generalization of the braid dynamics able to describe particle reactions in terms of the fusion and decay of braid strands. For some strange reason I had not realized that number theoretic braids naturally allow fusion and decay. Indeed, cusp catastrophe is a canonical representation for the fusion process: cusp region contains two minima (plus maximum between them) and the complement of cusp region single minimum. The crucial control parameter of cusp catastrophe corresponds to the time parameter of X_l^3. More concretely, two valleys with a mountain between them fuse to form a single valley as the two real roots of a polynomial become complex conjugate roots. The continuation of light-like surface to slicing of X^4 to light-like 3-surfaces would give the full cusp catastrophe.

In the catastrophe theoretic setting the time parameter of X_l^3 appears as a control variable on which the roots of the polynomial equation defining minimum of Higgs depend: the dependence would be given by a rational function with rational coefficients.

This picture means that particle reactions occur at several levels which brings in mind a kind of universal mimicry inspired by Universe as a Universal Computer hypothesis. Particle reactions in QFT sense correspond to the reactions for the number theoretic braids inside partons. This level seems to be the simplest one to describe mathematically. At parton level particle reactions correspond to generalized Feynman diagrams obtained by gluing partonic 3-surfaces along their ends at vertices. Particle reactions are realized also at the level of 4-D space-time surfaces. One might hope that this multiple realization could code the dynamics already at the simple level of single partonic 3-surface.

5.6.3 About 3-D minima of Higgs potential

The dominating contribution to the modulus of the Higgs field comes from $\delta M_\pm^4$ distance to the axis R_+ defining quantization axis. Hence in scales much larger than CP_2 size the geometric picture is quite simple. The orbit for the 2-D minimum of Higgs corresponds to a particle moving in the vicinity of R_+ and minimal distances from R_+ would certainly give a contribution to the Dirac determinant. Of course also the motion in CP_2 degrees of freedom can generate local minima and if this motion is very complex, one expects large number of minima with almost same modulus of eigenvalues coding a lot of information about X_l^3.

It would seem that only the most essential information about surface is coded: the knowledge of minima and maxima of height function indeed provides the most important general coordinate invariant information about landscape. In the rational category where X_l^3 can be characterized by a finite set of rational numbers, this might be enough to deduce the representation of the surface.

What if the situation is stationary in the sense that the minimum value of Higgs remains constant for some time interval? Formally the Dirac determinant would become a continuous product having an infinite value. This can be avoided by assuming that the contribution of a continuous range with fixed value of Higgs minimum is given by the contribution of its initial point: this is natural if one thinks the situation information theoretically. Physical intuition suggests that the minima remain constant for the maxima of Kähler function so that the initial partonic 2-surface would determine the entire contribution to the Dirac determinant.

5.6.4 How generalized braid diagrams relate to the perturbation theory?

The association of generalized braid diagrams to incoming and outgoing partonic legs and possibly also vertices of the generalized Feynman diagrams forces to ask whether the generalized braid diagrams could give rise to a counterpart of perturbation theoretical formalism via the functional integral over configuration space degrees of freedom.

The question is how the functional integral over configuration space degrees of freedom relates to the generalized braid diagrams. The basic conjecture motivated also number theoretically is that radiative corrections in this sense sum up to zero for critical values of Kähler coupling strength and Kähler function codes radiative corrections to classical physics via the dependence of the scale of M^4 metric on Planck constant. Cancellation occurs only for critical values of Kähler coupling strength α_K: for general values of α_K cancellation would require separate vanishing of each term in the sum and does not occur.

This would mean following.

1. One would not have perturbation theory around a given maximum of Kähler function but as a sum over increasingly complex maxima of Kähler function. Radiative corrections in the sense of perturbative functional integral around a given maximum would vanish (so that the expansion in terms of braid topologies would not make sense around single maximum). Radiative corrections would not vanish in the sense of a sum over 3-topologies obtained by adding radiative corrections as zero energy states in shorter time scale.

2. Connes tensor product with a given measurement resolution would correspond to a restriction on the number of maxima of Kähler function labelled by the braid diagrams. For zero energy states in a given time

scale the maxima of Kähler function could be assigned to braids of minimal complexity with braid vertices interpreted in terms of an addition of radiative corrections. Hence a connection with QFT type Feyman diagram expansion would be obtained and the Connes tensor product would have a practical computational realization.

3. The cutoff in the number of topologies (maxima of Kähler function contributing in a given resolution defining Connes tensor product) would be always finite in accordance with the algebraic universality.

4. The time scale resolution defined by the temporal distance between the tips of the causal diamond defined by the future and past light-cones applies to the addition of zero energy sub-states and one obtains a direct connection with p-adic length scale evolution of coupling constants since the time scales in question naturally come as negative powers of two. More precisely, p-adic primes near power of two are very natural since the coupling constant evolution comes in powers of two of fundamental 2-adic length scale.

There are still some questions. Radiative corrections around given 3-topology vanish. Could radiative corrections sum up to zero in an ideal measurement resolution also in 2-D sense so that the initial and final partonic 2-surfaces associated with a partonic 3-surface of minimal complexity would determine the outcome completely? Could the 3-surface of minimal complexity correspond to a trivial diagram so that free theory would result in accordance with asymptotic freedom as measurement resolution becomes ideal?

The answer to these questions seems to be 'No'. In the p-adic sense the ideal limit would correspond to the limit $p \rightarrow 0$ and since only $p \rightarrow 2$ is possible in the discrete length scale evolution defined by primes, the limit is not a free theory. This conforms with the view that CP_2 length scale defines the ultimate UV cutoff.

5.6.5 How p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

One can wonder how this picture relates to the earlier hypothesis that p-adic length coupling constant evolution is coded to the hypothesized $log(p)$ normalization of the eigenvalues of the modified Dirac operator D. There are objections against this normalization. $log(p)$ factors are not number theoretically favored and one could consider also other dependencies on p. Since the eigenvalue spectrum of D corresponds to the values of Higgs expectation at points of partonic 2-surface defining number theoretic braids, Higgs expectation would have $log(p)$ multiplicative dependence on p-adic length scale, which does not look attractive.

Is there really any need to assume this kind of normalization? Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5\ \mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3.

5.6.6 How quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or a boundary term of YM action associated with a particle carrying gauge charges of the quantum state. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography. The difference between Chern-Simons action characterizing quantum state and the fundamental Chern-Simons type factor associated with the Kähler form would be that the latter emerges as the phase of the Dirac determinant.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

6 Super-symmetries at space-time and configuration space level

The first difference between TGD and standard conformal field theories and string models is that super-symmetry generators acting as configuration space gamma matrices acting as super generators carry either lepton or quark number. Only the anti-commutators of quark like generators expressible in terms of Hamiltonians H_A of $X^3_l \times CP_2$ can contribute to the super-symmetrization of the Poisson algebra and thus to CH metric via Poisson central extension, whereas leptonic generators, which are proportional to $j^{Ak}\Gamma_k$ can contribute to the super-symmetrization of the function algebra of CH. Quarks correspond to N-S type representations and kappa symmetry of string models whereas leptons correspond to Ramond type representations and ordinary super-symmetry.

Also Super Kac-Moody invariance allows lepton-quark dichotomy. What forces to assign leptons with Ramond representation is that covariantly constant neutrino must correspond to one conformal mode $(z^n, n = 0)$. The p-adic mass calculations [6] carried for more than decade ago led to the same assignment on physical grounds: p-adic mass calculations also forced to include $SO(3,1)$ besides M^4 a tensor factor to super-conformal representations, which in recent context suggests that causal determinants $X^3_l \times CP_2$, $X^3_l \subset M^4$ an arbitrary light like 3-surface rather than just a translate of δM^4_+, must be allowed. Also now the lepton-quark, Ramond-NS and SUSY-kappa dichotomies correspond to one and same dichotomy so that the general structure looks quite satisfactory although it must be admitted that it is based on heuristic guess work.

Second deep difference is the appearance of the zeros of Riemann Zeta as conformal weights of the generating elements of the super-canonical algebra and the expected action of conformal algebra associated with 3-D CDS as a spectral flow in the space of super-canonical conformal weights inducing a mere gauge transformation infinitesimally and a braiding action in topological degrees of freedom.

In this section the relationship of Super Kac-Moody invariance to ordinary super-conformal symmetry and the interaction between Super-Kac Moody and super-canonical symmetries are discussed. For years the role of quaternions and octonions in TGD has been under an active speculation. These aspects are considered in [E2], where the number theoretic equivalent of spontaneous compactification is proposed. The conjecture states that space-time surfaces can be regarded either as 4-surfaces in $M^4 \times CP_2$ or as hyper-quaternionic 4-surfaces in the space $HO = M^8$ possessing hyper-octonionic structure (the attribute 'hyper' means that imaginary units are multiplied by $\sqrt{-1}$ in order to achieve number theoretic norm with Minkowskian signature).

6.1 Super-canonical and Super Kac-Moody symmetries

The proper understanding of super symmetries has turned out to be crucial for the understanding of quantum TGD and it seems that the mis-interpreted super-symmetries are one of the basic reasons for the difficulties of super string models too. At this moment one can fairly say that the construction of the configuration space spinor structure reduces to a purely group theoretical problem of constructing representations for the super generators of the super-canonical algebra of CP_2 localized with respect to $\delta M^4_\pm$ in terms of second quantized induced spinor fields.

6.1.1 Super canonical symmetries

One can imagine two kinds of causal determinants besides $\delta M^4_+ \times CP_2$. In principle all surfaces $X^3_l \times CP_2$, where X^3_l is a light like 3-surface of M^4, could act as effective causal determinants: the reason is that the creation of pairs of positive and negative energy space-time sheets is possible at these surfaces. There are good hopes that the super-canonical and super-conformal symmetries associated with δX^3_l allow to generalize the construction of the configuration space geometry performed at $\delta M^4_+ \times CP_2$. If X^3_l can be restricted to be unions of future and past light cone boundaries, the generalization is more or less trivial: one just forms a union of configuration spaces associated with unions of translates of δM^4_+ and δM^4_-.

As explained in the previous chapter, one can understand how the causal determinants $X^3_l \times CP_2$ emerge from the facts that space-time sheets with negative time orientation carry negative energy and that the most elegant theory results when the net quantum numbers and conserved classical quantities vanish for the entire Universe. Crossing symmetry allows consistency with elementary particle physics and the identification of gravitational 4-momentum as difference of conserved inertial (Poincare) 4-momenta for positive and negative energy matter provides consistency with macroscopic physics.

The emergence of these additional causal determinants means that super-canonical symmetries become microscopic, rather than only cosmological, symmetries commuting with Poincare transformations exactly for $M^4 \times CP_2$ and apart from small quantum gravitational effects for $M^4_+ \times CP_2$. Super-canonical symmetry differs in many respects from Kac-Moody symmetries of particle physics, which in fact correspond to the conformal invariance associated with the modified Dirac action and correspond to the product of Poincare, electro-weak and color groups. It seems that these symmetries are dually related.

6.1.2 Super Kac-Moody symmetries associated with the light like causal determinants

Also the light like 3-surfaces X^3_l of H defining elementary particle horizons at which Minkowskian signature of the metric is changed to Euclidian and boundaries of space-time sheets can act as causal determinants, and thus contribute to the configuration space metric. In this case the symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface X^2 determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results. Also the condition $\sqrt(g_3) = 0$ for the determinant of the induced metric seems to define a conformal symmetry associated with the light like direction. This conforms with duality since also the 7-D causal determinants $X^3_l \times CP_2$ allow both radial and transversal conformal symmetry.

Good candidate for the counterpart of this symmetry in the interior of space-time surface is hyper-quaternion conformal invariance [E2]. All that is needed for these symmetries to be equivalent that the spaces of super-gauge degrees of freedom defined by them are equivalent. Kac Moody generators and their super counterparts can be associated with the 3-D light like CDs.

If is enough to localize only the H-isometries with respect to X^3_l, the purely bosonic part of the Kac-Moody algebra corresponds to the isometry group $M^4 \times SO(3,1) \times SU(3)$. The physical interpretation of these symmetries is not so obvious as one might think. The point is that one can generalize the formulas characterizing the action of infinitesimal isometries on spinor fields of finite-dimensional Kähler manifold to the level of the configuration space. This gives rise to bosonic generators containing also a sigma-matrix term bilinear in fermionic oscillator operators. This representation is not equivalent with the purely fermionic representations provided by induced Dirac action. Thus one has two groups of local color charges and the challenge is to find a physical interpretation for them. The following arguments fix the identification.

1. The hint comes from the fact that $U(2)$ in the decomposition $CP_2 = SU(3)/U(2)$ corresponds in a well-defined sense electro-weak algebra identified as a holonomy algebra of the spinor connection. Hence one could argue that the $U(2)$ generators of either $SU(3)$ algebra might be identifiable as generators of local $U(2)_{ew}$ gauge transformations whereas non-diagonal generators would correspond to Higgs field. This interpretation would conform with the idea that Higgs field is a genuine scalar field rather than a composite of fermions.

2. Since X^3_l-local $SU(3)$ transformations represented by fermionic currents are characterized by central extension they would naturally correspond to the electro-weak gauge algebra and Higgs bosons. This is also consistent with the fact that both leptons and quarks define fermionic Kac Moody currents.

3. The fact that only quarks appear in the gamma matrices of the configuration space supports the view that action of the generators of X^3_l-local color transformations on configuration space spinor fields represents local color transformations. If the action of X^3_l-local $SU(3)$ transformations on configuration space spinor fields has trivial central extension term the identification as a representation of local color symmetries is possible.

The topological explanation of the family replication phenomenon is based on an assignment of 2-dimensional boundary to a 3-surface characterizing the elementary particle. The precise identification of this surface has remained open and one possibility is that the 2-surface X^2 defining the light light-like surface associated with an elementary particle horizon is in question. This assumption would conform with the notion of elementary particle vacuum functionals defined in the zero modes characterizing different conformal equivalences classes for X^2.

6.1.3 The relationship of the Super-Kac Moody symmetry to the standard super-conformal invariance

Super-Kac Moody symmetry can be regarded as $N = 4$ complex super-symmetry with complex H-spinor modes of H representing the 4 physical helicities of 8-component leptonic and quark like spinors acting as generators of complex dynamical super-symmetries. The super-symmetries generated by the covariantly constant right handed neutrino appear with *both* M^4 helicities: it however seems that covariantly constant neutrino does not generate any global super-symmetry in the sense of particle-sparticle mass degeneracy. Only righthanded neutrino spinor modes (apart from covariantly constant mode) appear in the expressions of configuration space gamma matrices forming a subalgebra of the full super-algebra.

$N=2$ real super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$ carrying $U(1)$ charge. Now $U(1)$ current would correspond to right-handed neutrino number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N=2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N=2$ super-conformal representations are modular invariant, and that $N=2$ algebra defines so called chiral ring defining a topological quantum field theory [21], lend a further support for the belief that $N=2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N=2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ \ l=0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ \ m=-l,-l+2,....,l-2,l \ . \end{aligned} \tag{70}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k=1$ one would have $q=0,1/3,-1/3$, which brings in mind anyons. $\Delta_{l=0,m=0}=0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [21].

For Ramond representation $L_0-c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta=c/24$ giving the condition $k=2\left[l(l+2)-m^2\right]$ (note that k must be even and that $(k,l,m)=(4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac}=\pm c/12=\pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators but this not necessary.

The conformal algebra defined as a direct sum of Ramond and NS $N=4$ complex sub-algebras associated with quarks and leptons might further extend to a larger algebra if lepto-quark generators acting effectively as half odd-integer Virasoro generators can be allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on configuration space Hamiltonians expressible in terms of Hamiltonians of $X^3_l\times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h=1$ whereas T and G have conformal weights $h=2$ and $h=3/2$.

The experience with $N=4$ complex super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h=1/2$ and their super-partners with $h=0$ and realized as fermion-antifermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

6.1.4 How could conformal symmetries of light like 3-D CDs act on super-canonical degrees of freedom?

An important challenge is to understand the action of super-conformal symmetries associated with the light like 3-D CDs on super-canonical degrees of freedom. The breakthrough in this respect via the algebraic formulation for the vision about vanishing loop corrections of ordinary Feynman diagrams in terms of equivalence of generalized Feynman diagrams with loops with tree diagrams [C6]. The formulation involves Yang-Baxter equations, braid groups, Hopf algebras, and so called ribbon categories and led to the following vision. The original formulation to be discussed in this sub-subsection is very heuristic and a more quantitative formulation follows in the next subsection.

1. Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of the homologically non-trivial geodesic sphere S^2 of CP_2 corresponds to the super-canonical conformal weights, and corresponds to a discrete set of points at the space-time surface. These points would also label mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. These points would belong to a "time=constant" section of 2-dimensional "space-time", presumably circle, defining physical states of a two-dimensional conformal field theory for which the scaling operator L_0 takes the role of Hamiltonian.

2. One could thus regard super-generators as conformal fields in space-time or complex plane having super-canonical conformal weights as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories [21] would induce by a pull-back a braid group action on the super-canonical conformal weights of configuration space gamma matrices (super generators) and corresponding isometry generators.

At the first sight the explicit realization of super-canonical and Kac Moody generators seems however to be in conflict with this vision. The interaction of the conformal algebra of X_l^3 on super-canonical algebra is a pure gauge interaction since the definition of super canonical generators is not changed by the action of conformal transformations of X_l^3. This is however consistent with the assumption that the action defined by the quantum-classical correspondence is also a pure gauge interaction locally. The braiding action would be analogous to the holonomies encountered in the case of non-Abelian gauge fields with a vanishing curvature in spaces possessing non-trivial first homotopy group.

Quantum classical correspondence would allow to map abstract configuration space level to space-time level.

1. The complex argument z of Kac Moody and Virasoro algebra generators $T(z) = \sum T_n z^n$ would be discretized so that it would have values on the set of supercanonical conformal weights corresponding to the space t in the Cartan decomposition $g = t + h$ of the tangent space of the configuration space. These points could be interpreted as punctures of the complex plane restricted to the lines $Re(z) = \pm 1/2$ and positive real axis if zeros of Riemann zeta define the conformal weights.

2. The vacuum expectation values of the enveloping algebra of the super-canonical algebra would reduce to n-point functions of a super-conformal quantum field theory in the complex plane containing infinite number of punctures defined by the super-canonical conformal weights, for which primary fields correspond to the representations of $SO(3) \times SU(3)$. These representations would combine to form infinite-dimensional representations of super-canonical algebra. The presence of the gigantic super-canonical symmetries raises the hope that quantum TGD could be solvable to a very high degree.

3. The Super Virasoro algebra and Super Kac Moody algebra associated with 3-D light like CDs would act as symmetries of this theory and the S-matrix of TGD would involve the n-point functions of this field theory. By 7–3 duality this indeed makes sense. The situation would reduce to that encountered in WZW theory in the sense that one would have space-like 3-surfaces X^3 containing two-dimensional closed surfaces carrying representations of Super Kac-Moody algebra.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors [28] having a close connection with the non-trivial representations of braid group and quantum groups. The sequence of non-trivial zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded an an infinite braid behind the von Neumann algebra [28]. Contrary to the expectations, also trivial zeros seem to be important. The finite braids defined by subsets of zeros, and also superpositions of non-trivial zeros of form $1/2 + \sum_i y_i$, could be seen as a hierarchy of completely integrable 1-dimensional spin chains leading to quantum groups and braid groups [20, 21] naturally.

It seems that not only Riemann's zeta but also polyzetas [29, 30, 31, 32] could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically. The physical justification for the hypothesis would be quantum criticality. Indeed, it has been found that the loop corrections of quantum field theory are expressible in terms of polyzetas [33]. If the arguments of polyzetas correspond to conformal weights of particles of many-particle bound state, loop corrections vanish when the super-canonical conformal weights correspond to the zeros of polyzetas including zeta.

6.2 The relationship between super-canonical and Super Kac-Moody algebras, Equivalence Principle, and justification of p-adic thermodynamics

The relationship between super-canonical algebra (SC) acting at light-cone boundary and Super Kac-Moody algebra (SKM) acting on light-like 3-surfaces has remained somewhat enigmatic due to the lack of physical insights. This is not the only problem. The question to precisely what extent Equivalence Principle (EP) remains true in TGD framework and what might be the precise mathematical realization of EP is waiting for an answer. Also the justification of p-adic thermodynamics for the scaling generator L_0 of Virasoro algebra -in obvious conflict with the basic wisdom that this generator should annihilate physical states- is lacking. It seems that these three problems could have a common solution.

Before going to describe the proposed solution, some background is necessary. The latest proposal for $SC - SKM$ relationship relies on non-standard and therefore somewhat questionable assumptions.

1. SKM Virasoro algebra (SKMV) and SC Virasoro algebra (SCV) (anti)commute for physical states.

2. SC algebra generates states of negative conformal weight annihilated by SCV generators L_n, $n < 0$, and serving as ground states from which SKM generators create states with non-negative conformal weight.

This picture could make sense for elementary particles. On other hand, the recent model for hadrons [F4] assumes that SC degrees of freedom contribute about 70 per cent to the mass of hadron but at space-time sheet different from those assignable to quarks. The contribution of SC degrees of freedom to the thermal average of the conformal weight would be positive. A contradiction results unless one assumes that there exists also SCV ground states with positive conformal weight annihilated by SCV elements L_n, $n < 0$, but also this seems implausible.

6.2.1 New vision about the relationship between SCV and $SKMV$

Consider now the new vision about the relationship between SCV and $SKMV$.

1. The isometries of H assignable with SKM are also symplectic transformations [B3] (note that I have used the term canonical instead of symplectic previously). Hence might consider the possibility that SKM could be identified as a subalgebra of SC. If this makes sense, a generalization of the coset construction obtained by replacing finite-dimensional Lie group with infinite-dimensional symplectic group suggests itself. The differences of SCV and $SKMV$ elements would annihilate physical states and (anti)commute with $SKMV$. Also the generators O_n, $n > 0$, for both algebras would annihilate the physical states so that the differences of the elements would annihilate automatically physical states for $n > 0$.

2. The super-generator G_0 contains the Dirac operator D of H. If the action of SCV and $SKMV$ Dirac operators on physical states are identical then cm of degrees of freedom disappear from the differences $G_0(SCV) - G_0(SKMV)$ and $L_0(SCV) - L_0(SKMV)$. One could interpret the identical action of the Dirac operators as the long sought-for precise realization of Equivalence Principle (EP) in TGD framework. EP would state that the total inertial four-momentum and color quantum numbers assignable to SC (imbedding space level) are equal to the gravitational four-momentum and color quantum numbers assignable to SKM (space-time level). Note that since super-canonical transformations correspond to the isometries of the "world of classical worlds" the assignment of the attribute "inertial" to them is natural.

3. The analog of coset construction applies also to SKM and SC algebras which means that physical states can be thought of as being created by an operator of SKM carrying the conformal weight and by a genuine SC operator with vanishing conformal weight. Therefore the situation does not reduce to that encountered in super-string models.

4. The reader can recognize $SC - SKM$ as a precise formulation for $7-3$ duality discussed in the section *About dualities and conformal symmetries in TGD framework* stating that 3-D light-like causal determinants and 7-D causal determinants $\delta M^4_\pm \times CP_2$ are equivalent.

6.2.2 Consistency with p-adic thermodynamics

The consistency with p-adic thermodynamics provides a strong reality test and has been already used as a constraint in attempts to understand the super-conformal symmetries in partonic level.

1. In physical states the p-adic thermal expectation value of the SKM and SC conformal weights would be non-vanishing and identical and mass squared could be identified to the expectation value of SKM scaling generator L_0. There would be no need to give up Super Virasoro conditions for $SCV - SKMV$.

2. There is consistency with p-adic mass calculations for hadrons [F4] since the non-perturbative SC contributions and perturbative SKM contributions to the mass correspond to space-time sheets labeled by different p-adic primes. The earlier statement that SC is responsible for the dominating non-perturbative contributions to the hadron mass transforms to a statement reflecting $SC - SKM$ duality. The perturbative quark contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SKM whereas non-perturbative contributions to hadron masses can be calculated most conveniently by using p-adic thermodynamics for SC. Also the proposal that the exotic analogs of baryons resulting when baryon looses its valence quarks [F5] remains intact in this framework.

3. The results of p-adic mass calculations depend crucially on the number N of tensor factors contributing to the Super-Virasoro algebra. The required number is $N = 5$ and during years I have proposed several explanations for this number. It seems that holonomic contributions that is electro-weak and spin contributions must be regarded as contributions separate from those coming from isometries. SKM algebras in electro-weak degrees and spin degrees of of freedom, would give 2+1=3 tensor factors corresponding to $U(2)_{ew} \times SU(2)$. $SU(3)$ and $SO(3)$ (or $SO(2) \subset SO(3)$ leaving the intersection of light-like ray with S^2 invariant) would give 2 additional tensor factors. Altogether one would indeed have 5 tensor factors.

There are some further questions which pop up in mind immediately.

1. Why mass squared corresponds to the thermal expectation value of the net conformal weight? This option is forced among other things by Lorentz invariance but it is not possible to provide a really satisfactory answer to this question yet. In the coset construction there is no reason to require that the mass squared equals to the integer value conformal weight for SKM algebra. This allows the possibility that mass squared has same value for states with different values of SKM conformal weights appearing in the thermal state and equals to the average of the conformal weight.

   The coefficient of proportionality can be however deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

2. The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations.

This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \quad (71)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which together with the presence of preferred plane M^2 would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \quad (72)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

3. Single particle super-canonical conformal weights can have also imaginary part, call it y. The question is what complex mass squared means physically. Complex conformal weights have been assigned with an inherent time orientation distinguishing positive energy particle from negative energy antiparticle (in particular, phase conjugate photons from ordinary photons). This suggests an interpretation of y in terms of a decay width. p-Adic thermodynamics suggest that y vanishes for states with vanishing conformal weight (mass squared) and that the measured value of y is a p-adic thermal average with non-vanishing contributions from states with mass of order CP_2 mass. This makes sense if y_k are algebraic or perhaps even rational numbers.

   For instance, if a massless state characterized by p-adic prime p has p-adic thermal average $y = psy_k$, where s is the denominator of rational valued $y_k = r/s$, the lowest order contribution to the decay width is proportional to $1/p$ by the basic rules of p-adic mass calculations and the decay rate is of same order of magnitude as mass. If the p-adic thermal average of y is of form $p^n y_k$ for massless state then a decay width of order $\Gamma \sim p^{-(n-1)/2} m$ results. For electron n should be rather large. This argument generalizes trivially to the case in which massless state has vanishing value of y.

6.2.3 Can SKM be lifted to a sub-algebra of SC?

A picture introducing only a generalization of coset construction as a new element, realizing mathematically Equivalence Principle, and justifying p-adic thermodynamics is highly attractive but there is a problem. SKM is defined at light-like 3-surfaces X^3 whereas SC acts at light-cone boundary $\delta H_\pm = \delta M^4_\pm \times CP_2$. One should be able to lift SKM to imbedding space level somehow. Also SC should be lifted to entire H. This problem was the reason why I gave up the idea about coset construction and $SC-SKM$ duality as it appeared for the first time.

A possible solution of the lifting problem comes from the observation making possible a more rigorous formulation of $HO-H$ duality stating that one can regard space-time surfaces either as surfaces in hyper-octonionic space $HO = M^8$ or in $H = M^4 \times CP_2$ [C1, E2]. Consider first the formulation of $HO-H$ duality.

1. Associativity also in the number theoretical sense becomes the fundamental dynamical principle if $HO-H$ duality holds true [E2]. For a space-time surface $X^4 \subset HO = M^8$ associativity is satisfied at space-time level if the tangent space at each point of X^4 is some hyper-quaternionic sub-space $HQ = M^4 \subset M^8$. Also partonic 2-surfaces at the boundaries of causal diamonds formed by pairs of future and past directed light-cones defining the basic imbedding space correlate of zero energy state in zero energy ontology and light-like 3-surfaces are assumed to belong to $HQ = M^4 \subset HO$.

2. $HO-H$ duality requires something more. If the tangent spaces contain the same preferred commutative and thus hyper-complex plane $HC = M^2$, the tangent spaces of X^4 are parameterized by the points s of CP_2 and $X^4 \subset HO$ can be mapped to $X^4 \subset M^4 \times CP_2$ by assigning to a point of X^4 regarded as point (m, e) of $M^4_0 \times E^4 = M^8$ the point (m, s). Note that one must also fix a preferred global hyper-quaternionic subspace $M^4_0 \subset M^8$ containing M^2 to be not confused with the local tangent planes M^4.

3. The preferred plane M^2 can be interpreted as the plane of non-physical polarizations so that the interpretation as a number theoretic analog of gauge conditions posed in both quantum field theories and string models is possible.

4. An open question is whether the resulting surface in H is a preferred extremal of Kähler action. This is possible since the tangent spaces at light-like partonic 3-surfaces are fixed to contain M^2 so that the boundary values of the normal derivatives of H coordinates are fixed and field equations fix in the ideal case X^4 uniquely and one obtains space-time surface as the analog of Bohr orbit.

5. The light-like "Higgs term" proportional to $O = \gamma_k t^k$ appearing in the generalized eigenvalue equation for the modified Dirac operator is an essential element of TGD based description of Higgs mechanism. This term can cause complications unless t is a covariantly constant light-like vector. Covariant constancy is achieved if t is constant light-like vector in M^2. The interpretation as a space-time correlate for the light-like 4-momentum assignable to the parton might be considered.

6. Associativity requires that the hyper-octonionic arguments of N-point functions in HO description are restricted to a hyperquaternionic plane $HQ = M^4 \subset HO$ required also by the $HO - H$ correspondence. The intersection $M^4 \cap int(X^4)$ consists of a discrete set of points in the generic case. Partonic 3-surfaces are assumed to be associative and belong to M^4. The set of commutative points at the partonic 2-surface X^2 is discrete in the generic case whereas the intersection $X^3 \cap M^2$ consists of 1-D curves so that the notion of number theoretical braid crucial for the p-adicization of the theory as almost topological QFT is uniquely defined.

7. The preferred plane $M^2 \subset M^4 \subset HO$ can be assigned also to the definition of N-point functions in HO picture. It is not clear whether it must be same as the preferred planes assigned to the partonic 2-surfaces. If not, the interpretation would be that it corresponds to a plane containing the over all cm four-momentum whereas partonic planes M_i^2 would contain the partonic four-momenta. M^2 is expected to change at wormhole contacts having Euclidian signature of the induced metric representing horizons and connecting space-time sheets with Minkowskian signature of the induced metric.

The presence of globally defined plane M^2 and the flexibility provided by the hyper-complex conformal invariance raise the hopes of achieving the lifting of SC and SKM to H. At the light-cone boundary the light-like radial coordinate can be lifted to a hyper-complex coordinate defining coordinate for M^2. At X^3 one can fix the light-like coordinate varying along the braid strands can be lifted to some hyper-complex coordinate of M^2 defined in the interior of X^4. The total four-momenta and color quantum numbers assignable to the SC and SKM degrees of freedom are naturally identical since they can be identified as the four-momentum of the partonic 2-surface $X^2 \subset X^3 \cap \delta M^4_\pm \times CP_2$. Equivalence Principle would emerge as an identity.

6.2.4 Questions about conformal weights

One can pose several non-trivial questions about conformal weights.

1. The negative SKM conformal weights of ground states for elementary particles [F3] remain to be understood in this framework. In the case of light-cone boundary the natural value for ground state conformal weight of a scalar field is $-1/2$ since this implies a complete analogy with a plane wave with respect to the radial light-like coordinate r_M with inner product defined by a scale invariant integration measure dr_M/r_M. If the coset construction works same should hold true for SKM degrees of freedom for a proper choice of the light-like radial coordinate. There are thus good hopes that negative ground state conformal weights could be understood.

2. Further questions relate to the imaginary parts of ground state conformal weights, which can be vanishing in principle. Do the ground state

conformal weights correspond to the zeros of some zeta function- most naturally the zeta function defined by generalized eigenvalues of the modified Dirac operator and satisfying Riemann hypothesis so that ground state conformal weight would have real part -1/2? Do SC and SKM have same spectrum of complex conformal weights as the coset construction suggests? Does the imaginary part of the conformal weight bring in a new degree of freedom having interpretation in terms of space-time correlate for the arrow of time with the generalization of the phase conjugation of laser physics representing the reversal of the arrow of geometric time?

3. The opposite light-cone boundaries of the causal diamond bring in mind the hemispheres of S^2 in ordinary conformal theory. In ordinary conformal theory positive/negative powers of z correspond to these hemispheres. Could it be that the radial conformal weights are of opposite sign and of same magnitude for the positive and negative energy parts of zero energy state?

6.2.5 Further questions

There are still several open questions.

1. Is it possible to define hyper-quaternionic variants of the superconformal algebras in both H and HO or perhaps only in HO. A positive answer to this question would conform with the conjecture that the geometry of "world of classical worlds" allows Hyper-Kähler property in either or both pictures [B3].

2. How this picture relates to what is known about the extremals of field equations [D1] characterized by generalized Hamilton-Jacobi structure bringing in mind the selection of preferred M^2?

3. Is this picture consistent with the views about Equivalence Principle and its possible breaking based on the identification of gravitational four-momentum in terms of Einstein tensor is interesting [D3]?

6.3 Brief summary of super-conformal symmetries in partonic picture

The notion of conformal super-symmetry is very rich and involves several non-trivial aspects, and as the following discussions shows, one could assign the attribute super-conformal to several symmetries. In the following I try to sum up what I see as important. What is new is that it is now possible to tie everything to the fundamental description in terms of the parton level action principle and provide a rigorous justification and precise realization for the claimed super-conformal symmetries.

6.3.1 Super-canonical symmetries

Super-canonical symmetries correspond to the isometries of the configuration space CH (the world of classical worlds) and are induced from the corresponding symmetries of $\delta H_{\pm} \equiv \delta M^4_{\pm} \times CP_2$. The explicit representations have been constructed for both 2-D and stringy options. The most stringent option having strong support from various considerations is that single particle conformal weights are of form $1/2 + i\sum_k n_k y_k$, where $s_k = 1/2 + iy_k$ is zero of Riemann zeta. The construction of many particle conformally bound states for poly-zetas leads to the same spectrum for bound states and predicts that only 2- and 3-parton bound states are irreducible. On the other hand, conformal weights are additive for the (anti)commutators of (super)Hamiltonians and gives thus all weights of form $s = n + i\sum_k n_k y_k$.

The interpretation of this picture is not obvious.

1. The first interpretation would be that also other conformal weights are possible but that the commutator and anti-commutator algebras of super-canonical algebra containing conformal weights $Re(s) = k/2$, $k > 1$, represent gauge degrees of freedom. The sub-Virasoro algebra generated by L_n, $n > 0$, would generate these conformal weights which would suggest that L_n, $n > 0$, but not L_0, must annihilate the physical states. The problem is that this makes p-adic thermodynamics impossible.

2. p-Adic mass calculations would suggest that Super Kac-Moody Virasoro (SKMV) generators L_n, $n > 0$, do not correspond to pure gauge degrees of freedom, and a more general interpretation would be that all these conformal weights are possible and represent genuine physical degrees of freedom. The extension of the algebra using the standard assumption $L_{-n} = L_n^{\dagger}$ would bring in also the conformal weights $Re(s) = -k/2$, $k \geq 1$. p-adic mass calculations would encourage to think that it is super-canonical (SC) generators L_{-n}, $n > 0$, which annihilate tachyonic ground states and stabilize them against tachyonic p-adic thermodynamics. The physical ground state with a vanishing conformal weight would be constructed from this tachyonic ground state and p-adic thermodynamics for $SKMV$ generators L_n, $n > 0$, would apply to it.

3. In the discrete variant of theory required by number theoretic universality all stringy sub-manifolds of X^2 corresponding to the inverse images of $z = \zeta(n/2 + i\sum_k n_k y_k) \in S^2 \subset CP_2$ would be realized so that one would have probability amplitude in the discrete set of these number theoretic strings. $SKMV$ generators L_n and G_r would excite $n > 0$ "shells" in this structure whereas SC generators would generate $n < 0$ shells.

4. Also the trivial zeros $s_n = -2n$, $n > 0$, of Riemann Zeta could correspond to physically interesting conformal weights for the super-canonical algebra (at least). In the region $r \geq r_0$ the function r^{-2n} approaches zero and these powers are square integrable in this region. The orthogonality with other

states could be achieved by arranging things suitably in other degrees of freedom [B2]. Since ζ is real also along real line, the set of even integers $\sum_k n_k s_k$, $n_k \in Z$ is mapped by ζ to the same real line of $S^2 \subset CP_2$ as non-trivial zeros of ζ. p-Adic mass calculations would suggest that states with conformal weight $s_{min} = -2n_{max}$ (at least these) could represent null states annihilated by L_{-n}, $n > 0$.

6.3.2 Bosonic super Kac-Moody algebra

The generators of bosonic super Kac-Moody algebra leave the light-likeness condition $\sqrt{g_3} = 0$ invariant. This gives the condition

$$\delta g_{\alpha\beta} Cof(g^{\alpha\beta}) = 0 \ , \tag{73}$$

Here Cof refers to matrix cofactor of $g_{\alpha\beta}$ and summation over indices is understood. The conditions can be satisfied if the symmetries act as combinations of infinitesimal diffeomorphisms $x^\mu \rightarrow x^\mu + \xi^\mu$ of X^3 and of infinitesimal conformal symmetries of the induced metric

$$\delta g_{\alpha\beta} = \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{74}$$

1. Ansatz as an X^3-local conformal transformation of imbedding space

Write δh^k as a super-position of X^3-local infinitesimal diffeomorphisms of the imbedding space generated by vector fields $J^A = j^{A,k}\partial_k$:

$$\delta h^k = c_A(x) j^{A,k} \ . \tag{75}$$

This gives

$$\begin{aligned} c_A(x) \quad & \left[D_k j^A_l + D_l j^A_k\right] \partial_\alpha h^k \partial_\beta h^l + 2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l \\ = \quad & \lambda(x) g_{\alpha\beta} + \partial_\mu g_{\alpha\beta}\xi^\mu + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \end{aligned} \tag{76}$$

If an X^3-local variant of a conformal transformation of the imbedding space is in question, the first term is proportional to the metric since one has

$$D_k j^A_l + D_l j^A_k = 2h_{kl} \ . \tag{77}$$

The transformations in question includes conformal transformations of $H_\pm$ and isometries of the imbedding space H.

The contribution of the second term must correspond to an infinitesimal diffeomorphism of X^3 reducible to infinitesimal conformal transformation ψ^μ:

$$2\partial_\alpha c_A h_{kl} j^{A,k} \partial_\beta h^l = \xi^\mu \partial_\mu g_{\alpha\beta} + g_{\mu\beta}\partial_\alpha \xi^\mu + g_{\alpha\mu}\partial_\beta \xi^\mu \ . \tag{78}$$

2. A rough analysis of the conditions

One could consider a strategy of fixing c_A and solving solving ξ^μ from the differential equations. In order to simplify the situation one could assume that $g_{ir} = g_{rr} = 0$. The possibility to cast the metric in this form is plausible since generic 3-manifold allows coordinates in which the metric is diagonal.

1. The equation for g_{rr} gives

$$\partial_r c_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{79}$$

   The radial derivative of the transformation is orthogonal to X^3. No condition on ξ^α results. If c_A has common multiplicative dependence on $c_A = f(r) d_A$ by a one obtains

$$d_A h_{kl} j^{Ak} \partial_r h^k \quad = \quad 0 \ . \tag{80}$$

   so that J^A is orthogonal to the light-like tangent vector $\partial_r h^k$ X^3 which is the counterpart for the condition that Kac-Moody algebra acts in the transversal degrees of freedom only. The condition also states that the components g_{ri} is not changed in the infinitesimal transformation.

   It is possible to choose $f(r)$ freely so that one can perform the choice $f(r) = r^n$ and the notion of radial conformal weight makes sense. The dependence of c_A on transversal coordinates is constrained by the transversality condition only. In particular, a common scale factor having free dependence on the transversal coordinates is possible meaning that X^3-local conformal transformations of H are in question.

2. The equation for g_{ri} gives

$$\partial_r \xi^i \quad = \quad \partial_r c_A h_{kl} j^{Ak} h^{ij} \partial_j h^k \ . \tag{81}$$

   The equation states that g_{ri} are not affected by the symmetry. The radial dependence of ξ^i is fixed by this differential equation. No condition on ξ^r results. These conditions imply that the local gauge transformations are dynamical with the light-like radial coordinate r playing the role of the time variable. One should be able to fix the transformation more or less arbitrarily at the partonic 2-surface X^2.

3. The three independent equations for g_{ij} give

$$\xi^\alpha \partial_\alpha g_{ij} + g_{kj}\partial_i \xi^k + g_{ki}\partial_j \xi^k = \partial_i c_A h_{kl} j^{Ak} \partial_j h^l \ . \tag{82}$$

These are 3 differential equations for 3 functions ξ^α on 2 independent variables x^i with r appearing as a parameter. Note however that the derivatives of ξ^r do not appear in the equation. At least formally equations are not over-determined so that solutions should exist for arbitrary choices of c_A as functions of X^3 coordinates satisfying the orthogonality conditions. If this is the case, the Kac-Moody algebra can be regarded as a local algebra in X^3 subject to the orthogonality constraint.

This algebra contains as a subalgebra the analog of Kac-Moody algebra for which all c_A except the one associated with time translation and fixed by the orthogonality condition depends on the radial coordinate r only. The larger algebra decomposes into a direct sum of representations of this algebra.

3. Commutators of infinitesimal symmetries

The commutators of infinitesimal symmetries need not be what one might expect since the vector fields ξ^μ are functionals c_A and of the induced metric and also c_A depends on induced metric via the orthogonality condition. What this means that $j^{A,k}$ in principle acts also to ϕ_B in the commutator $[c_A J^A, c_B J^B]$.

$$\left[c_A J^A, c_B J^B\right] = c_A c_B J^{[A,B]} + J^A \circ c_B J^B - J^B \circ c_A J^A \ , \tag{83}$$

where $\circ$ is a short hand notation for the change of c_B induced by the effect of the conformal transformation J^A on the induced metric.

Luckily, the conditions in the case $g_{rr} = g_{ir} = 0$ state that the components g_{rr} and g_{ir} of the induced metric are unchanged in the transformation so that the condition for c_A resulting from g_{rr} component of the metric is not affected. Also the conditions coming from $g_{ir} = 0$ remain unchanged. Therefore the commutation relations of local algebra apart from constraint from transversality result.

The commutator algebra of infinitesimal symmetries should also close in some sense. The orthogonality to the light-like tangent vector creates here a problem since the commutator does not obviously satisfy this condition automatically. The problem can be solved by following the recipes of non-covariant quantization of string model.

1. Make a choice of gauge by choosing time translation P^0 in a preferred M^4 coordinate frame to be the preferred generator $J^{A_0} \equiv P^0$, whose coefficient $\Phi_{A_0} \equiv \Psi(P^0)$ is solved from the orthogonality condition. This assumption is analogous with the assumption that time coordinate is non-dynamical in the quantization of strings. The natural basis for the algebra is obtained by allowing only a single generator J^A besides P^0 and putting $d_A = 1$.

2. This prescription must be consistent with the well-defined radial conformal weight for the $J^A \neq P^0$ in the sense that the proportionality of d_A to r^n for $J^A \neq P^0$ must be consistent with commutators. SU(3) part of the algebra is of course not a problem. From the Lorentz vector property of P^k it is clear that the commutators resulting in a repeated commutation have well-defined radial conformal weights only if one restricts $SO(3,1)$ to $SO(3)$ commuting with P^0. Also D could be allowed without losing well-defined radial conformal weights but the argument below excludes it. This picture conforms with the earlier identification of the Kac-Moody algebra.

   Conformal algebra contains besides Poincare algebra and the dilation $D = m^k\partial_{m^k}$ the mutually commuting generators $K^k = (m^r m_r \partial_{m^k} - 2m^k m^l \partial_{m^l})/2$. The commutators involving added generators are

$$\begin{array}{ll} \left[D, K^k\right] = -K^k \ , & \left[D, P^k\right] = P^k \ , \\ \left[K^k, K^l\right] = 0 \ , & \left[K^k, P^l\right] = m^{kl}D - M^{kl} \ . \end{array} \tag{84}$$

   From the last commutation relation it is clear that the inclusion of K^k would mean loss of well-defined radial conformal weights.

3. The coefficient dm^0/dr of $\Psi(P^0)$ in the equation

$$\Psi(P^0)\frac{dm^0}{dr} = -J^{Ak}h_{kl}\partial_r h^l$$

   is always non-vanishing due to the light-likeness of r. Since P^0 commutes with generators of $SO(3)$ (but not with D so that it is excluded!), one can *define* the commutator of two generators as a commutator of the remaining part and identify $\Psi(P^0)$ from the condition above.

4. Of course, also the more general transformations act as Kac-Moody type symmetries but the interpretation would be that the sub-algebra plays the same role as $SO(3)$ in the case of Lorentz group: that is gives rise to generalized spin degrees of freedom whereas the entire algebra divided by this sub-algebra would define the coset space playing the role of orbital degrees of freedom. In fact, also the Kac-Moody type symmetries for which c_A depends on the transversal coordinates of X^3 would correspond to orbital degrees of freedom. The presence of these orbital degrees of freedom arranging super Kac-Moody representations into infinite multiplets labelled by function basis for X^2 means that the number of degrees of freedom is much larger than in string models.

5. It is possible to replace the preferred time coordinate m^0 with a preferred light-like coordinate. There are good reasons to believe that orbifold singularity for phases of matter involving non-standard value of Planck constant corresponds to a preferred light-ray going through the tip of $\delta M^4_{\pm}$.

Thus it would be natural to assume that the preferred M^4 coordinate varies along this light ray or its dual. The Kac-Moody group $SO(3) \times E^3$ respecting the radial conformal weights would reduce to $SO(2) \times E^2$ as in string models. E^2 would act in tangent plane of $S^2_{\pm}$ along this ray defining also $SO(2)$ rotation axis.

4. Hamiltonians

The action of these transformations on Chern-Simons action is well-defined and one can deduce the conserved quantities having identification as configuration space Hamiltonians. Hamiltonians also correspond to closed 2-forms. The condition that the Hamiltonian reduces to a dual of closed 2-form is satisfied because X^2-local conformal transformations of $M^4_{\pm} \times CP_2$ are in question (X^2-locality does not imply any additional conditions).

5. Action on spinors

One can imagine two interpretations for the action of generalized Kac-Moody transformations on spinors.

1. Both SO(3) and SU(3) rotations have a standard action as spin rotation and electro-weak rotation allowing to define the action of the Kac-Moody algebra J^A on spinors. This action is not consistent with the generalized eigenvalue equation unless one restricts it to X^2 at $\delta H_{\pm}$.

2. Since Kac-Moody generator performs a local spinor rotation and increases the conformal weight by n units, the simplest possibility is that the action of transformation adds to Ψ_λ with $\lambda = 1/2 + i\sum_k n_k y_k$, a term with eigenvalue $\lambda + n$ and having $J^A \Psi_\lambda$ as initial values at X^2. This would make natural the interpretation as a gauge transformation apart from the effects caused by the possible central extension term.

6. How central extension term could emerge?

The central extension term of Kac-Moody algebra could correspond to a symplectic extension which can emerge from the freedom to add a constant term to Hamiltonians as in the case of super-canonical algebra. The expression of the Hamiltonians as closed forms could allow to understand how the central extension term emerges.

In principle one can construct a representation for the action of Kac-Moody algebra on fermions a representations as a fermionic bilinear and the central extension of Kac-Moody algebra could emerge in this construction just as it appears in Sugawara construction.

6.3.3 Fermionic Kac-Moody algebra in spin and electro-weak degrees of freedom

The action of spin rotations and electro-weak rotations can be identified in terms of the group $SU(2) \times SU(2) \times U(1)$ associated inherently with $N =$

4 super-conformal symmetry. The action on zero modes and eigen modes Ψ is straightforward to write as multiplication on the initial values at X^2 and assuming that λ in the generalized eigenvalue equation is replaced by $\lambda + n$.

Fermionic super-generators correspond naturally to zero modes and eigen modes of the modified Dirac operator labelled by the radial conformal weights $\lambda = 1/2 + i\sum_k n_k yk$ and by the quantum numbers labelling the dependence on transversal degrees of freedom. The real part of the conformal weight would corresponds for $D\Psi = 0$ to ground state conformal weight $h = 0$(Ramond) and to $h = 1/2$ for $\lambda \neq 0$ (N-S). That also bosonic super-canonical Hamiltonians can have half odd integer conformal weight is however in conflict with the intuition that half-odd integer conformal weights correspond to states with odd fermion number.

For Ramond representations the lines $\zeta(Re(s) = n) \subset S^2$, $n \geq 0$, would represent the conformal weights at space-time level and for N-S representations the lines would correspond to $\zeta(Re(s) = n + 1/2) \subset S^2$. If also trivial zeros are possible they would correspond to the lines $\zeta(Re(s) = n - 2k) \subset S^2$, $k = 1, 2, ...$

6.3.4 Radial Super Virasoro algebras

The radial Super Virasoro transformations act on both $\delta H_{\pm}$ and partonic 3-surface X^3 and are consistent with the freedom to choose the basis of $H_{\pm}$ Hamiltonians and the eigenmode basis of the modified Dirac operator by a re-scaling the light-like vector (t^k or more plausibly, its dual n^k) appearing in the definition of the generalized eigenvalue equation.

In the partonic sector a possible interpretation is as local diffeomorphisms of X^3. These transformations do not however leave X^3 invariant as a whole, which brings in some delicacies. In the case of $\delta H_{\pm}$ the tip of the future light-cone remains invariant only for $n \geq 0$ and $r = \infty$ only for $n \leq 0$. These facts could explain why only the generators L_n, $n < 0$ (or $n < 0$ depending on whether positive or negative energy component of zero energy state is in question) annihilate the ground states.

One can assign to the Virasoro algebra of $H_{\pm}$ Hamiltonians as Noether charges defined by current $\Pi^0_k j^{Ak}$ which reduces to a dual of a closed 2-form in the case of $H_{\pm}$ because its symplectic form annihilates j^{Ak}. The transformations associated with X^3 correspond to a unique shift of X^2 in the light-like direction by $\delta h^k = r^n \partial_r h^k$ so that the Hamiltonian is well-defined and reduces to a value of a closed 2-form so that the stringy picture emerges.

The corresponding fermionic super Hamiltonians $G_r = \overline{\nu} r^n \Gamma_r \Psi$ anti-commute to these as is easy to see by noticing that the light-like radial gamma matrices Γ_r appear in the combination $\Gamma_r \gamma^0 \Gamma_r = \gamma_0$ in the anti-commutator so that it does not vanish. One can consider also more general fermionic generators obtained by replacing right-handed neutrino spinor with an arbitrary solution of $D\Psi = 0$ which is eigen spinor of $J^{kl}\Sigma_{kl}$ appearing in the fermionic anti-commutation relations. This would give rise to a full $N = 4$ super-conformal symmetry of Ramond type but having infinite degeneracy due to the dependence on transversal coordinates of X^3. If one allows also the solutions of $D\Psi = \lambda\Psi$ one obtains counterparts of N-S type representations with a similar degeneracy.

It must be emphasized that four-momentum does not appear neither in the representations of Super Virasoro generators as it does in string models and this is consistent with the Lorentz invariant identification of mass squared as vacuum expectation value of the net conformal weight. Also the problems with tachyons are avoided. Four-momentum could creep in if one had Sugawara type representation of Super Virasoro generators in terms of Kac-Moody generators which indeed contain also translation generators now. Note also that the stringy conformal weight would be associated with partonic 2-surface, whereas radial conformal weight is associated with its light-like orbit. Furthermore, the origin of the radial super-conformal symmetries is light-likeness rather than stringy character. It is not clear whether it is useful to assign the usual conformal weights with the conformal fields at X^2 and whether the stringy anti-commutation relations for Ψ force this kind of assignment.

6.3.5 Gauge super-symmetries associated with the generalized eigenvalue equation for D

Zero modes which are annihilated by the operator $T = t^k\gamma_k$ or $N = n^k\gamma_k$. t^k (n^k) is the light-like appearing in the generalized eigenvalue equation for the modified Dirac operator. t^k is parallel to X^3 and n^k, which corresponds to the more plausible option, is obtained by changing the direction of the spatial part of t^k in the preferred M^4 coordinate frame associated with the space-time sheet (the rest system or number theoretically determined M^4 time). n^k defines inwards directed tangent vector to the space-time sheet containing X^3. The zero modes of the modified Dirac operator annihilated by T (N) act as super gauge symmetries for the generalized eigen modes of the generalized Dirac operator. They do not depend on r and thus have a vanishing conformal weight.

The freedom to choose the scaling of t^k (n^k) rather freely gives rise to a further symmetry which does not affect the eigenvalue spectrum but modifies the eigen modes. This symmetry is definitely a pure gauge symmetry.

6.3.6 What about ordinary conformal symmetries?

Ordinary conformal symmetries acting on the complex coordinate of X^2 have not yet been discussed. These symmetries involve the dependence on the induced metric through the moduli of characterizing the conformal structure of X^2. Stringy picture would suggest in the case of a spherical topology that the zero modes and eigen modes of Ψ are proportional to z^n at X^2. Only $n \geq 0$ mode would be non-singular at the northern hemisphere and $n \leq 0$ at the southern hemisphere and the eigen modes are non-normalizable.

One cannot glue these modes together at equator unless one assumes the behavior z^n, $n \geq 0$, on the northern hemisphere and $\overline{z}^{-n}$, $n \geq 0$, on the southern hemisphere. The identification $\Psi_+(z) = \Psi_-^\dagger(z)$ ($z \rightarrow \overline{z}$ in Hermitian conjugation) at equator would state that "positive energy" particle at the northern hemisphere corresponds to a negative energy antiparticle at the southern hemisphere. The assumption that energy momentum generators $T_+(z)$ and $T_-(z)$ are related in the same manner at equator gives $L_n = L^\dagger_{-n}$ as required. Second candidate for the basis are spherical harmonics which are eigenstates of $L_0 - \overline{L_0}$ defining angular momentum operator L_z but they do not possess well defined conformal weights.

The radial time evolution for the Kac-Moody generators does not commute with L_0 whereas well-defined radial conformal weights are possible. This would support the view that the conformal weight associated with X^2 degrees of freedom does not contribute to the mass squared. If this picture is correct, L_0 would label different SKM representations and play a role similar to that in conformal field theories for critical systems.

6.3.7 How to interpret the overall sign of conformal weight?

The overall sign of conformal weight can be changed by replacing r with $1/r$ and the region $r > r_0$ with $r < r_0$ of $\delta H_\pm$ or of partonic 3-surface. The earlier idea that the conformal weights associated with the super-conformal algebras assignable to $\delta H_\pm$ and to light-like partonic 3-surfaces have opposite signs would allow to construct representations of super-canonical algebra by constructing a tachyonic ground state using super-canonical generators and its excitations using super Super-Kac Moody generators as in super string models.

There is however an objection against this idea. The partons at $\delta H_\pm$ would have a finite distance from the tip of the light cone at all points where they correspond to non-vacuum extremals, so that the phase transitions changing the value of Planck constant should always occur via vacuum extremals. This would not allow the leakage of Kähler magnetic flux between different sectors of imbedding space. The cautious conclusion is that at least in the super-canonical sector both $r > r_0$ and $r < r_0$ sectors related by the conformal transformation $r \rightarrow 1/r$ must be allowed and correspond to positive and negative values for the radial super-conformal weights.

In zero energy ontology particle reactions correspond to zero energy states which at space-time level carry positive energy particles at the end of world in geometric past and negative energy particles at the end of world in the geometric future. Also conformal weights are of opposite sign so that vanishing of the net conformal weights holds true only for zero energy states in accordance with the spirit of p-adic mass calculations. If the states of geometric past correspond to positive (negative) super Kac-Moody (super-canonical) conformal weights, the scattering could be regarded as a process leading from the region $r > r_0$ at δM^4_+0 to the region $r < r_0$ at δM^4_-. At partonic level the incoming partons would correspond to the region $r < r_0$ and outgoing partons to the region $r > r_0$, which conforms with the idea that the final state can partons can be arbitrary far in the geometric future.

In certain sense this picture would reproduce big ban-big crush picture at the level of super-canonical algebra. $r < r_0$ means that partons can be arbitrarily near to the tip of δM^4_- representing the final singularity whereas $r > r_0$ for δM^4_+ would be the counterpart for big bang.

6.3.8 Absolute extremum property for Kähler action implies dynamical Kac-Moody and super conformal symmetries

The identification of the criterion selecting the preferred extremal of Kähler action defining space-time surface as a counterpart of Bohr orbit has been a long standing challenge. The first guess was that an absolute minimum is in question. The number theoretic theoretic picture, in particular $HO-H$ duality [E2] resolves the problem by assigning to each point of X^4 a preferred plane M^2, which also fixes the boundary conditions for the field equations at light-like partonic 3-surfaces. The still open questions are whether the H images of hyper-quaternionic 4-surfaces of $HO = M^8$ are indeed extremals of Kähler action and whether these preferred extremals satisfy absolute extremum property. Be as it may, the following argument suggests that absolute extremum property gives rise to additional symmetries.

The extremal property for Kähler action with respect to variations of time derivatives of initial values keeping h^k fixed at X^3 implies the existence of an infinite number of conserved charges assignable to the small deformations of the extremum and to H isometries. Also infinite number of local conserved super currents assignable to second variations and to covariantly constant right handed neutrino are implied. The corresponding conserved charges vanish so that the interpretation as dynamical gauge symmetries is appropriate. This result provides strong support that the local extremal property is indeed consistent with the almost-topological QFT property at parton level.

The starting point are field equations for the second variations. If the action contain only derivatives of field variables one obtains for the small deformations δh^k of a given extremal

$$\begin{aligned} \partial_\alpha J_k^\alpha &= 0 \ , \\ J_k^\alpha &= \frac{\partial^2 L}{\partial h_\alpha^k \partial h_\beta^l}\delta h_\beta^l \ , \end{aligned} \tag{85}$$

where h_α^k denotes the partial derivative $\partial_\alpha h^k$. A simple example is the action for massless scalar field in which case conservation law reduces to the conservation of the current defined by the gradient of the scalar field. The addition of mass term spoils this conservation law.

If the action is general coordinate invariant, the field equations read as

$$D_\alpha J^{\alpha,k} = 0 \tag{86}$$

where D_α is now covariant derivative and index raising is achieved using the metric of the imbedding space.

The field equations for the second variation state the vanishing of a covariant divergence and one obtains conserved currents by the contraction this equation with covariantly constant Killing vector fields j_A^k of M^4 translations which means that second variations define the analog of a local gauge algebra in M^4 degrees of freedom.

$$\begin{aligned} \partial_\alpha J_n^{A,\alpha} &= 0 \ , \\ J_n^{A,\alpha} &= J_n^{\alpha,k} j_k^A \ . \end{aligned} \tag{87}$$

Conservation for Killing vector fields reduces to the contraction of a symmetric tensor with $D_k j_l$ which vanishes. The reason is that action depends on induced metric and Kähler form only.

Also covariantly constant right handed neutrino spinors Ψ_R define a collection of conserved super currents associated with small deformations at extremum

$$\begin{aligned} J_n^\alpha &= J_n^{\alpha,k}\gamma_k \Psi_R \ , \\ &. \end{aligned} \tag{88}$$

Second variation gives also a total divergence term which gives contributions at two 3-dimensional ends of the space-time sheet as the difference

$$\begin{aligned} Q_n(X_f^3) - Q_n(X^3) &= 0 \ , \\ Q_n(Y^3) &= \int_{Y^3} d^3x J_n \ , \ \ J_n = J^{tk} h_{kl} \delta h_n^l \ . \end{aligned} \tag{89}$$

The contribution of the fixed end X^3 vanishes. For the extremum with respect to the variations of the time derivatives $\partial_t h^k$ at X^3 the total variation must vanish. This implies that the charges Q_n defined by second variations are identically vanishing

$$Q_n(X_f^3) = \int_{X_f^3} J_n = 0 \ . \tag{90}$$

Since the second end can be chosen arbitrarily, one obtains an infinite number of conditions analogous to the Virasoro conditions. The analogs of unbroken loop group symmetry for H isometries and unbroken local super symmetry generated by right handed neutrino result. Thus extremal property is a necessary condition for the realization of the gauge symmetries present at partonic level also at the level of the space-time surface. The breaking of super-symmetries could perhaps be understood in terms of the breaking of these symmetries for light-like partonic 3-surfaces which are not extremals of Chern-Simons action.

7 Appendix

7.1 Representations for the configuration space gamma matrices in terms of super-canonical charges at light cone boundary

This appendix represents the earlier version of the construction of configuration gamma matrices and definitely differs from the recent, much more elegant construction to be discussed in the main text and based on the realization of that quantum TGD reduces to almost topological QFT at parton level. This approach, which relies on number theoretical considerations and von Neumann algebras, supports strongly stringy picture meaning that configuration space metric and gamma matrices are coded by data at 1-dimensional sub-manifolds of partonic 2-surfaces. The motivation for including the older representation is that it perhaps gives some general idea about the geometric aspects of the problem.

A priori one can consider two representations for the gamma matrices and canonical supercharges in terms of the leptonic (Ramond) and quark like (N-S) oscillator operators. This degeneracy might relate to the degeneracy associated with the magnetic and electric representations of the configuration space Hamiltonians and also to the ordinary and kappa super-symmetries. The representations differ only by the replacement of Kähler magnetic flux with Kähler electric flux in the defining formulas. In the following arguments are developed to support the view that quark like N-S super generators indeed provide simple representations of configuration space gamma matrices. This is in contrast to the representation in terms of leptonic oscillator operators deduced in the next section: whether also quark like representation exists remains an open problem.

7.1.1 Magnetic flux representation of the canonical algebra

In order to derive representation of the configuration space gamma matrices and super charges it is good to restate the basic facts about the magnetic flux representation of the configuration space gamma matrices.

1. Kähler magnetic invariants

The Kähler magnetic fluxes defined both the normal component of the Kähler magnetic field and by its absolute value

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \ , \\ Q_m^+(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \ , \end{aligned} \tag{91}$$

over two-surfaces X^2 are invariants under both Lorentz isometries and the canonical transformations of CP_2 and can be calculated once X^3 is given. Assuming 7–3 duality, the surfaces X^2 corresponds to an intersections of the 3-D light like CDs with the $X^3_l \times CP_2$ and 2-sub-manifolds of the space-like 3-surface X^3. In this case no additional data about the extremal of Kähler action is needed.

For a closed surface $Q_m(X^2)$ vanishes unless the homology equivalence class of the surface is nontrivial in CP_2 degrees of freedom. In this case the flux is quantized. $Q^+_M(X^2)$ is non-vanishing for closed surfaces, too. Signed magnetic fluxes over non-closed surfaces depend on the boundary of X^2 only:

$$\begin{array}{l} \int_{X^2} J = \int_{\delta X^2} A \quad . \\ J = dA \quad . \end{array}$$

Un-signed fluxes can be written as sum of similar contributions over the boundaries of regions of X^2 in which the sign of J remains fixed.

$$\begin{aligned} Q_m(X^2) &= \int_{X^2} J_{CP_2} = J_{\alpha\beta}\epsilon^{\alpha\beta}\sqrt{g_2}d^2x \quad , \\ Q^+_m(X^2) &= \int_{X^2} |J_{CP_2}| \equiv \int_{X^2} |J_{\alpha\beta}\epsilon^{\alpha\beta}|\sqrt{g_2}d^2x \quad , \end{aligned} \tag{92}$$

There are also canonical invariants, which are Lorentz covariants and defined as

$$\begin{aligned} Q_m(K,X^2) &= \int_{X^2} f_K J_{CP_2} \quad , \\ Q^+_m(K,X^2) &= \int_{X^2} f_K |J_{CP_2}| \quad , \\ f_{K\equiv(s,n,k)} &= e^{is\phi} \times \frac{\rho^{n-k}}{(1+\rho^2)^k} \times (\frac{r_M}{r_0})^k \end{aligned} \tag{93}$$

These canonical invariants transform like an infinite-dimensional unitary representation of Lorentz group.

2. Generalized magnetic fluxes

Isometry invariants are just a special case of fluxes defining natural coordinate variables for the configuration space. Canonical transformations of CP_2 act as $U(1)$ gauge transformations on the Kähler potential of CP_2 (similar conclusion holds at the level of $\delta M^4_+ \times CP_2$).

One can generalize these transformations to local canonical transformations by allowing the Hamiltonians to be products of the CP_2 Hamiltonians with the real and imaginary parts of the functions $f_{s,n,k}$ defining the Lorentz covariant function basis H_A, $A \equiv (a,s,n,k)$ at the light cone boundary: $H_A = H_a \times f(s,n,k)$, where a labels the Hamiltonians of CP_2.

One can associate to any Hamiltonian H^A of this kind both signed and unsigned magnetic flux via the following formulas:

$$
\begin{aligned}
Q_m(H_A|X^2) &= \int_{X^2} H_A J \ , \\
Q_m^+(H_A|X^2) &= \int_{X^2} H_A |J| \ .
\end{aligned}
\tag{94}
$$

Both signed and unsigned magnetic fluxes and their superpositions

$$
Q_m^{\alpha,\beta}(H_A|X^2) \ = \ \alpha Q_m(H_A|X^2) + \beta Q_m^+(H_A|X^2) \ , \ \ A \equiv (a,s,n,k) \tag{95}
$$

provide representations of Hamiltonians. Note that canonical invariants $Q_m^{\alpha,\beta}$ correspond to $H^A = 1$ and $H^A = f_{s,n,k}$. $H^A = 1$ can be regarded as a natural central term for the Poisson bracket algebra. Therefore, the isometry invariance of Kähler magnetic and electric gauge fluxes follows as a natural consequence.

7.1.2 Expressions for the canonical supercharges

The natural hypothesis generalizing the identification of the configuration space Hamiltonians as classical canonical charges, is that quark like super-canonical generators S_q^A are obtained as the supercharges, which are in a well-defined sense square roots of flux Hamiltonians. There are two possibilities.

1. The quantities J or $|J|$ appearing in the definition of the flux Hamiltonians are replaced with their square roots. The requirement that super charge is real favors the use of unsigned flux $|J|$.

2. The anti-commutation relations of the induced fermion fields at Y^3 are proportional to J or to $|J|$. This conforms with the fact that modified Dirac equation is related to Kähler action by super-symmetry. In the following only this option is discussed.

   This guarantees that the definitions of the flux Hamiltonians and corresponding super-charges is consistent with the basic properties of the Kähler action. For 3-surfaces which reduce to vacuum extremals at light cone boundary the super charges should vanish identically and this is indeed the case since the Kähler magnetic field is identically vanishing. Thus one can say that although Kähler action need not appear explicitly in the definition of configuration space Hamiltonians, only the dynamics defined by the Kähler action conforms with the basic properties of the super-canonical algebra.

   These supercharges do not correspond to the symmetries of the Kähler action but to the isometries of the configuration space. The requirement that these currents are conserved on the light cone boundary would presumably mean that Kähler function is stationary with respect to the canonical variations: obviously this does not make sense.

The representation of super-canonical generators are derived from following requirements.

1. Super generators and their "hermitian conjugates" correspond to the positive and negative energy space-time sheets meeting at space-like 3-surface at $X_l^3 \times CP_2$.

2. The solutions of the modified Dirac equation generate gauge super symmetries and thus the addition of a solution of the modified Dirac equation to $\Psi_{\pm}$ does not affect the configuration space super generator and gamma matrix. This is guaranteed D_+ *resp.* D_-^{-1} appears in the definition of the super generator $S^{A,+}$ and its "hermitian conjugate" $S^{A,-}$.

3. A further assumption crucial for anticommutation relations of gamma matrices is the condition $D_+D_-^{-1} = 1$. Hence D_+ and D_- act only as projectors eliminating the super gauge degrees of freedom. This implies that Dirac determinant defining the exponent for the difference of the Kähler action for the maximal deterministic space-time regions meeting at X^3 equals to one. This means that space like 3-surfaces at 7-D CDs do not code information about configuration space Kähler function, which is thus determined by the data at 3-D light like CDs.

A priori there are two candidates for the super-canonical sub-algebra defining gamma matrices.

1. A candidate for a quark representation of the super-canonical generators is obtained as

$$
\begin{aligned}
S_q^{A+} &= \frac{1}{2i}\int_{Y^3} \overline{\nu}_R j_{\bar{k}}^A \Gamma^{\bar{k}} D_+ \Psi_+ dV \quad , \\
S_q^{A-} &= -\frac{1}{2i}\int_{Y^3} \overline{\Psi}_- \overleftarrow{D}_-^{-1} j_k^A \Gamma^k \nu_R dV \quad , \\
dV &= f(h) \times \sqrt{g} d^2xper , \\
j^{Ak} &= J^{kl}\partial_l H_A \quad .
\end{aligned} \tag{96}
$$

   The effective two-dimensionality implied by 7-3 duality allows to drop from the earlier formula for dV the factor dr_M/r_M.

   It is essential that complex coordinates and holomorphic half of the canonical current $j_{\bar{k}}^A$ of CP_2 are used. For the Hermitian conjugate S^{A+} j_k^A appears in the formula. Ψ denotes the second quantized free induced quark field at $X^4(Y^3)$. The function $f(h)$ is some function of the coordinates of $\delta M_+^4 \times CP_2$ needed to take care that the anti-commutations are precisely correct. It is important to notice that j_k^A are restricted to the light cone boundary in these formulas and the selection of preferred extremals as generalized Bohr orbits defines the continuation of the canonical deformations to the entire space-time surface $X^4(Y^3)$. One can extend this algebra by allowing arbitrary H-spinors harmonics instead of only ν_R so that generators carrying both quark and lepton number become possible.

2. One might criticize the super-algebra in which contraction of second quantized quark field and classical lepton spinor harmonics occur, and argue that only vector currents make sense. One can indeed define a set of leptonic vector currents by replacing D_+ and D_-^{-1} in the defining formulas by the projector P into spinor modes with non-vanishing eigenvalue so that the local $N = 4$ super symmetry is not lost. If D^+ and D^- have same eigen value spectrum, the resulting super generators anti-commute to same expressions as the super generators already defined. If this argument is accepted then only the leptonic super-algebra with D replaced by a projector P can be used to define the configuration space gamma matrices. It must be re-emphasized that the covariantly constant right-handed neutrino can be replaced by any spinor harmonic of H so that the algebra extends to a much larger super algebra which can be used to construct physical states.

   These algebras can be assumed to be Ramond type algebras. The reason is that constant Hamiltonian does not contribute to the algebra and one thus avoids the un-desired global $N = 1$ super-symmetry generated by the right handed neutrino.

It is also possible to define super-algebras which anti-commute to the function algebra defined by the super-canonical generators.

1. A vectorial leptonic super-algebra is obtained by generalizing the super-symmetry $\Psi \to \Psi + \epsilon\nu_R$ to canonical super-symmetries $\Psi \to \Psi + \epsilon H_A \nu_R$. This would give rise to super-canonical generators

$$
\begin{aligned}
S_l^{A,+} &= \frac{1}{2i}\int_{Y^3} \overline{\nu}_R H_A D_+ \Psi_+ dV \ , \\
S_l^{A,-} &= -\frac{1}{2i}\int_{Y^3} \overline{\Psi}_- \overleftarrow{D}_-^{-1} H_A \nu_R dV
\end{aligned} \tag{97}
$$

   depending on the leptonic oscillator operators only. If the representation is of Ramond type, constant Hamiltonian has a non-vanishing super-counterpart whereas for the quark representation it vanishes. This phenomenon also occurs for the electric and magnetic flux representation of the configuration space Hamiltonians but need not have any special significance. If one assumes that this algebra is N-S type algebra with Hamiltonians possessing half-odd integer conformal weight with respect to the radial coordinate, one can avoid non-vanishing generator associated with a constant Hamiltonian and generating a global super-symmetry. Also this algebra extends by allowing all spinor harmonics.

2. The replacement of $D_\pm$ with P gives a set of generators which have quantum numbers of lepto-quark. This option is subject to the same criticism as the quark like algebra for $D^\pm$. The expression for super-Virasoro algebra seems however to require both lepto-quark like and vectorial representations of the configuration space gamma matrices.

7.1.3 Is second quantization performed for imbedding space spinor fields or for induced spinors?

Whether the second quantization should be performed for imbedding space spinors or for induced spinor fields is key question concerning the construction of the configuration space spinor structure. The simplest option would certainly be that the restriction of the second quantized imbedding space spinor field to Y^3 appears in the definition of super-canonical charges. An alternative option is that the second quantization is done for the induced spinor field. It seem that the latter option is the correct one.

The anti-commutators of super-canonical charges can give canonical super charges only if the anti-commutators of leptonic and and quark fields are proportional to a 2-dimensional delta function

$$\begin{array}{l} \delta_3 = \frac{\delta_2(x,y)}{X}\sqrt{g_2} \ , \\ X = J \ \text{ or } \ |J| \ . \end{array}$$

where x and y refer to the coordinates of X_i^2 (intersection of X_l^3 with X^3). This is what second quantized induced spinor fields should give as anti-commutation relations with normal direction of X_i^2 playing the role of the time coordinate in the ordinary quantization. The appearance of J or —J— is strongly favored by the fact that induced spinor fields should represent vacuum degrees of freedom when 3-surface becomes vacuum extremal.

On the other hand, the anti-commutation relations for the imbedding space spinor fields are expected to produce delta function of $\delta M_+^4 \times CP_2$ rather than that of light cone boundary:

$$\{\Psi^\dagger(h^1), \Psi(h^2)\} = \gamma^0 \delta_7(h_1, h_2) = \gamma^0 \delta^3(m_1, m_2)\delta^4(s_1, s_2) \ . \tag{98}$$

The problem is that the anti-commutator would be proportional to δ_7 rather than δ_3. The formal introduction of $f(h) = \sqrt{\delta_2/\delta_7}$ together with the assumption that $\sqrt{|J|}$ is included in the definition of S_q and S_l certainly extremely non-aesthetic and artificial manner to resolve the problem.

This result supports the number theoretical vision encouraging to think that space-time surfaces as hyper-quaternionic manifolds of M^8 (recall the conjecture about number theoretic spontaneous compactification [E2]) induce "polarizations" of the imbedding space $M^4 \times CP_2$ such that anti-commutation relations for the imbedding space spinor fields can be given only inside that polarization. The associativity requirement would allow only quantization in the hyper-quaternionic sub-manifold of $X^4 \subset M^8$ just as Uncertainty Principle forces to restrict Schrödinger amplitudes to n-dimensional Legendre sub-manifolds of $2n$-dimensional phase space. What is nice that the anti-commutation relations are fixed uniquely for induced spinor fields from the anti-commutations of super-canonical algebra and are very nearly what one might naively expect.

7.1.4 Anti-commutation relations for super-canonical charges

Assuming that the induced spinor fields at the surfaces X_i^2 satisfy the 2-dimensional anti-commutation relations

$$\begin{aligned} \{\Psi^\dagger(x),\Psi(y)\} &= \frac{\delta_2(x,y)X}{\sqrt{g_2}} \ , \\ X &= \alpha J + \beta|vertJ| \ \text{ or } \ |J| \end{aligned} \tag{99}$$

the anti-commutator $\{S^{A+},S^{B-}\}$ gives the term

$$\begin{aligned} \{S^{A-},S^{B+}\} &= \int XY\sqrt{g_2}d^2x \ , \\ Y &= \{j_{A\bar{k}}\Gamma^{\bar{k}},j_{Bl}\Gamma^l\} = J^{\bar{k}l}j_{A\bar{k}}j_{Bl} \\ &= J^{\bar{k}l}\partial_{\bar{k}}H_A\partial_l H_B \equiv \{H_A,H_B\}_{-+} \ . \end{aligned} \tag{100}$$

The result is just the half commutator of the imbedding space Hamiltonians with respect to CP_2 symplectic structure:

$$\{S_q^{A-},S_q^{B+}\} = Q_m^{\alpha,\beta}(\{H_A,H_B\}_{-+}) \ . \tag{101}$$

This means that the real and imaginary parts of the resulting anti-commutator give the metric and Kähler form of the configuration space and that quark super charges correspond to configuration space gamma matrices.

In the case of leptonic super-canonical charges one obtains the product H_AH_B of the Hamiltonians rather than their Poisson bracket.

$$\{S_l^{A-},S_l^{B+}\} = Q_m(H_AH_B) \ . \tag{102}$$

Therefore it seems that in the case of leptons it is not possible to obtain the required kind of super-algebra and that leptons cannot define the representation of configuration space gamma matrices. This resolves the problem whether quarks (N-S representation) or leptons (Ramond representation) should define configuration space gamma matrices. Also the coset space structure of the configuration space based on the generalization of conformal algebras to contain also generators with half-odd integer conformal charge allows only N-S type representation for gamma matrices.

This does not of course mean that leptonic super algebra would be something artificial: leptonic super algebra can be seen as defining the super-symmetrization of the function algebra of $\delta M^4_+ \times CP_2$ so that both symplectic and function algebras are super-symmetrized. In string model context this degeneracy corresponds to the ordinary and kappa super-symmetries. One can also assign to leptons Ramond type super-conformal algebra and corresponding Super Virasoro conditions define leptonic variant of configuration space Dirac equation so that two different Super Virasoro conditions emerge.

Besides magnetic flux Hamiltonians one can also define electric flux Hamiltonians and the conjecture is that the selection of preferred extremals of Kähler action as genearalized Bohr orbits implies that these two representations for the configuration space Hamiltonians are equivalent. This conjecture is nothing but electric-magnetic duality at the level of the configuration space geometry. It seems that the generalization of the electric magnetic duality to the case of super-charges requires only the trivial replacement of magnetic flux J with the electric flux in the defining formulas.

7.2 Self referentiality as a possible justification for $\lambda = \zeta^{-1}(z)$ hypothesis

The development of TGD has involved many ad hoc conjectures having nothing to do with the fundamental structure of TGD. One of them was that the spectrum of the modified Dirac operator could correspond to the spectrum for the inverse of some Zeta function at some preferred points, say zero. The original candidate for the Zeta function was Riemann Zeta, and this led to the proposal of some highly non-trivial number theoretic conjectures stating essentially that the zeros of zeta correspond to algebraic numbers: many of these conjectures have turned out to be wrong. It is now clear that the replacement of Riemann Zeta with the zeta function defined by the values of the generalized eigenvalues of modified Dirac operator at the points of number theoretic braid (minima of Higgs field) is the only reasonable option and there is no need to make the number theoretical conjectures. One can however try to see whether the proposal $\lambda = -zeta^{-1}$ for points of braid might make sense and poses additional and certainly very powerful constraint to the theory.

A real mathematical justification for the hypothesis $\lambda(z) = -\zeta^{-1}(z)$ or its generalization allowing some overall scaling factor is lacking. Note that scaling factor -1 is suggested by the requirement that $\lambda = -1/2 - iy$ is very natural in the case of radial conformal weights.

The justification should relate to the fact that both eigenvalues of D and generalized ζ function in question (perhaps a generalization of $\zeta_p(z) = 1/(1 - p^{-s})$) would code for data about the properties of the partonic 2-surface so that the relationship $\lambda = -\zeta^{-1}$ would make sense. ζ^{-1} has several branches so that it is not clear whether they contain same information. Perhaps the different branches of ζ^{-1} correspond to different branches for number theoretic braids associated with points z. This would suggest that the number of branches is same as the number of points of the braid.

7.2.1 Is the spectrum of D expressible in terms of branches of an inverse of some zeta function?

$\hat{\Gamma}^r$ vanishes as X^3_l approaches to a vacuum extremal so that its inverse fails to exist. This requires that $N\Psi$ approaches zero in such a manner that the action of O on Ψ given by

$$\begin{aligned} O\Psi &= \frac{\Phi}{a^k a^l h_{kl}} a^k \partial_r h^l \gamma_k \gamma_l \Psi = \frac{\Phi}{\sqrt{a^k a^l h_{kl}}}(A+B)\Psi \ , \\ A &= e^k h_{kl} \partial_r h^l \ , \\ B &= \frac{1}{2} e^k \partial_r h^l \Sigma_{kl} \ , \\ e^k &= \frac{a^k}{\sqrt{a^k a^l h_{kl}}} \end{aligned} \tag{103}$$

gives a finite and well-defined result. This poses conditions on Φ already fixed by the requirement that zero modes induces super- conformal symmetries.

The expectation is that the condition

$$\frac{\Phi}{\sqrt{a^k a^l h_{kl}}}(A+B)\Psi = K\Psi \ , \tag{104}$$

where K is constant, can be posed asymptotically so that it becomes possible to speak about asymptotic eigen-states of the "Hamiltonian" O. In the non-asymptotic region O depends on r so that global eigen-states are not possible in general. An interpretation in terms of interactions is natural. In the asymptotic region Ψ would behave as $\Psi \propto exp(\lambda K r)$. Since the sigma matrices defined by the commutators of time like M^4 gamma matrices and CP_2 gamma matrices are antihermitian, the eigenvalues of B are expected to be imaginary. If the surface approaches asymptotically $X^1 \times X^2 \subset M^4 \times CP_2$, only B with two opposite, and in general complex, eigenvalues contributes to the asymptotic condition so that $K = \pm i K_1$ become the eigenvalues of O. In a more general case one has $K = K_0 \pm i K_1$.

Concerning the spectrum of λ, an interesting possibility suggested by the number theoretic considerations [E8, E1] is that $u = e^{kr/r_0}$, where k is some suitably chosen numerical factor, is the natural coordinate variable so that the exponents of r would transform to powers of u. If so, the numbers

$$\Delta = \frac{1}{k}(K_0 \pm i K_1)\lambda$$

could be interpreted as dynamical conformal weights having also complex values. This is however not the only possible interpretation (see below).

By the earlier number theoretic speculations the allowed eigenvalues could relate in a simple manner to the zeros of Riemann Zeta or of polyzeta in the case that X^4 contains several partonic 3-surfaces so that it becomes possible to introduce the notion of bound state conformal weight [E1]. For $k = 2K_0$ one has $Re(\Delta) = 1/2$ guaranteing that the conformal weights are at the critical line. This spectrum has been suggested earlier for the conformal weights associated with the super-canonical representations defined at $\delta M^4_\pm$ on basis of number theoretical considerations [E8]. Asymptotia would in this case correspond to the intersection of $X^3_l \cup M^4_\pm$ and need not mean to an exact vacuum extremal. The recent proposal would relate this hypothesis directly to the dynamics of modified Dirac operator.

It is however quite possible that the spectrum is determined by the requirement that p-adicization is possible rather than by some finiteness condition or boundary conditions as in the case of ordinary Schrödinger equation. If the exponents q^{iy} are algebraic numbers for y the imaginary part of zero of Riemann Zeta and for q prime (and therefore for any rational), spinor modes exist p-adically for a given rational value of the coordinate u in a suitable algebraic extension of the p-adic numbers for given p. Also the number theoretical building blocks $n^{1/2+iy}$ of Riemann Zeta exist in suitable algebraic extensions of p-adic numbers at zeros of Zeta and their integer multiples.

One can consider also more general zeta function coding number theoretical data about partonic 2-surface and number theoretic arguments suggest that a zeta expressible as rational function might be a better choice.

7.2.2 Self-referentiality hypothesis coupled with the identification of Higgs

The geometric interpretation of the position dependent eigenvalues of D as Higgs field would mean $H(s) = \zeta^{-1}(s)$. $H(s)$ should be now many-valued Higgs field in S^2_{II} *resp.* S^2_r or ζ should have a preferred branch for which self-referentiality holds true. This kind of branch indeed exists in the case of Riemann Zeta and corresponds to the branch satisfying $n \rightarrow \zeta(n)$.

On the other hand, $H(s)$, $s \in S^2_{II}$ would be many-valued since $H(w)$ is single valued only as a field in X^2 and in general several points of X^2 project to the same point of S^2_{II}. Self-referentiality might be considered even in the strong sense of the word if the number of branches of ζ^{-1} is same as the number of branches of X^2 over S^2_{II}. Same holds true in the case of S^2_r. This would require that at each branch of X^2 only single branches of ζ^{-1} is selected. X^2 would define covering of S^2 gluing various branches of ζ^{-1} to single-valued function.

A possible interpretation of self-referentiality would would be in terms of super-symmetry in the sense that super-canonical conformal weights and eigenvalues of the modified Dirac operator can be identified.

7.2.3 Self referentiality as a defining property of ζ function

One can find endless variety of zeta functions. The physicist's manner to define zeta function is as a characterizer of the eigenvalue spectrum:

$$\zeta(s) \quad = \quad \sum_{\lambda} \lambda^{-s} \quad . \tag{105}$$

Just for perverse fun one can ask what this definition could mean in the recent situation?

For a given point $z = \zeta(s_k)$ one would have spectrum $\lambda_k(z) = -\zeta_k^{-1}(z)$. Let us make the somewhat questionable identification

$$s \quad = \quad -c\lambda_k(z) \ , \tag{106}$$

where c is assumed to be positive constant and look what one obtains.

Substituting to the defining equation one obtains the conditions

$$\sum_k [-c\lambda_k(z)]^{c\lambda_{k_0}(z)} \quad = \quad z \ \text{ for any choice of } k_0. \tag{107}$$

A weaker form of the condition would involve a choice of preferred branch λ_{k_0} of ζ^{-1} and restriction to real axis or even to the set $\{\zeta(n)\}$ and $\lambda_{k_0}(\zeta(n)) = n$.

Could these strangely self-referential equations determine possible spectra $\lambda_k(z)$ and could these spectra have also a geometric meaning? Denoting by N the number of branches of ζ^{-1} there would be N equations at each point z for N functions in the most stringent case so that these equations might have solutions.

7.2.4 Simple examples

To see what happens it is instructive to look for a couple of examples first.

1. For single eigenvalue one would have

   $$[-c\lambda(z)]^{c\lambda(z)} = z \ ,$$

   and this equation has a solution. This equation can be reduced by redefinition $c\lambda \rightarrow \lambda$ to that for $c = 1$. Differentiation gives ordinary differential equation $d\lambda/dz = [-\lambda]^{-\lambda}/[log(-\lambda) + 1]$, which can be solved. At point $z = 1$ one obtains $\lambda(1) = -1$ and at the limit $z \rightarrow 0$ one must have $\lambda(z) \rightarrow -\infty$ and at the limit $z \rightarrow \infty$ one must have $\lambda(z) \rightarrow 0_-$. This fixes the solution uniquely as a function which is non-positive everywhere. Allowing complex numbers, one must consider different branches of logarithm and in this case one obtains infinite family of conditions as $[-\lambda(z)]^{\lambda(z)} exp[in2\pi\lambda(z)] = z$.

2. In the case there are N identical eigenvalues one would have $[-\lambda(z)]^{\lambda(z)} = z/N$ and differential equation $d\lambda/dz = [-\lambda]^{-\lambda}/[N(log(\lambda)+1]$, which reduces to the previous equation by the change $u = z/N$ of variables.

7.2.5 Real domain

In the general case one obtains N differential equations for N functions λ_k (factor c has been absorbed to the definition of λ_k).

$$\sum_k [\frac{d\lambda_k}{dz} \times \frac{\lambda_{k_0}}{\lambda_k} + log(-\lambda_k)\frac{d\lambda_{k_0}}{dz}][-\lambda_k]^{\lambda_{k_0}(z)} = 1 \quad , \quad k_0 = 1, ...N \ . \tag{108}$$

In general this set of equations has a solution with initial values of λ_k fixed in arbitrary manner at some point z_0 so that the hypothesis would solve the generalized eigenvalue spectrum more or less completely. The condition that critical line contains zeros would give $\lambda_k(0) = 1/2 + iy_k$.

On the other hand, there are consistency conditions coming from the fact that differential equations are consistent with the replacement of right hand sides of the consistency condition of equation for $k_0 = k$ with $z + c_k$. This means that one must pose N additional consistency conditions which would suggest that the only solution is just the symmetric solution $\lambda_k(z) = \lambda(z)$ already discussed. This is certainly not consistent with what is known about Riemann Zeta which suggests that conditions should be weakened.

Note that one can also consider modification of the consistency conditions by replacing them with differential equations. This would be equivalent to the replacement $z \rightarrow z + c_{k_0}$ at the right hand side of the original consistency conditions. This option looks however rather tricky.

7.2.6 What happens in the complex domain?

One can wonder whether the situation might change in complex domain. Due to the non-uniqueness of the logarithm the situation is not so simple in the complex domain. The replacement $log(-\lambda_k) = log(-\lambda_k) + in_k 2\pi$ transforms the basic equation to the form

$$\sum_k [-\lambda_k(z)]^{\lambda_{k_0}(z)} exp\,[in_k 2\pi\lambda_{k_0}(z)] \quad = \quad z \ . \tag{109}$$

Even if one chooses $n_k = n_0$, there is an infinite number of manners to choose the branch of logarithm so that infinite number of solutions labelled by N integers are obtained. It will be found that in the case of Riemann Zeta rather a complex situation occurs for non-trivial zeros so that this option does not look very attractive.

7.2.7 Restriction of self-referentiality conditions to integers or real zeros of ζ

The non-uniqueness of logarithm does not affect the condition associated with $\lambda_k(\zeta(n)) = n$ but all other conditions are plagued by the non-uniqueness in the complex domain. This raises the question whether one should restrict the conditions to the points $z_n = \zeta(n)$ and to the number theoretically preferred inverse $\lambda_k(z_n) = n$ for z_n. This would mean selection of a unique branch of ζ^{-1} and restriction to integer values of λ. Number theoretically this option would be very natural.

One could also consider the restriction of the conditions to the case when all branches of ζ^{-1} are integer valued. In the case of Riemann Zeta this would leave only $z = 0$ and $s = -2m$, $n = 1, 2, ...$, or more generally self referentiality only for zeros of ζ. As will be found these conditions are satisfied for Riemann Zeta in this case for $c = 2r$. This situation is very interesting since it corresponds physically to maximal quantum criticality.

7.2.8 Trying to kill the self referentiality hypothesis

One might try to kill the self referentiality hypothesis for finite values of N by using the condition that $\lambda_k(z)$ is algebraic number at the points corresponding to the number theoretic braid. Also the condition that the functions λ_k correspond to inverses of the same function $\zeta(s)$ could kill the hypothesis. If λ_k are inverses of a rational function these conditions would be satisfied and this is indeed what one expects on basis of facts about generalized zeta functions.

Self consistency equation at $z = 0$ gives a consistency condition for the zeros $\lambda_k \equiv \lambda_k(0)$ of ζ:

$$\sum_k [-c\lambda_k]^{c\lambda_{k_0}} = 0 \ . \tag{110}$$

It is probably easy to kill the general hypothesis by checking whether this consistency condition is satisfied for known zeta functions even in the case that the conditions are restricted to a preferred branch of ζ^{-1}.

The facts that $\lambda_n = n$ is naturally the spectrum characterized by Riemann Zeta at points $\zeta(n)$ and that integers are in very special position number theoretically raises the question whether Riemann zeta is consistent with the weakest form of self referentiality conditions for some choice of the constant c.

1. The restriction of the conditions to $z_n = \zeta(n)$ and to $\lambda_{k_0} = n$ for z_n (no trouble from the non-uniqueness of logarithm and number theoretical attractiveness) would give the conditions

$$\sum_m \left[-c \times \zeta_m^{-1}(\zeta(n))\right]^{cn} = \zeta(n) = \sum_m m^{-n} \ . \tag{111}$$

For $n=-2m$ the conditions are satisfied for $c=2r$.

This would suggest that the restriction of zeta functions on the real axis satisfies the general consistency condition implying the consistency conditions for real zeros:

$$\zeta(s) \quad = \quad \sum_m \left[-\zeta_m^{-1}(0)/2\right]^{-s} \quad . \tag{112}$$

This could be used as a general definition of ζ fixing the real zeros from internal consistency. It would be interesting to find whether numerical iteration starting from a guess for a finite number of real zeros could converges to a zeta with finite number of real zeros.

2. Since Rieman ζ is real on real axis there are infinite number of points $\zeta^{-1}(x)$ for real values of x. As found, the most stringent form of consistency conditions for real or complex values of argument implies that the branches of the inverse Zeta are identical for finite value of N: one could of course check whether the infinite value of N could change the situation. A weaker condition would be obtained by restriction on the branches for which $\zeta^{-1}(\zeta(n))=n$. It should be relatively easy to kill numerically this option.

3. Consider next Riemann Zeta in complex plane for $c=2r$ and at origin $z=0$. For the zeros $z=-2m$, $m=1,2,...$ one obtains the condition

$$\sum_k \left[2z_k\right]^{-2rm} \quad = \quad 0 \ , \ \ z_k = 1/2+iy_k \ \ ,m=1,2,.... \ . \tag{113}$$

Non-trivial zeros at z_k give the conditions

$$\begin{aligned} &(-4r)^{2rz_k}\sum_n n^{2rz_{k_0}} + \sum_k \left[-2rz_k\right]^{2rz_{k_0}} \\ &= (-4r)^{2rz_k}\zeta(-2z_{k_0}) + (-2r)^{2rz_{k_0}}\sum_k z_k^{2rz_{k_0}} = 0 \ . \end{aligned} \tag{114}$$

The definition of the sum is problematic due to the problems related to the definition of logarithm.

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