Construction of Configuration Space Kähler Geometry from Symmetry Principles: part II

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Abstract

In this chapter some further aspects of the group theoretical construction of the configuration space geometry are considered. If Kähler action were strictly deterministic, the construction of the configuration space geometry would reduce to $\delta H = \delta M^4_+ \times CP_2$. Mathematically this would be simple and elegant but physically a catastrophe. The classical non-determinism of Kähler action however destroys the hopes/fears about quantum gravitational holography in the simplest sense of the word.

The failure of the classical non-determinism forces to introduce two kinds of causal determinants (CDs). 7-D light like CDs are unions of the boundaries of future and past directed light cones in M^4 at arbitrary positions (also more general light like surfaces $X^7 = X^3_l \times CP_2$ might be considered). CH is a union of sectors associated with these 7-D CDs playing in a very rough sense the roles of big bangs and big crunches. The time reflection of negative energy space-time sheets to positive energy space-time sheets occurs at $X^3 \subset X^7$.

Also 3-D light like causal determinants $X^3_l \subset X^4$ must be introduced: elementary particle horizons provide a basic example of this kind of CDs. For the light like 3-D CDs $X^3_l \subset X^4$ the conformal symmetries correspond to the isometries of the imbedding space localized with respect to the complex coordinate of the 2-surface determining the light like 3-surface X^3_l so that Kac-Moody type symmetry results. The notion of quantum gravitational holography suggests that the data about configuration space geometry and even quantum TGD is coded to these light like CDs.

7–3 duality which be seen as the analog of field particle duality realizes quantum gravitational holography: particle aspect would correspond to the spinor shock waves restricted at X^3_l and field aspect to the dynamics of the interior of X^4. 7–3 duality states that the two CDs play a dual role in the construction of the theory and implies that 3-surfaces are effectively two-dimensional with respect to the CH metric in the sense that all relevant data about CH geometry is contained by the two-dimensional intersections of 7-D and 3-D CDs. This is due to the additional invariance due to the degeneracy of the metric with respect to the deformations of light like X^3_l which preserve its intersections with X^7 and very much analogous to conformal symmetry.

This duality has deep implications for quantum TGD. For instance, the super-canonical isometry algebras associated with 7-D CDs *resp.* super Kac-Moody algebras associated with 3-D CDs can be seen as defining non-zero mode *resp.* zero mode sectors of the tangent space of CH defining quantal *resp.* classical degrees of freedom in 1-1 correspondence by the basic postulate of TGD inspired quantum measurement theory. Accordingly, the two super algebras play dual roles in the construction of quantum theory. The most dramatic implication effective 2-dimensionality is the equivalence of the generalized Feynman diagrams represented by 3-D CDs to tree diagrams since only the end points X^2_i at 7-D CDs matter. This means a resolution of the problem caused by the perturbative divergences in quantum theory. 7-3 duality has also practical implications. For instance, one can deduce Kähler function easily in terms of Dirac determinants associated with the 3-D CDs and Kähler metric easily from the data at 7-D CDs.

Divergences, which have plagued quantum field theories since their discovery, are basically due to the micro-locality of quantum field theories. In TGD framework 3-surface becomes the basic dynamical object instead of point like particle and physics is local only at the level of configuration space whereas Kähler function is a non-local functional of 3-surface. This does not however eliminate all sources of divergences. The cancellation of metric and Gaussian determinants in the configuration space functional integral eliminates the TGD counterparts of the standard divergences of quantum field theories. In higher orders divergence cancellation implies Ricci flatness. The conditions guaranteing Ricci flatness are discussed and it is shown that the basic Lie-algebraic properties implied by the symmetric space metric property imply Ricci flatness.

The so called Hyper Kähler property meaning the existence of quaternionic structure in the tangent space of the configuration space would imply Ricci flatness and the quaternion structure of space-time surface forces to take seriously the possibility of Hyper Kähler structure. Contrary to the earlier expectations, it seems that Hyper Kähler property means that sphere

S^2 labels the possible complexifications. The choice of the imaginary unit reduces basically to the choice of the quantization axis for the rotation group $SO(3)$ for $r_M = constant$ sphere at the light cone boundary so that S^2 parameterizes the possible choices. Analogous argument applies also to the quaternion(!) conformal contribution to the configuration space metric.

1 Introduction

In this chapter the discussion related to the group theoretical construction of the configuration space geometry is continued. If Kähler action were strictly deterministic, the construction of the configuration space geometry would reduce to $\delta H = \delta M^4_+ \times CP_2$. The failure of classical non-determinism of the Kähler action does not however allow to realize quantum gravitational holography in the simplest sense of the word. One can however consider a generalization of the notion of both quantum classical determism and the quantum holography.

1.1 The challenges posed by the non-determinism of Kähler action

The vision discussed in this chapter is far from a complete solution to the problem of constructing the configuration space geometry. The non-determinism of Kähler action means that the reduction of the construction of the configuration space geometry to the light cone boundary fails. Besides degeneracy of the absolute minima of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary. One can imagine two kinds of causal determinants.

1. Elementary particle horizons and light-like boundaries $X^3_l \subset X^4$ of 4-surfaces. In this case only Super-Kac-Moody type conformal algebra makes sense.

2. Causal determinants could also correspond to light like 7-surfaces of $X^3_l \times CP_2 \subset H = M^4 \times CP_2$ or perhaps even more general light like 7-surfaces of H. It would be a pity if Nature would have not expressed physically the extremely elegant mathematic associated with the light like surfaces. Hence the intuitive and somewhat irrational expectation is that super-canonical algebra emerges in some natural manner also for these more general light like surfaces.

3. The light-like 7-surfaces $X^3_l \times CP_2$ allow super-canonical symmetries whereas for completely general light like surfaces $X^7_l \subset H$ this symmetry is broken. Hence mathematical elegance does not favor these surfaces. The requirement that that $X^3_l \subset M^4$ is non-singular and allows Lorentz group as isometries, leaves only the option $X^4_l = \delta M^4_\pm$ with $\pm$ telling whether a boundary of a future or past light cone is in question.

4. An elegant looking manner to take into account the non-determinism is motivated by TGD inspired cosmology: replace the configuration space CH associated with the future light cone with the union of configuration spaces associated with all possible unions of future and past light cones $M^4_\pm \times CP_2$ with dips at arbitrary points of M^4. This would make the theory Poincare invariant and super-canonical algebra allows string mass formula with translations realized as translations of an entire sector of configuration space. An attractive hypothesis is that this is enough to take into account the non-determinism of Kähler action and that there exists a duality in the sense that the use of 7-D and 3-D light like causal determinants (imbedding space level and space-time level) provide dual approaches to the construction of the configuration space geometry. The dual construction utilizing 3-D light like causal determinants will be discussed in the next chapter.

1.2 Category theory and configuration space geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of the configuration space is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of the configuration space geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions. So called ribbon categories discussed in [C5] seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories.

1.3 Super-conformal symmetries and duality

There are two types of causal determinants (CDs) corresponding to 7-surfaces $X^3_l \times CP_2$ of imbedding space and light like 3-surfaces of space-time surface. The basic question is whether both of them contribute separately to the configuration space geometry or whether they provide descriptions which are in some sense dual.

Duality would allow to organize various super conformal symmetries to dual pairs.

1. In [B4] it will be found that 3-D light like CDs allow genuinely 3-D solutions of the modified Dirac equation, kind of spinorial shock waves besides four-dimensional solutions. The interpretation of elementary particles as this kind of spinorial shock waves is an extremely attractive option. By the metric 2-dimensionality of light like 3-D CDs a slight generalization of ordinary 2-D super-conformal invariance is involved. Note that the role of light like 3-D CDs are very much like closed string world sheets.

2. Number theoretic argument lead to the hypothesis that also the interior of space-time surface allows conformal invariance in some sense. The first identification is as quaternion conformal invariance acting as a gauge invariance. Second, one might hope equivalent, identification is as Abelian 4-D conformal invariance based on the generalization of hyper-complex numbers whose action is realized in terms of sigma matrices. Both super conformal symmetries would act as pure gauge symmetries and give rise to $N = 4$ super gauge symmetries. A natural interpretation would be in terms of quantum holography: the super-conformal gauge symmetries in the interior and at 3-D causal determinants and correspond to field and particle aspects of field particle duality.

3. One can assign to the conformal gauge symmetries of 7-D CDs what I have used to call super-canonical invariance and to the conformal symmetries of 3-D light like CDs super Kac-Moody algebra crucial for p-adic mass calculations. Duality would mean that the two descriptions of quantum would be dual. This would resolve the difficult question like "Do both 3-D and 7-D causal determinants contribute separately to the configuration space geometry?". In [B4] it will be found that the Dirac determinant associated with the modified Dirac action at 3-D light like CDs indeed allows to construct configuration space geometry: this description would be dual to the construction of the previous chapter.

1.4 Divergence cancellation and configuration space geometry

Divergences, which have plagued quantum field theories since their discovery, are basically due to the micro-locality of quantum field theories. In TGD framework 3-surface becomes the basic dynamical object instead of a point like particle and physics is local only at the level of configuration space whereas Kähler function is a non-local functional of the 3-surface: this eliminates the loop divergences resulting from the local interaction vertices in quantum fluctuating degrees of freedom. The localization occurring in each quantum jump in zero modes and having interpretation as state function reduction saves from the divergences related to the lack of Gaussian integration in zero modes.

This does not however eliminate all sources of divergences. The cancellation of metric and Gaussian determinants in the configuration space functional integral eliminates the TGD counterparts of the standard divergences of quantum field theories. In higher orders divergence cancellation implies Ricci flatness. The conditions guaranteing Ricci flatness are discussed and it is shown that the basic Lie-algebraic properties implied by the symmetric space metric property imply Ricci flatness.

The so called Hyper Kähler property meaning the existence of hyper-quaternionic structure [E2] in the tangent space of the configuration space would imply Ricci flatness and the quaternion structure of space-time surface forces to take seriously the possibility of Hyper Kähler structure. Contrary to the earlier expectations, it seems that Hyper Kähler property means that sphere S^2 labels the possible complexifications. The choice of the imaginary unit reduces basically to the choice of the quantization axis for the rotation group $SO(3)$ for $r_M = constant$ sphere at the light cone boundary so that S^2 parameterizes the possible choices. Analogous argument applies also to the quaternion(!) conformal contribution to the configuration space metric.

Number theoretic constraints from p-adicization of the theory sharpen the requirement about divergence cancellation: loops are not only finite but vanish. For generalization Feynman diagrams analogous to tangles with chords this corresponds to the equivalence loop diagrams with tree diagrams: a generalization of the duality symmetry of string models is in question. In [C5] these ideas are developed in detail.

2 How to generalize the construction of configuration space geometry to take into account the classical non-determinism?

If the imbedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of configuration space geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [16, 17] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction. The following considerations support the notion of duality stating that configuration space geometry and quantum TGD can be constructed either by using 7-D light like causal determinants of the imbedding space as done in [B2] or 3-D light like causal determinants of space-time surface as done in [B4].

2.1 Quantum holography in the sense of quantum gravity theories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture [16] which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [17], quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of the configuration space consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between configuration space spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of the configuration space geometry. One might however hope that the notion of quantum holography generalizes.

2.2 How the classical determinism fails in TGD?

The failure of classical determinism seem to have very many aspects and in the following I only try to list some of the most obvious aspects of the classical non-determinism.

2.2.1 Enumerable degeneracy of absolute minima is probably too much to hope

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of absolute minima $X^4(Y^3)$ of Kähler action so that one would get at most enumerably infinite number of replicas of a given configuration space region and the construction would still reduce to the light cone boundary.

1. This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

2. CP_2 type extremals are different vacuum extremals since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

3. The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial for the ordinary elementary particle physics. It seems that the conformal symmetries

responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the configuration space metric line element.

4. The possibility of space-time sheets with a negative time orientation with ensuing negative sign of classical energy is a further blow against δM^4_+ reductionism. Space-time sheets can be created as pairs of positive and negative energy space-time sheet from vacuum and this forces to modify radically the ontology of physics. Crossing symmetry allows to interpret particle reactions as a creation of zero energy states from vacuum, and the identification of the gravitational energy as the difference between positive and negative energies of matter supports the view that the net inertial (conserved Poincare-) energy of the universe vanishes both in quantal and classical sense. This option resolves unpleasant questions about net conserved quantum numbers of Universe, and provides an elegant interpretation of the vacuum extremals as correlates for systems with vanishing Poincare energy. This option is the only possible alternative from the point of view of TGD inspired cosmology where Robertson-Walker metrics are vacuum extremals with respect to inertial energy. In particular, super-canonical invariance transforms to a fundamental symmetry of elementary particle physics besides the conformal symmetry associated with 3-D light like causal determinants which means a dramatic departure from string models unless it is somehow equivalent with the super-canonical symmetry.

2.2.2 3-D light like causal determinants

Quite generally, elementary particle horizons and light-like boundaries of 4-surfaces are light-like 3-surfaces. At least elementary particle horizons seem to serve as causal determinants (CDs) by the non-determinism of Kähler action. Also the 3-surfaces separating two maximal deterministic regions of a given space-time sheet are expected to be light like. In the original δM^4_+ reductionistic scenario this forces to ask whether one must include these light-like 3-surfaces to the configuration space besides the 3-surfaces at light like boundary $\delta M^4_+ \times CP_2$ and whether these degrees of freedom correspond to zero modes or contribute also to the configuration space metric.

Light like 3-D CDs would provide an elegant and unique manner to fix the gauge for general coordinate invariance. This would mean taking the light like 3-surfaces X^3_l as kind of reference points in the configuration space and consider their canonical deformations localized with respect to the coordinates of X^3_l as representations of 3-surfaces X^3 for which $X^4(X^3)$ has deformed light like boundary or elementary particle horizon.

A formal generalization of the construction of configuration space metric and spinor structure at 7-D causal determinants so that it would apply at 3-D light like causal determinants is not expected to make sense. A second objection against this idea is that these light like surfaces can carry only classical conserved currents parallel to them and one cannot thus pose the initial values of induced spinor fields freely at them as one can in case of 3-surfaces at δH.

The most plausible solution of this problem is that 3-D light like CDs determine Kähler function indirectly in terms of the fermionic determinants associated with the modified Dirac action. From Kähler function it is in principle (not in practice) possible to deduce the Kähler metric and even configuration space spinor structure. 7-D causal determinants would in turn naturally determine the Kähler metric and gamma matrices but the construction of Kähler function would be difficult.

2.2.3 Lightlike 3-surfaces as vacuum solutions of 3-D vacuum Einstein equations and Witten's approach to quantum gravitation

There is an interesting relationship to the recent yet unpublished work of Witten related to 3-D quantum blackholes [43] which allows to get additional perspective.

1. The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [44, 43] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [45]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that gravitational radiation is not possible. HenceAdS_3 theory in Witten's sense might define this conformal field theory.

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory.

2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.

3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD [C3], gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.

4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.

2. Also the fermionic counterpart of Chern-Simons action for the induced spinors whose form is dictated by the super-conformal symmetry is present. Furthermore, partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.

3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.

4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.

5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [C9, C2, C3]. Compact groups and their quantum counterparts would closely relate to a hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the imbedding space related to the hierarchy of Planck constants [C9]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [C3, E11].

6. To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   i) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   ii) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   iii) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D lightlike surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom [C3]. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about S-matrix implied by the zero energy ontology [C2, C3].

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices [C2]: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure [C2, C3]. Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because S-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D lightlike surfaces as submanifolds (analogs of blackhole horizons and lightlike boundaries). Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the modified Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [B4].

5. The counterpart of the ordinary unitary S-matrix in mathematical sense is between zero energy states. I call it U-matrix [C2]. It has nothing to do with particle reactions. It is crucial for understanding consciousness via *moment of consciousness as quantum jump* identification.

2.3 Could classical non-determinism be described in terms of 7-D causal determinants $X^3_l \times CP_2$?

The non-determinism of Kähler action implies the presence of 3-D (most naturally) light like causal determinants $X^3_l \subset X^4$. It is difficult to imagine how to deduce the possible direct contribution of these degrees of freedom to the configuration space metric. A possible solution of the problems is based on the notion of duality.

Perhaps the allowance of sufficiently general family of surfaces $X^3_l \times CP_2$ as 7-D CDs instead of using only $\delta M^4_+ \times CP_2$ could be equivalent with using 3-dimensional light like CDs $X^3_l \subset X^4$. Duality would mean that one could construct X^4 either by giving space-like 3-surfaces X^3 at 7-D causal determinants or by giving light like 3-D causal determinants $X^3_l \subset X^4$. The duality should apply to the entire quantum TGD.

In its strongest form duality requires that stringy mass formula is realized both in super-canonical and Super Kac-Moody sense and thus also Poincare invariance. Hence the replacement $H_+ \rightarrow H = M^4 \times CP_2$ would be necessary. The configuration space would become the union of the configuration spaces associated with $X^3_l \times CP_2 \subset M^4 \times CP_2$. Most naturally the light like $X^3_l \subset M^4$ would correspond to unions of future and past light cones of the entire Minkowski space M^4. Duality would have dramatic consequences: in particular, the super-canonical and super-conformal symmetries of light cone boundary $\delta M^4_+ \times CP_2$ would be transmuted from cosmological to microscopic symmetries.

The new view about the relationship of inertial and gravitational energy encourages strongly this option and leads to what might called zero (inertial) energy cosmology. A given space-time sheet can have both positive and negative time orientation, and therefore also positive and negative classical charges. Hence one can consider pair creation in which a pair of 3-surfaces with themselves are not light-like and have opposite classical charges emerges at the light like 7-surfaces $X^3_l \times CP_2$. The process can be also interpreted as a time reflection of a negative energy space-time sheet as a positive energy space-time sheet.

The net Poincare quantum numbers associated with $X^3 \subset \delta M^4_+ \times CP_2$ vanish since the positive and negative energy branches of space-time surface must have opposite quantum numbers in general. The stringy mass formula should thus apply separately to both branches. The alternative possibility is to give up the stringy mass formula altogether and consider a more general form duality relating representations of theory relying on different quantum number spectrum. Indeed, super-canonical and Super Kac-Moody conformal weights have quite different spectra. In this case the mass squared in the stringy mass formula would be replaced with the Casimir operator of the Lorentz group. For this option 7–3 duality would relate descriptions based on different quantum number spectra. The states inside super-canonical representations would decompose into gigantic multiplets of almost degenerate states (ideal for representing biologically relevant information). These representations could be seen as unitary representations of Lorentz group carrying no four-momentum but only Lorentz and color quantum numbers.

The simplicity of the construction of the configuration space geometry at 7-D CDs allows to understand the general aspects of super-canonical conformal symmetries in a more general situation if the generalization involves only unions of future and past light cone boundaries. In particular, the zeros of Riemann Zeta determine the spectrum of conformal weights labelling configuration space tangent vectors and physical states so that Riemann hypothesis would have a direct physical content.

Even in the case that $\delta M^4_\pm \times CP_2$ are replaced with more general light like 7-surfaces $X^3_l \times CP_2$, it might be possible to construct super-canonical representations. Denote by r the light-like coordinate and by X^2 the $r = constant$ section which in turn has coordinates $(z, \bar{z})$. One can fix the local complex coordinates of X^2 in such a manner that the induced metric has the standard form $F(z, \overline{z}) dz d\bar{z}$: the choice of the local coordinates is unique apart from conformal transformations of X^2. In the case of the light cone boundary one has $F = \frac{1}{(1+\rho^2)^2}$. The representations could have the same general analytic form as the two types of representations of the Lorentz group at δM^4_+ corresponding to $SO(3)$ and $SO(2)$ subgroups of $SO(3,1)$, which were discussed in [B2]. From the general form of the representations one can deduce the general form of the generators taking the role of the generators of Lorentz group, whose algebra is indeed contained as a sub-algebra to conformal algebra and the construction of canonical charges and super-charges would generalize as such to the general context.

7–3 duality, or equivalently quantum classical correspondence, would be realized via the mapping of super-canonical conformal weights to the points of the light like CDs $X^3_l \subset X^4$ much like quantal momenta have direction vectors in 3-space as their classical correlates [C5]. Super conformal gauge transformations of $X^3_l \subset X^4$ would induce braiding operations in the space of complex super-canonical conformal weights inducing a spectral flow equivalent with a gauge change. Also Super Kac-Moody algebra would have a natural commutator action on super-canonical algebra. Hence super-canonical algebra would define something highly analogous to the primary fields of a conformal field theory defined in a complex plane containing super-canonical conformal weights as punctures along the lines defined by negative real axis and the line $Re(z) = 1/2$ containing the non-trivial zeros and their superpositions plus lines $Re(z) = -n + 1/2$ parallel to this line.

2.4 Could all light like 7-surfaces $X^3_l \times CP_2$ act as causal determinants?

The following arguments show that light like 7-surfaces $X^3_l \times CP_2$, where X^3_l is any light like 3-surface of M^4, cannot be excluded as 7-D causal determinants and that they might be even necessary.

1. p-Adic mass calculations necessitated to include Kac Moody algebra of Lorentz group. The Kac Moody algebra associated with $Y^3_l \subset X^4$ acts on $X^2 \subset X^3_l \times CP_2$. For $X^3_l = M^4_\pm$ local Lorentz transformations respecting the dip of the light cone would in general map the points of X^2 inside 7-D CD . Thus it is not necessary to assume $X^3_l \times CP_2$ to be more general than $M^4_\pm$ or union of them.

2. Conservation laws suggest that the 3-surfaces in question should approach vacuum extremals at the boundary of the light like surface. This however would mean that the magnetic flux Hamiltonians and super charges vanish. A given space-time sheets can however have both positive and negative time orientation, and therefore also positive and negative classical charges. Hence one can consider pair creation in which pair of 3-surfaces with opposite classical charges emerges from the boundary of a light like surface.

3. The classical non-determinism of Kähler action implies that the same 3-surface in general corresponds to several space-time surfaces $X^4(X^3)$ with the same value of absolute minimum of Kähler action and same values of classical conserved charges. Hence the creation of pairs of this kind of positive and negative energy space-time sheets at the boundary of the light like surfaces of M^4_+ or even H is possible. The inclusion of all possible pairs of 3-surfaces with vanishing net conserved charges would force to extend configuration space with additional vacuum sectors. It must be emphasized that these negative energy space-time sheets and their pair creation have become a corner stone of TGD inspired theory of consciousness and of the model of quantum biology.

These arguments would suggest that configuration space geometry and spinor structure involves the cosmological super-canonical algebra at $\delta M^4_+ \times CP_2$ and completely analogous algebras at the light like 7-surfaces $X^3_l \times CP_2$ of H or even more general light-like 7-surfaces X^7_l describing zero energy sectors of the configuration space.

The simplicity of the construction of the configuration space geometry at light cone boundary allows to understand the general aspects of super-canonical conformal symmetries in the more general situation. The representations of super-conformal algebra are expected to have a form very much similar to that encountered in the case of the light cone boundary, and certainly so for the light cone boundaries inside light cone.

Denote by r the light-like coordinate and by X^2_i the intersection of light like 3-D CD with $X^3_{l,i} \subset X^4$ with $X^3_l \times CP_2$. Assume that X^2_i has coordinates $(z, \bar{z})$. One can fix the local complex coordinates of X^2 in such a manner that the induced metric has the standard form $F(z, \overline{z})dzd\bar{z}$: the choice of the local coordinates is unique apart from conformal transformations of X^2_i. In the case of the light cone boundary one has $F = \frac{1}{(1+\rho^2)^2}$.

In the case of the light cone boundary the super-canonical algebra decomposes to two parts $g = g_{nt} + g_t$ corresponding to non-trivial and trivial zeros of Riemann Zeta. g_{nt} and g_t have both their own Cartan decompositions and they generate together a larger algebra having Cartan decomposition.

1. The algebra g_{nt} corresponding to non-trivial zeros correspond to the function basis associated with $SO(3)$ subroup of $SO(3,1)$:

$$f_{m,l,k} = Y_{lm}(\theta,\phi) \times (r_M/r_0)^{k_1+i\rho} \quad .$$

   For $k_1 = -1/2$ the representation is unitary for any value of ρ for the natural inner product reducing to the inner product of plane waves in r_M degrees of freedom. The unitarity with respect to Lorentz group is not however absolutely essential just as it is not essential in the case of Poincare group. The hypothesis is that for the generating elements of the super-canonical algebra g_{nt} the conformal weight $h = -k_1 - i\rho$ corresponds to non-trivial zeros of Zeta. If the factor $(r_M/r_0)^{-1/2}$ accompanies Poisson bracket g_{nt} closes under Poisson brackets to an algebra for which the imaginary parts of conformal weights are linear combinations of the imaginary parts of zeros of Zeta.

2. The function basis consisting of eigen states of rotation and boost generator is given by $f(m,n,k) = e^{\pm i(k-1)\phi}\rho^{k-1}(1+\rho^2)^{-2k}(r/r_0)^k$, $k > 0$, defines an orthogonal function basis orthogonal also with the basis of a). The Poisson bracket between generators of g_t and between the generators of g_t and g_{nt} involves the factor $(r_M/r_0)^{-1}$. The integers $k = 2m$ correspond to trivial zeros of Zeta and define t_t whereas odd values of k correspond to h_t in the Cartan decomposition $g_t = t_t + h_t$.

3. The general conclusion is that the subspace t for the algebra generated b g_{nt} and g_t contains elements with conformal weights $n - 1/2 - i\sum_i n_i y_i$ such that $\sum_i n_i = N$ is odd (even) for n even and even (odd) for n odd for elements of t (h). Also the elements of t_t belong to t. Orthogonality with respect to the inner product of δM^4_+ is achieved if one assumes Virasoro conditions which means that the values of $n = 0,1$ actually contribute.

2.5 The category of light cones, the construction of the configuration space geometry, and the problem of psychological time

Light-like 7-surfaces of imbedding space are central in the construction of the geometry of the world of classical worlds. The original hypothesis was that space-times are 4-surfaces of $H_+ = M^4_+ \times CP_2$, where M^4_+ is the future light cone of Minkowski space with the moment of big bang identified as its boundary $\delta H_+ = \delta M^4_+ \times CP_2$: "the boundary of light-cone". The naive quantum holography would suggest that by classical determinism everything reduces to the light cone boundary. The classical non-determinism of Kähler action forces to give up this naive picture which also spoils the full Poincare invariance.

The new view about energy and time forces to conclude that space-time surfaces approach vacua at the boundary of the future light cone. The world of classical worlds, call it CH, would consist of classical universes having a vanishing inertial 4-momentum and other conserved quantities and being created from vacuum: big bang would be replaced with a "silent whisper amplified to a big bang". The net gravitational mass density can be non-vanishing since gravitational momentum is difference of inertial momenta of positive and negative energy matter: Einstein's Equivalence Principle is exact truth only at the limit when the interaction between positive and negative energy matter can be neglected [D5]. This option would maximize the symmetries and cure the philosophical head aches caused by question "What are the net quark and lepton numbers of the Universe". M^4 and M^4_+ options might be indistinguishable at the practical level of observations and M^4 would be preferred because of its simplicity and exact Poincare invariance.

Poincare invariant theory results if one replaces CH with the union of its copies $CH(a)$ associated with the light cones $M^4_+(a)$ with a specifying the position of the dip of $M^4_+(a)$ in M^4. Also past directed light-cones $M^4_-(a)$ are allowed. The unions of the light cones with inclusion as a basic arrow would form category analogous to the category Set with inclusion defining the arrow of time. This category formalizes the ideas that cosmology has a fractal Russian doll like structure, that the cosmologies inside cosmologies are singularity free, and that cosmology is analogous to an organic evolution and organic evolution to a mini cosmology [D5].

The view also unifies the proposed two explanations for the arrow of psychological time [K1]

1. The mind like space-time sheets representing conscious self drift quantum jump by quantum jump towards geometric future whereas the matter like space-time sheets remain stationary. The self of the organism presumably consisting mostly of topological field quanta, would be like a passenger in a moving train seeing the changing landscape. The organism would be a mini cosmology drifting quantum jump to the geometric future. Also selves living in the reverse direction of time are possible.

2. Psychological time corresponds to a phase transition front in which intentions represented by p-adic space-time sheets transform to actions represented by real space-time sheets moving to the direction of geometric future. The motion would be due to the drift of $M^4_+(a)$. The very fact that the mini cosmology is created from vacuum, implies that space-time sheets of both negative and positive field energy are abundantly generated as realizations of intentions. The intentional resources are richest near the boundary of $M^4_+(a)$ and depleted during the ageing with respect to subjective time as asymptotic self-organization patterns are reached. Interestingly, mini cosmology can be seen as a fractally scaled up variant of quantum jump. The realization of intentions as negative energy signals (phase conjugate light) sent to the geometric past and inducing a positive energy response (say neural activity) is consistent with the TGD based models for motor action and long term memory [K1].

2.6 Duality of 3-D and 7-D causal determinants as particle-field duality

TGD predicts two kinds of super-conformal symmetries corresponding to 7-D and 3-D causal determinants and that their duality would generalize the age-old field-particle particle duality so that quantum gravitational holography and YM-gravitational duality could be seen as particular aspects of field particle duality. Without exaggerating, 7–3 duality means breakthrough not only in the understanding of the implications of the non-determinism of Kähler action but also in the construction of quantum TGD.

2.6.1 7–3 duality requires efective 2-dimensionality

The super-canonical algebra associated with 7-D CDs and Super Kac-Moody algebra associated with 3-D CDs are dual if they correspond to two preferred basis for the tangent space of CH and thus two preferred choices of CH coordinates.

The assumption that that all relevant data about configuration space geometry is contained by the intersections X^2_i of the 3-D CDs X^3_l with 7-D CDs defining 2-sub-manifolds of X^3 concretizes the idea about duality. Duality would thus imply effective 2-dimensionality of 3-surfaces and the task is to understand what this could mean.

1. By duality both X^2_i-local H-isometries and the Hamiltonians of $\delta M^4_+ \times CP_2$ restricted to X^2_i would span the tangent space of CH. A highly non-trivial implication would be a dramatic simplification of the construction of the configuration space Hamiltonians, Kähler metric, and gamma matrices since one could just sum only over the flux integrals over the sub-manifolds X^2_i. The best that one might hope is that it is possible to fix both 3-D light like CDs and their sub-manifolds X^2_i and 7-D CDs freely.

2. The reduction to dimension 2 could be understood in terms of the impossibility to choose X^3 freely once light like 3-D CDs are fixed but this does not remove the air of paradox. The resolution of the paradox comes from the following observation. The light likeness condition for 3-D CD can be written in the coordinates for which the induced metric is diagonal as a vanishing of one of the diagonal components of the induced metric, say g_{11}:

$$g_{1i}h_{kl}\partial_1 h^k \partial_i h^l \quad = \quad 0 \ , \ \ i = 1,2,3 \ . \tag{1}$$

The condition $g_{11} = 0$ is exactly like the light likeness condition for the otherwise random M^4 projection of CP_2 type extremals [D1]. When written in terms of the Fourier expansion this conditions gives nothing but classical Virasoro conditions. This analog of the conformal invariance is different from the conformal invariance associated with transversal degrees of freedom and and from quaternion conformal invariance and its commutative version. This symmetry conforms nicely with the duality idea since also the boundary of the light cone allows conformal invariance in both light like direction and transversal degrees of freedom. Ironically, the presence of this degeneracy should have been obvious from the beginning since only the end points of CP_2 type extremal should matter. Since wormhole contacts can be modelled as pieces of CP_2 type extremals, also they are expected to possess this degeneracy.

One can consider two interpretations of this symmetry.

i) The degrees of freedom generating different light like 3-D CDs X^3_l with a given intersection X^2 with 7-D CD correspond to zero modes. Physically this would mean that in each quantum jump a complete localization occurs in these degrees of freedom so that particles behave effectively classically. With this interpretation these degrees of freedom could perhaps be seen as dual for the zero mode degrees of freedom associated with the space-like 3-surfaces X^3 at 7-D CDs: deformation of X^3_l would induce deformation of X^3.

ii) Gauge degrees of freedom could be in question so that one can make a gauge choice fixing the orbits within certain limits.

At the level of configuration space geometry the result would mean that one can indeed code all data using only two-dimensional surfaces X^2_i of X^3. This brings in mind a number theoretic realization for the quantum measurement theory. That only mutually commuting observables can be measured simultaneously would correspond to the assumption that all data about configuration space geometry and quantum physics must be given at 2-dimensional surfaces of H for which the tangent space at each point corresponds to an Abelian sub-algebra of octonions. Quantum TGD would reduce to something having very high resemblance with WZW model. One cannot deny the resemblance with M-theories with M interpreted as a membrane.

2.6.2 7-3 duality and quantum measurement theory

The action of Super Kac-Moody generators on configuration space Hamiltonians is well defined and one might hope that as a functional of 2-surface it could give rise to a unique superposition of super-canonical Hamiltonians. Same should apply to the action of super-canonical algebra on Kac Moody algebra. At the level of gamma matrices the question is whether the configuration space metric can be defined equivalently in terms of anti-commutators of super-canonical and Super Kac-Moody generators. If the answer is affirmative, then 7–3 duality would be nothing but a transformation between two preferred coordinates of the configuration space.

TGD inspired quantum measurement theory suggests however that the two super-conformal algebras correspond to each other like classical and quantal degrees of freedom. Super Kac-Moody algebra and super conformal algebra would act as transformations preserving the conformal equivalence class of the partonic 2-surfaces X^2 associated with the maxima of the Kähler function whereas super-canonical algebra in general changes conformal moduli and induces a conformal anomaly in this manner. Hence Kac-Moody algebra seems to act in the zero modes of the configuration space metric. In TGD inspired quantum measurement zero modes correspond to classical non-quantum fluctuating dynamical variables in 1-1 correspondence with quantum fluctuating degrees of freedom like the positions of the pointer of the measurement apparatus with the directions of spin of electron. Hence Kac-Moody algebra would define configuration space coordinates in terms of the map induced by correlation between classical and quantal degrees of freedom induced by entanglement.

Duality would be also realized in a well-defined sense at the level of configuration space conformal symmetries. The idea inspired by Olive-Goddard-Kent coset construction is that the generators of Super Virasoro algebra corresponds to the differences of those associated with Super Kac-Moody and super-canonical algebras. The justification comes from the miraculous geometry of the light cone boundary implying that Super Kac-Moody conformal symmetries of X^2 can be compensated by super-canonical local radial scalings so that the differences of corresponding Super Virasoro generators annihilate physical states. If the central extension parameters are same, the resulting central extension is trivial. What is done is to construct first a state with a non-positive conformal weight using super-canonical generators, and then to apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and thus mass.

2.6.3 7–3 duality and the equivalence of loop diagrams with tree diagrams

The 3-D light like CDs are expected to define analogs of Feynman diagrams. In the simplest case there would be past of future and past directed 7-D CDs $X^7_\pm = \delta M^4_\pm \times CP_2$, and the lines of the generalized Feynman diagram would begin from X^7_+ and terminate to X^7_-. In [C5] the generalization of duality symmetry of string models stating that generalized Feynman diagrams with loops are equivalent with tree diagrams is discussed. By quantum-classical correspondence this would mean that the conformal equivalence for Feynman diagrams defined by 3-D light like CDs generalizes to a topological equivalence. This is indeed as it should be since it is the intersections X^2_i with $X^7_\pm$ which should code for physics and these intersections do not contain information about loops.

Interesting questions relate to the interpretation of the negative energy branches of the space-time surface. It would seem that also the surfaces X^2_i are accompanied by negative energy branch. The branching brings in mind a space-time correlate for bra-ket dichotomy. The two branches would represent Feynman diagrams which are equivalent but correspond to different sign of Kähler coupling strength if the generalization of electric-magnetic duality is accepted.

3 The association of the modified Dirac action to Chern-Simons action and explicit realization of super-conformal symmetries

Super Kac-Moody symmetries should correspond to solutions of modified Dirac equation which are in some sense holomorphic. The discussion below is based on the same general ideas but differs radically from the previous picture at the level of details. The additional assumption inspired by the considerations of this section is that the action associated with the partonic 3-surfaces is non-singular and therefore Chern-Simons action for the induced Kähler gauge potential.

This means that TGD is at the fundamental level almost-topological QFT: only the light-likeness of the partonic 3-surfaces brings in the induced metric and gravitational and gauge interactions and induces the breaking of scale and super-conformal invariance. The resulting theory possesses the expected super Kac-Moody and super-canonical symmetries albeit in a more general form than suggested by the considerations of this section. A connection of the spectrum of the modified Dirac operator with the zeros or Riemann Zeta is suggestive and provides support for the earlier number theoretic speculations concerning the spectrum of super-canonical conformal weights. One can safely say, that if this formulation is correct, TGD could not differ less from a physically trivial theory.

3.1 Zero modes and generalized eigen modes of the modified Dirac action

1. The modified gamma matrices appearing in the modified Dirac equation are expressible in terms of the Lagrangian density L assignable to the light-like partonic 3-surface X^43_l as

$$\hat{\Gamma}^\alpha = \frac{\partial L}{\partial_\alpha h^k}\Gamma_k \ , \tag{2}$$

   where Γ_k denotes gamma matrices of imbedding space. The modified Dirac operator is defined as

$$D = \hat{\Gamma}^\alpha D_\alpha \ , \tag{3}$$

   where D_α is the covariant derivative defined by the induced spinor connection. Modified gamma matrices satisfy the condition

$$D_\alpha \hat{\Gamma}^\alpha = 0 \tag{4}$$

   if the field equations associated with L are satisfied. This guarantees that one indeed obtains the analog of the massless Dirac equation. Zero modes of the modified Dirac equation should define the conformal super-symmetries.

2. The generalized eigenvalues and eigen solutions of the modified Dirac operator are defined as

$$\begin{aligned} D\Psi &= \lambda N\Psi \ , \\ N &= n^k\Gamma_k \ . \end{aligned} \tag{5}$$

   Here n^k denotes a light-like vector which must satisfy the integrability condition

$$\left[D, n^k\Gamma_k\right] = 0 \ . \tag{6}$$

   if the analog $D^2\Psi = 0$ for the square of massless Dirac equation is to hold true. n^k should be determined by the field equations associated with L somehow and commutativity condition could fix n more or less uniquely.

   If the commutativity condition holds true then any generalized eigen mode Ψ_λ gives rise to a zero mode as $\Psi = N\Psi_\lambda$. One can add to a given non-zero mode any superposition of zero modes without affecting the generalized eigen mode property.

3. The hypothesis is that Kähler function is expressible in terms of the Dirac determinant of the modified Dirac operator defined as the product of the generalized eigenvalues. The Dirac determinant must carry information about the interior of the space-time surface determined as preferred extremal of Kähler action or (as the hypothesis goes) as hyper-quaternionic or co-hyper-quaternionic 4-surface of M^8 defining unique 4-surface of $M^4 \times CP_2$. The assumption that X_L^3 is light-like brings in an implicit dependence on the induced metric. The simplest assumption is that n^k is a light-like vector field tangential to X_l^3 so that the knowledge of X_l^3 fixes completely the dynamics.

4. If the action associated with the partonic light-like 3-surfaces contains induced metric, the field equations become singular and ill-defined unless one defines the field equations at X_l^3 via a limiting procedure and poses additional conditions on the behavior of Ψ at X_l^3. Situation changes if the action does not contain the induced metric. The classical field equations are indeed well-defined at light-like partonic 3-surfaces for Chern-Simons action for the induced Kähler gauge potential

$$L = L_{C-S} = k\epsilon^{\alpha\beta\gamma} J_{\alpha\beta} A_\gamma \ . \tag{7}$$

One obtains the analog of WZW model with gauge field replaced with the induced Kähler form. This action does not depend on the induced metric explicitly so that in this sense a topological field theory results. There is no dependence on M^4 gamma matrices so that local Lorentz transformations act as super-conformal symmetries of both classical field equations and modified Dirac equation and $SL(2,C)$ defines the analog of the $SU(2)$ Kac-Moody algebra for $N = 4$ SCA.

The facts that the induced metric is light-like for X_l^3, that the modified Dirac equation contains information about this and therefore about induced metric, and that Dirac determinant is the product of the non-vanishing eigen values of the modified Dirac operator, imply the failure of topological field theory property at the level of Kähler function identified as the logarithm of the Dirac determinant.

A more complicated option would be that the modified Dirac action contains also interior term corresponding to the Kähler action. This alternative would break super-conformal symmetries explicitly and almost-topological QFT property would be lost. This option is not consistent with the idea that quantum-classical correspondence relates the partonic dynamics at X_l^3 with the classical dynamics in the interior of space-time providing first principle justification for the basic assumptions of the quantum measurement theory.

The classical field equations defined by L_{C-S} read as

$$\begin{aligned} D_\mu \frac{\partial L_{C-S}}{\partial_\mu h^k} &= 0 \ , \\ \frac{\partial L_{C-S}}{\partial_\mu h^k} &= \epsilon^{\mu\alpha\beta} \left[2J_{kl} \partial_\alpha h^l A_\beta + J_{\alpha\beta} A_k) \right] \ . \end{aligned} \tag{8}$$

From the explicit form of equations it is obvious that the most general solution corresponds to a X_l^3 with at most 2-dimensional CP_2 projection.

Although C-S action vanishes, the color isometry currents are in general non-vanishing. One can assign currents also to super-Kac Moody and super-canonical transformations using standard formulas and the possibility that the corresponding charges define configuration space Hamiltonians and their super-counterparts must be considered seriously.

Suppose that the CP_2 projection is 2-dimensional and not a Lagrange manifold. One can introduce coordinates for which the coordinates for X^2 are same as those for CP_2 projection. For instance, complex coordinates $(z,\overline{z})$ of a geodesic sphere could be used as local coordinates for X^2. One can also assign one M^4 coordinate, call it r, with M^4 projection X^1 of X^3_l. Locally this coordinate can be taken to be one of the standard M^4 coordinates. The remaining five H-coordinates can be expressed in terms of $(r,z,\overline{z})$ and light-likeness condition boils down to the vanishing of the metric determinant:

$$det(g_3) = 0 . \tag{9}$$

All diffeomorphisms of H respecting the light-likeness condition are symmetries of the solution ansatz.

Consider some special cases serve as examples.

1. The simplest situation results when X^4_l is of form $X^1 \times X^2$, where X^1 is light-like random curve in M^4 as for CP_2 type vacuum extremals. In this case light-likeness boils down to Virasoro conditions with real parameter r playing the role analogous to that of a complex coordinate: this conformal symmetry is dynamical and must be distinguished from conformal symmetries assignable to X^2. A plausible guess is that light-likeness condition quite generally reduces to the classical Virasoro conditions.

2. A solution in which CP_2 projection is dynamical is obtained by assuming that for a given value of M^4 time coordinate CP_2- and M^4- projections are one-dimensional curves. For instance, CP_2 projection could be the circle $\Theta = \Theta(m^0 \equiv t)$ whereas M^4 projection could be the circle $\rho = \sqrt{x^2+y^2} = \rho(m^0)$. Light-likeness condition reduces to the condition $g_{tt} = 1 - R^2\partial_t\Theta^2 - \partial_t\rho^2 = 0$.

3.2 Classical field equations for the modified Dirac equation defined by Chern-Simons action

The modified Dirac operator is given by

$$\begin{aligned} D &= \frac{\partial L_{C-S}}{\partial_\mu h^k}\Gamma_k D_\mu \\ &= \epsilon^{\mu\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_\mu \ , \\ \hat{\epsilon}^{\alpha\beta\gamma} &= \epsilon^{\alpha\beta\gamma}\sqrt{g_3} \ . \end{aligned} \tag{10}$$

Note $\hat{\epsilon}^{\alpha\beta\gamma} =$ does not depend on the induced metric. The operator is non-trivial only for 3-surfaces for which CP_2 projection is 2-dimensional non-Lagrangian sub-manifold. The modified Dirac operator reduces to a one-dimensional Dirac operator

$$D = \hat{\epsilon}^{r\alpha\beta}\left[2J_{kl}\partial_\alpha h^l A_\beta + J_{\alpha\beta}A_k\right]\Gamma^k D_r \ . \tag{11}$$

The solutions of the modified Dirac equation are obtained as spinors which are covariantly constant with respect to the coordinate r:

$$D_r\Psi = 0 \ . \tag{12}$$

Non-vanishing spinors $\Psi_1 = \partial_r \Psi$ satisfying $\Gamma_r \Psi_1 = 0$ are not possible. Ψ defines super-symmetry for the generalized eigen modes if the additional condition

$$\Psi = N\Psi_0 \tag{13}$$

is satisfied. The interpretation as super-conformal symmetries makes sense if the Fourier coefficients of zero modes and their conjugates are anticommuting Grassmann numbers. The zero modes which are not of this form do not generate super-conformal symmetries and might correspond to massless particles. TGD based vision about Higgs mechanism suggest the interpretation of n^k as a non-conserved gravitational four-momentum whose time average defines inertial four-momentum of parton. The sum of the partonic four-momenta would be identified as the classical four-momentum associated with the interior of the space-time sheet.

The covariant derivatives D_α involve only CP_2 spinor connection and the metric induced from CP_2. D_r involves CP_2 spinor connection unless X_l^3 is of form $X^1 \times X^2 \subset M^4 \times CP_2$. The eigen modes of D are correspond to the solutions of

$$D\Psi = \lambda N\Psi \tag{14}$$

The first guess is that $N = n^k \gamma_k$ corresponds to the tangential light-like vector $n^k = \Phi \partial_r h^k$ where Φ is a normalization factor which can depend on position.

The obvious objection is that with this assumption it is difficult to understand how Dirac determinant can correspond to an absolute extremum of Kähler action for 4-D space-time sheet containing partonic 3-surfaces as causal determinants ($\sqrt{g_4} = 0$). However, if one can select a unique M^4 time coordinate, say as that associated with the rest system for the average four-momentum defined as Chern-Simons Noether charge, then one can assign to n^k a unique dual obtained by changing the sign of its spatial components. The condition that this vector is tangential to the 4-D space-time sheet would provide information about the space-time sheet and bring in 4-dimensionality. At this stage one must however leave the question about the choice of n^k open.

One should be able to fix Φ apart from overall normalization. First of all, the requirement that zero modes defines super symmetries implies the condition $[D, n^k \Gamma_k]\Psi = 0$ for zero modes. This condition boils down to the requirement

$$D_r(\Phi \partial_r h^k \Gamma_k)\Psi = 0 \ . \tag{15}$$

This in turn boils down to a condition

$$D_r \partial_r h^k + \frac{\partial_r \Phi}{\Phi} \partial_r h^k = 0 \ . \tag{16}$$

These conditions in turn guarantee that the condition

$$D_r(h_{kl} \partial_r h^k \partial_r h^l) = 0 \tag{17}$$

implied by the light-likeness condition are satisfied. Since Φ is determined apart from a multiplicative constant from the light-likeness condition the system is internally consistent. The conditions above are not general coordinate invariant so that the coordinate r must correspond to a physically preferred coordinate perhaps defined by the conditions above.

One can express the eigenvalue equation in the form

$$\begin{aligned}\partial_r \Psi &= \lambda O \Psi \ , \\ O &= (\hat{\Gamma}^r)^{-1} N \ , \\ (\hat{\Gamma}^r)^{-1} &= \frac{\hat{\Gamma}^r}{a^k a^l h_{kl}} \ , \ \hat{\Gamma}^r \equiv a^k \Gamma_k \ . \end{aligned} \tag{18}$$

This equation defines a flow with r in the role of a time parameter. The solutions of this equation can be formally expressed as

$$\Psi(r, z, \overline{z}) = Pe^{\lambda \int O(r,z,\overline{z})dr} \Psi_0(z, \overline{z}) \ . \tag{19}$$

Here P denotes the ordered exponential needed because the operators $O(r, z\overline{z})$ need not commute for different values of r.

3.3 Can one allow light-like causal determinants with 3-D CP_2 projection?

The standard quantum field theory wisdom would suggest that light-like partonic 3-surfaces which are extremals of the Chern-Simons action correspond only to what stationary phase approximation gives when vacuum functional is the product of exponent of Kähler function resulting from Dirac determinant and an imaginary exponent of Chern-Simons action whose coefficient is proportional to the central charge of Kac-Moody algebras associated with CP_2 degrees of freedom.

One cannot exclude the possibility that 3-D light-like causal determinants might be required by the general consistency of the theory. The identification of the exponent of Kähler function as Dirac determinant remains a viable hypothesis for this option. "Off mass shell" breaking of super-conformal symmetries is implied since modified Dirac equation implies the conservation of super conformal currents only when CP_2 projection is at most 2-dimensional.

3.4 Some problems of TGD as almost-topological QFT and their resolution

There are some problems involved with the precise definition of the quantum TGD as an almost-topological QFT at the partonic level and the resolution of these problems leads to an unexpected connection between cosmology and parton level physics.

1. Three problems

The proposed view about partonic dynamics is plagued by three problems.

1. The definition of supercanonical and super-Kac-Moody charges in M^4 degrees of freedom poses a problem. These charges are simply vanishing since M^4 coordinates do not appear in field equations.

2. Classical field equations for the C-S action imply that this action vanishes identically which would suggest that the dynamics does not depend at all on the value of k. The central extension parameter k determines the over-all scaling of the eigenvalues of the modified Dirac operator. $1/k$- scaling occurs for the eigenvalues so that Dirac determinant scales by a finite power k^N if the number N of the allowed eigenvalues is finite for the algebraic extension considered. A constant $Nlog(k)$ is added to the Kähler function and its effect seems to disappear completely in the normalization of states.

3. The general picture about Jones inclusions and the possibility of separate Planck constants in M^4 and CP_2 degrees of freedom suggests a close symmetry between M^4 and CP_2 degrees of freedom at the partonic level. Also in the construction of the geometry for the world of classical worlds the symplectic and Kähler structures of both light-cone boundary and CP_2 are in a key role. This symmetry should be somehow coded by the Chern-Simons action.

2. A possible resolution of the problems

A possible cure to the above described problems is based on the modification of Kähler gauge potential by adding to it a gradient of a scalar function Φ with respect to M^4 coordinates.

1. This implies that super-canonical and super Kac-Moody charges in M^4 degrees of freedom are non-vanishing.

2. Chern-Simons action is non-vanishing if the induced CP_2 Kähler form is non-vanishing. If the imaginary exponent of C-S action multiplies the vacuum functional, the presence of the central extension parameter k is reflected in the properties of the physical states.

3. The function Φ could code for the value of $k(M^4)$ via a proportionality constant

$$\Phi = \frac{k(M^4)}{k(CP_2)}\Phi_0 \ , \tag{20}$$

   Here $k(CP_2)$ is the central extension parameter multiplying the Chern-Simons action for CP_2 Kähler gauge potential. This tricks does just what is needed since it multiplies the Noether currents and super currents associated with M^4 degrees of freedom with $k(M^4)$ instead of $k(CP_2)$.

   The obvious breaking of $U(1)$ gauge invariance looks strange at first but it conforms with the fact that in TGD framework the canonical transformations of CP_2 acting as $U(1)$ gauge symmetries do not give to gauge degeneracy but to spin glass degeneracy since they act as symmetries of only vacuum extremals of Kähler action.

3. How to achieve Lorentz invariance?

Lorentz invariance fixes the form of function Φ uniquely as the following argument demonstrates.

1. Poincare invariance would be broken in any case for a given light-cone in the decomposition $CH = \cup_m CH_m$ of the configuration space to sub-configuration spaces associated with light-cones at various locations of M^4 but since the functions Φ associated with various light cones would be related by a translation, translation invariance would not be lost.

2. The selection of Φ should not break Lorentz invariance. If Φ depends on the Lorentz proper time a only, this is partially achieved. Momentum currents would be proportional to m^k and become light like at the boundary of the light-cone. This fits very nicely with the interpretation that the matter emanates from the tip of the light cone in Robertson-Walker cosmology.

Lorentz invariance poses even stronger conditions on Φ.

1. Partonic four-momentum defined as Chern-Simons Noether charge is definitely not conserved and must be identified as gravitational four-momentum whose time average corresponds to the conserved inertial four-momentum assignable to the Kähler action [D3, D5]. This identification is very elegant since also gravitational four-momentum is well-defined although not conserved.

2. Lorentz invariance implies that mass squared is constant of motion. Hence it is interesting to look what expression for Φ results if the gravitational mass defined as Noether charge for C-S action is conserved. The components of the four-momentum for Chern-Simons action are given by

   $$P^k = \frac{\partial L_{C-S}}{\partial(\partial_\alpha a)} m^{kl}\partial_{m^l} a \ \ .$$

   Chern-Simons action is proportional to $A_\alpha = A_a \partial_\alpha a$ so that one has

   $$P^k \propto \partial_a \Phi \partial_{m^k} a = \partial_a \Phi \frac{m^k}{a} \ \ .$$

   The conservation of gravitational mass gives $\Phi \propto a$. Since CP_2 projection must be 2-dimensional, M^4 projection is 1-dimensional so that mass squared is indeed conserved.

   Thus one could write

   $$\Phi = \frac{k(M^4)}{k(CP_2)} x\theta(a)\frac{a}{R} \ \ , \tag{21}$$

   where R the radius of geodesic sphere of CP_2 and x a numerical constant which could be fixed by quantum criticality of the theory. Chern-Simons action density does not depend on a for this choice and this independence guarantees that the earlier ansatz satisfies field equations. The presence of the step function $\theta(a)$ tells that Φ is non-vanishing only inside light-cone and gives to the gauge potential delta function term which is non-vanishing only at the light-cone boundary and makes possible massless particles.

3. If M^4 projection is 1-dimensional, only homologically charged partonic 3-surfaces can carry gravitational four-momentum. This is not a problem since M^4 projection can be 2-dimensional in the general case. For CP_2 type extremals, ends of cosmic strings, and wormhole contacts the non-vanishing of homological charge looks natural. For wormhole contacts 3-D CP_2 projection suggests itself and is possible only if one allows also quantum fluctuations around light-like extremals of Chern-Simons action. The interpretation could be that for a vanishing homological charge boundary conditions force X^4 to approach vacuum extremal at partonic 3-surfaces.

   This picture does not fit completely with the picture about particle massivation provided by CP_2 type extremals. Massless partons must correspond to 3-surfaces at light-cone boundary in this picture and light-likeness allows only linear motion so that inertial mass defined as average must vanish.

5. Comment about quantum classical correspondence

The proposed general picture allows to define the notion of quantum classical correspondence more precisely. The identification of the time average of the gravitational four-momentum for

C-S action as a conserved inertial four-momentum associated with the Kähler action at a given space-time sheet of a finite temporal duration (recall that we work in the zero energy ontology) is the most natural definition of the quantum classical correspondence and generalizes to all charges.

In this framework the identification of gravitational four-momentum currents as those associated with 4-D curvature scalar for the induced metric of X^4 could be seen as a phenomenological manner to approximate partonic gravitational four-momentum currents using macroscopic currents, and the challenge is to demonstrate rigorously that this description emerges from quantum TGD.

For instance, one could require that at a given moment of time the net gravitational four-momentum of $Int(X^4)$ defined by the combination of the Einstein tensor and metric tensor equals to that associated with the partonic 3-surfaces. This identification, if possible at all, would certainly fix the values of the gravitational and cosmological constants and it would not be surprising if cosmological constant would turn out to be non-vanishing.

3.5 The eigenvalues of D as complex square roots of conformal weight and connection with Higgs mechanism?

An alternative interpretation for the eigenvalues of D emerges from the TGD based description of particle massivation. The eigenvalues could be interpreted as complex square roots of conformal weights in the sense that $|\lambda|^2$ would have interpretation as a conformal weight. There is of course the possibility of numerical constant of proportionality.

The physical motivation for the interpretation is that λ is in the same role as the mass term in the ordinary Dirac equation and thus indeed square root of mass squared proportional to the conformal weight. The vacuum expectation of Higgs would correspond to that for λ and Higgs contribution to the mass squared would correspond to the p-adic thermodynamical expectation value $\langle|\lambda|^2\rangle$ [C9]. Additional contributions to mass squared would come from super conformal and modular degrees of freedom. The interpretation of the generalized eigenvalue as a Higgs field is also natural because the generalized eigen values of the modified Dirac operator can depend on position.

3.6 Is the spectrum of D expressible in terms of branches of an inverse of some zeta function?

$\hat{\Gamma}^r$ vanishes as X^3_l approaches to a vacuum extremal so that its inverse fails to exist. This requires that $N\Psi$ approaches zero in such a manner that the action of O on Ψ given by

$$\begin{aligned}
O\Psi &= \frac{\Phi}{a^k a^l h_{kl}} a^k \partial_r h^l \gamma_k \gamma_l \Psi = \frac{\Phi}{\sqrt{a^k a^l h_{kl}}}(A+B)\Psi \ , \\
A &= e^k h_{kl} \partial_r h^l \ , \\
B &= \frac{1}{2} e^k \partial_r h^l \Sigma_{kl} \ , \\
e^k &= \frac{a^k}{\sqrt{a^k a^l h_{kl}}}
\end{aligned} \tag{22}$$

gives a finite and well-defined result. This poses conditions on Φ already fixed by the requirement that zero modes induces super- conformal symmetries.

The expectation is that the condition

$$\frac{\Phi}{\sqrt{a^k a^l h_{kl}}}(A+B)\Psi = K\Psi \ , \tag{23}$$

where K is constant, can be posed asymptotically so that it becomes possible to speak about asymptotic eigen-states of the "Hamiltonian" O. In the non-asymptotic region O depends on r so that global eigen-states are not possible in general. An interpretation in terms of interactions is natural. In the asymptotic region Ψ would behave as $\Psi \propto exp(\lambda K r)$. Since the sigma matrices defined by the commutators of time like M^4 gamma matrices and CP_2 gamma matrices are antihermitian, the eigenvalues of B are expected to be imaginary. If the surface approaches asymptotically $X^1 \times X^2 \subset M^4 \times CP_2$, only B with two opposite, and in general complex, eigenvalues contributes to the asymptotic condition so that $K = \pm i K_1$ become the eigenvalues of O. In a more general case one has $K = K_0 \pm i K_1$.

Concerning the spectrum of λ, an interesting possibility suggested by the number theoretic considerations [E8, E1] is that $u = e^{kr/r_0}$, where k is some suitably chosen numerical factor, is the natural coordinate variable so that the exponents of r would transform to powers of u. If so, the numbers

$$\Delta = \frac{1}{k}(K_0 \pm i K_1)\lambda$$

could be interpreted as dynamical conformal weights having also complex values. This is however not the only possible interpretation (see below).

By the earlier number theoretic speculations the allowed eigenvalues could relate in a simple manner to the zeros of Riemann Zeta or of polyzeta in the case that X^4 contains several partonic 3-surfaces so that it becomes possible to introduce the notion of bound state conformal weight [E1]. For $k = 2K_0$ one has $Re(\Delta) = 1/2$ guaranteing that the conformal weights are at the critical line. This spectrum has been suggested earlier for the conformal weights associated with the super-canonical representations defined at $\delta M^4_\pm$ on basis of number theoretical considerations [E8]. Asymptotia would in this case correspond to the intersection of $X^3_l \cup M^4_\pm$ and need not mean to an exact vacuum extremal. The recent proposal would relate this hypothesis directly to the dynamics of modified Dirac operator.

It is however quite possible that the spectrum is determined by the requirement that p-adicization is possible rather than by some finiteness condition or boundary conditions as in the case of ordinary Schrödinger equation. If the exponents q^{iy} are algebraic numbers for y the imaginary part of zero of Riemann Zeta and for q prime (and therefore for any rational), spinor modes exist p-adically for a given rational value of the coordinate u in a suitable algebraic extension of the p-adic numbers for given p. Also the number theoretical building blocks $n^{1/2+iy}$ of Riemann Zeta exist in suitable algebraic extensions of p-adic numbers at zeros of Zeta and their integer multiples.

One can consider also more general zeta function coding number theoretical data about partonic 2-surface and number theoretic arguments suggest that a zeta expressible as rational function might be a better choice.

3.7 Super-conformal symmetries

The topological character of the solutions spectrum makes possible the expected and actually even larger conformal symmetries in X^2 degrees of freedom. Arbitrary diffeomorphisms of CP_2, including local SU(3) and its holomorphic counterpart, act as symmetries of the non-vacuum solutions. Also the canonical transformations of CP_2 inducing a U(1) gauge transformation are symmetries. More generally, the canonical transformations of $\delta M^4_\pm \times CP_2$ define configuration space symmetries.

Diffeomorphisms of M^4 respecting the light-likeness condition define Kac-Moody symmetries. In particular, holomorphic deformations of X^3_l defined in E^2 factor of $M^2 \times E^2$ compensated by a hyper-analytic deformation in M^2 degrees taking care that light-likeness is not lost, act as symmetry transformations. This requires that M^2 and E^2 contributions of the deformation to the induced metric compensate each other.

The fact that the modified Dirac equation reduces to a one-dimensional Dirac equation allows the action of Kac-Moody algebra as a symmetry algebra of spinor fields. In M^4 degrees of freedom X^2-local SL(2,C) acts as super-conformal symmetries and extends the SU(2) Kac-Moody algebra of $N = 4$ super-conformal algebra to $SL(2, C)$. The reduction to SU(2) occurs naturally. These symmetries act on all spinor components rather than on the second spinor chirality or right handed neutrinos only. Also electro-weak $U(2)$ acts as X^2-local Kac-Moody algebra of symmetries. Hence all the desired Kac-Moody symmetries are realized.

The action of Super Kac-Moody symmetries corresponds to the addition of a linear combination of zero modes of D to a given eigen mode. This defines a symmetry if zero modes satisfy the additional condition $N\Psi = 0$ implied by $\Psi = N\Psi_0$ in turn guaranteed by the already described conditions. These symmetries are super-conformal symmetries with respect to z and $\overline{z}$.

The radial conformal symmetries generalize the dynamical conformal symmetries characterizing CP_2 type vacuum extremals and could be regarded as dynamical conformal symmetries defining the spectrum of super-canonical conformal weights assigned originally to the radial light-like coordinate of $\delta M^4_{\pm}$. It deserves to be emphasized that the topological QFT character of TGD at fundamental level broken only by the light-likeness of X^3_l carrying information about H metric makes possible these symmetries.

$N = 4$ super-conformal symmetry corresponding to the maximal representation with the group $SU(2)\times SU(2)\times U(1)$ acting as rotations and electro-weak symmetries on imbedding space spinors is in question. This symmetry is broken for light-like 3-surfaces not satisfying field equations. It seems that rotational SU(2) can be extended to the full Lorentz group.

3.8 How the super-conformal symmetries of TGD relate to the conventional ones?

The representation of super-symmetries as an addition of anticommuting zero modes to the second quantized spinor field defined by the superposition of non-zero modes of the modified Dirac equation differs radically from the standard realization based on the replacement of the world sheet or target space coordinates with super-coordinates. Also the fundamental role of the generalized eigen modes of the modified Dirac operator is something new and absolutely essential for the understanding of how super-conformal invariance is broken: the breaking of super-symmetries is indeed the basic problem of the super-string theories.

Since the spinor fields in question are not Majorana spinors the standard super-field formalism cannot work in TGD context. It is however interesting to look to what extent this formalism generalizes and whether it allows some natural modification allowing to formally integrate the notions of the bosonic action and corresponding modified Dirac action.

1. One can consider the formal introduction of super fields by replacing of X^3_l coordinates by super-coordinates requiring the introduction of anti-commuting parameters θ and $\overline{\theta}$ transforming as H-spinors of definite chirality, which is not consistent with Majorana condition. Using real coordinates x^α for X^3_l, one would have

   $$x^\alpha \rightarrow X^\alpha = x^\alpha + \overline{\theta}\hat{\Gamma}^\alpha\Psi + \overline{\Psi}\hat{\Gamma}^\alpha\theta \quad ,$$

   Super-conformal symmetries would add to θ a zero mode with Grassmann number valued coefficient. The replacement $z^\alpha \rightarrow X^\alpha$ for the arguments of CP_2 and M^4 coordinates would super-symmetrize the field C-S action density. As a matter fact, the super-symmetrization is non-trivial only in radial degree of freedom since only $\hat{\Gamma}^r$ is non-vanishing.

2. Also imbedding space coordinates could be formally replaced with super-fields using a similar recipe and super-symmetries would act on them. The topological character of Chern-Simons action would allow the super-symmetries induced by the translation of θ by an anticommuting zero mode as formal symmetries at the level of the imbedding space. In both cases it is however far from clear whether the formal super-symmetrization has any real physical meaning.

3. The notion of super-surface suggests itself and would mean that imbedding space Θ parameters are functions of single θ parameter assignable with X_l^3. A possible representation of super-part of the imbedding is a generalization of ordinary imbedding in terms of constraints $H_{i)}(h^k) = 0$, $i = 1, 2....$ Symmetries allow only linear functions so that one would have

$$c_{i)}^{\alpha}(r, z, \overline{z})\Theta_{\alpha} = 0 \ .$$

   A hyper-plane in the space of theta parameters is obtained. Since only single theta parameter is possible in integral the number of constraints is seven and one obtains the modified Dirac action from the super-space imbedding.

Consider next the basic difficulty and its resolution.

1. The super-conformal symmetries do not generalize to the level of action principle in the standard sense of the word and the reason is the failure of the Majorana property forced by the separate conservation of quark and lepton numbers so that the standard super-space formalism remains empty of physical content.

2. One can however consider the modification of the integration measure $\prod_i d\theta_i d\overline{\theta}_i$ over Grassmann parameters by replacing the product of bilinears with

$$d\overline{\theta}\gamma_1 d\theta d\overline{\theta}\gamma_2 d\theta ...$$

   analogous to the product $dx^1 \wedge dx^2 ...$ (where γ^k would be gamma matrices of the imbedding space) transforming like a pseudoscalar. It seems that the replacement of product with wedge product leads to a trivial theory. This formalism could work for super fields obeying Weyl condition instead of Majorana condition and it would be interesting to find what kind of super-symmetric field theories it would give rise to.

   The requirement that the number of Grassmann parameters given by $2D$ is the number of spinor components of definite chirality (counting also conjugates) given by $2 \times 2^{D/2-1}$ gives critical dimension $D = 8$, which suggest that this kind of quantum field theory might exist. As found, the zero modes which are not of form $\Psi = N\Psi_0$ do not generate super-conformal symmetries in the strict sense of the word and might correspond to light particles. One could ask whether chiral SUSY in $M^4 \times CP_2$ might describe the low energy dynamics of corresponding light parton states. General arguments do not however support space-time super-symmetry.

3. Because of the light-likeness the super-symmetric variant of C-S action should involve the modified gamma matrices $\hat{\Gamma}^{\alpha}$ instead of the ordinary ones. Since only $\hat{\Gamma}^r$ is non-vanishing for the extremals of C-S action and since super-symmetrization takes place for the light-like coordinate r only, the integration measure must be defined as $d\overline{\theta}\hat{\Gamma}_r d\theta$, with θ perhaps assignable to a fixed covariantly constant right-handed neutrino spinor and $\hat{\Gamma}_r$ the inverse of $\hat{\Gamma}^r$. This action gives rise to the modified Dirac action with the modified gamma matrices emerging naturally from the Taylor expansion of the C-S action in powers of super-coordinate.

3.9 Absolute extremum property for Kähler action implies dynamical Kac-Moody and super conformal symmetries

The extremal property for Kähler action with respect to variations of time derivatives of initial values keeping h^k fixed at X^3 implies the existence of an infinite number of conserved charges assignable to the small deformations of the extremum and to H isometries. Also infinite number of local conserved super currents assignable to second variations and to covariantly constant right handed neutrino are implied. The corresponding conserved charges vanish so that the interpretation as dynamical gauge symmetries is appropriate. This result provides strong support that the local extremal property is indeed consistent with the almost-topological QFT property at parton level.

The starting point are field equations for the second variations. If the action contain only derivatives of field variables one obtains for the small deformations δh^k of a given extremal

$$\begin{aligned} \partial_\alpha J_k^\alpha &= 0 \ , \\ J_k^\alpha &= \frac{\partial^2 L}{\partial h_\alpha^k \partial h_\beta^l} \delta h_\beta^l \ , \end{aligned} \tag{24}$$

where h_α^k denotes the partial derivative $\partial_\alpha h^k$. A simple example is the action for massless scalar field in which case conservation law reduces to the conservation of the current defined by the gradient of the scalar field. The addition of mass term spoils this conservation law.

If the action is general coordinate invariant, the field equations read as

$$D_\alpha J^{\alpha,k} = 0 \tag{25}$$

where D_α is now covariant derivative and index raising is achieved using the metric of the imbedding space.

The field equations for the second variation state the vanishing of a covariant divergence and one obtains conserved currents by the contraction this equation with covariantly constant Killing vector fields j_A^k of M^4 translations which means that second variations define the analog of a local gauge algebra in M^4 degrees of freedom.

$$\begin{aligned} \partial_\alpha J_n^{A,\alpha} &= 0 \ , \\ J_n^{A,\alpha} &= J_n^{\alpha,k} j_k^A \ . \end{aligned} \tag{26}$$

Conservation for Killing vector fields reduces to the contraction of a symmetric tensor with $D_k j_l$ which vanishes. The reason is that action depends on induced metric and Kähler form only.

Also covariantly constant right handed neutrino spinors Ψ_R define a collection of conserved super currents associated with small deformations at extremum

$$\begin{aligned} J_n^\alpha &= J_n^{\alpha,k} \gamma_k \Psi_R \ , \\ . \end{aligned} \tag{27}$$

Second variation gives also a total divergence term which gives contributions at two 3-dimensional ends of the space-time sheet as the difference

$$\begin{aligned} Q_n(X_f^3) - Q_n(X^3) &= 0 \ , \\ Q_n(Y^3) &= \int_{Y^3} d^3x J_n \ , \ \ J_n = J^{tk} h_{kl} \delta h_n^l \ . \end{aligned} \tag{28}$$

The contribution of the fixed end X^3 vanishes. For the extremum with respect to the variations of the time derivatives $\partial_t h^k$ at X^3 the total variation must vanish. This implies that the charges Q_n defined by second variations are identically vanishing

$$Q_n(X^3_f) = \int_{X^3_f} J_n = 0 \ . \tag{29}$$

Since the second end can be chosen arbitrarily, one obtains an infinite number of conditions analogous to the Virasoro conditions. The analogs of unbroken loop group symmetry for H isometries and unbroken local super symmetry generated by right handed neutrino result. Thus extremal property is a necessary condition for the realization of the gauge symmetries present at partonic level also at the level of the space-time surface. The breaking of super-symmetries could perhaps be understood in terms of the breaking of these symmetries for light-like partonic 3-surfaces which are not extremals of Chern-Simons action.

4 Ricci flatness and divergence cancellation

Divergence cancellation in configuration space integration requires Ricci flatness and in this section the arguments in favor of Ricci flatness are discussed in detail.

4.1 Inner product from divergence cancellation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of the configuration space over the reduced configuration space containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ('lightcone boundary') using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \ , \\ \overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \ . \end{aligned} \tag{30}$$

The degeneracy for the absolute minima of Kähler action implies additional summation over the degenerate minima associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by configuration space integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of the configuration space integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the noncompact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [26]. The scalar product for two complex valued representation functions is defined as

$$(f,g) \quad = \quad \int \overline{f} g exp(nK)\sqrt{g} dV \quad . \tag{31}$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancellation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancellation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the configuration space into sectors D_P labelled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x, \alpha \rightarrow y, \beta) = \sum_{r,s} |S(r, \alpha \rightarrow s, \beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \quad ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r, m \rightarrow s, n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since configuration space metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in configuration space integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems (Duistermaat-Hecke theorem) stating that semiclassical approximation is exact for certain systems (for example for integrable systems [27]). Symmetric space property suggests that Kähler function might possess the properties guaranteing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving configuration space spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that configuration space integrals are continuable to p-adic number fields requires this kind of reduction.

4.2 Why Ricci flatness

It has been already found that the requirement of divergence cancellation poses extremely strong constraints on the metric of the configuration space. The results obtained hitherto are the following:
a) If the vacuum functional is the exponent of Kähler function one gets rid of the divergences resulting from the Gaussian determinants and metric determinants: determinants cancel each other.
b) The non-locality of the Kähler action gives good hopes of obtaining divergence free perturbation theory.

The following arguments show that Ricci flatness of the metric is a highly desirable property.

1. Dirac operator should be a well defined operator. In particular its square should be well defined. The problem is that the square of Dirac operator contains curvature scalar, which need not be finite since it is obtained via two infinite-dimensional trace operations from the curvature tensor. In case of loop spaces [36] the Kähler property implies that even Ricci tensor is only conditionally convergent. In fact, loop spaces with Kähler metric are Einstein spaces (Ricci tensor is proportional to metric) and Ricci scalar is infinite.

   In 3-dimensional case situation is even worse since the trace operation involves 3 summation indices instead of one! The conclusion is that Ricci tensor had better to vanish in vibrational degrees of freedom.

2. For Ricci flat metric the determinant of the metric is constant in geodesic complex coordinates as is seen from the expression for Ricci tensor [37]

$$R_{k\bar{l}} \;=\; \partial_k \partial_{\bar{l}} ln(det(g)) \qquad (32)$$

in Kähler metric. This obviously simplifies considerably functional integration over the configuration space: one obtains just the standard perturbative field theory in the sense that metric determinant gives no contributions to the functional integration.

3. The constancy of the metric determinant results not only in calculational simplifications: it also eliminates divergences. This is seen by expanding the determinant as a functional Taylor series with respect to the coordinates of the configuration space. In local complex coordinates the first term in the expansion of the metric determinant is determined by Ricci tensor

$$\delta\sqrt{g} \propto R_{k\bar{l}} z^k \bar{z}^l \quad . \qquad (33)$$

In configuration space integration using standard rules of Gaussian integration this term gives a contribution proportional to the contraction of the propagator with Ricci tensor. But since the propagator is just the contravariant metric one obtains Ricci scalar as result. So, in order to avoid divergences, Ricci scalar must be finite: this is certainly guaranteed if Ricci tensor vanishes.

4. The following group theoretic argument suggests that Ricci tensor either vanishes or is divergent. The holonomy group of the configuration space is a subgroup of $U(n = \infty)$ ($D = 2n$ is the dimension of the Kähler manifold) by Kähler property and Ricci flatness is guaranteed if the $U(1)$ factor is absent from the holonomy group. In fact Ricci tensor is proportional to the trace of the $U(1)$ generator and since this generator corresponds to an infinite dimensional unit matrix the trace diverges: therefore given element of the Ricci tensor is either infinite or vanishes. Therefore the vanishing of the Ricci tensor seems to be a mathematical necessity. This naive argument doesn't hold true in the case of loop spaces, for which Kähler metric with finite non-vanishing Ricci tensor exists [36]. Note however that also in this case the sum defining Ricci tensor is only conditionally convergent.

There are indeed good hopes that Ricci tensor vanishes. By the previous argument the vanishing of the Ricci tensor is equivalent with the absence of divergences in configuration space integration. That divergences are absent is suggested by the non-locality of the Kähler function as a functional of 3-surface: the divergences of local field theories result from the locality of interaction vertices. Ricci flatness in vibrational degrees of freedom is not only necessary mathematically. It is also appealing physically: one can regard Ricci flat configuration space as a vacuum solution of Einstein's equations $G^{\alpha\beta} = 0$.

4.3 Ricci flatness and Hyper Kähler property

Ricci flatness property is guaranteed if configuration space geometry is Hyper Kähler [38, 39] (there exists 3 covariantly constant antisymmetric tensor fields, which can be regarded as representations of quaternionic imaginary units). Hyper Kähler property guarantees Ricci flatness because the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(n)$ generators instead of $U(n)$ generators so that the traces vanish. In the case of the loop spaces left invariance implies that Ricci tensor in the vibrational degrees is a multiple of the metric tensor so that Ricci scalar has an infinite value. This is basically due to the fact that Kac-Moody algebra has $U(1)$ central extension.

Consider now the arguments in favor of Ricci flatness of the configuration space.

1. The canonical algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the canonical generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-canonical generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property.

1. The dimensions of the imbedding space and space-time are 8 and 4 respectively so that the dimension of configuration space in vibrational modes is indeed multiple of four as required by Hyper Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of the configuration space. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold manners.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of the configuration space. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at δM^4_+ can be chosen in S^2-fold manners. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

If these naive arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and canonical symmetries would also imply Hyper Kähler property of the configuration space and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the imbedding space.

In the sequel we shall show that Ricci flatness is guaranteed provided that the holonomy group of the configuration space is isomorphic to some subgroup of $SU(n = \infty)$ instead of $U(n = \infty)$ (n is the complex dimension of the configuration space) implied by the Kähler property of the metric. We also derive an expression for the Ricci tensor in terms of the structure constants of the isometry algebra and configuration space metric. The expression for the Ricci tensor is formally identical with that obtained by Freed for loop spaces: the only difference is that the structure constants of the finite-dimensional group are replaced with the group $Can(\delta H)$. Also the arguments in favor of Hyper Kähler property are discussed in more detail.

4.4 The conditions guaranteing Ricci flatness

In the case of Kähler geometry Ricci flatness condition can be characterized purely Lie-algebraically: the holonomy group of the Riemann connection, which in general is subgroup of $U(n)$ for Kähler manifold of complex dimension n, must be subgroup of $SU(n)$ so that the Lie-algebra of this group consists of traceless matrices. This condition is easy to derive using complex coordinates. Ricci tensor is given by the following expression in complex vielbein basis

$$R^{A\bar{B}} = R^{A\bar{C}B}_{\quad\ \bar{C}} \ , \tag{34}$$

where the latter summation is only over the antiholomorphic indices $\bar{C}$. Using the cyclic identities

$$\sum_{cycl\ \bar{C}B\bar{D}} R^{A\bar{C}B\bar{D}} = 0 \ , \tag{35}$$

the expression for Ricci tensor reduces to the form

$$R^{A\bar{B}} = R^{A\bar{B}C}_{\quad\ C} \ , \tag{36}$$

where the summation is only over the holomorphic indices C. This expression can be regarded as a trace of the curvature tensor in the holonomy algebra of the Riemann connection. The trace is taken over holomorphic indices only: the traces over holomorphic and anti-holomorphic indices cancel each other by the antisymmetry of the curvature tensor. For Kähler manifold holonomy algebra is subalgebra of $U(n)$, when the complex dimension of manifold is n and Ricci tensor vanishes if and only if the holonomy Lie-algebra consists of traceless matrices, or equivalently: holonomy group is subgroup of $SU(n)$. This condition is expected to generalize also to the infinite-dimensional case.

We shall now show that if configuration space metric is Kähler and possesses infinite-dimensional isometry algebra with the property that its generators form a complete basis for the tangent space (every tangent vector is expressible as a superposition of the isometry generators plus zero norm vector) it is possible to derive a representation for the Ricci tensor in terms of the structure constants of the isometry algebra and of the components of the metric and its inverse in the basis formed by the isometry generators and that Ricci tensor vanishes identically for the proposed complexification of the configuration space provided the generators $\{H_{A,m\neq 0}, H_{B,n\neq 0}\}$ correspond to zero norm vector fields of configuration space.

The general definition of the curvature tensor as an operator acting on vector fields reads

$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z \ . \tag{37}$$

If the vector fields considered are isometry generators the covariant derivative operator is given by the expression

$$\begin{aligned} \nabla_X Y &= (Ad_X Y - Ad^*_X Y - Ad^*_Y X)/2 \ , \\ (Ad^*_X Y, Z) &= (Y, Ad_X Z) \ , \end{aligned} \tag{38}$$

where $Ad_X Y = [X,Y]$ and Ad^*_X denotes the adjoint of Ad_X with respect to configuration space metric.

In the sequel we shall assume that the vector fields in question belong to the basis formed by the isometry generators. The matrix representation of Ad_X in terms of the structure constants $C_{X,Y:Z}$ of the isometry algebra is given by the expression

$$\begin{aligned} Ad^m_{Xn} &= C_{X,Y:Z}\hat{Y}_n Z^m \ , \\ [X,Y] &= C_{X,Y:Z}Z \ , \\ \hat{Y} &= g^{-1}(Y,V)V \ , \end{aligned} \tag{39}$$

where the summation takes place over the repeated indices and $\hat{Y}$ denotes the dual vector field of Y with respect to the configuration space metric. From its definition one obtains for Ad^*_X the matrix representation

$$\begin{aligned} Ad^{*m}_{Xn} &= C_{X,Y:Z}\hat{Y}^m Z_n \ , \\ Ad^*_X Y &= C_{X,U:V}g(Y,U)g^{-1}(V,W)W = g(Y,U)g^{-1}([X,U],W)W \ , \end{aligned} \tag{40}$$

where the summation takes place over the repeated indices.

Using the representations of ∇_X in terms of Ad_X and its adjoint and the representations of Ad_X and Ad^*_X in terms of the structure constants and some obvious identities (such as $C_{[X,Y],Z:V} = C_{X,Y:U}C_{U,Z:V}$) one can by a straightforward but tedious calculation derive a more detailed expression for the curvature tensor and Ricci tensor. Straightforward calculation of the Ricci tensor has however turned to be very tedious even in the case of the diagonal metric and in the following we shall use a more convenient representation [36] of the curvature tensor applying in case of the Kähler geometry.

The expression of the curvature tensor is given in terms of the so called Toeplitz operators T_X defined as linear operators in the "positive energy part" G_+ of the isometry algebra spanned by the $(1,0)$ parts of the isometry generators. In present case the positive and negative energy parts and cm part of the algebra can be defined just as in the case of loop spaces:

$$\begin{aligned} G_+ &= \{H^{Ak} | k > 0\} \ , \\ G_- &= \{H^{Ak} | k < 0\} \ , \\ G_0 &= \{H^{Ak} | k = 0\} \ . \end{aligned} \tag{41}$$

Here H^{Ak} denote the Hamiltonians generating the canonical transformations of δH. The positive energy generators with non-vanishing norm have positive radial scaling dimension: $k \geq 0$, which corresponds to the imaginary part of the scaling momentum $K = k_1 + i\rho$ associated with the factors $(r_M/r_0)^K$. A priori the spectrum of ρ is continuous but it is quite possible that the spectrum of ρ is discrete and $\rho = 0$ does not appear at all in the spectrum in the sense that the flux Hamiltonians associated with $\rho = 0$ elements vanish for the maximum of Kähler function which can be taken to be the point where the calculations are done.

T_X differs from Ad_X in that the negative energy part of $Ad_X Y = [X,Y]$ is dropped away:

$$\begin{aligned} T_X : G_+ &\rightarrow G_+ \ , \\ Y &\rightarrow [X,Y]_+ \ . \end{aligned} \tag{42}$$

Here "+" denotes the projection to "positive energy" part of the algebra. Using Toeplitz operators one can associate to various isometry generators linear operators $\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ acting on G_+:

$$\begin{aligned}\Phi(X_0) &= T_{X_0} \ , \ X_0 \varepsilon G_0 \ , \\ \Phi(X_-) &= T_{X_-} \ , \ X_- \varepsilon G_- \ , \\ \Phi(X_+) &= -T^*_{X_-} \ , \ X_+ \varepsilon G_+ \ . \end{aligned} \tag{43}$$

Here "*" denotes hermitian conjugate in the diagonalized metric: the explicit representation $\Phi(X_+)$ is given by the expression [36]

$$\begin{aligned}\Phi(X_+) &= D^{-1} T_{X_-} D \ , \\ DX_+ &= d(X) X_- \ , \\ d(X) &= g(X_-, X_+) \ . \end{aligned} \tag{44}$$

Here $d(X)$ is just the diagonal element of metric assumed to be diagonal in the basis used. denotes the conformal factor associated with the metric.

The representations for the action of ,$\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ in terms of metric and structure constants of the isometry algebra are in the case of the diagonal metric given by the expressions

$$\begin{aligned}\Phi(X_0)Y_+ &= C_{X_0,Y_+:U_+} U_+ \ , \\ \Phi(X_-)Y_+ &= C_{X_-,Y_+:U_+} U_+ \ , \\ \Phi(X_+)Y_+ &= \frac{d(Y)}{d(U)} C_{X_-,Y_-:U_-} U_+ \ . \end{aligned} \tag{45}$$

The expression for the action of the curvature tensor in positive energy part G_+ of the isometry algebra in terms of the these operators is given as [36]:

$$R(X,Y)Z_+ = \{[\Phi(X),\Phi(Y)] - \Phi([X,Y])\}Z_+ \ . \tag{46}$$

The calculation of the Ricci tensor is based on the observation that for Kähler manifolds Ricci tensor is a tensor of type $(1,1)$, and therefore it is possible to calculate Ricci tensor as the trace of the curvature tensor with respect to indices associated with G_+.

$$Ricci(X_+, Y_-) = (\hat{Z}_+, R(X_+, Y_-)Z_+) \equiv Trace(R(X_+, Y_-)) \ , \tag{47}$$

where the summation over Z_+ generators is performed.

Using the explicit representations of the operators Φ one obtains the following explicit expression for the Ricci tensor

$$\begin{aligned}Ricci(X_+, Y_-) &= Trace\{[D^{-1}T_{X_+}D, T_{Y_-}] - T_{[X_+,Y_-]_{|G_0+G_-}} \\ &- D^{-1}T_{[X_+,Y_-]_{|G_+}} D\} \ . \end{aligned} \tag{48}$$

This expression is identical to that encountered in case of loop spaces and the following arguments are repetition of those applying in the case of loop spaces.

The second term in the Ricci tensor is the only term present in the finite-dimensional case. This term vanishes if the Lie-algebra in question consists of traceless matrices. Since canonical transformations are volume-preserving the traces of Lie-algebra generators vanish so that this term is absent. The last term gives a non-vanishing contribution to the trace for the same reason.

The first term is quadratic in structure constants and does not vanish in case of loop spaces. It can be written explicitly using the explicit representations of the various operators appearing in the formula:

$$\begin{aligned} Trace\{[D^{-1}T_{X_-}D, T_{Y_-}]\} &= \sum_{Z_+,U_+} [C_{X_-,U_-:Z_-} C_{Y_-,Z_+:U_+} \frac{d(U)}{d(Z)} \\ &- C_{X_-,Z_-:U_-} C_{Y_-,U_+:Z_+} \frac{d(Z)}{d(U)}] \ . \end{aligned} \tag{49}$$

Each term is antisymmetric under the exchange of U and Z and one might fail to conclude that the sum vanishes identically. This is not the case. By the diagonality of the metric with respect to radial quantum number, one has $m(X_-) = m(Y_-)$ for the non-vanishing elements of the Ricci tensor. Furthermore, one has $m(U) = m(Z) - m(Y)$, which eliminates summation over $m(U)$ in the first term and summation over $m(Z)$ in the second term. Note however, that summation over other labels related to canonical algebra are present.

By performing the change $U \rightarrow Z$ in the second term one can combine the sums together and as a result one has finite sum

$$\begin{aligned} \sum_{0<m(Z)<m(X)} [C_{X_-,U_-:Z_-} C_{Y_-,Z_+:U_+} \frac{d(U)}{d(Z)} &= C \sum_{0<m(Z)<m(X)} \frac{m(X)}{m(Z)-m(X)} \ , \\ C &= \sum_{Z,U} C_{X,U:Z} C_{Y,Z:U} \frac{d_0(U)}{d_0(Z)} \ . \end{aligned} \tag{50}$$

Here the dependence of $d(X) = |m(X)| d_0(X)$ on $m(X)$ is factored out; $d_0(X)$ does not depend on k_X. The dependence on $m(X)$ in the resulting expression factorizes out, and one obtains just the purely group theoretic term C, which should vanish for the space to be Ricci flat.

The sum is quadratic in structure constants and can be visualized as a loop sum. It is instructive to write the sum in terms of the metric in the canonical degrees of freedom to see the geometry behind the Ricci flatness:

$$C = \sum_{Z,U} g([Y,Z],U) g^{-1}([X,U],Z) \ . \tag{51}$$

Each term of this sum involves a commutator of two generators with a non-vanishing norm. Since tangent space complexification is inherited from the local coset space, the non-vanishing commutators in complexified basis are always between generators in $Can_{\neq 0}$; that is they do not not belong to rigid $su(2) \times su(3)$.

The condition guaranteing Ricci flatness at the maximum of Kähler function and thus everywhere is simple. All elements of type $[X_{\neq 0}, Y_{\neq 0}]$ vanish or have vanishing norm. In case of CP_2 Kähler geometry this would correspond to the vanishing of the $U(2)$ generators at the origin of CP_2 (note that the holonomy group is $U(2)$ in case of CP_2). At least formally stronger condition is that the algebra generated by elements of this type, the commutator algebra associated with $Can_{\neq 0}$, consist of elements of zero norm. Already the (possibly) weaker condition implies that adjoint map $Ad_{X\neq 0}$ and its hermitian adjoint $Ad^*_{X_{\neq 0}}$ create zero norm states. Since isometry conditions involve also adjoint action the condition also implies that $Can_{\neq 0}$ acts as isometries. More concrete form for the condition is that all flux factors involving double Poisson bracket and three generators in $Can_{\neq 0}$ vanish:

$$Q_e(\{H_A,\{H_B,H_C\}\}) \quad = \quad 0 \ , \ \text{ for } \ H_A,H_B,H_C \ \text{ in } \ Can_{\neq 0} \ . \tag{52}$$

The vanishing of fluxes involving two Poisson brackets and three Hamiltonians guarantees isometry invariance and Ricci flatness and, as found in [B2], is implied by the $[t,t] \subset h$ property of the Lie-algebra of coset space G/H having symmetric space structure.

The conclusion is that the mere existence of the proposed isometry group (guaranteed by the symmetric space property) implies the vanishing of the Ricci tensor and vacuum Einstein equations. The existence of the infinite parameter isometry group in turn follows basically from the condition guaranteing the existence of the Riemann connection. Therefore vacuum Einstein equations seem to arise, not only as a consequence of a physically motivated variational principle but as a mathematical consistency condition in infinite dimensional Kähler geometry. The flux representation seems to provide elegant manner to formulate and solve these conditions and isometry invariance implies Ricci flatness.

4.5 Is configuration space metric Hyper Kähler?

The requirement that configuration space integral integration is divergence free implies that configuration space metric is Ricci flat. The so called Hyper-Kähler metrics [39, 38, 40] are particularly nice representatives of Ricci flat metrics. In the following the basic properties of Hyper-Kähler metrics are briefly described and the problem whether Hyper Kähler property could realized in case of $M^4_+ \times CP_2$ is considered.

4.5.1 Hyper-Kähler property

Hyper-Kähler metric is a generalization of the Kähler metric. For Kähler metric metric tensor and Kähler form correspond to the complex numbers 1 and i and therefore define complex structure in the tangent space of the manifold. For Hyper Kähler metric tangent space allows three closed Kähler forms I,J,K, which with respect to the multiplication obey the algebra of quaternionic imaginary units and have square equal to - 1, which corresponds to the metric of Hyper Kähler space.

$$I^2 \quad = \quad J^2 = K^2 = -1 \ \ IJ = -JI = K, \text{ etc. } \ . \tag{53}$$

To define Kähler structure one must choose one of the Kähler forms or any linear combination of I,J and K with unit norm. The group $SO(3)$ rotates different Kähler structures to each other playing thus the role of quaternion automorphisms. This group acts also as coordinate transformations in Hyper Kähler manifold but in general fails to act as isometries.

If K is chosen to define complex structure then K is tensor of type $(1,1)$ in complex coordinates, I and J being tensors of type $(2,0)+(0,2)$. The forms $I+iJ$ and $I-iJ$ are holomorphic and anti-holomorphic forms of type $(2,0)$ and $(0,2)$ respectively and defined standard step operators I_+ and I_- of $SU(2)$ algebra. The holonomy group of Hyper-Kähler metric is always $Sp(k)$, $k \leq dimM/4$, the group of $k \times k$ unitary matrices with quaternionic entries. This group is indeed subgroup of SU(2k), so that its generators are traceless and Hyper Kähler metric is therefore Ricci flat.

Hyper Kähler metrics have been encountered in the context of 3-dimensional super symmetric sigma models: a necessary prerequisite for obtaining $N=4$ super-symmetric sigma model is that target space allows Hyper Kähler metric [41, 40]. In particular, it has been found that Hyper Kähler property is decisive for the divergence cancellation.

Hyper-Kähler metrics arise also in monopole and instanton physics [39]. The moduli spaces for monopoles have Hyper Kähler property. This suggests that Hyper Kähler property is characteristic for the configuration (or moduli) spaces of 4-dimensional Yang Mills types systems. Since YM action appears in the definition of configuration space metric there are hopes that also in present case the metric possesses Hyper-Kähler property.

4.5.2 Does the 'almost' Hyper-Kähler structure of CP_2 lift to a genuine Hyper-Kähler structure in configuration space?

The Hyper-Kähler property of configuration space metric does not seem to be in conflict with the general structure of TGD.

1. In string models the dimension of the "space-time" is two and Weyl invariance and complex structures play a decisive role in the theory. In present case the dimension of the space-time is four and one therefore might hope that quaternions play a similar role. Indeed, Weyl invariance implies YM action in dimension 4 and as already mentioned moduli spaces of instantons and monopoles enjoy the Hyper Kähler property.

2. Also the dimension of the imbedding space is important. The dimension of Hyper Kähler manifold must be multiple of 4. The dimension of configuration space is indeed infinite multiple of 8: each vibrational mode giving one "8".

3. The complexification of the configuration space in canonical degrees of freedom is inherited from $S^2 \times CP_2$ and CP_2 Kähler form defines the symplectic form of configuration space. The point is that CP_2 Weyl tensor has 3 covariantly constant components, having as their square metric apart from sign. One of them is Kähler form, which is closed whereas the other two are non-closed forms and therefore fail to define Kähler structure. The group $SU(2)$ of electro-weak isospin rotations rotate these forms to each other. It would not be too suprising if one could identify the configuration space counterparts of these forms as representations of quaternionic units at the level of configuration space. The failure of the Hyper Kähler property at the level of CP_2 geometry is due to the electro-weak symmetry breaking and physical intuition (in particular, p-adic mass calculations [6]) suggests that electro-weak symmetry might not be broken at the level of configuration space geometry).

A possible topological obstruction for the Hyper Kähler property is related to the cohomology of the configuration space: the three Kähler forms must be co-homologically trivial as is clear from the following argument. If any of 3 quaternionic 2-form is cohomologically nontrivial then by $SO(3)$ symmetry rotating Kähler forms to each other all must be co-homologically nontrivial. On the other hand, electro-weak isospin rotation leads to a linear combination of 3 Kähler forms and the flux associated with this form is in general not integer valued. The point is however that Kähler form forms only the $(1,1)$ part of the symplectic form and must be co-homologically trivial whereas the zero mode part is same for all complexifications and can be co-homologically nontrivial. The co-homological non-triviality of the zero mode part of the symplectic form is indeed a nice feature since it fixes the normalization of the Kähler function apart from a multiplicative integer. On the other hand the hypothesis that Kähler coupling strength is analogous to critical temperature provides a dynamical (and perhaps equivalent) manner to fix the normalization of the Kähler function.

Since the properties of the configuration space metric are inherited from $M^4_+ \times CP_2$ then also the Hyper Kähler property should be understandable in terms of the imbedding space geometry. In particular, the complex structure in CP_2 vibrational degrees of freedom is inherited from CP_2.

Hyper Kähler property implies the existence of a continuum (sphere S^2) of complex structures: any linear superposition of 3 independent Kähler forms defines a respectable complex structure. Therefore also CP_2 should have this continuum of complex structures and this is certainly not the case.

Indeed, if we had instead of CP_2 Hyper Kähler manifold with 3 covariantly constant 2-forms then it would be easy to understand the Hyper Kähler structure of configuration space. Given the Kähler structure of the configuration space would be obtained by replacing induced Kähler electric and magnetic fields in the definition of flux factors $Q(H_{A,m})$ with the appropriate component of the induced Weyl tensor. CP_2 indeed manages to be very nearly Hyper Kähler manifold!

How CP_2 fails to be Hyper Kähler manifold can be seen in the following manner. The Weyl tensor of CP_2 allows three independent components, which are self dual as 2-forms and rotated to each other by vielbein rotations.

$$\begin{aligned}
W_{03} &= W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
W_{01} &= W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
W_{02} &= W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ .
\end{aligned} \tag{54}$$

The component I_3 is just the Kähler form of CP_2. Remaining components are covariantly constant only with respect to spinor connection and not closed forms so that they cannot be interpreted as Maxwell fields. Their squares equal however apart from sign with the metric of CP_2, when appropriate normalization factor is used. If these forms were covariantly constant Kähler action defined by any linear superposition of these forms would indeed define Kähler structure in configuration space and the group $SO(3)$ would rotate these forms to each other. The projections of the components of the Weyl tensor on 3-surface define 3 vector fields as their duals and only one of these vector fields (Kähler magnetic field) is divergenceless. One might regard these 3 vector fields as counter parts of quaternion units associated with the broken Hyper Kähler structure.

One cannot exclude the possibility that the canonical invariance of the induced Kähler electric field implies that the electric parts of the other two components of induced Weyl tensor are canonically invariant. This is the minimum requirement. What is however obvious is that the magnetic parts cannot be closed forms for arbitrary 3-surfaces at light cone boundary. One counter example is enough and CP_2 type extremals seem to provide this counter example: the components of the induced Weyl tensor are just the same as they are for CP_2 and clearly not canonically invariant.

Thus it seems that configuration space could allow an 'almost' quaternionic structure broken by electro-weak interactions but it cannot be inherited from CP_2. An open question is whether it allows genuine quaternionic structure. Good prospects for obtaining quaternionic structure are provided by the quaternionic counterpart QP_2 of CP_2, which is 8-dimensional and has coset space structure $QP_2 = Sp(3)/Sp(2) \times Sp(1)$. This choice does not seem to be consistent with the symmetries of the standard model. Note however that the over all symmetry group is obtained by replacing complex numbers with quaternions on the matrix representation of the standard model group.

4.5.3 Could different complexifications for M^4_+ and light like surfaces induce Hyper Kähler structure for configuration space?

Quaternionic structure means also the existence of a family of complex structures parameterized by a sphere S^2. The complex structure of the configuration space is inherited from the complex structure of some light like surface.

In the case of the light cone boundary δM^4_+ the complex structure corresponds to the choice of quantization axis of angular momentum for the sphere $r_M = constant$ so that the coordinates orthogonal to the quantization axis define a complex coordinate: the sphere S^2 parameterizes these choices. Thus there is a temptation to identify the choice of quantization axis with a particular imaginary unit and Hyper Kähler structure would directly relate to the properties rotation group. This would bring an additional item to the list of miraculous properties of light like surfaces of 4-dimensional space-times.

This might relate to the fact that configuration space geometry is not determined by the canonical algebra of CP_2 localized with respect to the light cone boundary as one might first expect but consists of $M^4_+ \times CP_2$ Hamiltonians so that infinitesimal canonical transformation of CP_2 involves always also M^4_+-canonical transformation. M^4_+ Hamiltonians are defined by a function basis generated as products of the Hamiltonians H_3 and $H_1 \pm iH_2$ generating rotations with respect to three orthogonal axes, and two of these Hamiltonians are complexified.

Also the light like 3-surfaces X^3_l associated with quaternion conformal invariance are determined by some 2-surface X^2 and the choice of complex coordinates and if X^2 is sphere the choices are labelled by S^2. In this case, the presence of quaternion conformal structure would be almost obvious since it is possible to choose some complex coordinate in several manners and the choices are labelled by S^2. The choice of the complex coordinate in turn fixes 2-surface X^2 as a surface for which the remaining coordinates are constant. X^2 need not however be located at the elementary particle horizon unless one poses additional constraint. One might hope that different choices of X^2 resulting in this manner correspond to all possible different selections of the complex structure and that this choice could fix uniquely the conformal equivalence class of X^2 appearing as argument in elementary particle vacuum functionals. If X^2 has a more complex topology the identification is not so clear but since conformal algebra SL(2,C) containing algebra of rotation group is involved, one might argue that the choice of quantization axis also now involves S^2 degeneracy. If these arguments are correct one could conclude that Hyper Kähler structure is implicitly involved and guarantees Ricci flatness of the configuration space metric.

5 Consistency conditions on metric

In this section various consistency conditions on the configuration space metric are discussed. In particular, it will be found that the conditions guaranteeing the existence of Riemann connection in the set of all(!) vector fields (including zero norm vector fields) gives very strong constraints on the general form of the metric and that these constraints are indeed satisfied for the proposed metric.

5.1 Consistency conditions on Riemann connection

To study the consequences of the consistency conditions, it is most convenient to consider matrix elements of the metric in the basis formed by the isometry generators themselves. The consistency conditions state the covariant constancy of the metric tensor

$$\nabla_Z g(X,Y) \quad = \quad g(\nabla_Z X, Y) + g(X, \nabla_Z Y) = Z \cdot g(X,Y) \ . \tag{55}$$

$Z \cdot g(X,Y)$ vanishes, when Z generates isometries so that conditions state the covariant constancy of the matrix elements in this case. It must be emphasized that the ill defined-ness of the inner products of form $g(\nabla_Z X, Y)$ is just the reason for requiring infinite-dimensional isometry group. The point is that $\nabla_Z X$ need not to belong to the Hilbert space spanned by the tangent vector fields since the terms of type $Zg(X,Y)$ do not necessarily exist mathematically [36]. The elegant solution to the problem is that all tangent space vector fields act as isometries so that these quantities vanish identically.

The conditions of Eq. (55) can be written explicitly by using the general expression for the covariant derivative

$$
\begin{aligned}
g(\nabla_Z X, Y) &= [Zg(X,Y) + Xg(Z,Y) - Yg(Z,X) \\
&+ g(Ad_Z X - Ad^*_Z X - Ad^*_X Z, Y)]/2 \ .
\end{aligned} \tag{56}
$$

What happens is that the terms depending on the derivatives of the matrix elements (terms of type $Zg(X,Y)$) cancel each other (these terms vanish for the metric invariant under isometries), and one obtains the following consistency conditions

$$
g(Ad_Z X - Ad^*_Z X - Ad^*_X Z, Y) + g(X, Ad_Z Y - Ad^*_Z Y - Ad^*_Y Z) = 0 \ . \tag{57}
$$

Using the explicit representations of $Ad_Z X$ and $Ad *_Z X$ in terms of structure constants

$$
\begin{aligned}
Ad_Z X &= [Z,X] = C_{Z,X:U} U \ . \\
Ad^*_Z X &= C_{Z,U:V} g(X,V) g^{-1}(U,W) W = g(X,[Z,U]) g^{-1}(U,W) W \ .
\end{aligned} \tag{58}
$$

where the summation over repeated "indices" is performed, one finds that consistency conditions are identically satisfied provided the generators X and Y have a non-vanishing norm. The reason is that the contributions coming from $\nabla_Z X$ and $\nabla_Z Y$ cancel each other.

When one of the generators, say X, appearing in the inner product has a vanishing norm so that one has $g(X,Y) = 0$, for any generator Y, situation changes! The contribution of $\nabla_Z Y$ term to the consistency conditions drops away and using Eqs. (57) and (58) one obtains the following consistency conditions

$$
C_{Z,X:U} g(U,Y) + C_{X,Y:U} g(U,Z) = -X \cdot g(Z,Y) \ . \tag{59}
$$

Note that summation over U is carried out. If X is isometry generator (this need not be the case always) the condition reduces to a simpler form:

$$
C_{X,Z:U} g(U,Y) + C_{X,Y:U} g(Z,U) = g([X,Z].Y) + g(Z,[X,Y]) = 0 \ . \tag{60}
$$

These conditions have nice geometric interpretation. If the matrix elements are regarded as ordinary Hilbert space products between the isometry generators the conditions state that the metric defining the inner product behaves as a scalar in the general case.

5.2 Consistency conditions for the radial Virasoro algebra

The action of the radial Virasoro in nontrivial manner in the zero modes. Therefore isometry interpretation is excluded and consistency conditions do not make sense in this case. One can however consider the possibility that metric is invariant or suffers only an overall scaling under the action of the radial scaling generated by $L_0 = r_M d/dr_M$. Since the radial integration measure is scaling invariant and only powers of r_M/r_0 appear in Hamiltonians, the effect of the scaling $r_M \rightarrow \lambda r_M$ on the matrix elements of the metric is a scaling by $\lambda^{k_a + \bar{k}_b}$). One can interpret this by saying that scaling changes the values of zero modes and hence leads outside the symmetric space in question.

Invariance of reduced matrix element obtained by dividing away the powers of the scaling factor is achieved if the metric contains the conformal factor

$$S = \frac{1}{\Delta u} f(\frac{r_i}{r_j}) \ , \tag{61}$$

where r_i are the extrema of r_M interpreted as height function of X^3 and f is a priori arbitrary positive definite function. Since the presence of f presumably gives rise to renormalization corrections depending on the size and shape of 3-surface by scaling the propagator defined by the contravariant metric, the dependence on the ratios r_i/r_j should be slow, logarithmic dependence. Also the dependence on the Fourier components of the solid angles $\Omega(r_M)$ associated with the $r_M = constant$ sections is possible.

5.3 Explicit conditions for the isometry invariance

The identification of the Lie-algebra of isometry generators has been proposed but cannot provide any proof for the existence of the infinite parameter symmetry group at this stage. What one can do at this stage is to formulate explicitly the conditions guaranteing isometry invariance of the metric and try to see whether there are any hopes that these conditions are satisfied. It has been already found that the expression of the metric reduces for light cone alternative to the sum of two boundary terms coming from infinite future and from the boundary of the light cone. If the contribution from infinitely distant future vanishes, as one might expect, then only the contribution from the boundary of the light cone remains.

A tedious but straightforward evaluation of the second variation (see Appendix of the book) for Kähler action implies the following form for the second variation of the Kähler action

$$\delta^2 S = \int_{a=0}^{a=\infty} I^{n\beta}_{kl} \delta h^k D_\beta \delta h^l \ , \tag{62}$$

where the tensor $I^{\alpha\beta}_{kl}$ is defined as partial derivatives of the Kähler Lagrangian with respect to the derivatives $\partial_\alpha h^k$

$$I^{\alpha\beta}_{kl} = \partial_{\partial_\alpha h^k} \partial_{\partial_\beta h^l} L_M \ . \tag{63}$$

If the upper limit $a = \sqrt{(m^0)^2 - r_M^2} = \infty$ in the substitution vanishes then one can calculate second variation and therefore metric from the knowledge of the time derivatives $\partial_n h^k$ and $\partial_n \delta h^k$ on the boundary of the light cone only.

Kähler metric can be identified as the $(1,1)$ part of the second variation. This means that one can express the deformation as an element of the isometry algebra plus a arbitrary deformation in radial direction of the light cone boundary interpretable as conformal transformation of the light cone boundary. Radial contributions to the second variation are dropped (by definition of Kähler metric) and what remains is essentially a deformation in S^2 degrees of freedom.

The left invariance of the metric under the deformations of the isometry algebra implies an infinite number of conditions of the form

$$J^C g(J^A, J^B) = 0 \ , \tag{64}$$

where J^A, J^B and J^C denote the generators of the isometry group. These conditions ought to fix completely the time derivatives of the coordinates h^k for each 3-surface at light cone boundary and therefore in principle the whole minimizing four-surface provided the initial value problem associated with the Kähler action possesses unique solution. What is nice that the requirement of isometry invariance in principle provides solution to the problem of finding absolute minima of the Kähler action.

These conditions, when written explicitly give infinite number of conditions for the time derivative of the generator J^C (we assume for a moment that C is held fixed and let A and B run) at the boundary of the light cone. Time derivatives are in principle determined also by the requirement that deformed surface corresponds to an absolute minimum of the Kähler action. The basis of δH scalar functions respecting color and rotational symmetries is the most promising one.

5.4 Direct consistency checks

If duality holds true, the most general form of the configuration space metric is defined by the fluxes $Q_m^{\alpha,\beta}$, where α and β are the coefficients of signed and unsigned magnetic fluxes. Present is also a conformal factor depending on those zero modes, which do not appear in the symplectic form and which characterize the size and shape of the 3-surface. $[t,t] \subset h$ property implying Ricci flatness and isometry property of canonical transformations, requires the vanishing of the fluxes $Q_m^{\alpha,\beta}(\{H_{A,m\neq 0},\{H_{B,n\neq 0},H_{C,p\neq 0}\}\})$ associated with double commutators and poses strong consistency conditions on the metric. If n labelling canonical generators has half integer values then the conditions simply state conformal invariance: generators labelled by integers have vanishing norm whereas half-odd integers correspond to non-vanishing norm. Isometry invariance gives additional conditions on fluxes $Q_m^{\alpha,\beta}$. Lorentz invariance strengthens these conditions further. It could be that these conditions fix the initial values of the imbedding space coordinates completely.

5.5 Why some variant of absolute minimization might work?

Posing the invariance under canonical transformations and radial Virasoro to the initial values of the time derivatives of the imbedding space coordinates at δH associates more or less unique space-time surface $X^4(X^3)$ to given X^3 on δH as an extremal of Kähler action (or of any action, as a matter of fact!). Furthermore, in principle Kähler function can calculated and one can also understand how to integrate over the zero modes in the functional integral over all 3-surfaces.

Absolute minimization has served for one and half decades as an educated guess for the condition selecting the preferred extremals of Kähler action. Only quite recently (I am writing this in the beginning of 2005) emerged the idea that space-time surfaces could be regarded as hyper-quaternionic sub-manifolds of hyper-octonionic space $HO = M^8$ with the property that hyper-complex structure is fixed in a local manner at each point of space-time surface [E2]. This means a selection of a preferred hyper-octonionic imaginary unit. The automorphisms leaving this selection invariant form group $SU(3)$ identifiable as color group. The selections of hyper-quaternionic sub-space under this condition are parameterized by CP_2. This means that each 4-surface in HO defines a 4-surface in $M^4 \times CP_2$ and one can speak about a number-theoretic analog of spontaneous compactification having of course nothing to do with dynamics. It would be possible to make physics in two radically different geometric pictures: HO picture and $H = M^4 \times CP_2$ picture.

This assumption is consistent with the requirement that space-time surface is an extremal of Kähler action provided that the notion of Kähler calibration makes sense [E2]. The implication is however that absolute minimization is replaced with the requirement that the absolute value of Kähler action is minimized separately in each region where Kähler Lagrangian has definite sign. Obviously the extremals are as near as possible to vacuum extremals. There are good reasons to expect that this is guaranteed if the extremals minimize rest energy and possibly also other conserved quantities and it would become possible to deduce the initial values of the time derivatives of the imbedding space coordinates at space-like causal determinant X^3 from energy minimization numerically and hence also construct the space-time surfaces from data at X^3

The basic question is whether the absolute minimization of Kähler action or some variant of this condition indeed leads to the proposed realization of canonical and radial conformal symmetries, implies electric-magnetic duality and gives same Kähler function as the group theoretical construction. There are indeed some hopes since there is remarkable consistency between these approaches.

1. The two constructions predict complete degeneracy of the metric for 3-surfaces having zero-dimensional CP_2 projection and the dynamical group of vacuum symmetries for Kähler action corresponds to the zero modes of constructed configuration space metric.

2. Whatever the rule associating to X^3 the space-time surface $X^4(X^3)$ as a particular extremal of the Kähler action is, the rule must be such that the second variation of the Kähler action around the extremal reduces to a boundary term expressible as an integral over X^3 belonging to δH. The second variation around absolute minimum indeed has this property. Quite generally, the set of minima of function tends to possess symmetries as a function of its parameters and therefore absolute minima are the most promising candidates for the extrema with the required symmetries.

3. The requirement that configuration space metric is Euclidian in vibrational degrees of freedom requires Diff^4 degeneracy of the configuration space metric and absolute minimization gives this degeneracy according to the previous argument. The quadratic form defined by the $(1,1)$ part of the Kähler function should be negative definite. It seems that isometry invariance and Diff degeneracy together guarantee the Euclidian signature of the metric (line element is negative for vibrational deformations).

   (a) The second variation of the Kähler function is certainly negative for the maximum of Kähler function (most probable 3-space).

   (b) Negativity requirement for the second variation implies that second variation reduces to $(1,1)$ form at the maximum of the Kähler function as is easy to find by elementary study of the properties of quadratic forms in one complex dimension (general situation reduces to 1-dimensional situation by considering deformations in single complex direction only).

   (c) Isometry invariance clearly guarantees the negativity of vibrational line element everywhere.

Even more, the group theoretical construction of the configuration space metric relies heavily on the basic properties of Kähler action and is actually guided by these properties.

1. The Kähler structure of the light cone boundary not unique on purely group theoretical grounds and different Kähler structures are parametrized by $SO(3,1)/SO(3)$. The Kähler structure for given Y^3 is fixed uniquely by the requirement that the subgroup $SO(3)$ characterizing the structure for given Y^3 acts as the isotropy group of the classical 4-momentum assigned to Y^3 by absolute minimization of the Kähler action.

2. Canonical transformations of $S^2 \times CP_2$ correspond to zero modes and act also as dynamical symmetries in vacuum sector; the canonical invariance of Kähler electric field is realized in excellent approximation for the nearly flat space-time surfaces; the metric deduced from symmetry considerations has same peculiar vacuum degeneracy as the Kähler metric associated with Kähler action.

3. Coset space decomposition $\cup_i G/H_i$ implies spin glass analogy, which also follows directly from the vacuum degeneracy of the Kähler action. Vacuum degeneracy is crucial for many applications of TGD.

6 Appendix: General coordinate invariance and Poincare invariance for $H = M^4_+ \times CP_2$ option

$H = M^4 \times CP_2$ option is consistent both with the new about the relationship between inertial and gravitational energy and cosmology, and allows both Poincare invariance and general coordinate invariance realizing thus the original TGD dream. For the sake of completeness also the notion of Diff^4 invariant Poincare transformations stimulated by the $M^4_+ \times CP_2$ option is discussed below.

6.1 Diff^4 invariant representation of M^4 translations in $C(\delta H)$

Concerning the definition of U-matrix it is important to define what one exactly means with four-momentum eigen states. The factorization of U-matrix to a tensor product of cosmological U-matrix and local U-matrices solves this problem trivially since Poincare invariance is excellent approximation outside the light cone boundary. This factorization became obvious only after the emergence of the TGD as a generalized number theory vision and it was necessary to attack the problem of defining what $Diff^4$ invariant Poincare transformations might mean. The following argument describes a modification of Poincare algebra allowing to achieve this goal.

In fact, the breaking of Poincare symmetry caused by the light cone boundary turns out to play a crucial role in the definition of S-matrix and the physical interpretation of the theory. The crucial observation is that Poincare transformations realized as transformations just moving the 4-surface like rigid body do not commute with Diff^4. The reason is simply that in general the translated 4-surface is not an absolute minimum of the Kähler action although it is an extremal of the Kähler action: commutativity holds true for the Lorentz group of the future light cone only. It is therefore natural to ask whether one could realize translations in a Diff^4 invariant manner. These transformations should in general deform space-time surface and should reduce to ordinary translations for some suitably chosen set of 3-surfaces.

The problem is how to choose the subset of the 3-surfaces in question. If one requires that the set of 3-surfaces is Lorentz invariant with respect to the Lorentz group associated with the dip of the light cone acting as exact symmetries of the theory, so that also the energy momentum eigen state basis is Lorentz invariant, then the choice becomes unique: the 3-surfaces are obtained as the intersections of the space-time surfaces X^4 with the set $H_a \times CP_2$, where H_a is the $\sqrt{m_{kl}m^km^l} = a$ hyperboloid of the future light cone.

One can realize M^4 translations as transformations on the light cone boundary $C(\delta H)$ as follows.

1. Consider the unique space-time surface $X^4(X^3)$ going through X^3 and denote the intersection of this surface with light cone boundary by Y^3.
2. Consider the intersection $Z^3(a)$ of this surface with $a = constant$ (proper time of M^4_+) hyperboloid of H. Perform an infinitesimal translation for this surface. This translation induces unique deformation of $X^4(X^3)$ and of Y^3 so that the action of infinitesimal translations on $C(\delta H)$ is uniquely defined provided one restricts oneself to a given $a = constant$ surface.

This $Diff^4$ invariant representation for the translations differs from the ordinary representations in two important respects.

1. The representations of the infinitesimal translations associated with different values of cosmic time a are not identical and do not commute. This implies the breaking of Poincare invariance in a cosmic scale!

2. One can associate to any basis of energy momentum eigen-states a unique value of cosmic time a. The situation is quite contrary to that encountered in a Poincare invariant theory, where it is not possible to associate any value of the time coordinate to an energy momentum eigen state. Since quantum jumps occur between quantum histories in TGD framework, one encounters the problem of explaining the origin of psychological time. The original interpretation of the time parameter a was as the subjective time experienced by a conscious (and sufficiently intelligent) observer. The construction of TGD inspired theory of consciousness [10] has however led to a radical rethinking of the concept of psychological time and shown that this naive interpretation is not correct. In fact, the value of a is most naturally infinite just like in ordinary quantum field theories.

The recent work suggests a more concrete identification of the $Diff^4$ invariant Poincare. One can identify the translation generators at a given point of the future light cone as the time translation with respect to a, as the radial translation with respect to r_M and as two non-vanishing rotation generators generating rotations in the directions orthogonal to r_M. Since the third generator generating rotations around the direction defined by r_M, vanishes at r_M, one can say that the remaining rotation generators commute approximately and thus represent approximately Poincare translations. Only the time translation generator takes 3-surface out of $a =$ constant hyperboloid and must be defined in $Diff^4$ invariant manner. This could spoil the closure of the Poincare algebra or, at best, could lead to a deformation of the Poincare algebra.

The requirement that S-matrix is Poincare invariant, requires that the momentum generators p_k appearing in Super Virasoro generators and in time evolution operator correspond to $Diff^4$ invariant momentum generators $p_k(a)$ at the limit, when the value of the light cone proper time approaches infinity. At this limit $Diff^4$ invariant generators generate an algebra isomorphic to the ordinary Poincare algebra and one can construct momentum conserving S-matrix provided that the time parameter t defining time evolution operator $U = U(-t, t)$ varies in the entire range $(-\infty, \infty)$.

6.2 Diff invariant Poincare algebra as a deformation of Poincare algebra?

In the following the possibility that Diff invariant Poincare algebra at the limit $a \to \infty$ might correspond to a nontrivial deformation of Poincare algebra, is discussed.

Recall first, that the concept of $Diff^4$ invariant Poincare algebra arises in the following manner.

1. The configuration space of the TGD consists of all 3-surfaces in the Cartesian product of the future light cone M^4_+ (points m^k of M^4 satisfying $m_{kl}m^km^l = (m^0)^2 - (\overline{m})^2 \geq 0, m^0 \geq 0$).

2. Kähler geometry for this space is defined in terms of the Kähler function, which corresponds to the absolute minimum for Kähler action, which is $Diff^4$ invariant. This definition associates to each 3-surface X^3 a unique space-time surface $X^4(X^3)$, the classical history associated with the 3-surface. In the first part of the book it was explained how this concept defines the quantum counterpart of the Thom's catastrophe theory: in this theory discontinuous jumps take place along 'Maxwell line' rather than along the 'fold line': this is what is known to happen in phase transitions.

3. Kähler action is same for 4-surface and its Poincare translate but Poincare transformation does not in general map absolute minimum to an absolute minimum: exception is formed by the Lorentz transformations mapping light cone to itself. Therefore Poincare invariance is broken and ordinary representations of Poincare group are not $Diff^4$ invariant.

4. One can overcome the problem by modifying the concept of Poincare invariance. By $Diff^4$ invariance of state functions, one can consider instead of 3-surface X^3 the 3-surface X^3_a, the

intersection $X^4(X^3)$ with the cartesian product of light cone proper time constant hyperboloid $H_a = \{m^k | m_{kl} m^k m^l = a^2\}$ with CP_2. X^3_a is invariant under Lorentz group and one can define the action of an infinitesimal Poincare transformation by requiring that the action on X^3_a is ordinary infinitesimal Poincare transformation: the action on other 3-surfaces on $X^4(X^3)$ is fixed by the requirement that $X^4(X^3)$ is replaced with absolute minimum surface associated with the infinitesimal Poincare translate of X^3_a. The result is that absolute minimum surface suffers in general a nontrivial deformation and even its topology can change.

5. This $Diff$ invariant realization of Poincare algebra depends on the value of the proper time parameter a and gives rise to a continuous family of nonidentical unitarily related energy momentum eigen state basis. For finite values of a one expects that the resulting algebra is not closed but at the limit $a \rightarrow \infty$ one expects that $Diff^4$ invariant Poincare algebra is isomorphic with ordinary Poincare algebra. The generators $P_k(a \rightarrow \infty)$ are assumed to appear in the Super Virasoro conditions defining also time development operator and S-matrix commuting with Poincare transformations.

There are several open questions related to the $Diff^4$ invariant Poincare algebra at the limit $a \rightarrow \infty$. If algebra is closed, do commutation relations get deformed? If the algebra does not close, should one try to extend the Poincare algebra in order to get a closed algebra? Could quantum groups have some relevance in the problem? The answers to these questions seem to be beyond calculational capacities since it is difficult to imagine how one could deduce analytic expressions for the action of $Diff^4$ invariant Poincare transformations for such a complicated structure as the space of all possible 3-surfaces in $M^4_+ \times CP_2$ is.

Quite a surprise in this respect was the paper of Kehagias and Meessen [42], where it was shown that Poincare group allows deformation with an exact Lorentz algebra: the structure might raise the concept of $Diff^4$ invariant Poincare transformations at the limit $a \rightarrow \infty$ on a sound footing and even predict nontrivial effects. What happens in the deformation is a modification for the expression of energy $P_0(diff)$: the new energy is certain function $\beta(P_0)$ of the 'old' energy P_0. The old energy corresponds to the energy associated with ordinary Poincare transformations and new energy to the energy associated with $Diff^4$ invariant Poincare transformations. Lorentz invariance corresponds to the Lorentz transformations leaving the future light cone invariant. The only thing, which one cannot calculate at this stage is the explicit form of the function $\beta(P_0)$ (it could be also trivial!). The preferred time like direction implied by the dependence of the commutators relations on energy corresponds naturally to the special time direction defined by the classical momentum of 3-surface Y^3.

It is worthwhile to write the explicit expressions for the deformed commutation relations

$$
\begin{aligned}
[J_i, J_j] &= i\epsilon_{ijk} J_k \ , \\
[J_i, K_j] &= i\epsilon_{ijk} K_k \ , \\
[J_i, P_j] &= i\epsilon_{ijk} P_k \ , \\
[K_i, K_j] &= i\epsilon_{ijk} J_k \ , \\
[J_i, P_0] &= 0 \ , \\
[P_i, P_j] &= 0 \ , \\
[K_i, P_0] &= i\alpha(P_0) P_i \ , \\
[K_i, P_j] &= i\beta(P_0)\delta_{ij} \ , \\
\alpha(P_0)\frac{d\beta(P_0)}{dP_0} &= 1 \ . \qquad (65)
\end{aligned}
$$

Deformation is trivial at the limit

$$\begin{aligned} \alpha(P_0) &= 1 \ , \\ \beta(P_0) &= P_0 \ . \end{aligned} \tag{66}$$

The deformed algebra leaves invariant the lenghts of the deformed four-momentum vector and Pauli-Lubanski vector

$$\begin{aligned} m^2 &= m_{kl}P^k(d)P^l(d) \ , \\ W^2(d) &= m_{kl}W^k(d)W^l(d) \ , \\ P_k(d) &\leftrightarrow (\beta(P_0), P_k) \ , \\ W_i(d) &\leftrightarrow (W_0 = \overline{J}\cdot P, W_i = \beta(P_0)J_i + \epsilon_{ijk}P_jK_k) \ . \end{aligned} \tag{67}$$

The effect of the deformation is clearly to replace the expression of the energy with a more general one.

Consider first the possible application of the concept to Quantum TGD at particle physics length scales. The non-triviality of S-matrix in Quantum TGD follows from the parametric dependence of $\beta(P_0)$ on the light cone proper time a. There are good reasons to expect that this dependence is extremely weak at CP_2 length scale. Although the deviation from triviality might be extremely small, the criticality of TGD Universe at quantum level is expected to imply initial value sensitivity and large deviations of S-matrix from unit matrix in particle physics length scales. The essential point is the instability of the 3-surface (particle) to topological decay into several 3-surfaces: only a small deformation (say small time translation) can cause this decay.

In long length scales (macroscopic, astrophysical, cosmological) the difference between Poincare energy and $Diff^4$ invariant Poincare energy could be large.

1. $Diff^4$ invariant energy vanishes, when absolute minimum 4-surface is static since time translations correspond to $Diff^4$ transformations and must leave the state invariant. By the same argument, Diff invariant energy is just a multiple for the frequency of the oscillation for small periodic deformations and one obtains just a harmonic oscillator. For non-periodic surfaces near these simple surfaces the previous equation makes possible to define $Diff^4$ invariant energy. These observations suggest that for simple systems $Diff^4$ invariant energy is roughly the vacuum subtracted energy, vacuum energy defined as the ordinary Poincare invariant energy of the static configuration.

2. The function $\beta(P_0)$, the deformed energy, can depend on 3-surface via the isometry invariant parameters describing the general geometric properties like the shape and the size of the 3-surface.

3. At the classical limit of the theory it should be generally possible to identify the eigen value of the quantum energy $\beta(P_0)$ with its classical counterpart $\beta(P_0)$, where $P_0(X^3)$ is the classical conserved Poincare energy associated with the absolute minimum surface $X^4(X^3)$. For state functionals dispersed in a set of 3-surfaces with different values of $P_0(X^3)$ the identity of classical and quantum energies requires that the functional form of the classical counterpart for the Poincare invariant energy $\beta(P_0(X^3))$ must depend on 3-surface in the manner dictated by the condition

$$\beta(P_0(X^3), X^3) = \beta_0 \equiv \beta(P_0(X_0^3), X_0^3) \ . \tag{68}$$

The parametric dependence is only on those parameters, which are Poincare invariant and the two dependences on X^3 must compensate each other in the expression of the energy. One must fix the value of β_0 by some criterion for some surface. A reasonable choice is a surface, which corresponds to a periodic oscillation around static ground state, for which $Diff^4$ invariance implies oscillator spectrum.

4. An interesting (but perhaps purely formal) possibility is that the dispersion relation $P_0 = m+\frac{\bar{p}^2}{2m}$, characteristic to the Galilei invariance and resulting approximately at non-relativistic limit, is in fact an exact relationship implied by a suitable deformation of the Poincare algebra

$$\begin{aligned} \beta^2(P_0) &= 2mP_0 - m^2 \geq 0 \ , \\ \alpha(P_0) &= \frac{\sqrt{\beta}}{m} \ . \end{aligned} \tag{69}$$

Effective Galilei invariance would result from the deformation of the Poincare group. The alarming feature is that this deformation depends on the particle mass but this is in accordance with the dependence of β on 3-surface since each particle corresponds to certain classical surface at semiclassical limit of TGD. The reduction of the ordinary Poincare invariance to effective Galilei invariance (but "cosmological" Lorentz transformations acting as exact symmetries!) would be implied basically by the absolute minimization of the Kähler action.

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