Possible Role of p-Adic Numbers in Bio-Systems

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Contents

Abstract

The identification of some p-adic length scales predicted by the p-adic length scale hypothesis as biologically relevant length scales is suggested. p-Adic ultrametricity, the non-determinism of the p-adic differential equations, the special features of the p-adic dynamical flows, the delicacies of the p-adic probabity concept and the special features of p-adic entanglement are also discussed briefly and possible implications for biosystems are pointed out. Also ideas, which are only marginally consistent with the interpretation of p-adic physics as physics of cognition, are discussed.

In particular, some speculations about the possible role of so called exotic representations of quaternion conformal algebra are included. These speculations rely heavily on the assumption that canonical correspondence between p-adic and real masses holds true in full generality. The prediction is the existence of a hierarchy of p-adic states for which p-adic masses have having extremely small real counterparts whereas the corresponding real states have super-astronomical masses. These strange states have huge degeneracies and the original speculation was that they are crucial for the understanding of biological life. Later however came the realization that the states of the supercanonical representations associated with the lightlike boundaries of massless extremals (MEs) have also gigantic almost-degeneracies. In particular, there is no need to assume the highly questionable p-adic–real correspondence at the level of masses for them. Therefore the cautious conclusion is that biology can do without the exotic quaternion conformal representations.

1 Introduction

In this chapter p-adic length scale hypothesis and the special features of p-adic numbers are discussed from the point of view of biosystems. The identification of p-adic physics as physics of cognition understood as a simulation of real physics is the basic philosophical quide line. The characteristic features of p-adic physics are due to p-adic ultrametricity, p-adic non-determinism, and to some exotic properties of p-adic probability and are expected to characterize also cognition. It is however too early to take too strong views concerning the interpretation of p-adics. Therefore also speculative ideas about the role of p-adic numbers in biology, which are only marginally consistent with the cognitive interpretation, are discussed in the sequel.

Also some speculations about possible role of so called exotic representations of quaternion conformal algebra are included. These speculations rely heavily on the assumption that canonical correspondence between p-adic and real masses holds true in full generality. The prediction is the existence of a hierarchy of p-adic states for which p-adic masses have extremely small real counterparts whereas the corresponding real states have super-astronomical masses. These strange states have huge degeneracies and the original speculation was that they are crucial for the understanding of biological life. Later however came the realization that the states of the supercanonical representations associated with the lightlike boundaries of massless extremals (MEs) have also gigantic almost-degeneracies. In particular, there is no need to assume the highly questionable p-adic–real correspondence at the level of masses for them. Therefore the cautious conclusion is that biology can do without the exotic quaternion conformal representations.

2 p-Adic length scale hypothesis and biosystems

2.1 Biologically relevant p-adic length scales

The following table lists the p-adic length scales L_p. p near prime power of 2, which might be interesting as far as biosystems are considered. Some overall scaling factor r of order one is present in the definition of the length scale and it is interesting to look whether with a suitable choice of r it is possible to identify p-adic length scales as biologically important length scales. The requirement that $L(151)$ corresponds to the thickness of the cell membrane about 10^{-8} meters gives $r \simeq 1.2$.

k	127	131	137	139	149
$L_p/10^{-10}m$	.025	.1	.8	1.6	50
k	151	157	163	167	169
$L_p/10^{-8}m$	1	8	64	256	512
k	173	179	181	191	193
$L_p/10^{-4}m$	.2	1.6	3.2	100	200
k	197	199	211	223	227
L_p/m	.08	.16	10	640	2560

Table 1. p-Adic length scales $L_p = 2^{k-151}L_{151}$, $p \simeq 2^k$, k prime, possibly relevant to bio physics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L(151)$ is take to be thickness of cell scale, which is 10^{-8} meters in good approximation.

The study of the table supports the idea that p-adic length scale hypothesis might have explanatory power in biology. What is remarkable is the frequent occurrence of twin length scales related by a factor 2 in the range of biologically interesting p-adic length scales: only 3 of 15 primes in the range do not belong to a twin pair! The fact that these length scales seem to correspond to biologically interesting length scales suggests that twins might be related to replication phenomenon and to the possible 2-adicity in biology: for a given twin pair the smaller length scale would define basic 2-adic length scale. In the following the scales denoted by $\hat{L}(n)$ are related by a factor $r = 1.2$ to the lengthscales $L(n)$ appearing in the table above.

1. $\hat{L}(137) \simeq 7.84E - 11\ m$, $\hat{L}(139) \simeq 1.57E - 10m$ form a twin pair. This length scales might be associated with atoms and small molecules.

2. The secondary p-adic length scale $L(\hat{71,2}) \simeq .44\ nm$ corresponds to the thickness of the DNA strand which is about .5 nm. Both DNA strand and double helix must correspond to this length scale. The secondary p-adic length scale $L(\hat{73,2}) \simeq 1.77\ nm$ is longer than the thickness of DNA double strand which is roughly 1.1 nm. Whether one could interpret this length scale as that associated with DNA double strand remains an open question. Alpha helix, the basic building block of proteins provides evidence for has radius 1.81 $nm \sim \hat{L}(139)$ and the height of single step in the helix is .544 nm.

3. $\hat{L}(149) \simeq .5.0\ nm$ and $\hat{L}(151) \simeq 10.0\ nm$ form also a twin pair. The thickness of cell membrane of order $10^{-8}\ m \sim \hat{L}(151)$. Cell membrane consists of two separate membranes and the thickness of single membrane therefore corresponds to $\hat{L}(149)$. Microtubules, which are basic structural units of the cytoskeleton, are hollow cylindrical surfaces having thickness $d \sim 11\ nm$, which is not too far from the length scale $\hat{L}(151)$. It has been suggested that microtubules might play key role in the understanding of biosystem as macroscopic quantum system [37, 35].

4. If neutrinos have masses of order one eV as suggested by recent experiments then the primary condensation level of neutrinos could correspond to $k_Z = 167$ or $k_Z = 13^2 = 169$ and would be the level at which nuclei feed their Z^0 gauge charges. This level is many particle quantum system in p-adic sense and p-adic effects are expected to important at this condensation level. Chirality selection should take place via the breaking of neutrino superconductivity at this level and involve the generation of Z^0 magnetic fields at some level $k < k_Z$, too. $k = 151$ is a good candidate for the level in question.

5. In the previous version of this chapter it was stated that $\hat{L}(167) = 2.73\ \mu m$, $\hat{L}(169 = 13^2) = 5.49\ \mu m$ form a twin pair and correspond to typical length scales associated with cellular structures. Neutrino mass calculations give best predictions for $k = 169$ and this suggests that the generalization of '$k =$ prime' to '$k =$ power of prime' should be considered: generalization would allow also $k = 169$ as basic length scale. Also blackhole elementary particle analogy suggests the generalization of the length scale hypothesis. Furthermore, only $k = 169$ would appear as a new length scale between electron length scale and astrophysical length scales ($k = 3^5, 2^8, 17^2$)! This suggests that the length scales $L(167)$ and $L(169)$ might form effective twin pair. That this could be the case is suggested by the fact that so called epithelial sheets appearing in skin, glands, etc., consisting of two layers of cells play in biosystems same role as cell membranes and are generally regarded as a step of bioevolution analogous to the formation of cell membrane.

6. $\hat{L}(173) = 2.20 \cdot 10^{-5}\ m$ might correspond to a size of some basic cellular structure (A structure consisting of 64 cell layers?). $\hat{L}(179) = 1.75 \cdot 10^{-5}\ m$ and $\hat{L}(181) = 3.52 \cdot 10^{-4}\ m$ form a twin pair. Later it will be found that the pair $k = 179, 181$ might correspond to basic structures associated with cortex.

7. Length scales $\hat{L}(191) = 1.12\ cm, \hat{L}(193) = 2.24\ cm$ and $\hat{L}(197) = 9.0\ cm$. $\hat{L}(199) = 18.0\ cm$ are again twins.

The proposed structure of topological condensate at condensed matter length scales makes it possible to make guesses about the general structure and mechanisms of information processing in biosystems.

1. There are three fundamental condensate levels. The nuclear condensation level $k = 113$, the atomic condensation level $k = k_{em} = 131$ and the condensation level $k = k_Z = 167$ or 169 at which nuclei feed their Z^0 gauge fluxes. $k = k_Z$ p-adic cube is genuine many particle quantum system in p-adic sense in accordance with the basic hypothesis that biosystems are macroscopic quantum systems. Neutrinos are condensed also at level $k = k_Z$ and they might play important role in biosystems as already the proposed model for the chirality selection in vivo suggests.

2. The basic mechanism of information transfer is the penetration of gauge fields from level k to upper or lower condensate level. Fields can be induced: the motion of k_{prev} # throats in the gauge fields at level $k > k_{prev}$ induces currents at level k_{prev}, which in turn create gauge fields at this level. Since # throats are located only on the p-adic blocks situated at the boundary of k_{prev} block their motion is very limited and the mechanism favors magnetic fields. Second mechanism is via formation of # contacts with quantized gauge and magnetic charges. In super conductors of type II (at least) magnetic field probably penetrates at higher condensate level in this manner.

3. The information can proceed from $k = k_Z$ level directly via the nuclear $k = 113$ level to atomic $k = 131$ level since the motion of Z^0 # throats in Z^0 magnetic fields at level $k = k_Z$ induces motion of electromagnetic # throats at level $k = 113$. Of course, the information can proceed also from some electromagnetic level $k < k_Z$ up to $k = 113$ level and directly down to cellular $k = k_Z$ level via same mechanism. This makes possible complicated feedback loops between $k = k_Z$ level and electromagnetic condensation levels.

Super conductivity was proposed to be the basic feature of biosystems already quite early. There are indeed arguments in favor of super conductivity at levels $k > k_{em} = 131$. The basic observation is that the density of # throats carrying electromagnetic charge is small and scales as $1/L^3(k_{prev})$ if the simple scaling law for ϵ_{em} holds true. On the average there is something like nuclear charge inside p-adic cube $L(k)$. # throats are located at the boundaries of condensed blocks so that spherical block creates Coulomb field only in the exterior of the block. Scaling law suggest that the density of electrons is essentially the same as the average density of # throats so that it would not be totally surprising if electrons would form Cooper pairs making higher condensate levels superconducting in the length scales $L > L_{upper}(k_{prev})$. For twin primes $k = k_{prev} + 2$ the length scale $L_{upper}(k_{prev})$ is especially large as compared with $L(k)$ and this perhaps explains why the length scales associated with twin primes seem to be important in biosystems.

If condensate levels are super conductors of type I, the magnetic field can penetrate from lower level to higher level in the form of complicated cylindrically symmetric regions [32] characterized by integers n_1 and n_2 related to the phases of CP_2 coordinates as found in [D7]. Also the magnetic fluxes are quantized and flux quanta are expressible in terms of the integers n_i [D7]. These integers are good candidates for coding biological information. Induction mechanism is the most natural mechanism for the penetration of magnetic fields if super conductor of type I is in question: the motion of # throats along the boundaries of the join along boundaries network of condensate blocks blocks destroys super conductivity in the interior of the region defined by network. Networks are dynamical since join along boundaries contacts can be split. Recall that join along boundaries networks are macroscopic quantum systems in accordance with basic ideas of TGD. In the simplest scenario the basic superconducting level would be $k = k_Z$ level and all condensate levels $k = 167, 163, 157, 151, 149, 139, 137$ could in principle be superconducting.

3 p-Adic ultrametricy and biosystems

3.1 Spin glasses and ultrametricity

Spin glasses [20, 29, 31] are spin systems with the property that the couplings J_{kl} between neighboring spins σ_k and σ_l are random variables although the characteristic scale of time variation of J_{kl} is very long as compared to the corresponding time scale associated with the dynamics of the spins. The characteristic property of spin glasses is their infinite ground state degeneracy. More precisely, the dynamics of the spin glasses is nonergodic and there is infinite number of pure states, which correspond to the local minima of free energy. For the purposes of comparison it should be recalled that for ferromagnet above critical temperature only one pure state exists and below the critical temperature there are two pure states corresponding to two possible directions of magnetization.

The space of pure states possesses a very general property called ultrametricity, which means that one can define in this space distance function $d(x, y)$ with the property

$$d(x, z) \quad \leq \quad max\{d(x, y), d(y, z)\} \ . \tag{1}$$

The properties of the distance function make it possible to decompose the space into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \quad \leq \quad D \ . \tag{2}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

These properties of ultrametric spaces suggest several biological applications.

1. Parisi [31] has suggested that ultrametricity might be used in taxonomy. Individuals of various species correspond to points of the ultrametric space and ultrametric distance gives mathematical description for the classicification criterion: in practice ultrametric distance might correspond to some genetic measure for the difference between individuals.

2. The representation of biological information seems to take place using a hierarchy of categories. Lowest and most important categories are very rough (friend/ enemy?, black/white?, etc...). Higher levels correspond more refined classifications (what kind of enemy?, does enemy move or not?,....). This kind of representation has obvious value in the struggle for survival. The hypothesis that biosystems save information into variables, which define points of ultrametric space, leads automatically to a hierarchical structure of information storage. The simplest model assumes that states of brain or at least memories correspond to free energy minima of a spin glass like statistical system [29].

3. Statistical models of memory and learning process, which share with spin glasses the property that the minima of free energy form ultrametric space are proposed [29], the main idea being that memories correspond to the minima of free energy. Learning takes place in these models via a slow change (slow as compared to the time scale of the dynamics associated with the spin variables) of the field J_{kl} associated with bond connecting k:th and l:th cell.

3.2 p-Adic ultrametricity

p-Adic numbers [18] are a natural candidate for a basic tool in the description of higher dimensional critical systems since the distance function defined by p-adic norm is ultrametric. The verification of the ultrametricity is elementary task using the definition of the p-adic norm [18]

$$|x|_p = |\sum_{k\geq k_0} x_k p^k| = p^{-k_0} \quad . \tag{3}$$

Since p-adic norm possesses discrete set of values, the values of the parameter D in the classification criterion $|x-y|_p \leq D$ can be chosen to be belong to the set $\{D_k = p^{-k}\}$, k integer. p-Adic numbers belonging to same class have same k:th pinary digit p-adic cutoff

$$\begin{aligned} x &= x_0 + x_1 \quad , \\ x_0 &= \sum_{k_1<m<k} x_m p^m \quad , \\ x_1 &= \sum_{m\geq k} x_m p^m \quad , \end{aligned} \tag{4}$$

so that the set of classes corresponds to p-adic numbers with cutoff in k:th pinary digit. In this picture the p-adic power expansion of any p-adic observable defines a tree. The levels of the tree correspond to various pinary digits of p-adic number and each branching point gives rise to p branches. In p-adic case one can regard the root of the either as the highest cutoff pinary digit or lowest non-vanishing pinary digit of the p-adic number. In the first case, the tree is infinite: in the latter case the tree is always finite.

For x, y with same p-adic norm (same class) and z with different p-adic norm as x, y (different class) the distance function satisfies the condition

$$\begin{aligned} d_p(x,y) &\leq d_p(x,z) \quad ,p>2 \ , \\ d_2(x,y) &< d_2(x,z) \ , \end{aligned} \tag{5}$$

so that there is important difference between $p = 2$ and $p > 2$ cases. Ultrametricity (or non-Archimedean property as it is called in p-adic context) holds also true for the algebraic extensions of p-adic numbers with distance defined by the canonical norm [18].

It has become clear that p-adicity emerges in TGD at the level of space-time topology and that one can identify p-adic space-time regions as cognitive representations of matter regions. Thus p-adic dynamics is predicted to be the dynamics of cognition and thus p-adic ultrametricity, the exotic features of p-adic probability concept, and non-determinism of p-adic differential equations are predicted to characterize the physics of cognition.

There is however also a second manner how p-adic ultrametricity might emerge in the description of biological systems. TGD Universe is quantum critical and critical systems [28] are characterized typically by a large degeneracy of metastable states and resemble in this respect spin glasses. The vacuum degeneracy of the Kähler action defining the Kähler function in the configuration space of 3-surfaces is highly analogous to the ground state degeneracy of the spin glasses in [B1, D1]. One cannot therefore exclude the possibility that p-adicity, in particular, small-p p-adicity for which there is also evidence, could emerge at the level of energy landscape of spin glass. According to the arguments of quantum TGD the reduced configuration space CH_{red}, consisting of the maxima of Kähler function as a function of zero modes characterizing the shape, size and induced Kähler fields on 3-surface, can be regarded as a spin glass energy landscape. Hence one can define ultrametric distance function, and it is possible that this distance function could be regarded as being induced from p-adic norm.

3.3 p-Adic ultrametricity and information processing in biosystems

This picture suggest a general model for the information processing in biosystems. Observations made by biosystem correlate with the variables characterizing the possibly conscious knowledge of the system about itself. The p-adic expansions of these variables give intrinsically hierarchical coding of the information associated with the observation. The lowest pinary digit is the most significant pinary digit and gives the roughest description for the observation. Higher pinary digits add details to the observation. The number of pinary digits in the p-adic representation of the observation measures the amount of information associated with it. The time order in which the information is stored or retrieved is from lowest to highest pinary digit. The Slaving Hierarchy associated with the topological condensation might have counterpart at the level of biosystems: this would mean the existence of a hierarchy $p_1 < p_2 < ..p_n < ..$ of p-adic dynamics each with its own characteristic time scale satisfying $T_1 < T_2 < .... < T_n < ...$ and the relationship between two consecutive dynamics is that of master and slave. The higher the level p_n in the p-adic Slaving Hierarchy the higher is the intelligence associated with that level as measured in the number of possible conceptual categories.

A highly nontrivial prediction is that the number of conceptual categories is same at all levels and equal to prime p. This means that $p = 2$ case provides most primitive (but much used!) classification of type black/white. A well known mystery of cognitive science is the so called 7 ± 2 rule [30]: human mind tends to classify observations into $p = 7$ categories and the classification using more categories than this is difficult. One possibility to test applicability of the p-adic ideas to biosystems is to check whether biossystems obey small-p p-adic rather than ordinary statistics. The nondeterminism and fractality of biosystems might be in better accordance with p-adic rather than ordinary statistics.

The analogy with spin glass models of learning suggests a microscopic TGD inspired physical model for learning. Short term learning is believed to correspond to slow changes in synaptic connections between neighboring cells. Long term learning probably involves the formation of new contacts between neighoring cells and according to suggestion of [D7] topological storage of information. In TGD inspired model for brain it was suggested that cells correspond to 'topological field quanta', 3-surfaces possessing outer boundary and having size of cell One mechanism for the formation of macroscopic quantum systems is as a formation of bonds connecting boundaries of neighbouring 'topological field quanta' (now 3-surfaces associated with cells). A possible identification for the counterpart of the spin glass coupling parameter J_{kl} is as Kähler electric interaction energy between neighbouring topological field quanta associated with this kind of bond. Therefore it would be the Kähler electric fluxes through the bonds, which change primarily in the short term learning. This change can be partially nondeterministic process since p-adic dynamics allows partial non-determinism and this nondeterminism is related to freedom to choose the low pinary digits of the dynamical variables arbitrarily.

4 p-Adic non-determinism and biosystems

The non-determinism of quantum jump, the classical non-determinism associated with the maximization of Kähler action, and p-adic non-determinism form a trinity of independent non-determinisms. Classical non-determinism of Kähler action can be assigned with volition whereas p-adic non-determinism is naturally the geometric correlate of imagination.

4.1 Could p-adic differential equations simulate quantum jump sequence?

In practical applications one must idealize the biosystem with a system of differential or partial differential equations. Since cognition is basic aspect of living systems one might expect that the general properties of p-adic differential equations might be useful for modelling not only cognition but also the behavior of living matter.

1. The non-determinism associated with p-adic differential and partial differential equations is due to the presence of arbitrary functions depending on finite number of pinary digits of p-adic coordinates, which are in the role of the integration constants. p-Adic integration constants are actual constants below some p-adic time scale. Solution of field equations typically consists of regions which are deterministic in the ordinary sense of the world glued to each other. Various conserved quantities are pseudo constants. This means that p-adic reality is somewhat like the reality of dreams consisting of fragments which could be realized also in everyday reality.

2. p-Adic space-time sheets could provide a simulation for the time development occurring via quantum jumps. p-Adic space-time surface would consist of fragments for which p-adic integration constants are ordinary constants. These pieces would represent the conscious information obtained about various real space-time surfaces in the sequence of quantum jumps (the space-time surfaces appearing in the quantum superpositions defined by the final states of quantum jumps are macroscopically equivalent). The lack of well-orderedness of the p-adic topology could reflect the fact that the arrow of time associated with t is only statistical.

3. p-Adic realization of the Slaving Hierarchy [25] roughly means that there is a hierarchy $.. < p_1 < p_2 < ...$ of p-adic dynamics and that the integration constants at level p_1 (slave) obey some dynamic equations at some higher p-adic level $p_2 > p_1$ (master) and are actual constants below length scale $L_{p_2} > L_{p_1}$ for each pair in the sequence. This hierarchy of dynamics need not be completely deterministic. If Kähler action allows nonunique classical histories, the p-adic integration constants can be chosen to some degree freely at each level of the Slaving Hierarchy. The free choice of p-adic integration constants has interpretation as a plan of an intelligent system for its future behavior. At p- adic length and time scales (macroscopic!) it is possible to "break physical laws": Universe learns engineering skills and begins to plan its own future!

4. The real counterparts for the solutions of p-adic differential equations have characteristic large jumps followed by small scale zig-zag type behavior. This zig-zag behavior is observed also for analytic solutions containing only ordinary integration constants, say for $x = At^2, y = Bt$ at values $t = 2^n$. Since p-adic integration constants are actual constants only for time scales smaller than $\Delta t = 2^{-n}$, the nondeterminism appears also as sudden jumps concentrated at multiples of Δt: Δt defines clearly a natural unit of time and therefore biological clock. In [30] the generality of this zig-zag motion in all length scales was emphasized as one of the characteristics of biosystems and the sudden jumps were identified as jumps from strange attractor to another and small scale motion as motion along attractor. A good example of this kind of motion is the motion of eye [30]. The fractal property of solutions of p-adic differential equations implies an infinite number of time scales corresponding to $\Delta_m = p^{-m}$, $m \leq n$. This in turn implies characteristic $1/f$ spectrum for the Fourier transform of orbit, which is quite general feature of biosystems [30].

5. An important property of p-adic differential equations suggested by the iteration of simpls p-adic maps (say $Z \rightarrow Z^2$) in algebraic extensions of R_p is that critical orbits form a set ($|Z| = 1$), which possesses same dimension as the configuration space so that critical metastable orbits are therefore not rare occurrences like in ordinary dynamics based on real topology. The small scale zig-zag motion between large jumps in the motion desribed by p-adic differential equations could correspond to motion near metastable orbit and be analogous to the motion along strange attractor in the strange attractor model of information processing proposed in [30].

4.2 Information filtering and p-adics

Intelligent systems are extremely effective information filters. Only an extremely small amount of information is absorbed from the incoming information. As far as visual observations are considered it is the angles and boundaries, which receive most attention [30]. An interesting possibility is that intelligent system concentrates its attention to p-adic quaternion conformal invariants such as angles. This would apply quite generally: any observation correspond to an orbit in some internal configuration space simulating the observed system. The correspondence between the observation and simulation is determined only modulo p-adic quaternion conformal transformations of the configuration space.

One can even consider a simple model for the coupling between internal configuration space and outer world using 'Newton's equations' assuming that acceleration corresponds to the sensory experience:

$$\frac{d^2x^k}{dt^2} = F^k(t) - k\frac{dx^k}{dt} \quad . \tag{6}$$

$F^k(t)$ describes observation as an external force acting on system. In the absence of $F^k(t)$ motion is linear (no angles!) and only when $F^k(t)$ is non-vanishing the direction of motion changes direction (angle). The friction term guarantees that constant stimulus leads to no sensory experience situation (adaptation). Idealized case corresponds to delta pulses causing zig-zag type motion. Note that p-adic indeterminacy brings in certain degree of 'subjectivity' and could provide a phenomenological model for the quantum nondeterminacy.

This kind of model could serve as a model of language analogous to that considered in [30]. Individual phonemes correspond to linear part on the orbit in some configuration space and the duration of phonem doesn't matter. The change of phoneme to another one corresponds to angle on the orbit (external force) and different angles correspond to different phonem pairs. The hierarchy of structures (phonemes, hyphens, worlds, sentences,..) might correspond to p-adic slaving hierarchy (say, $p = 2, 3, 7, 127..$) associated with the differential equations governing the orbit in internal configuration space.

5 p-Adic probabilities and biosystems

p-Adic probabilities can be defined in a manner analogous to that used to define ordinary probabilities [23]. One can consider sufficiently large number of observations N chosen by some criterion since conditional probabilities are considered in practice and observe possible mutually exclusive outcomes N_i labeled by integer i. The relative frequencies N_i/N are estimates for p-adic probabilities. Probability conservation corresponds to the condition $\sum_i N_i = N$. The feature, which differentiates between ordinary and p-adic probabilities is related to the large N limit, which must exist in p-adic rather than ordinary sense. This means that the values of N, which differ by large powers of p are p-adically near to each other. For example, N and $N+1$ are in general not near each other p-adically! For large values of p say $p = M_{127} \simeq 10^{38}$ the value of N rarely exceeds the critical value p and there is no practical difference between p-adic and ordinary probabilities. For small values of p the situation changes.

5.1 Does p-adic probability apply only to cognition?

p-Adic probality concept is expected to apply in quantum statistical models of cognition. If p-adic space-time sheets indeed model sequences of quantum jumps by replacing consciously observed pieces of real space-time appearing in the sequence of quantum jumps by finite space-time regions glued to each other in p-adically continuous manner, then p-adic statistics might apply as a model of self-organization resulting from a dissipative time development by quantum jumps.

p-Adic probalities might be natural in the statistical description of fractal structures resulting in the self-organization, and which by definition can contain same structural detail with all possible sizes.

1. Consider counting of conformally invariant structural details of a p-adic fractal. A simple biologically interesting example is the solution curve of p-adic differential equations in some configuration space associated with biosystem (say the space of average chemical concentrations). The angles associated with the kinks of the curve measured with some finite precision are the structural details in question.

2. One can count how many times i:th structural detail appears in a finite region of the fractal structure: although this number is infinite as real number it might possess (and probably does so!) finite norm as p-adic number and provides a useful p-adic invariant of the fractal. One can calculate also the total number of structural details defined as $N = \sum_i N_i$ and also define p-adic probability for the appearance of i:th structural detail as relative frequency

$p_i = N_i/N$. The real, 'renormalized' counterparts of N_i and P_i obtained via the canonical correspondence define real valued invariants of the fractal structure.

3. The evaluation of the p-adic probalities of occurrence can be done by evaluating the required numbers N_i and N in a given resolution. Better estimate is obtained by increasing resolution and counting the numbers of the hitherto unobserved structural details. The increase in the resolution greatly increases the number of observations in case of p-adic fractal and the fluctuations in the values of N_i and N increase with resolution so that N_i/N has no well defined limit as real number although one can define the probabilities of occurrence as resolution dependence concept. In p-adic sense the increase in the values of N_i and fluctuations is small and the procedure should converge rapidly so that reliable estimates should result with quite a reasonable resolution.

5.2 Is small-p p-adic statistics possible?

There is a distinct possibility that p-adic statistics with small p might be a unique testable signature of intelligent systems! The replication property of biosystems suggests that the lowest level in topological condensate of biosystem has $p = 2$. The quantization of the number of observations in biological experiment could be understood in the following manner. A natural choice for N in biosystem corresponds to all individuals that have existed or exist in the biosystem during some time interval. For an ideally replicating biosystem this number develops during time in the following manner. $N = 1$ for zeroth generation, $N = 1 + 2 = 3$ for the second generation,$N = 1 + 2 + ...2^k = 2^{k+1} - 1$ for $k + 1$:th generation. The expression for the relative frequency is

$$P = \frac{\sum_k N_k}{\sum_k 2^k} . \tag{7}$$

The dominating contribution to p-adic probability comes from the lowest generations. For p-adic probability to make sense the behavior of the system must be sufficiently deterministic during the earliest stages of the development. Non-determinism becomes possible for large of N. The development of the embryo during the first cell divisions is indeed highly deterministic process.

An interesting feature of the ideally replicating biosystem is that $N = 2^{k+1} - 1$ is Mersenne prime for certain values of the generation number $k + 1$. If the topological condensate associated with biosystem contains also higher levels p then these values of N might mean the emergence of something new since the value of N exceeds the critical value $p = M_{k+1}$, when the number generations becomes $k + 1$ and p-adic probability concept begins to apply at p:th level. This suggests that the values of the total cell number $N_{cell} = 2^{k-1}$ associated with the Mersenne primes M_k are critical cell numbers. Some of the lowest critical generation numbers are $k = 2$: $N_{cell} = 2$, $k = 3$: $N_{cell} = 4$,$k = 7$: $N_{cell} = 64$,...

5.3 The concept of monitoring

In p-adic quantum theory expected to provide a model for cognition one must somehow associate real probabilities to p-adic probabilities. This problem has been already discussed and leads to the conclusion that the transition probabilities of p-adic quantum system depend on how it is monitored. p-Adic sum of transition probabilities corresponds to the experimental situation, when one does not monitor individual transitions but using some common experimental signature only looks whether the transition leads to this set of finals states or not. When one looks each transition separately or effectively performs different experiment by considering only one transition channel in each experiment one must use the sum of real probabilities. More precisely, the choice of experimental signatures divides the set U of the final states to disjoint union $U = \cup_i U_i$ and one must define the real counterparts for transition probabilities P_{iU_k} as

$$\begin{aligned} P_{iU_k} &= \sum_{j \in U_k} P_{ij} \ , \\ P_{iU_k} &\rightarrow (P_{iU_k})_R \ , \\ (P_{iU_k})_R &\rightarrow \frac{(P_{iU_k})_R}{\sum_l (P_{iU_l})_R} \equiv P^R_{iU_k} \ . \end{aligned} \tag{8}$$

Similar resolution can be defined also for initial states by decomposing them into a union disjoint subsets. The assumption means deep difference with respect to the ordinary probability theory.

p-Adic probability conservation implies that the lowest order terms for p-adic probabilities satisfy the condition $\sum_j P^0_{ij} = 1 + O(p)$. The general solution to the condition is $P^0_{ij} = n_{ij}$. If the number of the final states is much smaller than p this alternative implies that real transition rates are enormous: typically of order p! Therefore it seems that one must assume

$$P^0_{ij} = \delta(i,j) \ . \tag{9}$$

As a consequence the probability for anything to happen (no monitoring of different events) is given by

$$\sum_j (P_{ij} - \delta(i,j)) = 0 \ , \tag{10}$$

and vanishes identically! This is not so peculiar as it looks first since there must be some signature for anything to happen in order that it can be measured and signature always distinguishes between two different events at least: it is difficult to imagine what the statement 'anything did not happen' might mean! Of course, in real context this philosophy would imply the triviality of S-matrix.

If biosystems are indeed quantum systems and p-adic probabilities apply to their description then the unavoidable prediction is that the behavior of biosystems depends on how it is monitored (remembering all anecdotes about experimentation with living matter, one might somewhat light-heartedly argue that this is just the case!). For small values of p, in particular for $p = 2$, the deviations from the standard probability theory are especially large. In particular, the resolution of the monitoring is essential factor. It must be stressed that this peculiar behavior seems not to be related with the predictions of standard quantum measurement theory and this supports the view that p-adic probabilities apply only to the statistical modelling of cognition.

6 Is small-p p-adicity possible?

A longstanding problem of TGD inspired theory of consciousness and p-adic TGD in general has been whether small-p p-adicity is present in macroscopic length scales. The basic form of p-adic length scale hypothesis suggests that small-p p-adicity should be present only in length scales near CP_2 size about 10^4 Planck lengths, which defines the fundamental p-adic length scale. On the other hand, p-adic fractality suggests that also the scaled up versions of entire p-adic length scale hierarchy might be possible in the sense that CP_2 size is effectively replaced with p-adic length scale L_p for any prime p and most probably for primes $p \simeq 2^k$, k power of prime. In particular, the realization of genetic code, which corresponds to p-adic prime $p = 127$, at the level of DNA molecules suggests strongly, that small-p p-adicity is realized in Nature and involves transmutation of the fundamental p-adic length scale to atomic length scale. This expectation conforms also with the idea that Universe is infinite-sized self-organizing quantum computer emulating itself in all possible scales and building scaled up simulations of the lower levels. Even science could be regarded as one such emulation.

6.1 Small-p p-adicity and hydrodynamics

Hydrodynamic turbulence in the atmosphere involves generation of coherent macroscopic structures which are typically structures appearing in excitable media. One example are spiral waves which represent spiral like convective roll pattern such that the radius of the rolling vortex increases exponentially when one moves away from the apex of the spiral wave. Tornadoes and hurricanes are also well known self-sustaining structures. The generation of these structures is difficult to understand in ordinary hydrodynamics and Indian meteorologists Mary Selvam [43] takes as her challenge to understand the microscopic mechanism leading to the generation of these structures. TGD suggests quite generally the reduction of the hydrodynamical turbulence and chaos in excitable media to magnetic or Z^0 magnetic turbulence. The work of Selvam related to the turbulent atmospheric flows inspires also additional very interesting insight to p-adic length scale hypothesis and suggests that n-ary p-adic length scales $L(n,k)$ corresponding to very large values of n are realized in hydrodynamical turbulence, and that hydrodynamical vortices could be regarded as elementary particle like objects on the space-time sheets at which they are condensed topologically.

6.1.1 Spiral waves and magnetic turbulence

Self-sustaining spiral waves are known to be characteristic for all excitable media [39] and typical results of self-organization. The growth of plants leads quite generally to the generation of logarithmic spirals; spiral Ca_{++} waves are known to be crucial for intracellular communications [41]; spiral waves appear also in heart failure [38, 40].

1. Logarithmic spiral and Penrose tilings

Spiral waves (say roll-vortices with vortex core along spiral) are waves for which the the center of the wave defined by logarithmic spiral

$$\frac{R}{r} = exp(b\theta) \ .$$

The values of R/r are Fibonacci numbers $F(n+1) = F(n) + F(n-1)$ for certain values of the angular variable θ. At the limit of large Fibonacci numbers one has $F_n \simeq \tau^n$ and substituting to the equation one obtains $\theta \simeq n\theta_0$, $\theta_0 = log(\tau)/b$, $\tau = \frac{1+\sqrt{5}}{2}$.

Logarithmic spirals form a one-parameter family and especially interesting is the logarithmic spiral for which the line connecting the points $r = F_n$ and $r = F(n+1)$ has length F_n. In this case

$$\theta_0 = \frac{2\pi}{10} = 36 \ \text{degrees} \ .$$

This particular logarithmic spiral leads to a generation of Penrose tiling [33]: this occurs in both 2- and 3-dimensional case. This particular logarithmic spiral is very general in botany. Rather interestingly, the angle of 36 degrees happens to be the angle between two subsequent DNA nucleotides in DNA helix, which encourages to consider the possibility that the helical structure of DNA rather concretely codes is in some sense fractal growth defined by the logarithmic spiral with this value of b. Note that this kind of growth preserves shape and this is probably one reason for why logarithmic spirals appear so often in botany. In fact, the notion of manysheeted DNA

[L2] suggests that genes in DNA helix in some sense represent contracted versions of the organism preserving 1-dimensional homology: perhaps the contraction preserves also spiral structure. A further interesting point to notice is that the shortest sequence of DNA:s for which the net winding angle along helix is multiple of 2π and which codes for an entire protein consisting of 30 DNA nucleotides, has thickness of cell membrane as already found.

2. Reduction of chaos to magnetic turbulence?

TGD suggests that quite generally spiral waves are accompanied by the underlying magnetic and Z^0 magnetic flux tube structures. Spiral wave would correspond to Z^0 flux tube around which ordinary matter rotates so that rolling vortex results. At the apex magnetic flux tube apparently ends. The conservation of (Z^0) magnetic flux requires that flux tube leaves the space-time sheet at the apex and continues at the second space-time sheet. This suggests the fascinating possibility that macroscopic structures in hydrodynamic turbulence could correspond to wormhole magnetic fields [J5] associated with pairs of space-time sheets and be generated by rotating wormholes at the boundaries of the structure. If time orientation is negative at second space-time sheet, this space-time sheet carries negative energy density which can be very small if only the energy of Z^0 magnetic field is in question. If wormhole magnetic fields (besides MEs) represent mindlike space-time sheets of finite time duration, one could perhaps (rather loosely) speak about interaction of matter and mind. The same mechanism might be at work also at cell level.

3. Magnetic turbulence and loss of macroscopic quantum coherence

For superconductors quantization conditions imply that the increment of the phase of the complex order parameter of the supra phase around the circuit along boundary of the flux tube equals to the magnetic flux through the tube. Thus magnetic turbulence implies turbulence of superconductor and probably destruction of the supraphase. If ionic superconductors are responsible for biocontrol, then magnetic turbulence would be reflected as chaotic functioning of organ. This loss of quantum coherence would be caused by the leakage oft he supra currents from flux tubes via join along boundaries bonds. This in turn would imply dissipation at the non-superconducting space-time sheets by particle collisions. This leakage would be forced by the inertia when the local curvature of the flux tube becomes too large: this is indeed expected to occur in a chaotic situation when flux tubes have very Brownian shapes.

Heart failure, known to involve the generation of decaying spiral waves modellable using Hodkin-Huxley equations or their variants [38, 40], might be one example of this mechanism. The reduction of this model to quantum level is required by internal consistency if one takes seriously TGD based model of nerve pulse activity in terms of ionic and electronic superconductors relying crucially on Josephson junctions associated with axons [M2]. In case of heart, normal situation would in ideal case correspond to spatially constant phase wave of Josephson current oscillating in time with basic frequency (there is precise analogy with a rotating mathematical pendulum) so that the Josephson currents associated with all heart cells oscillate in unisono, perhaps at the the rhytm of heart beat. During heart failure magnetic turbulence destroys this coherence. Interestingly, the time period of fibrillation is .1 seconds, the time scale of the memetic code [38].

4. Atmosphere as cortex of Mother Gaia?

In TGD framework self-organization means the presence of conscious selves and suggests that even atmosphere is in some sense part of Mother Gaia. Perhaps it is of some significance that the ratio of the thickness of atmosphere (10 km) to the radius of Earth radius is of order 1/100 and is same as the ratio of cell membrane thickness to cell size. Fractality indeed suggests this ratio if atmosphere is regarded as scaled-up version of the cell membrane. Note however that the thickness of flora is about 10 m: in case of cell membrane this would suggest a layer of thickness of order 10^{-11} meters, which happens to correspond to the p-adic length scale $L_{M_{127}}$ associated with electron. The p-adic prime associated with the memetic code pops up again and one could wonder whether the MEs with length of $L_2(127)$ could have thickness equal to $L(127)$ and form structure analogous to biosphere at surface of Earth. The fact, that the frequency distribution of so called sferics, em perturbations induced by lightnings resemles at low frequencies delta band in EEG [42], suggests that these exotic levels of life might be there and interact with animal brains.

6.1.2 Selvam's model and claims

Selvam studies a model for hydrodynamical spiral waves by assuming that these waves are vortices with core at logarithmic spiral

$$z \equiv \frac{R}{r} = b \times exp(b\theta) \ \ .$$

Selvam assumes also that the radius ρ of rolling convective vortex grows with z and that also this growth obeys similar law: that is $\rho = exp(b\theta)$. Selvam assumes that the parameter θ_0 corresponds to the angle of 36 degree associated with equilateral Fibonacci triangle having short sides F_n and long side $F(n+1)$ at the limit $n \rightarrow \infty$. As noticed, this logarithmic spiral gives rise to Penrose tiling.

Selvam does not specify precisely this growth law: for instance, whether there is phase lag between R characterizing position of growing vortex and r characterizing its size. Selvam does not either clearly specify how R develops with time: for instance, whether growth occurs linearly in which case θ would grow logarithmically. One possible manner to obtain the proposed growth is to assume that the growth is analogous to biological growth such that turbulent eddies are in the role of cells and replicate. If the growing vortex decomposes of radius $\rho(n)$ to an inner cylinder of thickness $\rho(n-2)$ and outer annulus of thickness $r(n-1)$ such that outer annulus replicates to annulus of same thickness at $n+1$:th step of growth process one indeed obtains $\rho(n+1) = \rho(n) + \rho(n-1)$ giving rise to Fibonacci sequence asymptotically.

Selvam claims that the dominating temporal periodicities T_n of flow are Fibonacci numbers in suitable units:

$$T_n = F(n) \simeq \tau^n \ \ , \ \ \ \tau = \tfrac{1+\sqrt{5}}{2} \ \ .$$

This claim can be understood if vortex structures with radius F_n form special structures and if there are standing waves moving with constant velocity v along these structures: this gives

$$T_n = \frac{F_n r}{v}$$

for the periodicities of these waves. Selvam argues that Fibonacci numbers reflect also the periodicities of prime number distribution but I find it difficult to understand the motivations for this claim.

Selvam also studies the distribution for the ratio $z = R/r$ of large vortex radius R to smallest vortex radius r, and, as far as I have understood correctly, claims that this distribution is the same as the distribution of primes in region of rather small primes. This could be understood if vortex radii are prime multiples of r

$$R = kr \ \ , \ \ k \ \ \text{prime} \ \ ,$$

and if each prime appears with the same probability. This assumption can be actually loosened: one can also interpret r as the p-adic length scale associated with minimum size vortex interpreted as space-time sheet. Even the assumption that vortices sizes are given by primes might be too strong: only one-one correspondence with the distribution of primes is needed. Selvam also argues that vortex dynamics has quantal features and that vortices could in some aspects be regarded as quantum objects: this is certainly what TGD approach strongly suggests.

It must be emphasized that the arguments of Selvam do not satisfy the requirement of mathematical rigour and it is only my personal feeling that something deep is involved and I just take Selvam's claims as inspiration for studying whether small-p p-adicity suggested strongly by fractality might be realized in hydrodynamical flows. Certainly, TGD predicts p-adic evolution and this evolution should reflect itself directly in biological growth and perhaps even in hydrodynamical self-organization. Also Matthew Watkins has proposed a connection between evolution and prime numbers [24].

p-Adic evolution andontogeny recapitulates phylogeny principle in its generalized form (one of the basic guiding principles of quantum TGD stating that classical dynamics should provide a Bohr orbit type representation for quantum dynamics) suggests that growth processes quite generally corresponds to p-adic evolution. First pop-up structures with $p = 2$, then structures with $p = 3$, and so on. In hydrodynamics case these structures correspond to stable vortices with prime-valued radius $R/r = p$. If the growth of spiral wave is linear in time then vortices with prime valued radio pop-up for the first time at time values which are prime multiples of basic time unit. If the emergence of these vortices reflects itself as somekind of distinguishable feature in the temporal behavior of dynamical quantities, as one might expect, the Fourier spectrum should reflect the properties of the spectrum of prime numbers. This is clearly a strong and testable prediction.

6.1.3 Why vortices with prime radii are stable?

The first question to be answered is why vortices with radii which are prime valued are stable. Suppose that there is fundamental length scale r identifiable as the radius of turbulent eddy. This radius would result from the quantization of Z^0 magnetic flux if one assumes that there is a preferred value for the strength of the Z^0 magnetic field. Flux quantization would imply that the radii of the vortices are quantized as $r \propto \sqrt{n}$, n integer. The problem is to understand why n is square of prime rather than arbitrary integer.

One could however correspond the possibility that prime valued radii correspond to secondary p-adic lengths scales with a scaled-up fundamental p-adic length scale defined by the Z^0 magnetic flux quantization (a possible mechanism leading to transmutation of the fundamental p-adic length scale will be discussed later). This implies that all vortices (cylindrical and annular) have radius which is integer multiple of this length scale: $z = n$. Vortices consists of turbulent eddies or tend to decay to vortices $z = 1 < m < n$. The wavelengths of the radial perturbations tending to induce the decay of the vortices to smaller ones, are integer multiples of r. One has effectively aperiodic lattice, Penrose tiling known to be associated with logarithmic spirals [33]. Also in the periodic lattice only integer multiples of the basic wave vector propagate. Turbulent eddy defines the equivalent of fundamental lattice cell.

As a consequence, only vortices with prime-valued radii are stable. For instance, $n = p_1 \times p_2$, p_1 and p_2 primes, the vortex can decompose to p_1 cylindrical or annular vortices with radius p_2 or vice versa by a perturbation with wavelength $\lambda = p_1 r$ $(p_2 r)$. The impossibility to generate radial periodic perturbations with wavelength which is nontrivial multiple of the fundamental length, explains why prime vortices are stable against decay. Note that in [H3] precisely the same argument was used to explain why some retarded persons are able to 'see' factorization of 8-digit numbers into prime factors (see the book 'The man who mistook his wife for hat' of Oliver Sacks [36]). Mental images representing number n, is represented by some structure, perhaps vortex(!), and if n is not prime it has tendency to decay to some number of identical smaller structures! Thus non-primeness is directly visible property: perhaps higher levels selves spend their time by monitoring the factorization of very large integers.

6.1.4 How the transmutation of fundamental p-adic length scale to macroscopic length scale could occur?

What might be the mechanism effectively leading to the transmutation of the fundamental p-adic length scale $l \simeq 10^4$ Planck lengths to a macroscopic length scale? The most plausible solution to this problem emerged, when I discovered a solution to a second equally long-standing problem of quantum TGD: How to associate to a given real space-time sheet a unique p-adic prime (or possibly several of them) as required by the p-adic length scale hypothesis?

1. Information concept in TGD

The notion of information in TGD framework differs in some respects from the standard notion and these differences lead to a solution of the above mentioned problems.

1. The definition of the entropy in p-adic context is based on the notion p-adic logarithm depending on the p-adic norm of the argument only ($Log_p(x) = Log_p(|x|_p) = n$) [H2]. For rational-valued probabilities this entropy can be regarded as a real number. The entropy defined in this manner can be negative so that the entanglement can carry genuine positive information. Rationally entangled p-adic system has a positive information content only if the number of the entangled state pairs is proportional to a positive power of the p-adic prime p.

2. This kind of definition of entropy works also in the real-rational case and makes always sense for finite ensembles. I have proposed in [E1] that the physics at Hilbert space level is rational number based.

3. Quantum-classical correspondence suggests that the notion of information is well defined also at the space-time level. In the presence of the classical non-determinism of Kähler action and p-adic non-determinism one can indeed define ensembles, and therefore also probability distributions and entropies. For a given space-time sheet the natural ensemble consists of the deterministic pieces of the space-time sheet regarded as different states of the same system. The information measure for this ensemble can be non-negative only for those primes, which divide the number N of the deterministic regions of the space-time sheet and these primes are preferred candidates for the p-adic primes associated with the space-time sheet in question.

2. How space-time sheet represents factorization of an integer?

Consider now the problem how to associate to a given real space-time sheet a unique p-adic prime (or possibly several of them) as required by the p-adic length scale hypothesis.

1. The simplest hypothesis is that a real space-time sheet consisting of N deterministic pieces corresponds to p-adic prime defining the largest factor of N.

2. One can also consider a more general possibility. If N contains p^n as a factor, then the real fractality above n-ary p-adic length scale $L_p(n) = p^{(n-1)/2}L_p$ corresponds to smoothness in the p-adic topology. This option is more attractive since it predicts that the fundamental p-adic length scale L_p for a given p can be effectively replaced by any integer multiple NL_p, such that N is not divisible by p. For instance, genetic code and the appearance of binary pairs like cell membrane consisting of liquid layers suggests 2-adicity in nano length scales. This view means that the fractal structure of a given real space-time sheet represents both an integer N and its decomposition to prime factors physically so that prime factorization, which is extremely slow process using classical computation, could be deduced by representing integer N as a space-time sheet and performing physical measurements. This obviously conforms with the physics as a generalized number theory vision [E1].

3. How small-p p-adicity in macroscopic length scales emerges?

The hypothesis that real space-time sheets represent integers and their prime factorizations in their fractal structure implies that, for a given space-time sheet allowing p-adic topology, the fundamental p-adic length scale is effectively replaced by any of the length scales $\sqrt{N/p^n}l_0$, where l_0 is the fundamental p-adic length scale and N/p^n is integer not divisible by p. This means that small-p p-adicity is achieved also in macroscopic length scales. This option allows practically any length scale to replace the fundamental p-adic length scale. One expects that also the elementary p-article black-hole analogy generalizes to this case in a straightforward manner.

6.1.5 Very speculative speculations

The following speculations represent an archeological layer of p-adic TGD preceding the realization of the mechanism for how the p-adic primes associated with a real space-time sheet are determined, and of how this mechanism allows to understand the emergence of the small-p p-adicity in macroscopic length scales. I have not had heart to throw this material away, and it might have some entertainment value besides demonstrating how strange and tortuous are the routes to a discovery.

1. p-Adic length scale hypothesis from elementary particle blackhole analogy

One can try to understand results on basis of the p-adic length scale hypothesis $p \simeq 2^{k^m}$, k prime, m positive integer.

1. At quantum level p-adic length scale hypothesis follows from the generalization of Hawking-Bekenstein law for the radius of elementary particle horizon defined as the surface at which the Euclidian signature of the induced metric of the space-time sheet containing topologically condensed particle changes to Minkowskian signature of the metric in regions faraway from particle. Ordinary elementary particles corresponds to CP_2 type extremals condensed on larger space-time sheet with size of order $L_p = \sqrt{p}l$, $l \simeq 10^4$ Planck lengths. Generalized Hawking-Bekenstein law implies that the p-adic entropy of elementary particle characterized by p-adic prime p is proportional to the surface area of the elementary particle horizon. Since entropy is proportional to $log(p)$, the radius r of the elementary particle horizon satisfies $r^2 \propto log(p)$.

2. The idea is to require that the radius of the elementary particle horizon itself is m-ary p-adic length scale. For $p \simeq 2^{k^m}$ this is indeed the case if generalized Hawking-Bekenstein law holds and one has

$$r = \sqrt{k^m} \times l \ \ , \ \ k \ \ \text{prime} \ \ .$$

   For $m = 2$ one has

$$r = kl \ \ .$$

   This is the same law as holds true for the vortex radii except that l corresponds to Planck length scale rather than macroscopic size of the minimal vortex. Therefore a generalization replacing l with the size of the minimal vortex is needed.

2. Does a generalization of Hawking-Bekenstein hold true also for vortices regarded as elementary particles?

One should be able to generalize the notion of elementary particle by allowing also larger space-time surfaces than CP_2 extremals as models of particle and to assume that the metric of the space-time sheet at which particle is condensed has Euclidian metric signature inside the particle region, now inside the region covered by vortex.

1. A more general situation allowed by the p-adic length scale hypothesis corresponds to vortices topologically condensed at space-time sheets with size of order of n-ary p-adic length scale

$$L_p(n) = p^{n/2}L_p \ , \qquad p \simeq 2^{k^m} \ .$$

   In this case generalized Hawking-Bekenstein law implies that the radius of the elementary particle horizon is given by

$$r = k^m \times L \ , \quad L = \frac{n}{2} \times l \ .$$

   $m = 2$ applies in the situation studied by Mary Selvam. Also the values of k can be small in this case. What is important is that the fundamental p-adic length scale l has been effectively replaced by $L = nl/2$. This is in accordance with the idea of fractality.

2. The requirement that r is also now p-adic length scale would imply that the length scale $k^m \times \frac{n}{2} \times l$ is p-adic length scale. This does not make sense except possibly as an approximation. p-Adic length scale hypothesis however suggests that the new fundamental length scale L itself is some n-ary p-adic length scale (this is not required by the general hypothesis that space-time sheet can represent any integer). The simplest possibility is that $n/2$ is large prime p_1 so that one has

$$n = 2p_1 \ , \quad r = p_1 l \ .$$

   $L = p_1 l$ and clearly corresponds to the secondary p-adic length scale associated with p_1 satisfying itself p-adic length scale hypothesis $p_1 \simeq 2^{k_1^{m_1}}$. This assumption provides the scenario with strong predictive power since the number of the secondary p-adic length scales is not very high.

3. Does atmospheric turbulence provide a fractally scaled version of elementary particle physics?

In the length scale range between .1 meters and Earth circumference the following p-adic primes $p_1 = n/2$ are possible:

$$p_1 \simeq 2^{k_1^{m_1}} \ , \quad k_1^{m_1} = 101, 103, 107, 109, 113, 11^2 = 121, 5^3 = 125, 127 \ .$$

There would be only 8 minimal vortex sizes in this length scale range, which is very strong and testable prediction. What is fascinating is that these secondary length scales correspond to the p-adic primes associated with quarks, atomic nuclei, and leptons so that the physics of vortices in atmosphere might in some sense be regarded as a fractal copy of elementary particle and nuclear physics! Note that the length scale $L(n, k)$ giving the size of the space-time sheet at which vortex is condensed, is given by

$$L(n, k^2) \simeq 2^{2^{k_1-1} \times k^2} \ ,$$

and is completely super-astronomical already for small values of k. Note that similar preferred minimal vortex sizes are predicted also in length scales below $L(199) = 16$ cm so that the prediction could be tested in laboratory. These vortex sizes might be important even for self-organization of living matter.

4. Does the space-time region at which vortex is condensed have Euclidian metric signature?

What this model implies is that the induced metric at the space-time sheet at which vortex is condensed, should have Euclidian signature inside radius r. TGD indeed allows huge number of vacuum extremals with Euclidian signature: signature becomes Euclidian if the dependence of the CP_2 coordinates on M^4_+ coordinates is too fast. The simplest situation is encountered when the angle coordinate ϕ associated with CP_2 geodesic circle satisfies the condition $\phi = \omega t$, $\omega \geq 1/R$, where $2\pi R$ is the length of the CP_2 geodesic circle and t is Minkowski time coordinate. From this it is clear that time gradients must be typically larger than $1/R$, where R is CP_2 size, for Euclidization to happen. Also absolute minimization of the Kähler action is consistent with the formation of Euclidian regions. Thus field equations support the idea that space-time sheets can contain Euclidian regions of even macroscopic size. Inside the region covered by the vortex light would not propagate at all and Euclidian regions would be in some respects analogous to black holes. Vortex space-time sheets itself would obey good old Minkowskian physics.

6.2 2-adic psychophysics?

Music metaphor has turned out to be of crucial importance for the theory of qualia. The most natural explanation for this is that music metaphor reflects underlying 2-adicity of our sensory experience. Perhaps at least some aspects of our experience result from a mimickry of the lowest level of the p-adic self-hierarchy. Taking 2-adicity seriously, one is forced to ask for the possible consequences of 2-adicity. For instance, could it be that at the level of primary qualia the intensity of sensation as function of stimulus depends on the 2-adic norm of the 2-adic counterpart of the stimulus and is thus a piecewise constant function if sensory input?

An observation supporting this speculation is following. When over-learning occurs in tasks involving temporal discrimination, the intensity of sensation as a function of stimulus deviates from smooth logarithmic form in small scales by becoming piecewise continuous function [22] such that the plateaus where response remains constant are octaves of each other. This observation suggests a generalization inspired by 2-adic version of music metaphor. Primary quale has multiple of cyclotron frequency as its correlate and, being integer valued, is essentially 2-based logarithm of the 2-adic norm for the 2-adic counterpart of the intensity of the sensory input. Hence the increase of intensity of the sensory input by octave correspond to a jump-wise replacement of the n:th harmonic by n+1:th one and should be seen in EEG. Our experience usually corresponds to the average over a large number of this kind of primary experiences so that underlying 2-adicity is smoothed out. In case of over-learning or neurons involved act unisono and the underlying 2-adicity is not masked anymore. At the level of ELF selves this would mean generation of higher harmonic when the number of nerve pulses per unit of time achieves threshold value allowing the amplification of corresponding frequency by the mechanism discussed already earlier.

6.3 Small-p p-adicity in biosystems and psychophysics

There are several hints for small-p p-adicity in macroscopic length and time scales from biology and psychophysics besides this decisive result of Selvam.

1. 2-Adicity of music experience suggests that 2-adicity present in macro-temporal scales [H3]. Also the general form of the p-adic length scale hypothesis and the concrete appearance of 2-adic fractals [E4] suggests that 2-adicity is realized also in macroscopic length scales. The topological model for thoughts as association sequences suggests strongly small-p p-adicity and this idea was in fact one of the first ones relating p-adic numbers with consciousness. The 2-adicity of music experience is relatively easy to understand if any p-adic time scale can serve as effective fundamental time scale for 2-adicity of music experience. Note however

that by p-adic length scale hypothesis the fundamental time scales come as powers of 2. The apparently complete freedom to choose the fundamental time scale can be understood if practically any p-adic time scale L_p replacing l can serve as effective fundamental time scale.

2. Genetic code corresponds to $p = 127 = 2^7 - 1$ in TGD inspired model of abstraction process predicting infinite hierarchy of 'genetic codes' [L1]. It should be however realized in macrotemporal scales rather than near CP_2 time scale and if the proposed mechanism scales l to p-adic length scale of order atomic length scale this is indeed realized.

3. Memetic code corresponds to $p = 2^{127} - 1$ and to a unique p-adic time scale of .1 seconds [L1]. Codeword has 126 bits and single bit corresponds to the time scale of nerve pulse. What is disturbing that this would make time scale of human brain unique. Situation changes if any p-adic time scale can take the role of fundamental p-adic time scale so that .1 seconds would become lower limit for time duration of memetic code word. Hence brain would represents the first step in the evolution creating memetic codes in longer time scales. In light of p-adic fractality the idea that the time scale associated with M_{127} is the only possible duration of memetic codon, does not sound plausible. One can indeed imagine a hierarchy of scaled-up versions of M_{127} code. This would suggest that M_{127} could be also realized at time scales $k \times T_2(127)$, k prime, $T_2(127) = .1$ s. $T_2(127)$ would be the smallest p-adic time scale, where memetic code is possible and the distribution of longer time scales would obey distribution of primes. This distribution should reflect itself in the EEG spectrum at very low frequencies.

7 $L_0\ mod\ p^m = 0$ excitations of Super Virasoro algebra as higher forms of life?

Topological field quanta can have all possible sizes. Uncertainty Principle suggests that the size of the topological field quantum corresponds to the p-adic length scale of the corresponding 3-surface. This would mean that the vibrational excitations of even macroscopic 3-surfaces could correspond to Super Virasoro representations. Indeed, the states of real supercanonical representations associated with the lightlike boundaries of MEs have gigantic almost-degeneracies and provide excellent candidates for representing biological information [J4]. These representations realize the idea of quantum hologram in the sense of quantum gravity and quantum information theory concretely and emerge naturally also in the TGD based theory of qualia ([K3].

Besides this there are also what might be called exotic p-adic representations of quaternion conformal Super Virasoro algebra for which the real counterparts of the p-adic masses are extremely small although the masses of the corresponding real states are super-astronomical. These states have enormous quaternion-conformal (rather than only supercanonical degeneracies) degeneracies and this raises the question about the possible biological relevance of these states. Thus it seems (at least now when I am writing this!) that the exotic states are not relevant for the understanding of biosystems. Despite this, and also because I ended up with supercanonical representations via exotic p-adic representations, I do not have heart to throw away the discussion of the properties and possible biological significance of these representations. The reader can however safely skip this section if she wishes.

7.1 Exotic p-adic quaternion conformal representations

The eigenvalues of Super Virasoro generator L_0 are non-negative integers n. In p-adic context one can naturally decompose these eigenvalues into classes such that in class m eigen-values are of form $n = kp^m$, $k = 1, 2, ..., k\ mod\ p \neq 0$. In class m the real counterpart of the mass squared is of order $1/p^m$ and hence extremely small for large values of m. Does this predict the existence of light excitations for all particles, even fermions?

1. The answer 'No' suggested by the fact that p-adic representations of quaternion conformal algebras should describe the physics of cognition rather than real physics so that these exotic states need not correspond to real physics states.

2. One might argue that p-adic and real states correspond to each other in case that the states can be transformed to each other by a p-adic–real phase transition respecting conservation laws. This is achieved if the exotic states appear in pairs having opposite quantum numbers: this is the case if the otherwise identical states of the pair are associated with space-time sheets having opposite time orientations. The exotic p-adic states would represent vacuum polaron realized as a pair of real states with superastronomical energies which however cancel each other exactly. Obviously the gigantic masses of the real space-time sheets would make virtually impossible to separate them from each other. It is of course questionable if the real counterparts of these vacuum polarization states can have any significance. p-Adic counterparts might however have relevance for the physics of cognition.

3. One might of course argue that every every p-adic state (imaginable state!) must have a real counterpart with essentially the same real physics properties. In recent case the real counterparts of the p-adic masses obtained by canonical identification are extremely small whereas the masses of the corresponding real states are superastronomical if the value of the string tension is formally the same and of order $O(p^0)$. String tension is however a dynamical quantity and one can consider the possibility that the real counterpart of the p-adic string tension for the quaternion conformal representations is such that the real and p-adic mass scales are mutually consistent. Admittedly, this argument does not satisfy the requirement of mathematical elegance.

7.2 Elementary particles cannot correspond to exotic quaternion conformal states

If the real counterparts of the exotic states are created in pairs with vanishing total quantum numbers and having super-astronomical real masses, they certainly cannot have any relevance for elementary particle physics. If one assumes that string tension for real states is such that real masses of exotic states are of same order as p-adic mass situation can change. For instance, intermediate gauge bosons would have also excitations with mass $1/\sqrt{p}$ and one can wonder whether these excitations could correspond to the observed intermediate gauge bosons. One could even consider the possibility of understanding the entire elementary particle mass spectrum in terms of these $n = 0$ and $n = p$ excitations assuming that the vacuum weight of the Super Virasoro representations is vanishing. There are quite a number of consistency conditions, which definitely exclude this possibility.

1. Photon, graviton and gluon correspond to a ground state created by vanishing conformal weight. This happens to be the case. By a suitable choice for the coefficient of modular contribution and with a suitable choice of mass scale one might be able to reproduce charged lepton mass ratios correctly.

2. All states with non-vanishing ground state vacuum weight should correspond to $n = p$ states and would have same non-vanishing mass equal to $1/\sqrt{p}$ in natural units for given p. For quarks no mass splitting would result in first order approximation and the experience with CKM matrix suggests very strongly that it is not possible to achieve correct CKM matrix for mass degenerate u and d quarks.

3. A strong counter argument against the scenario is the huge ground state degeneracy of the states expected. As well known the degeneracy of states with eigenvalue n of L_0 increases exponentially as a function of n. For instance, huge number of color, electro-weak and spin excitations would have same mass and this does not seem to make sense. Thus it seems that p-adic thermodynamics giving extremely small probability for all large n excitations must be correct for elementary particles at least. Again there is however loophole involved. Low energy hadron physics corresponds to non-perturbative QCD like theory and one might wonder whether these exotic states of Super Virasoro algebra could become important at low hadron momentum transfers and whether some kind of phase transition from the dominance of the ordinary Super Virasoro representations to that of exotic Super Virasoro representations might take place. Amazingly, this hypothesis predicts the mass of pion and Regge slope correctly as fundamental constants of Nature [6].

7.3 Could exotic p-adic counterparts of elementary particles be relevant for living systems?

Previous arguments do not exclude the appearance of $n\ mod\ p^k = 0$ p-adic states. Also their zero energy pairs could appear as real states. If the couplings of these excitations obey the conservation of L_0 charge (conformal weight), the states in class m couple only to the states in same class or to $n = 0$ massless states and therefore these particles could probably emit and absorb ordinary $n = 0$ elementary particles. The possibility of pair creation seems to be excluded (it would require that antiparticles have negative spectrum of L_0, which looks peculiar). If this is true then $m = 1$ states are not be created in ordinary elementary particle reactions. It must be emphasized that the matrix elements for emission of exotic states could be small for other reasons: for instance, because the conformal weights of states involved differ so much.

An interesting possibility is that $m > 1$ excitations of known elementary particles could be present in macroscopic length scales.

1. For hadrons $m = 2$ excitations correspond by Uncertainty Principle to the length scale $L(k = 2 \times 107) \sim .4$ meters whereas for electron one has length scale $L(k = 2 \times 127) \sim 10^7$ meters. The corresponding time scale is .1 seconds, which is the fundamental time scale of brain consciousness defining the duration of psychological moment. This time scale is crucial in the TGD based model of memetic code. The model derives from a model of abstraction process leading to a hierarchy of 'genetic codes' labelled by Mersenne numbers: $M(n) = M_{M(n-1)}$. $M_7 = 127$ corresponds to genetic code and M_{127}, which is the next level of the hierarchy, corresponds to the memetic code.

2. For $m = 2$ excitations of Z^0 and W (also other states could be present) the corresponding length scale is $L(k = 2 \times 89 = 178) \sim 10^{-4}$ meters, which is $2^{4.5}$ times larger than the p-adic length scale $L(k = 169)$ associated with neutrinos. Is this a pure accident or could it be that there are exotic Z^0 bosons in cell length scale and that this explains the primary condensation level of neutrinos? In this picture it would be perhaps easier to understand also why classical Z^0 fields appear dominantly above cell length scale as required by the arguments based on the smallness of parity breaking effects. It should be mentioned that $k = 178$ corresponds to the size of the largest neurons.

The super astronomical degeneracy $D \sim exp(p)$, $p = M_{89}$ (!) associated with these excitations plus Negentropy Maximization Principle could make biosystems with size larger that the critical size of 10^{-4} meters something quite special, to put it very mildly! The same argument applies to the $p = M_{127}$ associated with the memetic code. The p-adic length scale nearest to $L(178)$ corresponds to the secondary condensation level for the $m = 2$ particles. It is $k = 179$ and in fact forms twin prime with $k = 181$. As a rule, twin primes in bio-systems seem to be associated with two-layered structures and this particular twin prime corresponds to ocular dominance columns, the largest known two-layered structure in the cortex (in fact this twin prime is the first one in the series of three twin primes $(179, 181)$, $(191, 193)$, $(197, 199)$!).

This raises the question whether the physics based explanation for the huge qualitative and quantitative differences in the behavior of higher primates and more primitive life forms could be based on the huge entanglement entropy resources provided by these exotic particles? It seems that this question becomes more or less obsolete with the realization that the immense supercanonical almost-degeneracies for the massless states of quaternion conformal representations explain very naturally the huge information resources of biosystems without need to introduce exotic representations.

One can end up to the similar speculations via a different route by starting from the TGD based reduction of the notion of potential energy to space-time topology (potential energy unlike kinetic energy does not allow any visualization in standard physics and thus remains a fictive concept).

1. In TGD framework the sign of energy depends on the time orientation of the space-time sheet and can be negative. Topological field quanta of negative energy represent negative energy virtual particles. The generation of negative potential energy corresponds to the emission of negative energy virtual bosons condensing on larger space-time sheets and in this manner one can understand potential energy as the total energy emitted by particle in form of low energy topological field quanta condensed on larger space-time sheets. In particular, the huge energy densities in strong gravitational fields of early cosmology result via the emission of negative energy virtual gravitons: only in this manner one can understand in TGD framework how conservation of energy can be consistent with gravitational interaction. For instance, gravitational redshift, which in GRT means nonconservation of energy, results in TGD framework from the absorption of negative energy virtual gravitons.

2. An objection against this interpretation is provided by long range classical Z^0 fields: attractive classical Z^0 potential energy should also correspond to topological field quanta of negative energy at larger space-time sheets. This is certainly possible. These topological field quanta cannot however correspond to the ordinary quanta of Z^0 field which are extremely massive and propagate only over range of order 10^{-17} meters. Thus the correspondence *quanta↔topological quanta* seems to fail.

3. There is however a loophole allowed by p-adic mathematics. As already noticed, the secondary almost massless excitations $n\ mod\ p = 0$ of Super Virasoro algebra have mass of order $m(CP_2)/p$ and possess huge exponential degeneracy of states characteristic for the Super Virasoro algebra. For $p = M_{89} = 2^{89} - 1$ the mass of these excitations is of order $m \sim m_W 2^{-89/2} \sim 10^{-2}$ eV, which happens to be rather near to the thermal energy associated with the room temperature, which is the critical temperature for the higher forms of biological life. The corresponding length scale is by Uncertainty Principle 10^{-4} meters and would represent the range of the Z^0 forces based on the exchange of the secondary quanta. Thus the exchange of these quanta between nuclei and neutrinos could be an essential element of what it is to be biosystem. These excitations having huge ground state degeneracy could also provide a quantum level description for the huge degeneracy of states certainly characteristic for biosystems. This degeneracy might also explain dynamically why neutrinos topologically condense on cell length scale.

4. A further objection is that classical Z^0 force seems to be not restricted to biological length scales but is present also in the planetary length scales. This objection can be circumvented too. Higher secondary excitations of Super Virasoro algebra satisfying $n\ mod\ p^3 = 0$ with mass of order $m(CP_2)/p^{3/2}$ should be also present. This mass would correspond to

$m \sim 2^{-89} m_W$ and to the length scale of 2×10^9 meters characterizing solar system. The corresponding time scale is 8 seconds, which is also an important length scale in biosystems as is also the time scale of .1 seconds associated with the second power of $p = M_{127}$, which is the p-adic length scale of electron and characterizes memetic code. This hypothesis is consistent with the idea that ELF em and Z^0 fields give rise to a new form of life, 'culture', living in symbiosis with biological life.

5. This would suggest a hierarchy of lifeforms whose intelligence quotient is roughly characterized by the degeneracy of the Super Virasoro states involved and thus by the power and value of the p-adic prime p to which they correspond. Since Mersenne primes are fundamental for elementary particle physics, one expects that the powers of the Mersenne primes M_{89}, M_{107} and $M127$ should label the most important higher lifeforms. M_{89} would give rise to two higher levels already discussed whereas M_{127} gives rise to the menetic code. The $n\ mod\ p^2 = 0$ excitations associated with M_{107}, the Mersenne prime characterizing hadrons, would correspond to the length scale of about 25 meters and time scale of order 10^{-7} seconds. $n\ mod\ p^3 = 0$ excitations associated with M_{107} would in turn correspond to the time scale of 10^9 seconds, or 30 years in more natural units: this is of the same order as human life span!

6. A further observation of possible relevance is that if Super Algebra representation has vanishing conformal vacuum weight, the subalgebra consisting of generators having conformal weights n proportional to p^m forms sub-algebra of entire Super algebra. Thus the exotic states correspond to sub-algebra of Super Virasoro and become therefore even more interesting in light of fractality suggesting strongly hierarchical breaking of supersymmetry to subalgebras of Super Virasoro algebra isomorphic with the entire algebra.

Because of their physical properties MEs provide excellent candidate for a model of mindlike space-time sheets and one can assign to the lightlike boundaries of MEs supercanonical representations defining quantum holograms. Thus MEs could carry also exotic p-adic Super Virasoro representations but as already noticed, they are not needed in order to understand the information sources associated with living matter.

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